Comparison of Low-Complexity Diversity Schemes For Dual-Hop AF Relaying Systems

Fakhreddine Gaaloul, *Student Member, IEEE,* Redha Radaydeh, *Member, IEEE,*
and Mohamed-Slim Alouini, *Fellow, IEEE*

***Abstract*—This correspondence investigates the performance of two low-complexity combining schemes, which are based on one-phase or two-phase observation, to mitigate multipath fading in dual-hop amplify-and-forward (AF) relaying systems. For the one-phase based combining, a single-antenna station is assumed to relay information from multiple-antenna transmitter to multiple-antenna receiver, and the activation of the receive antennas is performed adaptively based on the second hop statistics, regardless of the first hop conditions. On the other hand, the two-phase based combining suggests using multiple single-antenna stations between the multiple-antenna transmitter and the single-antenna receiver, where the suitable set of active relays is identified according to the pre-combining end-to-end fading conditions. To facilitate comparisons between the two schemes, formulations for the statistics of the combined signal-to-noise ratio (SNR) and some performance measures are presented. Numerical and simulation results are shown to clarify the tradeoff between the achieved diversity-array gain, the processing complexity, and the power consumption.**

***Index Terms*—AF protocol, dual-hop relaying, low-complexity diversity combining, multiple-antenna systems, transmit diversity, receive diversity, performance measures.**

I. Introduction

COOPERATIVE techniques can extend the coverage and improve the performance of wireless networks [1], [2]. Further performance improvements can be achieved with the use of multiple-antenna systems [3], [4]. Various diversity combining schemes have been analyzed in the context of multiple-antenna relaying systems [5]–[6]. However, an important aspect to consider herein is to maintain the processing complexity and power consumption as low as possible, while satisfying a target performance. This correspondence focuses on this aspect and investigates two low-complexity diversity schemes for dual-hop relaying networks employing the amplify-and-forward (AF) protocol. This protocol may be useful when data transfer is time sensitive, and its performance can be improved when channel-state-information (CSI) of the preceding hop is used to control the relaying gain [7], [8].

For the dual-hop AF relaying system under consideration, the relaying stations use single antenna in each direction due to space limitation and/or processing power constraints. For the case when the direct link between the communication ends is infeasible, one approach to improve the end-to-end performance is by employing multiple antennas at the transmitter, multiple antennas at the receiver, and/or parallel deployment of relaying stations. The use of transmit antenna array can provide transmit diversity-array gain given that the CSI is known to the transmitter. This gain reduces the multiplexing capability and needs accurate first hop CSI feedback. In addition, the improvement achieved on the first hop may not be sufficient, particularly when the second hop signal quality is poor. The use of multiple receive antennas can improve the second hop without requiring feedback and affecting the multiplexing gain. However, it requires sufficient spacing between receive antennas. On the other hand, the end-to-end performance can be enhanced when multiple relaying stations are coordinated to serve the desired receiver. This can be achieved at the expense of reducing the system throughput and increasing the complexity. It is therefore desirable to activate just enough receive antennas or relaying stations to satisfy the target performance, while reducing the processing complexity and power consumption.

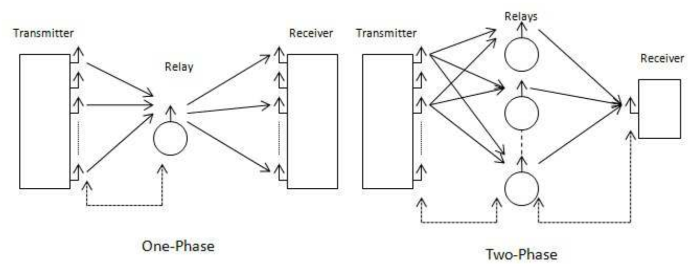

Fig. 1. System model

This correspondence investigates two possible schemes to improve the performance of dual-hop relaying, while reducing the numbers of activated receive antennas or relaying stations separately. Specifically, the first scheme is referred to as one-phase based combining where a single relaying station is used in conjunction with multiple receive antennas, whereas the second scheme is referred to as two-phase based combining in which multiple relaying stations are employed with single-antenna receiver (see Fig. 1). For these two schemes, the extreme scenarios of the single transmit antenna with no diversity-array gain and the maximal ratio transmission (MRT) for optimal diversity-array combining on the first hop are considered [9], [10].

Manuscript received April 9, 2011, revised June 19, 2011. The associate editor coordinating the review of this paper and approving it for publication was Prof. Jingxian Wu.

The material in this paper was presented in part at the IEEE 73rd Vehicular Technology Conference (VTC Spring), Budapest, Hungary, May 2011.

The authors are with the Electrical Engineering Program, Division of Physical Sciences and Engineering, King Abdullah University of Science and Technology (KAUST), Thuwal, Makkah Province, Saudi Arabia e-mail: ({fakhreddine.gaaloul, redha.radaydeh, slim.alouini}@kaust.edu.sa).

The one-phase based combining may be applicable when the physical space at the receiver is sufficient to employ multiple uncorrelated antennas, such as work stations or personal computers. On the other hand, the two-phase based combining may be suitable for space-limited receiver when multiple relaying stations can be coordinated to transfer the desired information, such as the case when small-size cellular devices are operating indoors. An important practical issue that may affect the achieved performance of the one-phase based combining is the accuracy of CSI feedback on the first hop. On the other hand, the two-phase based combining is sensitive to both the first hop CSI feedback and the ability of the receiver to synchronize transmissions from multiple active relaying stations, as highlighted in Fig. 1. One of the main objectives of this work is to show the advantages of employing the two-phase based combining over the one-phase based combining under the ideal assumptions of perfect first hop CSI feedback and perfect synchronization between relaying stations for the case of two-phase based combining. The work can be also extended to consider the effects of these impairments, but further results are not shown herein due to space limitations (see [11]–[16] for related works).

For the one-phase based combining, the gain in performance is possible through an adaptive combining of the signal replicas on the second hop. Therefore, the numbers of estimated and combined receive branches are independent of the first hop statistics. On the other hand, for the two-phase based combining, the improvement is achieved based on the end-to-end communication links and the minimum possible number of active relaying stations can be set accordingly while inducing as low effect on the aggregate throughput as possible. The power-efficient scheme in [17] is adopted for both schemes to reduce the power drain from the battery by reducing the number of active receive antennas (based on the second link statistics) or relaying stations (based on pre-combining end-to-end links quality). To enable performance-complexity comparisons between the two considered schemes, analytical formulations for the statistics of the combined signal-to-noise ratio (SNR) are obtained, which can be used to study various performance measures. Numerical and simulation comparisons are provided to validate the analytical development and to clarify the systems behavior for different operating conditions.

The rest of the correspondence is organized as follows. Section II presents the system model. Sections III and IV discuss the one-phase and two-phase based combining schemes, respectively. Comparison results are presented in section V, which will be followed by conclusions in section VI.

II. SYSTEM MODEL

This section revisits the system models of the two schemes, and characterizes of the instantaneous SNRs on diversity branches, which will be used in the following sections.

A. One-Phase Based Combining

For the one-phase based combining, a dual-hop relayed system $T_x \rightarrow R \rightarrow R_x$ with single relaying station is considered. The node T_x acts as the transmitter with n_t antennas, which relays the desired signal through node R to the receiver R_x with n_r antennas. This scheme aims at reducing the number of active receive antennas by adaptively estimating and then combining the second hop SNRs such that the second hop combined SNR is raised above a specific threshold. The effect of this processing on the end-to-end performance can be then studied for different scenarios of the first hop combined SNR. Note that this scheme does not require feedback information from the receiver to the relaying station, which is assumed to be ready-to-serve when needed.

