On the Sum of Gamma Random Variates With Application to the Performance of Maximal Ratio Combining over Nakagami-m Fading Channels

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***Abstract*—The probability distribution function (PDF) and cumulative density function of the sum of L independent but not necessarily identically distributed gamma variates, applicable to maximal ratio combining receiver outputs or in other words applicable to the performance analysis of diversity combining receivers operating over Nakagami-m fading channels, is presented in closed form in terms of Meijer *G*-function and Fox $\bar{H}$-function for integer valued fading parameters and non-integer valued fading parameters, respectively. Further analysis, particularly on bit error rate via PDF-based approach, too is represented in closed form in terms of Meijer *G*-function and Fox $\bar{H}$-function for integer-order fading parameters, and extended Fox $\bar{H}$-function ($\hat{H}$) for non-integer-order fading parameters. The proposed results complement previous results that are either evolved in closed-form, or expressed in terms of infinite sums or higher order derivatives of the fading parameter m.**

***Index Terms*—Gamma variates, cellular mobile radio systems, non-integer parameters, diversity, maximal ratio combining (MRC), binary modulation schemes, bit error rate (BER), Fox *H*-function distribution, Meijer *G*-function distribution, Fox $\bar{H}$-function distribution, and Extended Fox $\bar{H}$-function ($\hat{H}$) distribution.**

I. Introduction

The probability distribution function (PDF) and cumulative distributive function (CDF) of the sum of n independent but not necessarily identical (i.n.i.d.) gamma random variables (RVs) has been investigated quite a couple of times earlier in [1]–[4] but in a rather complex manner that renders the given expressions therein highly intractable or they have been derived via complicated moment-generating function (MGF)-based approach. In [2], the author has proposed an infinite-series representation for the PDF of the sum of the i.n.i.d. gamma RVs and in [4], authors have extended the results of [2] for the case of arbitrarily correlated gamma RVs and studied the performance of MRC among other receivers as well as in the presence of cochannel interference (CCI). Moreover, in [5]–[9], the authors have followed the MGF-based approach or started from characteristic function (CF) for the performance analysis and derived analytical results in terms of either infinite sums and/or higher order derivatives of the fading parameter. This occurs as there are no simple closed-form expressions available in the open literature either for the PDF or the CDF of the sum of i.n.i.d. gamma RVs [3]. Recently, authors in [3] have derived closed-form expressions for the PDF and the CDF of the sum of nonidentical squared Nakgami-m RVs or equivalently gamma RVs with integer-order fading parameters but these results involve infinite series summations.

In this work, we offer novel closed-form expressions for the PDF and CDF of the sum of i.n.i.d. gamma RVs or equivalently squared Nakagami-m RVs with integer-order as well as non-integer-order fading parameters in terms of Meijer *G*-function [10] and Fox $\bar{H}$-function [11]–[13, App. (A.5)] respectively. It is noteworthy to mention that the bit error rate (BER) is one of the most important performance measures that forms the basis in designing wireless communication systems. Hence, we demonstrate closed-form expressions of the BER, as a performance metric, for binary modulation schemes, via PDF-based approach, of a L-branch MRC diversity receiver in the presence of gamma or Nakagami-m multipath fading, in terms of Meijer *G*-function and Fox $\bar{H}$-function for integer-order fading parameters, and in terms of extended Fox $\bar{H}$-function ($\hat{H}$) for non-integer-order fading parameters. This proves the importance and the simplicity in the employment of those earlier derived simple closed-form statistical PDF and CDF expressions. These resulting easy-to-evaluate expressions also give an alternative form for previously known/published results obtained via CF or MGF-based approaches. It should be noted that all our newly proposed results have been checked and validated by Monte Carlo simulations.

II. Nakagami-m Channel Model and Gamma Distribution

A MRC based communication system with a source and a destination is considered with L diversity paths undergoing i.n.i.d. Nakagami-m fading channels as follows

$$Y = \alpha X + n, \tag{1}$$

where Y is the received signal at the receiver end, X is the transmitted signal, α is the channel gain, and n is the additive white Gaussian noise (AWGN). In a Nakagami multipath fading channel, $\gamma = |\alpha|^2$ follows gamma distribution. Hence, the channel gains experience multipath fading whose statistics follow a gamma distribution given by

$$p_\gamma(\gamma) = \left(\frac{m}{\Omega}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} \, \exp\left(-\frac{m}{\Omega}\,\gamma\right), \tag{2}$$

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where $m > 0$ is the Nakagami-*m* multipath fading parameter, $\Omega > 0$ is the mean of the local power , $\gamma > 0$ is the gamma random variable (RV), and $\Gamma(\cdot)$ is the Gamma function as defined in [14, Eq. (8.310)]. The parameter m quantifies the severity of multipath fading, in the sense that small values of m indicates severe multipath fading and vice versa. The instantaneous SNR of the n^{th} branch is given by $\gamma_n = (E_b/N_0)\, {x_n}^2$ where x_n is the signal amplitude for the n^{th} branch, E_b is the average energy per bit and N_0 is the power spectral density of the AWGN.

III. Closed-Form Statistical Characteristics for the Sum of Gamma Random Variates

A. Probability Density Function

Theorem 1 (PDF of the sum of gamma or equivalently squared Nakagami-*m* RVs)**.** *Let* $\{\gamma_l\}_{l=1}^{L}$ *be a set of i.n.i.d. gamma variates with parameters* m_l *and* Ω_l*. Then, the closed-form PDF*[1] *of the sum*

$$Y = \sum_{l=1}^{L} \gamma_l \tag{3}$$

for both integer-order as well as non-integer-order fading parameters can be expressed in terms of Fox $\bar{H}$*-function as*

$$p_Y(y) = \prod_{l=1}^{L} \left(\frac{m_l}{\Omega_l}\right)^{m_l} \bar{\mathrm{H}}_{L,L}^{0,L}\left[\exp(y) \,\middle|\, \begin{matrix} \Xi_L^{(1)} \\ \Xi_L^{(2)} \end{matrix}\right], \tag{4}$$

where $y > 0$*, the coefficient sets* $\Xi_k^{(1)}$ *and* $\Xi_k^{(2)}$*,* $k \in \mathbb{N}$ *are defined as*

$$\Xi_k^{(1)} = \overbrace{\left(1-\frac{m_1}{\Omega_1}, 1, m_1\right), \ldots, \left(1-\frac{m_k}{\Omega_k}, 1, m_k\right)}^{k\text{-bracketed terms}}, \tag{5}$$

and

$$\Xi_k^{(2)} = \overbrace{\left(-\frac{m_1}{\Omega_1}, 1, m_1\right), \ldots, \left(-\frac{m_k}{\Omega_k}, 1, m_k\right)}^{k\text{-bracketed terms}} \tag{6}$$

respectively.

Further, it is worth mentioning that the closed-form expression in (4) simplifies to the following expression (7) for integer-order fading parameters via simple algebraic manipulations.

$$p_Y(y) = \prod_{l=1}^{L} \left(\frac{m_l}{\Omega_l}\right)^{m_l} \mathrm{G}_{\kappa,\kappa}^{\kappa,0}\left[\exp(-y) \,\middle|\, \begin{matrix} \Psi_\kappa^{(1)} \\ \Psi_\kappa^{(2)} \end{matrix}\right], \tag{7}$$

[1]For correlated diversity branches, the statistical characteristics derivation and the performance analysis can be carried out in a similar fashion as independent fading case. For an arbitrarily correlated Nakagami-*m* fading environment, assuming that the fading parameter is common to all the diversity branches, the desired MGF of the sum of correlated gamma RVs can be expressed as $\mathcal{M}_Y(s) = \det(I + sR\Lambda)^{-m} = \prod_{l=1}^{L} \mathcal{M}_{\gamma_l}(s) = \prod_{l=1}^{L} (1 + \lambda_l\ s)^{-m}$ where I is the $L \times L$ identity matrix. Λ is a positive definite matrix of dimension L (determined by the branch covariance matrix), R is a diagonal matrix as $R = \mathrm{diag}(\Omega_1/m, \ldots, \Omega_L/m)$, and λ_l is the l^{th} eigenvalue of matrix $R\Lambda$ where each eigenvalue is modeled as a gamma RV. Hence, on replacing γ_l's with λ_l's, in our presented work, we will achieve the results applicable to correlated diversity case.

where $\kappa = \sum_{l=1}^{L} m_l$ is an integer, the coefficient set $\Psi_k^{(1)}$, $k \in \mathbb{N}$ is as defined in (8) and the coefficient set $\Psi_k^{(2)}$, $k \in \mathbb{N}$ is defined as

$$\Psi_k^{(2)} = \overbrace{\underbrace{\left(\frac{m_1}{\Omega_1}\right), \ldots, \left(\frac{m_1}{\Omega_1}\right)}_{m_1\text{-times}}, \ldots, \underbrace{\left(\frac{m_K}{\Omega_K}\right), \ldots, \left(\frac{m_K}{\Omega_K}\right)}_{m_K\text{-times}}}^{k\text{-bracketed terms}}, \tag{9}$$

where K is the total number of gamma or equivalently squared Nakagami-m RVs i.e. L number of total branches for our specific wireless communication system being considered here.