B. Two-Phase Based Combining

For the two-phase based combining, a dual-hop relayed system $T_x \rightarrow \{R_j\}_{j=1}^{n_a} \rightarrow R_x$ with n_a spatially separated relaying stations is considered. The active nodes from the set $\{R_j\}_{j=1}^{n_a}$ captures the desired signal from node T_x during the first phase, and then relay the same signal to the single-antenna node R_x during the second phase. The scheme aims at reducing the number of active relaying stations by adaptively testing the combined end-to-end SNR against a specific threshold that guarantees a target performance. As the pre-combining end-to-end SNRs are related to the combined first hop SNR, it is of interest here to observe the effect of the number of transmit antennas on the average number of relaying stations that should be activated. In order for this scheme to work properly, feedback link is needed to keep the relaying stations updated with the decisions of the receiver.

C. Statistics of SNRs on Diversity Branches

For the sake of simplicity, it is assumed that nodes T_x and R_x know the CSI of the T_x-to-R and R-to-R_x links. The fading channels are assumed to be spatially uncorrelated, where fading envelopes follow Rayleigh distribution. Define $\gamma_{1,i,k}$, for $i = 1, 2, .., n_t$ and $k = 1, 2, \ldots, n_a$, as the instantaneous SNR observed at the kth relaying station when the ith transmit antenna is used, and $\gamma_{2,k,j}$, for $j = 1, 2, .., n_r$ as the instantaneous SNR observed at the jth receive antenna when the kth relaying station is active. The probability density functions (PDFs) of the instantaneous SNRs can be expressed as $f_{\gamma_{1,i,k}}(x) = \frac{1}{\bar{\gamma}_1}\, e^{-x/\bar{\gamma}_1}$ and $f_{\gamma_{2,k,j}}(x) = \frac{1}{\bar{\gamma}_2}\, e^{-x/\bar{\gamma}_2}$, where $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are the average SNRs on the first hop and the second hop, respectively. Note that $k = 1$ for one-phase based combining, whereas $j = 1$ for the two-phase based combining.

III. ONE-PHASE BASED COMBINING SCHEME

This section considers the one-phase based combining for two scenarios of the first hop combined SNR, namely the optimal MRT and the single transmit antenna. These two scenarios enable comprehensive understanding of the effect of the first hop statistics on the overall performance. Note that this case can be reformulated to study the effect of the one-phase combining scheme on the average number of active transmit antennas and/or the feedback load when the receiver employs either maximal ratio combining (MRC) or single receive antenna. However, the adopted formulation herein focuses on reducing the processing load and the power consumed from the receiver battery.

A. Preliminary Results

For the case under consideration, the end-to-end SNR, which is denoted by γ_c, can be generally expressed as $\gamma_c = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+b}$ (see, e.g., [1], [8]), where γ_1 and γ_2 are the combined SNRs on the first and second hops, respectively, and b is a constant. When $b = 0$, the end-to-end SNR can be upper bounded and lower bounded as $\gamma_{c,\text{UB}} = \min(\gamma_1, \gamma_2)$ and $\gamma_{c,\text{LB}} = 1/2 \min(\gamma_1, \gamma_2)$, respectively. The tightness of these bounds will be verified and compared to the exact results for the sake of accuracy in section V.

The cumulative distribution function (CDF) of γ_c can be obtained using the PDF of the first hop SNR γ_1 and the CDF of the second hop SNR γ_2 as shown in [8, Appendix]. The moment generating function (MGF) of γ_c, which can be used to obtain various performance measures [18], can be obtained using the CDF of γ_c as given in [8, eq. (6)]. Moveover, the CDFs of $\gamma_{c,\text{UB}}$ and $\gamma_{c,\text{LB}}$ can be expressed as $F_{\gamma_{c,\text{UB}}}(x) = \Pr\{\min(\gamma_1, \gamma_2) < x\} = 1 - \bar{F}_{\gamma_1}(x)\bar{F}_{\gamma_2}(x)$ and $F_{\gamma_{c,\text{LB}}}(x) = F_{\gamma_{c,\text{UB}}}(2x)$, respectively, where $\bar{F}_X(x)$ is the complementary cumulative distribution function (CCDF) of X.

B. Statistics of Combined SNRs on Two Hops

With the use of MRT, the first hop SNR $\gamma_1 = \sum_{i=1}^{n_t}\gamma_{1,i,1}$ follows a Gamma distribution with shape parameter of n_t and scale parameter of $\bar{\gamma}_1$ [9, eq. (22)].

For the second hop, the low complexity combining scheme in [17] is adopted in which the receiver tries to perform just enough processing complexity and power consumption to raise the second hop SNR above the target threshold, which is denoted by $\gamma_{2,T}$. This scheme suggests two sequential modes of operation, which are based on statistically unordered receive SNRs for single receive antenna switching and ordered receive SNRs with multiple-antenna combining, respectively. Specifically, during the first mode of operation, the receiver starts testing unordered diversity branches one by one, and the branch whose instantaneous SNR is found above the threshold $\gamma_{2,T}$ is directly used. If the SNRs on all diversity branches are found below $\gamma_{2,T}$, the receiver starts the second mode of operation, in which it orders the estimated SNRs and then tries to adaptively combine the minimum number of the best diversity branches as per the scheme in [19], [20]. In this case, a threshold is set on the maximum number of diversity branches that can be used in the second mode of operation, which is referred to as $L \leq n_r$. If the receiver fails to satisfy the inequality $\gamma_2 > \gamma_{2,T}$ when L receive antennas are activated, the receiver terminates the process and acts as the conventional generalized selection combiner of L active antennas. According to [17, eq. (6)], the CDF of the resulting SNR γ_2 can be written as

$$F_{\gamma_2}(x) = F_{\Gamma_{2,L}}(x)\Big(\text{U}(x) - \text{U}(x-\gamma_{2,T})\Big) + \Big(A_2(x) + \sum_{l=2}^{L} F_{\gamma_2}^{(l)}(x) + F_{\Gamma_{2,L}}(\gamma_{2,T})\Big)\text{U}(x-\gamma_{2,T}), \quad (1)$$

where $\text{U}(x)$ is the unit step function and $F_{\Gamma_{2,l}}(x)$ is the CDF of $\Gamma_{2,l} = \sum_{i=1}^{l}\gamma_{2,1,i:n_r}$, and $\gamma_{2,1,i:n_r}$ represents the SNR of the ith strongest receive diversity branch. In (1),

$$A_2(x) = \frac{1-\left(F_{\gamma_{2,1,\varsigma}}(\gamma_{2,T})\right)^{n_r}}{1-F_{\gamma_{2,1,\varsigma}}(\gamma_{2,T})} \times \left(F_{\gamma_{2,1,\varsigma}}(x) - F_{\gamma_{2,1,\varsigma}}(\gamma_{2,T})\right) \quad (2a)$$

$$F_{\gamma_2}^{(l)}(x) = g_2(x,l)\text{U}\left(\frac{l}{l-1}\gamma_{2,T} - \gamma_{2,T}\right) + \left(F_{\Gamma_{2,l-1}}(\gamma_{2,T}) - F_{\Gamma_{2,l}}(\gamma_{2,T})\right)\text{U}\left(\frac{l}{l-1}\gamma_{2,T}\right), \quad (2b)$$

where $F_{\gamma_{2,1,\varsigma}}(x)$ is the CDF of the SNR per a receive diversity branch, and

$$g_2(x,l) = \int_{\frac{l-1}{l}\gamma_{2,T}}^{\frac{l-1}{l}x}\int_{\gamma_{2,T}-y}^{\frac{y}{l-1}} f_{\gamma_{2,1,l:n_r},\Gamma_{2,l-1}}(z,y)dzdy + \int_{\frac{l-1}{l}x}^{\gamma_{2,T}}\int_{\gamma_{2,T}-y}^{x-y} f_{\gamma_{2,1,l:n_r},\Gamma_{2,l-1}}(z,y)dzdy, \quad (3)$$

$$f_{\gamma_{2,1,l:n_r},\Gamma_{2,l-1}}(x,y) = \frac{n_r!\left[F_{\gamma_{2,1,\varsigma}}(x)\right]^{n_r-l}\left[1-F_{\gamma_{2,1,\varsigma}}(x)\right]^{l-1}}{(n_r-l)!(l-1)!} \times f_{\gamma_{2,1,\varsigma}}(x) f_{\sum_{j=1}^{l-1}\gamma'_{2,1,j}}(y), \quad (4)$$

for $x > 0$ and $y > (l-1)x$, in which $f_{\gamma_{2,1,\varsigma}}(x)$ is the PDF of the SNR per a receive antenna and $\sum_{j=1}^{l-1}\gamma'_{2,1,j}$ is the sum of $l-1$ truncated received SNRs, such that $f_{\gamma'_{2,1,\varsigma}}(y) = f_{\gamma_{2,1,\varsigma}}(y)/(1-F_{\gamma_{2,1,\varsigma}}(x))$, for $y \geq x$. The PDF of γ_2, which is defined as $f_{\gamma_2}(x) = dF_{\gamma_2}(x)/dx$, can now be obtained as

$$f_{\gamma_2}(x) = f_{\Gamma_{2,L}}(x)\Big(\text{U}(x) - \text{U}(x-\gamma_{2,T})\Big) + \left(\sum_{l=2}^{L} h_2(x,l) \times \left(\text{U}(x-\gamma_{2,T}) - \text{U}\left(x - \frac{l}{l-1}\gamma_{2,T}\right)\right) + \frac{1-\left(F_{\gamma_{2,1,\varsigma}}(\gamma_{2,T})\right)^{n_r}}{1-F_{\gamma_{2,1,\varsigma}}(\gamma_{2,T})} f_{\gamma_{2,1,\varsigma}}(x)\right)\text{U}(x-\gamma_{2,T}), \quad (5)$$

where $f_{2,\Gamma_l}(x) = dF_{2,\Gamma_l}(x)/dx$,

$$h_2(x,l) = \int_{\frac{l-1}{l}x}^{\gamma_{2,T}} f_{\gamma_{2,1,l:n_r},\Gamma_{2,l-1}}(x-y,y)dzdy, \quad (6)$$

and $F_{\gamma_{2,1,\varsigma}}(x) = 1-e^{-x/\bar{\gamma}_2}$, $f_{\gamma_{2,1,\varsigma}}(x) = \frac{1}{\bar{\gamma}_2}e^{-x/\bar{\gamma}_2}$, as defined above. The results for $g_2(x,l)$ and $F_{\Gamma_{2,l}}(x)$ are provided in [20, eq. (30)] and [21, eq. (24)], respectively, with the parameters L and $\bar{\gamma}$ therein are replaced by n_r and $\bar{\gamma}_2$, respectively. Moreover, the results for $h_2(x,l)$ and $f_{\Gamma_{2,l}}(x)$ are given in [20, eqs. (32) and (33)] with L and $\bar{\gamma}$ therein are replaced by n_r and $\bar{\gamma}_2$, respectively. For the two-phase based combining that will be discussed in the next section, these results are no longer applicable.

C. CDF of End-to-End SNR

With the help of the results discussed in the previous subsection, and using the identity in [8, Appendix], the complementary CDF (CCDF) of the end-to-end SNR can be expressed as

$$\bar{F}_{\gamma_c}(x) = \Bigg(\mathcal{W}_1(x) + \frac{1-(F_{\gamma_{2,1,\varsigma}}(\gamma_{2,T}))^{n_r}}{1-F_{\gamma_{2,1,\varsigma}}(\gamma_{2,T})}\mathcal{P}_2(x,1,0,\gamma_{2,T}) + \sum_{l=2}^{L}\mathcal{W}_2(x,l)\Bigg)\Big(\mathrm{U}(x)-\mathrm{U}(x-\gamma_{2,T})\Big) + \Bigg(\frac{1-(F_{\gamma_{2,1,\varsigma}}(\gamma_{2,T}))^{n_r}}{1-F_{\gamma_{2,1,\varsigma}}(\gamma_{2,T})}\mathcal{P}_1(x,1,0) \quad (7)$$

$$+\sum_{l=2}^{L}\mathcal{W}_3(x,l)\mathrm{U}\left(\frac{l}{l-1}\gamma_{2,T}-x\right)\Bigg)\mathrm{U}(x-\gamma_{2,T}), \quad (8)$$

where the terms $\mathcal{W}_1(x)$, $\mathcal{W}_2(x,l)$ and $\mathcal{W}_3(x,l)$ are obtained as shown in (9)–(11), respectively, in which the terms $\mathcal{P}_1(x,p,d)$ and $\mathcal{P}_2(x,p,d,r)$ are expressed as given in (12), where $K_\nu(a)$ is the modified Bessel function of the second kind of order ν and $k_\nu^+(a,b)$ is the incomplete modified Bessel function of second kind of order ν (see [22]–[23]). The result in (8) includes many combinations of combining schemes on the first and second hops as limiting cases. For example, the result for single transmit antenna follows directly when $n_t = 1$. The result when generalized selection combining (GSC) is applied on the second hop can be deduced when $\gamma_{2,T} \to +\infty$. The result for single receive antenna case follows when $\gamma_{2,T}$ is replaced by zero.

D. MGF of End-to-End SNR

Substituting the CCDF of γ_c given in (8) into [8, eq.(6)], the MGF of γ_c can be obtained as shown in (13), in which $\mathcal{V}_1(s,p,d,u) = \mathcal{Q}_1^l(s,p,d,u) - \mathcal{Q}_2(s,p,d,u,u)$, $\mathcal{V}_2(s,p,d,u,v) = \mathcal{V}_1(s,p,d,v) - \mathcal{V}_1(s,p,d,u)$, $\mathcal{Q}_1^l(s,p,d,u) = \int_0^u e^{-sx}\mathcal{P}_1(x,p,d)dx$, $\mathcal{Q}_1^u(s,p,d,u) = \int_u^{+\infty} e^{-sx}\mathcal{P}_1(x,p,d)dx$, and $\mathcal{Q}_2(s,p,d,r,u) = \int_0^u e^{-sx}\mathcal{P}_2(x,p,d,r)dx$. To the best of our knowledge, no exact representations for these integrals can be obtained. In this case, numerical integration methods, such as Gauss-Chebyshev method [24], can be applied. It has been noted that this method provides relatively fast and accurate computations of the associated integrals as compared to extensive simulations, as will be shown in section IV.

It has been also noted that approximate results for the CDF and MGF of the end-to-end SNR using the bounds discussed in subsection III-A can be obtained in simple closed forms. These bounds can be readily used to obtain various performance measures, and their tightness will be verified in section V.

IV. Two-phase Based Combining

For the two-phase based combining, the receive combining will be performed on the independent components of the pre-combined end-to-end instantaneous SNRs observed from active relaying stations.