To our best knowledge, the Fox $\bar{H}$-function is not available in any standard mathematical packages. As such, we offer an efficient Mathematica® implementation of this function (similar to [15]–[17]) in order to give numerical results based on (4). With this implementation, the Fox $\bar{H}$-function can be evaluated fast and accurately. This computability, therefore, has been utilized for different scenarios and is employed to discuss the results in comparison to respective Monte Carlo simulation outcomes.

Fig. 1 presents the PDF of the output SNR obtained from the exact closed-form expression (4) and shows a perfect match between this obtained closed-form analytical result and the one obtained via Monte Carlo simulations for varying $L's$ (i.e. $L = 3, 4, 5$), and their respective fixed fading parameters $m_1 = 0.6$, $m_2 = 1.1$, $m_3 = 2$, $m_4 = 3.4$, and $m_5 = 4.5$.

Now let us consider some special cases in order to check the correctness and accuracy of (7). These special cases give a further insight to the above obtained results.

Special Case 1 (Sum of two exponential RVs)**.** Let us assume that we have two i.n.i.d. gamma RVs with fading figures $m_1 = 1$ and $m_2 = 1$ and average powers Ω_1 and Ω_2. Substituting these parameters in (7) results in

$$p_Y(y) = \frac{1}{\Omega_1\Omega_2} \mathrm{G}_{2,2}^{2,0}\left[\exp(-y) \,\middle|\, \begin{matrix} 1+\frac{1}{\Omega_1}, 1+\frac{1}{\Omega_2} \\ \frac{1}{\Omega_1}, \frac{1}{\Omega_2} \end{matrix}\right]. \tag{10}$$

Then, using the Meijer's G identity given in [13, Eq. (1.142)] and then using [18, Eq. (07.23.03.0227.01)], (10) readily reduces to [19, Sec. 5.2.4]

$$p_Y(y) = \frac{\exp(-\frac{y}{\Omega_1}) - \exp(-\frac{y}{\Omega_2})}{\Omega_1 - \Omega_2}. \tag{11}$$

Special Case 2 (Sum of L exponential RVs)**.** Let us assume that we have L i.n.i.d. gamma RVs with fading figures $m_l = 1$ for all $l \in \{1, 2, 3, \ldots, L\}$ and average powers $\Omega_l \neq \Omega_k$ for all $k, l \in \{1, 2, 3, \ldots, L\}$. Substituting these parameters in (7) results in

$$p_Y(y) = \frac{1}{\prod_{l=1}^{L} \Omega_l} \mathrm{G}_{L,L}^{L,0}\left[\exp(-y) \,\middle|\, \begin{matrix} 1+\frac{1}{\Omega_1}, \ldots, 1+\frac{1}{\Omega_L} \\ \frac{1}{\Omega_1}, \ldots, \frac{1}{\Omega_L} \end{matrix}\right]. \tag{12}$$

Then, performing some algebraic manipulations using the Meijer's G identity given in [18, Eq. (07.34.26.0004.01)] and

$$\Psi_k^{(1)} = \overbrace{\underbrace{\left(1+\frac{m_1}{\Omega_1}\right),\ldots,\left(1+\frac{m_1}{\Omega_1}\right)}_{m_1\text{-times}},\ldots,\underbrace{\left(1+\frac{m_K}{\Omega_K}\right),\ldots,\left(1+\frac{m_K}{\Omega_K}\right)}_{m_K\text{-times}}}^{k\text{-bracketed terms}} \tag{8}$$

[18, 07.31.06.0017.01], we simplify (12) to [19, Eq. (5.8)]

$$p_Y(y) = \sum_{l=1}^{L}\left(\prod_{k\neq l}\frac{\frac{1}{\Omega_k}}{\frac{1}{\Omega_k}-\frac{1}{\Omega_l}}\right)\frac{1}{\Omega_l}\exp\left(-\frac{y}{\Omega_l}\right). \tag{13}$$

It is appropriate to mention that the steps followed in Special Case 1 and Special Case 2 can also be readily used to obtain the CDF for the same special cases.