A. Preliminary Results

The end-to-end SNR when the kth relaying station is active can be expressed as $\gamma_{c,k} = \frac{\gamma_{1,k}\gamma_{2,k}}{\gamma_{1,k}+\gamma_{2,k}+b}$, where $\gamma_{1,k} = \sum_{i=1}^{n_t}\gamma_{1,i,k}$ and $\gamma_{2,k} = \gamma_{2,k,1}$. The CDF of the pre-combined SNR when the kth relaying station is active can be deduced from (8) when $\gamma_{2,T} = 0$, and the result is

$$F_{\gamma_{c,k}}(x) = 1 - 2\sum_{j=0}^{n_t-1}\frac{\binom{n_t-1}{j}}{(n_t-1)!\bar{\gamma}_1^{n_t}}\left(\frac{\bar{\gamma}_1}{\bar{\gamma}_2}\right)^{\frac{j+1}{2}} x^{n_t-\frac{j}{2}-\frac{1}{2}} \times(x+b)^{\frac{j+1}{2}}e^{-(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2})x}K_{j+1}\left(2\sqrt{\frac{x(x+b)}{\bar{\gamma}_1\bar{\gamma}_2}}\right). \quad (14)$$

The CDF of the post-combining end-to-end SNR, γ_c, can be written, as per the discussion in subsection III-B, as

$$F_{\gamma_c}(x) = F_{\Gamma_{c,L}}(x)\Big(\mathrm{U}(x)-\mathrm{U}(x-\gamma_{c,T})\Big) + \Big(A_c(x) + \sum_{l=2}^{L}F_{\gamma_c}^{(l)}(x) + F_{\Gamma_{c,L}}(\gamma_{c,T})\Big)\mathrm{U}(x-\gamma_{c,T}), \quad (15)$$

where $\gamma_{c,T}$ refers to the post-combining end-to-end SNR threshold. The terms in the preceding equation have similar definitions as described in (2)–(4) and (6), after taking care of the differences in notations and with the statistics of $\gamma_{c,k}$ are used instead of $\gamma_{2,1,\varsigma}$. It is seen that the terms in (15) can not be obtained in exact forms due to the complicated form of (14). Instead, the following subsections propose an approximate approach to model the statistics of γ_c analytically, which will be verified in the next section.

B. CDF of End-to-End SNR

Instead of using the complicated from in (14), it is proposed to use the upper bound $\gamma_{c,k,\mathrm{UB}} = \min(\gamma_{1,k},\gamma_{2,k})$. The CDF and PDF of $\gamma_{c,k,\mathrm{UB}}$ can be obtained as

$$F_{\gamma_{c,k,\mathrm{UB}}}(x) = 1 - \sum_{i=0}^{n_t-1}\frac{1}{i!\bar{\gamma}_1^i}x^i e^{-(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2})x} \quad (16)$$

$$f_{\gamma_{c,k,\mathrm{UB}}}(x) = \frac{1}{(n_t-1)!\bar{\gamma}_1^{n_t}}x^{n_t-1}e^{-(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2})x} + \sum_{i=0}^{n_t-1}\frac{1}{i!\bar{\gamma}_1^i\bar{\gamma}_2}x^i e^{-(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2})x}. \quad (17)$$

It is noted that the evaluation of (15) requires obtaining $F_{\Gamma_{c,l}}(x)$, where $\Gamma_{c,l} = \sum_{k=1}^{l}\gamma_{c,k:n_a}$. The term $F_{\Gamma_{c,l}}(x)$ can be efficiently evaluated using a fast convergent series [25], which gives

$$F_{\Gamma_{c,l}}(x) = \frac{1}{2} - \sum_{n=1,\, n\ \mathrm{odd}}\frac{2}{n\pi}\Im\left\{e^{-\mathrm{i}n\omega_0 x}M_{\Gamma_{c,l}}(-\mathrm{i}n\omega_0)\right\}, \quad (18)$$

where $\Im(z)$ denote the imaginary part of z, T is a parameter governing the sampling rate in frequency domain, $\omega_0 = 2\pi/T$, $\mathrm{i} = \sqrt{-1}$, and $M_{\Gamma_{c,l}}(s)$ is the MGF of $\Gamma_{c,l}$, which can be efficiently obtained using a finite-range single integral as [26]

$$M_{\Gamma_{c,l}}(s) \simeq L\binom{n_a}{L}\int_0^{\pi/2} e^{-s\tan(v)}f_{\gamma_{c,k,\mathrm{UB}}}(\tan(v))\sec^2(v) \times\left[F_{\gamma_{c,k,\mathrm{UB}}}(\tan(v))\right]^{n_r-L}\left[M_{\gamma_{c,k,\mathrm{UB}}}(s,\tan(v))\right]^{L-1}dv, \quad (19)$$

$$\mathcal{W}_1(x) = \frac{\binom{n_r}{L}}{(L-1)!\bar{\gamma}_2^{L-1}}\Big[\mathcal{P}_1\left(x,1,L-1\right) - \mathcal{P}_2\left(x,1,L-1,\gamma_{2,T}\right)\Big] + \binom{n_r}{L}\sum_{l=1}^{n_r-L}(-1)^{L+l-1}\binom{n_r-L}{l}$$
$$\times\Bigg[\mathcal{P}_1\left(x,\frac{l}{L}+1,0\right) - \mathcal{P}_2\left(x,\frac{l}{L}+1,0,\gamma_{2,T}\right) - \sum_{m=0}^{L-2}\frac{1}{m!}\left(-\frac{l}{L\bar{\gamma}_2}\right)^m\Big[\mathcal{P}_1\left(x,1,m\right) - \mathcal{P}_2\left(x,1,m,\gamma_{2,T}\right)\Big]\Bigg], \tag{9}$$

$$\mathcal{W}_2(x,l) = \frac{1}{(l-1)!\bar{\gamma}_2^{l-1}}\binom{n_r}{l}\sum_{u=0}^{l-1}\binom{l-1}{u}\frac{(-1)^u\left(l\gamma_{2,T}\right)^{l-1-u}}{(l-1)^{-u}}\left[\mathcal{P}_2\left(x,1,u,\gamma_{2,T}\right) - \mathcal{P}_2\left(x,1,u,\frac{l}{l-1}\gamma_{2,T}\right)\right]$$
$$+\sum_{j=1}^{n_r-l}\frac{n_r!(-1)^{j-l+1}}{(n_r-l-j)!l!j!}\left(\frac{l}{j}\right)^{l-1}\Bigg[\mathcal{P}_2\left(x,\frac{l+j}{l},0,\gamma_{2,T}\right) - \mathcal{P}_2\left(x,\frac{l+j}{l},0,\frac{l}{l-1}\gamma_{2,T}\right) - e^{\frac{j\gamma_{2,T}}{\bar{\gamma}_2}}\sum_{k=0}^{l-2}\sum_{u=0}^{k}$$
$$\frac{\binom{k}{u}j^k\left(-l\gamma_{2,T}\right)^{k-u}}{k!l^k\bar{\gamma}_2^k(l-1)^{-u}}\left(\mathcal{P}_2\left(x,\frac{j(l-1)+(l+j)}{l},u,\gamma_{2,T}\right) - \mathcal{P}_2\left(x,\frac{j(l-1)+(l+j)}{l},u,\frac{l}{l-1}\gamma_{2,T}\right)\right)\Bigg], \tag{10}$$

$$\mathcal{W}_3(x,l) = \frac{1}{(l-1)!\bar{\gamma}_2^{l-1}}\binom{n_r}{l}\sum_{u=0}^{l-1}\binom{l-1}{u}\frac{(-1)^u\left(l\gamma_{2,T}\right)^{l-1-u}}{(l-1)^{-u}}\left[\mathcal{P}_1\left(x,1,u\right) - \mathcal{P}_2\left(x,1,u,\frac{l}{l-1}\gamma_{2,T}\right)\right]$$
$$+\sum_{j=1}^{n_r-l}\frac{n_r!(-1)^{j-l+1}}{(n_r-l-j)!l!j!}\left(\frac{l}{j}\right)^{l-1}\Bigg[\mathcal{P}_1\left(x,\frac{l+j}{l},0\right) - \mathcal{P}_2\left(x,\frac{l+j}{l},0,\frac{l}{l-1}\gamma_{2,T}\right) - e^{\frac{j\gamma_{2,T}}{\bar{\gamma}_2}}\sum_{k=0}^{l-2}\sum_{u=0}^{k}$$
$$\frac{\binom{k}{u}j^k\left(-l\gamma_{2,T}\right)^{k-u}}{k!l^k\bar{\gamma}_2^k(l-1)^{-u}}\left(\mathcal{P}_1\left(x,\frac{j(l-1)+(l+j)}{l},u\right) - \mathcal{P}_2\left(x,\frac{j(l-1)+(l+j)}{l},u,\frac{l}{l-1}\gamma_{2,T}\right)\right)\Bigg], \tag{11}$$

$$\mathcal{P}_1(x,p,d) = \sum_{i=0}^{n_t-1}\sum_{k=0}^{i}\sum_{e=0}^{d}\frac{2\binom{i}{k}\binom{e}{d}}{i!\bar{\gamma}_1^i\bar{\gamma}_2}\left(\frac{\bar{\gamma}_2}{\bar{\gamma}_1 p}\right)^{\frac{k+e-i+1}{2}}x^{d+\frac{k-e+i+1}{2}}(x+b)^{\frac{i-k+e+1}{2}}e^{-(\frac{1}{\bar{\gamma}_1}+\frac{p}{\bar{\gamma}_2})x}K_{k-i+e+1}\left(2\sqrt{\frac{x(x+b)p}{\bar{\gamma}_1\bar{\gamma}_2}}\right), \tag{12a}$$
$$\mathcal{P}_2(x,p,d,r) = \sum_{i=0}^{n_t-1}\sum_{k=0}^{i}\sum_{e=0}^{d}\frac{4\binom{i}{k}\binom{e}{d}}{i!\bar{\gamma}_1^i\bar{\gamma}_2}\left(\frac{\bar{\gamma}_2}{\bar{\gamma}_1 p}\right)^{\frac{k+e-i+1}{2}}x^{d+\frac{k-e+i+1}{2}}(x+b)^{\frac{i-k+e+1}{2}}e^{-\left(\frac{1}{\bar{\gamma}_1}+\frac{p}{\bar{\gamma}_2}\right)x}$$
$$\times k^{+}_{k+e-i+1}\left(2\sqrt{\frac{x(x+b)p}{\bar{\gamma}_1\bar{\gamma}_2}},\log\left((r-x)\sqrt{\frac{p\bar{\gamma}_1}{x(x+b)\bar{\gamma}_2}}\right)\right), \tag{12b}$$

---

where $M_{\gamma_{c,k,\text{UB}}}(s,x)$ represents the incomplete MGF of $\gamma_{c,k,\text{UB}}$, which can be obtained as

$$M_{\gamma_{c,k,\text{UB}}}(s,x) = \frac{1}{(n_t-1)!\bar{\gamma}_1^{n_t}}\left(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2}+\omega\right)^{-n_t}$$
$$\times\Gamma\left(n_t,\left(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2}+\omega\right)x\right)$$
$$+\sum_{i=0}^{n_t-1}\frac{1}{i!\bar{\gamma}_1^i\bar{\gamma}_2}\left(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2}+\omega\right)^{-i-1}$$
$$\times\Gamma\left(i+1,\left(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2}+\omega\right)x\right). \tag{20}$$

For the term $A_c(x)$ in (15), it can be obtained as

$$A_c(x) \simeq \frac{1-(F_{\gamma_{c,k,\text{UB}}}(\gamma_{c,T}))^{n_a}}{1-F_{\gamma_{c,k,\text{UB}}}(\gamma_{c,T})}(F_{\gamma_{c,k,\text{UB}}}(x) - F_{\gamma_{c,k,\text{UB}}}(\gamma_{c,T})). \tag{21}$$

The evaluation of the term $F^{(l)}_{\gamma_c}(x)$ in (15) requires obtaining the joint PDF $f_{\gamma_{c,l:n_r},\Gamma_{c,l-1}}(x,y)$, which can be approximated as

$$f_{\gamma_{c,l:n_r},\Gamma_{c,l-1}}(x,y) \simeq \frac{n_a!}{(n_a-l)!(l-1)!}\left[F_{\gamma_{c,k,\text{UB}}}(x)\right]^{n_a-l}$$
$$\times\left[1-F_{\gamma_{c,k,\text{UB}}}(x)\right]^{l-1}f_{\gamma_{c,k,\text{UB}}}(x)f_{\sum_{k=1}^{l-1}\gamma'_{c,k}}(y), \tag{22}$$

for $x>0$ and $y>(l-1)x$. The remaining task is to find the PDF of the sum $\sum_{k=1}^{l-1}\gamma'_{c,k}$. In order to obtain the PDF of this sum, it is noted that $M_{\sum_{k=1}^{l-1}\gamma'_{c,k}}(s,x) = \prod_{k=1}^{l-1}M_{\gamma'_{c,k}}(s,x)$, where $M_{\gamma'_{c,k}}(s,x)$ can be obtained as

$$M_{\gamma'_{c,k}}(s,x) \simeq \frac{1}{1-F_{\gamma_{c,k,\text{UB}}}(x)}\Bigg[\frac{1}{(n_t-1)!\bar{\gamma}_1^{n_t}}\left(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2}+s\right)^{-n_t}$$
$$\times\Gamma\left(n_t,\left(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2}+s\right)x\right)+\sum_{i=0}^{n_t-1}\frac{1}{i!\bar{\gamma}_1^i\bar{\gamma}_2}$$
$$\times\left(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2}+s\right)^{-i-1}\Gamma\left(i+1,\left(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2}+s\right)x\right)\Bigg]. \tag{23}$$