B. Cumulative Distribution Function

Theorem 2 (CDF of the sum of gamma or equivalently squared Nakagami-m RVs)**.** *The CDF of Y for both integer-order as well as non-integer-order fading parameters can be closely expressed in terms of Fox $\bar{H}$-function as*

$$P_Y(y) = 1+\prod_{l=1}^{L}\left(\frac{m_l}{\Omega_l}\right)^{m_l}\bar{\mathrm{H}}_{L+1,L+1}^{0,L+1}\left[\exp(y)\,\middle|\,\begin{matrix}\Xi_L^{(1)},(1,1,1)\\ \Xi_L^{(1)},(0,1,1)\end{matrix}\right], \tag{14}$$

where the coefficient sets $\Xi_k^{(1)}$ and $\Xi_k^{(2)}$ are defined earlier in (5) *and* (6) *respectively.*

Fig. 2 presents the logarithmic CDF of the output SNR obtained from the exact closed-form expression (14) and shows a perfect match between this obtained closed-form analytical result and the one obtained via Monte Carlo simulations for varying $L's$ (i.e. $L = 2,3,4,5$), and their respective fixed fading parameters $m_1 = 0.6$, $m_2 = 1.1$, $m_3 = 2$, $m_4 = 3.4$, and $m_5 = 4.5$. The logarithmic plots were selected to display the accuracy of the matched results.

Further down the line, it is worth mentioning that via simple algebraic manipulations, the expression in (14) simplifies to

$$P_Y(y) = \prod_{l=1}^{L}\left(\frac{m_l}{\Omega_l}\right)^{m_l}\mathrm{G}_{1+\kappa,1+\kappa}^{1+\kappa,0}\left[\exp(-y)\,\middle|\,\begin{matrix}\Psi_\kappa^{(1)},1\\ \Psi_\kappa^{(2)},0\end{matrix}\right] \tag{15}$$

for integer-order fading parameters where the coefficient sets $\Psi_k^{(1)}$ and $\Psi_k^{(2)}$ are defined earlier in (8) and (9) respectively.

IV. Applications to the Performance of Diversity Combining Receiver Systems

A. Outage Probability

When the probability that the instantaneous MRC output SNR falls below a given threshold y_{th}, we encounter a situation labeled as outage and it is an important feature to study OP of a system. Hence, another important fact worth stating here is that the expression derived in (15) also serves the purpose for the expression of OP of MRC diversity combining receivers based wireless communication system that is experiencing i.n.i.d. Nakagami-*m* fading channels or in other words, when the desired user is subject to Nakagami-*m* fading, the probability that the SNR falls below a predetermined protection ratio y_{th} can be simply expressed, for both integer-order as well as non-integer-order fading parameters, by replacing y with y_{th} in (14) as

$$P_{\mathrm{out}}(y_{\mathrm{th}}) = P_Y(y_{\mathrm{th}}). \tag{16}$$

Employing similar substitutions, all the other respective expressions of CDF can be utilized for OP such as replacing y with y_{th} in (15).

B. Bit Error Rate

In MRC combining scheme, all the branches are selected at the output. In our case, for a L-branch MRC diversity receiver, the signal-to-noise ratio (SNR) y, is given by $y = \gamma_1+\cdots+\gamma_L$.

The most straightforward approach to obtain BER P_e for MRC is to average the conditional error probability (CEP) $P_e(\epsilon|y)$ for the given SNR given by $P_e(\epsilon|y) = \frac{\Gamma(p,qy)}{2\Gamma(p)}$ over the PDF of the combiner output SNR [20] i.e. $P_e = \int_0^\infty P_e(\epsilon|y)\,p_Y(y)\,dy$.

The $P_e(\epsilon|y)$ expression is a unified CEP expression for coherent and non-coherent binary modulation schemes over an AWGN channel [21]. $\Gamma(\cdot,\cdot)$ is the complementary incomplete gamma function [14, Eq. (8.350.2)]. The parameters p and q account for different modulation schemes. For an extensive list of modulation schemes represented by these parameters, one may look into [22] or refer to Table I.

TABLE I
Conditional Error Probability (CEP) Parameters

Modulation	p	q
Binary Frequency Shift Keying (BFSK)	0.5	0.5
Binary Phase Shift Keying (BPSK)	0.5	1
Differential Phase Shift Keying (DPSK)	1	1

Now, utilizing BER (P_e) equation given above with proper substitutions and performing some simple manipulations along with some simple rearrangements of $\Gamma(.)$ function terms, we get an exact closed-form result of the integral valid for both integer-order as well as non-integer-order fading parameters and for any binary modulation scheme including BFSK, BPSK, and DPSK, in terms of extended Fox $\bar{H}$-function ($\hat{H}$), as

$$P_e = \frac{q^p}{2}\prod_{l=1}^{L}\left(\frac{m_l}{\Omega_l}\right)^{m_l}\hat{\mathrm{H}}_{L+2,L+2}^{L+1,1}\left[1\,\middle|\,\begin{matrix}(1-q,1,p),\zeta_1\\ \zeta_2,(-q,1,p)\end{matrix}\right], \tag{19}$$

where $\zeta_1 = \Upsilon_L^{(1)},(1,1,1)$ and $\zeta_2 = (0,1,1),\Upsilon_L^{(2)}$, and where

$$\Delta_k^{(1)} = \overbrace{\underbrace{\overbrace{\left(1+\frac{m_1}{\Omega_1},1\right),\ldots,\left(1+\frac{m_1}{\Omega_1},1\right)}^{m_1\text{-times}},\ldots,\overbrace{\left(1+\frac{m_K}{\Omega_K},1\right),\ldots,\left(1+\frac{m_K}{\Omega_K},1\right)}^{m_K\text{-times}}}_{}}^{k\text{-bracketed terms}}, \tag{17}$$

$$\Delta_k^{(2)} = \overbrace{\overbrace{\left(\frac{m_1}{\Omega_1},1\right),\ldots,\left(\frac{m_1}{\Omega_1},1\right)}^{m_1\text{-times}},\ldots,\overbrace{\left(\frac{m_K}{\Omega_K},1\right),\ldots,\left(\frac{m_K}{\Omega_K},1\right)}^{m_K\text{-times}}}^{k\text{-bracketed terms}} \tag{18}$$

the coefficient sets $\Upsilon_k^{(1)}$ and $\Upsilon_k^{(2)}$, $k \in \mathbb{N}$ are defined as

$$\Upsilon_k^{(1)} = \overbrace{\left(1+\frac{m_1}{\Omega_1},1,m_1\right),\ldots,\left(1+\frac{m_k}{\Omega_k},1,m_k\right)}^{k\text{-bracketed terms}}, \tag{20}$$

and

$$\Upsilon_k^{(2)} = \overbrace{\left(\frac{m_1}{\Omega_1},1,m_1\right),\ldots,\left(\frac{m_k}{\Omega_k},1,m_k\right)}^{k\text{-bracketed terms}} \tag{21}$$

respectively.

The above presented BER expression in (19) is simplified, via simple algebraic manipulations, to the following closed-form expression when considering BFSK and BPSK binary modulation schemes with only integer-order fading parameters. It is represented in terms of Fox $\bar{\text{H}}$-function as

$$P_e = \frac{q^p}{2}\prod_{l=1}^{L}\left(\frac{m_l}{\Omega_l}\right)^{m_l}\bar{\text{H}}_{\kappa+2,\kappa+2}^{\kappa+1,1}\left[1\,\middle|\,\begin{matrix}(1-q,1,p),\chi_1\\ \chi_2,(-q,1,p)\end{matrix}\right], \tag{22}$$

where $\chi_1 = \Delta_\kappa^{(1)},(1,1)$ and $\chi_2 = (0,1),\Delta_\kappa^{(2)}$, and where the coefficient sets $\Delta_k^{(1)}$ and $\Delta_k^{(2)}$, $k \in \mathbb{N}$ are defined in (17) and (18) respectively. K is the total number of gamma or equivalently squared Nakagami-m RVs i.e. L number of total branches for our specific wireless communication system being considered here.

To our best knowledge, the extended Fox $\bar{H}$-function ($\hat{H}$) is not available in any standard mathematical packages. As such, we offer in [23] an efficient, simple and accurate Mathematica® implementation of this function in order to give numerical results based on (19) and/or (22). With this implementation, the extended Fox $\bar{H}$-function ($\hat{H}$) can be evaluated fast and accurately. This computability, therefore, has been utilized for different digital modulation schemes and is employed to discuss the computer results in comparison to respective Monte Carlo simulation outcomes.

The average SNR per bit in all the scenarios discussed is assumed to be equal. In addition, different digital modulation schemes are represented based on the values of p and q as displayed in Table I.