The PDF of $\sum_{k=1}^{l-1}\gamma'_{c,k}$ can be now written, using a fast

$$M_{\gamma_c}(s) = 1 - s\frac{1-(F_{\gamma_{2,1,\varsigma}}(\gamma_{2,T}))^{n_r}}{1-F_{\gamma_{2,1,\varsigma}}(\gamma_{2,T})}\left[\mathcal{Q}_1^u(s,1,0,\gamma_{2,T}) + \mathcal{Q}_2(s,1,0,\gamma_{2,T},\gamma_{2,T})\right] + \frac{s\binom{n_r}{L}}{(L-1)!\bar{\gamma}_2^{L-1}}$$
$$\times \mathcal{V}_1(s,1,L-1,\gamma_{2,T}) + s\binom{n_r}{L}\sum_{l=1}^{n_r-L}(-1)^{L+l-1}\binom{n_r-L}{l}\Bigg[\mathcal{V}_1(s,\frac{l}{L}+1,0,\gamma_{2,T}) - \sum_{m=0}^{L-2}\frac{1}{m!}\left(-\frac{l}{L\bar{\gamma}_2}\right)^m$$
$$\times \mathcal{V}_1(s,1,m,\gamma_{2,T})\Bigg] + s\sum_{l=2}^{L}\Bigg[\frac{\binom{n_r}{l}}{(l-1)!\bar{\gamma}_2^{l-1}}\sum_{u=0}^{l-1}\binom{l-1}{u}\frac{(-1)^u(l\gamma_{2,T})^{l-1-u}}{(l-1)^{-u}}\mathcal{V}_2\left(s,1,u,\gamma_{2,T},\frac{l}{l-1}\gamma_{2,T}\right)$$
$$+\sum_{j=1}^{n_r-l}\frac{n_r!(-1)^{j-l+1}}{(n_r-l-j)!l!j!}\left(\frac{l}{j}\right)^{l-1}\Bigg[\mathcal{V}_2\left(s,\frac{l+j}{l},0,\gamma_{2,T},\frac{l}{l-1}\gamma_{2,T}\right) - e^{\frac{j\gamma_{2,T}}{\bar{\gamma}_2}}$$
$$\times\sum_{k=0}^{l-2}\sum_{u=0}^{k}\frac{\binom{k}{u}j^k(-l\gamma_{2,T})^{k-u}(l-1)^u}{k!l^k\bar{\gamma}_2^k}\mathcal{V}_2\left(s,\frac{j(l-1)+(l+j)}{l},u,\gamma_{2,T},\frac{l}{l-1}\gamma_{2,T}\right)\Bigg]\Bigg], \tag{13}$$

---

convergent series, in the following form:

$$f_{\sum_{k=1}^{l-1}\gamma'_{c,k}}(x) = \frac{4}{T}\sum_{n=1,\,n\,\text{odd}}^{+\infty}\Re\{e^{-\mathrm{i}n\omega_0 x}\prod_{k=1}^{l-1}M_{\gamma'_{c,k}}(-\mathrm{i}n\omega_0,x)\}, \tag{24}$$

where $\Re(z)$ denotes the real part of the quantity z. After substituting the preceding result into the joint PDF $f_{\gamma_{c,l:n_r},\Gamma_{c,l-1}}(x,y)$, it follows that the integrals $\int_{\frac{l-1}{l}\gamma_{c,T}}^{\frac{l-1}{l}x}\int_{\gamma_{c,T}-y}^{\frac{y}{l-1}} f_{\gamma_{c,l:n_r},\Gamma_{c,l-1}}(z,y)dzdy$ and $\int_{\frac{l-1}{l}x}^{\gamma_{c,T}}\int_{\gamma_{c,T}-y}^{x-y} f_{\gamma_{c,l:n_r},\Gamma_{c,l-1}}(z,y)dzdy$, which are associated with the term $g_c(x,l)$ in (3) as part of $F_{\gamma_c}^{(l)}(x)$ in (15), are finite-range integrals; hence, they can be computed efficiently using Gauss-Chebyshev method.

C. MGF of The Output SNR

This subsection presents an analytical approach that can quantify the MGF of γ_c based on the pre-combining end-to-end SNRs upper bounds. Specifically, it can be shown that the MGF of γ_c in this case can be expressed as

$$M_{\gamma_c}(s) \simeq I_1(s) + \frac{1-(F_{\gamma c,k,\text{UB}}(\gamma_{c,T}))^{n_a}}{1-F_{\gamma c,k,\text{UB}}(\gamma_{c,T})}I_2(s) + \sum_{l=2}^{L}I_3(s,l), \tag{25}$$

where the terms $I_1(s)$, $I_2(s)$, and $I_3(s,l)$ in the preceding equation are given by

$$I_1(s) = \frac{4}{T}\sum_{n=1,\,n\,\text{odd}}\Re\Bigg\{\frac{M_{\Gamma_{c,L}}(-\mathrm{i}n\omega_0)}{(s+\mathrm{i}n\omega_0)} \times(1-e^{-(s+\mathrm{i}n\omega_0)\gamma_{c,T}})\Bigg\}, \tag{26a}$$

$$I_2(s) = M_{\gamma_{c,k,\text{UB}}}(s,\gamma_{c,T}), \tag{26b}$$

$$I_3(s,l) = \int_{\gamma_{c,T}}^{\frac{l\,\gamma_{c,T}}{l-1}}\int_{\frac{l-1}{l}x}^{\gamma_{c,T}} e^{-sx}f_{\gamma_{c,l:n_r},\Gamma_{c,l-1}}(x-y,y)dydx, \tag{26c}$$

in which the term $M_{\gamma_{c,k,\text{UB}}}(s,x)$ is given in (20). Again, the integral in (26c) can be efficiently computed using Gauss-Chebyshev method, where fast and accurate results can be obtained with only ten points.

V. Numerical Results and Discussions

This section shows numerical and simulation results for the outage probability, average bit error rate (BER) for B-DPSK modulation, and average numbers of estimated and activated branches of the two combining schemes. Note that, since we have the MGF, the BER can be generated for any differential or non-differential modulation using the MGF-approach [18].

Fig. 2 shows the outage probability against the outage threshold, γ_{th} for the one-phase based combining with $n_r = n_t = 4$ and $n_a = 1$, and two-phase based combining with $n_a = n_t = 4$ and $n_r = 1$. In Fig. 2, it is assumed that $\bar{\gamma}_j = 10$ dB, $\gamma_{2,T} = \gamma_{c,T} = 10$ dB, $L = 2$, and $b = 0$. The figure clarifies the tightness of the upper bounds of the two combining schemes for the outage performance measure. Moreover, it compares the outage performance of the two schemes under similar conditions. For the two-phase based combining, it is observed that the use of the upper bound for the pre-combining end-to-end SNR per each relaying station gives tight result to the exact one that has been obtained from simulations. On the other hand, for the one-phase based combining, the use of the upper bound for the second hop combined SNR provides tight result to the exact one, which has been obtained using the analytical results and verified by simulations, only for relatively low values of γ_{th}. The two-phase system outperforms the one-phase based combining, particularly for relatively large values of γ_{th}.

An important observation from the exact results in Fig. 2, which will be noted in the remaining figures, is that the effect of combining threshold is clearly seen for the two-phase combining as compared to the one-phase based combining. Specifically, for the two-phase based combining that is based on the pre-combining end-to-end statistics, the change in the outage slope occurs when $\gamma_{\text{th}} = \gamma_{c,T}$, at which the combining process at the receiver varies between the cases of conventional generalized selection when $\gamma_{\text{th}} < \gamma_{c,T}$ (i.e. choosing the best two relaying stations out of the four available ones) and the adaptive process of either an arbitrarily chosen relaying station or the one having the best end-to-end channel quality when $\gamma_{\text{th}} > \gamma_{c,T}$. On the other hand, the effect of γ_{th} is relatively less notable on the overall performance of the one-phase based combining due to the fact that the second hop contributes

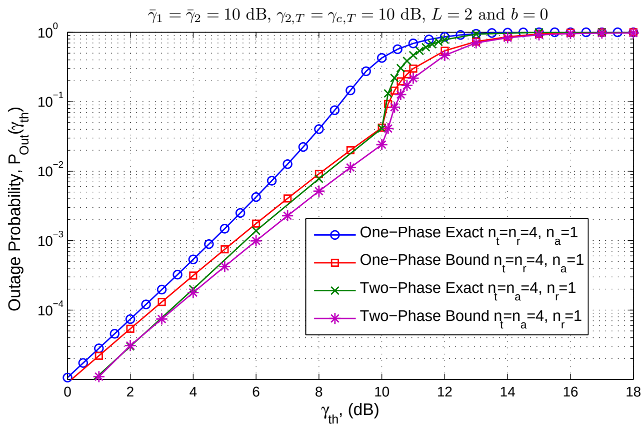

Fig. 2. Comparison between the exact results and upper bound approximate results (on the combined end-to-end SNR) for the outage probability of the one-phase and two-phase based combining. The results are shown for different combinations of n_t, n_r, n_a.

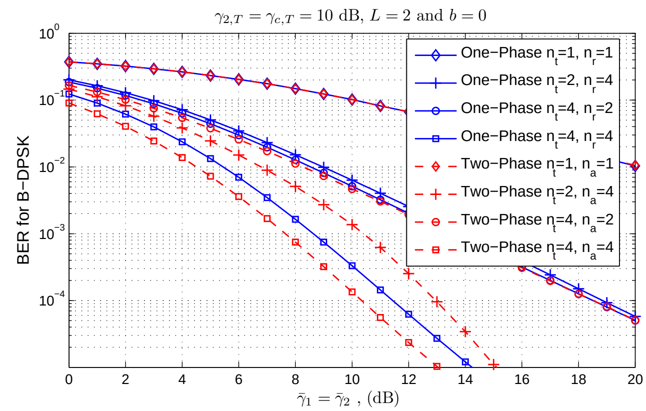

Fig. 4. Effect of the number of receive antennas for one-phase based combining and the number of relaying stations for two-phase based combining on the average BER of B-DPSK.

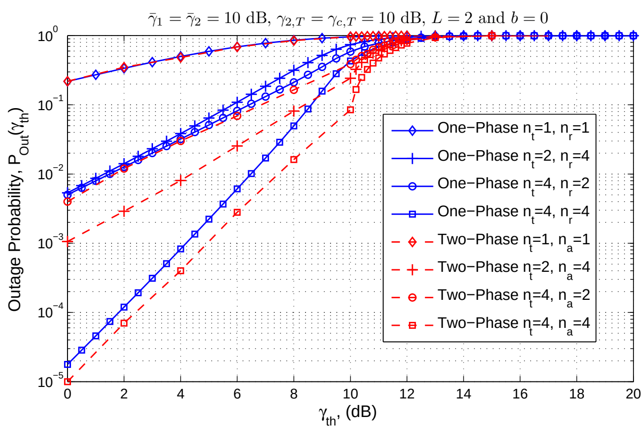

Fig. 3. Effect of the number of receive antennas for one-phase based combining and the number of relaying stations for two-phase based combining on the outage performance.

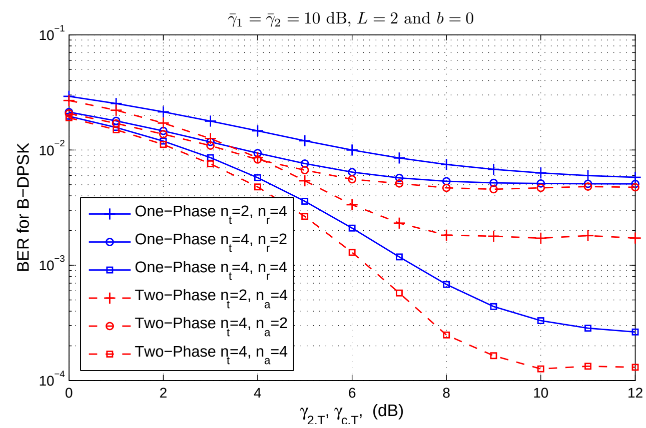

Fig. 5. Effect of combining thresholds, $\gamma_{2,T}$ and $\gamma_{c,T}$, for one-phase and two-phase based combining schemes, respectively, on the average BER of B-DPSK for different combinations of receive antennas (or relaying stations).

partially to this performance. Specifically, the quality of the combined SNR on the first hop can significantly vary the effect of the adaptive combining of receive antennas on the overall performance.

Fig. 3 and 4 show the effects on n_t, n_r, n_a on the outage probability, depicted against γ_{th}, and the average BER of B-DPSK, depicted against the average SNR per hop, respectively. The outage probability results are shown with $\bar{\gamma}_j = 10$ dB in Fig. 3, and $\gamma_{2,T} = \gamma_{c,T} = 10$ dB, $L = 2$, and $b = 0$ are used in both figures. It is observed that the two combining schemes perform similarly only when $n_t = n_r = n_a = 1$. The two-phase based combining provides better outage and BER performance than the one-phase based combining for any combinations of n_t and $n_r = n_a$. For the one-phase system, the case of $n_t = 4$, $n_r = 2$, and $n_a = 1$ provides slightly better performance than that when $n_t = 2$, $n_r = 4$, and $n_a = 1$, which reveals that increasing the number of receive antennas may not improve the end-to-end performance when the first hop is relatively weak. However, different conclusion is drawn for the two-phase based combining when the case of $n_t = 4$, $n_r = 1$, and $n_a = 2$ is compared to that of $n_t = 2$, $n_r = 1$, and $n_a = 4$, where the use of more relaying stations improves the performance substantially. In addition to the effects of n_t, n_r, n_a, the effects of $\bar{\gamma}_1$ and $\bar{\gamma}_2$ on the outage performance and average BER have been also studied. For one-phase based combining, it has been observed that the increase in the second hop average SNR may not guarantee improved performance as compared to the case with similar increase in the first hop average SNR. On the other hand, for the two-phase based combining, substantial gain is observed with the improvement in the second hop fading conditions.

Fig. 5 extends the results shown in Fig. 4 by studying the effect of the combining thresholds for one-phase and two-phase based combining on the average BER of B-DPSK for different combinations of n_t, n_r, n_a when $\bar{\gamma}_1 = \bar{\gamma}_2 = 10$ dB, $b = 0$, and $L = 2$. It is seen that the increase in the combining threshold improves the performance but at the expense of activating more receive antennas or relaying stations (see Figs. 6 and 7). However, further increase in the combining threshold does not affect the performance when the best possible combined SNR is achieved.

Figs. 6 and 7 show the average number of estimated and combined branches, respectively, against the combining

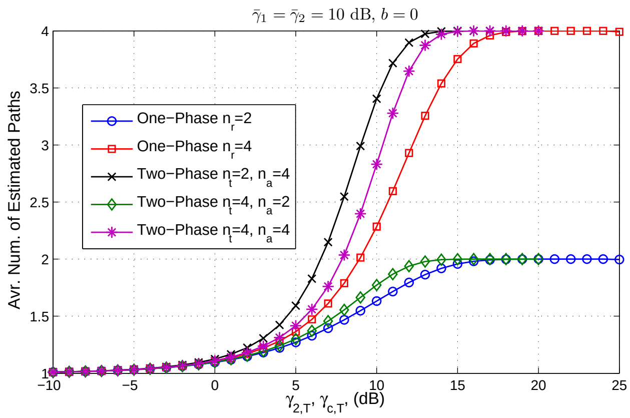

Fig. 6. Effect of the number of receive antennas (or relaying stations) on the average number of estimated branches.