We observe from Fig. 3 that this implemented computability of extended Fox $\bar{H}$-function ($\hat{H}$) provides a perfect match to the MATLAB simulated results and the results are as expected i.e. the BER decreases as the signal-to-noise ratio (SNR) increases. Its important to note here that these values for the parameters were selected randomly to prove the validity of the obtained results and hence specific values based on the standards can be used to obtain the required results.

Furthermore, it can be seen from Fig. 3 that, as expected, BPSK outperforms the other modulation schemes and DPSK outperforms BFSK. Similar results for any other values of $m's$ can be observed for the exact closed-form BER for L-diversity i.n.i.d. gamma or equivalently squared Nakagami-*m* channels presented in this work.

V. Concluding Remarks

We derived novel closed-form expressions for the PDF and the CDF of the sum of i.n.i.d. gamma or equivalently squared Nakagami-*m* RVs valid for any fading figure parameters. Further, an exact closed-form expression for the BER performance of different binary modulations with L-branch MRC scheme was also derived.

Our results complement previously published results that are either in the form of infinite sums or higher order derivatives of the fading parameter or recursive solutions. This approach provides a novel closed-form expression as compared to earlier work in the literature. Having the computability for these functions such as Fox *H*-function and extended Fox $\bar{H}$-function ($\hat{H}$) and/or the decomposition of these functions into univariate Meijer *G*-function, this approach will form the basis for further analysis in similar proceedings.

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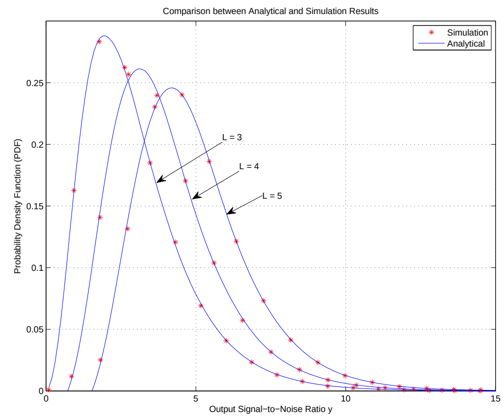

Fig. 1. Comparison between PDFs obtained analytically and via Monte Carlo simulations for varying branches L and respective fixed fading parameters for these channels as $m_1 = 0.6$, $m_2 = 1.1$, $m_3 = 2$, $m_4 = 3.4$, and $m_5 = 4.5$.

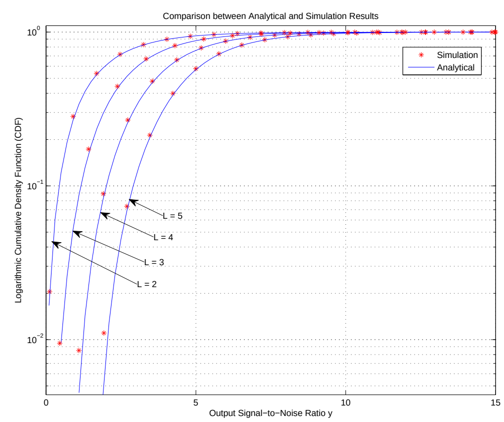

Fig. 2. Comparison between CDFs obtained analytically and via Monte Carlo simulations, on log scale, for varying branches L and respective fixed fading parameters for these channels as $m_1 = 0.6$, $m_2 = 1.1$, $m_3 = 2$, $m_4 = 3.4$, and $m_5 = 4.5$.

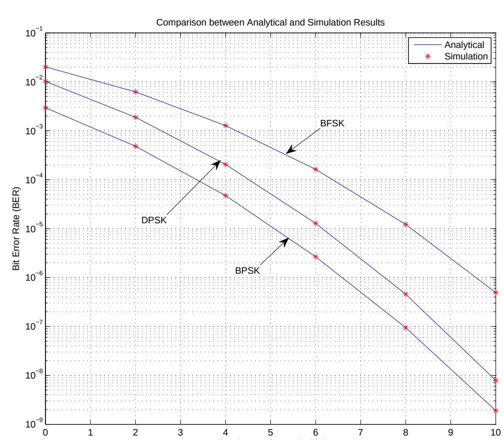

Fig. 3. Average BER of BPSK over i.n.i.d. gamma or equivalently squared Nakagami-m fading channels with $L = 5$-branch MRC and fading parameters for these channels as $m_1 = 0.6$, $m_2 = 1.1$, $m_3 = 2$, $m_4 = 3.4$, and $m_5 = 4.5$.