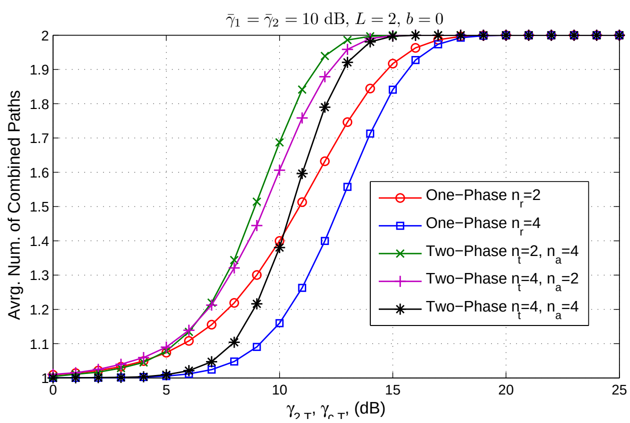

Fig. 7. Effect of the number of receive antennas (or relaying stations) on the average number of combined branches.

threshold for the case studies in the previous figures. The figures show the effect of different combinations of n_t, n_r, n_a for one-phase and two-phase based combining schemes when $\bar{\gamma}_1 = \bar{\gamma}_2 = 10$ dB, $L = 2$, and $b = 0$. It is observed that the two-phase based combining requires activating more relays when n_t is low, whereas the one-phase based combining operates independently of n_t. The one-phase based combining requires less number of estimations than that of the two-phase based combining, but may require more active receive antennas, particularly when n_r is low. In addition, the effects of the first and second hops average fading conditions on the average number of estimated and combined diversity branches have been investigated. For the one-phase based combining, it has been noted that the increase in $\bar{\gamma}_2$ reduces the average numbers of estimated and combined receive antennas substantially. On the other hand, the two-phase based combining needs less numbers of estimated and combined relaying branches when $\bar{\gamma}_2 > \bar{\gamma}_1$ and n_t is relatively large.

VI. Conclusions

This correspondence has proposed two low-complexity combining schemes to mitigate multipath fading on the performance dual-hop AF relaying systems in the absence of direct communication link. The two schemes, which have been referred to as one-phase and two-phase combining, vary in terms of their implementation requirements and configurations, and they can be applied for different conditions of receive station. Through the analysis, analytical models have been developed for the statistics of the end-to-end SNR of the two proposed schemes, from which some performance measures, such as outage performance and error rate, have been studied. Numerical and simulation results have been presented to clarify the performance-complexity tradeoff of the proposed schemes.

References

[1] M. O. Hasna and M.-S. Alouini, "Harmonic mean and end-to-end performance of transmission systems with relays," *IEEE Trans. on Commun.*, vol. 52, no. 1, pp. 130–135, Jan. 2004.

[2] M. O. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. on Wireless Commun.*, vol. 2, no. 6, pp. 1126–1131, Nov. 2003.

[3] H. Bölcskei and R. U. Nabar, "Capacity scaling laws in MIMO relay networks," *IEEE Trans. on Wireless Commun.*, vol. 5, no. 6, pp. 1433–1444, Jun. 2006.

[4] Y. Fan and J. Thompson, "MIMO configurations for relay channels: Theory and practice," *IEEE Trans. on Wireless Commun.*, vol. 6, no. 5, pp. 1774–1768, May 2007.

[5] H. Shin and J. B. Song, "MRC analysis of cooperative diversity with fixed-gain relays in Nakagami-m fading channels," *IEEE Trans. on Wireless Commun.*, vol. 7, no. 6, pp. 2069–2074, May 2007.

[6] S. Chen, W. Wang, X. Zhang, M. Peng, and D. Zhao, "Capacity performance of amplify-and-forward MIMO relay with transmit antenna selection and maximal-ratio combining," *in Proc. IEEE GLOBECOM, Miami, FA*, 2009.

[7] M. Di Renzo, F. Graziosi, and F. Santucci, "A unified framework for performance analysis of CSI-assisted cooperative communications over fading channels," *IEEE Trans. on Commun.*, vol. 57, no. 9, pp. 2551–2557, Sept. 2009.

[8] D. Senaratne and C. Tellambura, "Unified exact performance analysis of two hop amplify and forward relaying in Nakagami fading," *IEEE Trans. Veh. Technol.*, vol. 59, no. 3, pp. 1529–1534, Mar. 2010.

[9] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Trans. on Commun.*, vol. 47, no. 9, pp. 1458–1461, Oct. 1999.

[10] R. H. Y. Louie, Y. Li, H. A. Suraweera and B. Vucetic, "Performance analysis of beamforming in two hop amplify and forward relay networks with antenna correlation," *IEEE Trans. Wireless Commun.*, vol.8, no.6, pp. 3132–3141, Jun. 2009.

[11] R. M. Radaydeh and M.-S. Alouini, "Impact of co-Channel interference on the performance of adaptive generalized transmit beamforming," *IEEE Trans. Wireless Commun.*, vol. 10, no. 8, pp. 2616–2629, Aug. 2011.

[12] R. M. Radaydeh and M.-S. Alouini, "Adaptive transmit selection with interference suppression," *in Proc. 5th IEEE Int. Symp. Wireless Pervasive Comput. (ISWPC'2010)*, Modena, Italy, May 2010, pp. 33–38.

[13] R. M. Radaydeh, "Performance of non-ideal OT-MRC with co-Channel interference," *IEEE Trans. Commun.*, vol. 58, no. 12, pp. 3352–3357, Dec. 2010.

[14] R. M. Radaydeh, "Impact of delayed arbitrary transmit antenna selection on the performance of rectangular QAM with receive MRC in fading channels," *IEEE Commun. Lett.*, vol. 13, no. 6, pp. 390–392, Jun. 2009.

[15] R. M. Radaydeh, "Receive maximal-ratio combining with outdated arbitrary transmit antenna selection in Nakagami-m fading," *IET Commun.*, vol. 3, no. 10, pp. 1638–1648, Oct. 2009.

[16] R. M. Radaydeh and M.-S. Alouini,, "Transmit selection for imperfect threshold-based receive MRC in Rayleigh fading channels," *in Proc. IEEE 17th Int. Conf. Telecommun. (ICT'2010)*, Doha, Qatar, Apr. 2010, pp. 177–182.

[17] M.-S. Alouini and H. C. Yang, "Minimum estimation and combining generalized selection combining (MEC-GSC)," *IEEE Trans. on Wireless Commun.*, vol. 6, no. 2, pp. 526–532, Feb. 2007.

[18] M. K. Simon and M.-S. Alouini, *Digital Communications over Generalized Fading Channels: A Unified Approach to Performance Analysis*, New York, NY: John Wiley and Sons, 2000.

[19] S. W. Kim, D. S. Ha, and J. H. Reed, "Minimum selection GSC and adaptive low-power rake combining scheme," in *Proc. IEEE Int. Symp. on Circuits and Syst. (ISCAS2003)*, vol. 4, May 2003.

[20] H. C. Yang, "New results on ordered statistics and analysis of minimum-selection generalized selection combining (GSC)," *IEEE Trans. on Wireless Commun.*, vol. 5, no. 7, pp. 1876–1885, Jul. 2006.

[21] M.-S. Alouini and M. K. Simon, "An MGF-based performance analysis of generalized selective combining over Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 48, pp. 401–415, Mar. 2000.

[22] R. Cicchet and A. Faraone, "Incomplete Hankel and modified Bessel functions: A class of special functions for electro magnetics," *IEEE Trans. on Antennas and Propag.*, vol. 52, no. 12, pp. 3373–3389, Dec. 2004.

[23] D. S. Jones, "Incomplete Bessel functions. I," *in Proc. of the Edinburgh Math. Soci.*, vol. 50, pp. 173–183, 2007.

[24] A. Quarteroni, R. Sacco and F. Saleri, *Num. Math. (Texts in Applied Math.)*, Springer, 2006.

[25] C. Tellambura and A. Annamalai, "Further results on the Beaulieu series," *IEEE Trans. Commun.*, vol. 48, no.11, pp. 1774–1777, Nov 2000.

[26] A. Annamalai, G. Deora and C. Tellambura, "Analysis of generalized selection diversity systems in wireless channels," *IEEE Trans. Veh. Technol.* , vol.55, no.6, pp.1765–1775, Nov. 2006.