Time domain oscillating poles: Stability redefined in Memristor based Wien-oscillator

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***Abstract*— Traditionally, the necessary and sufficient condition for any system to be oscillating is that its poles are located on the imaginary (j*ω*) axis. In this paper, for the first time, we have shown that systems can oscillate with time-domain oscillating poles. The idea is verified using a Memristor based Wien oscillator. Sustained oscillations are observed without having the poles of the system fixed on the imaginary axis and the oscillating behavior of the system poles is reported. The oscillating resistance and triangular shape of FFT are also demonstrated with mathematical reasoning and simulation results to support the unusual and surprising characteristics.**

***Index Terms*—Memristor, oscillating poles, stable oscillation**

I. Introduction

In 1971 Leon O. Chua [1, 2] showed the existence of a new two-terminal circuit element called the 'Memristor'. He proposed the Memristor as the fourth basic circuit element which relates between the flux-linkage (φ) and the charge (q). The unconventional properties of Memristor have led to the successful modeling of a number of physical devices and systems [3-5]. Interest in the Memristor revived in 2008 when an experimental version was reported by HP lab [6]. That device neither uses magnetic flux, nor stores charge as a capacitor does, rather achieves a resistance dependent on the history of current. The Memristor has been investigated independently or with the other three passive components (R, L, and C) [7, 8]. Some patents related to Memristors appear to include applications in DRAM [9], programmable logic [9], signal processing [10], neural networks [11], and control systems [12].

Oscillators generally use some form of active device: opamp or BJT, surrounded by passive devices such as resistors, capacitors, and inductors, to generate a sinusoidal output [13]. They are widely used in generating signals to broadcast by radio and television transmitters, clock signals that regulate computers and quartz clocks. The Wien oscillator is one of the simplest and best known oscillators.

Poles of an oscillatory system play the major role in determining the stability of oscillation [13]. The location of the poles and the values of the real and imaginary parts determine the response of the system. The condition for any system to oscillate is achieved by forcing the poles to have only imaginary part and these poles should have a fixed position.

In this article we will describe a new phenomenon caused by employing a Memristor in a conventional Wien oscillator circuit. A mathematical model is proposed to analyze Wien Memristor-oscillator. The simulation results and mathematical models presented here demonstrate the surprising findings in the field of oscillation which represent a new paradigm in circuit theory.

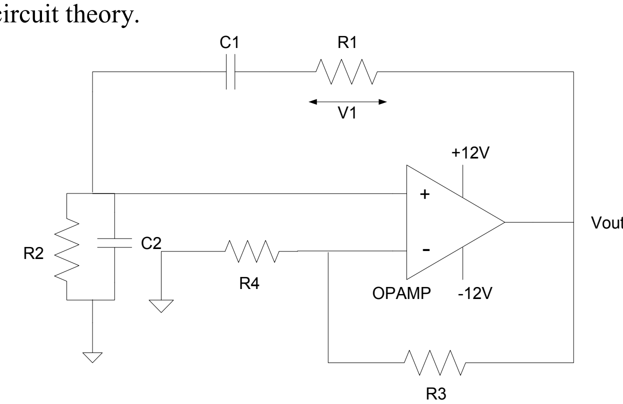

Fig. 1: Schematics of conventional Wien oscillator

II. Overview of Wien Oscillator

A. Wien Oscillator

The Wien bridge oscillator is one of the various types of electronic oscillators that can generate sine waves. It can generate a large range of frequencies. The main components of this oscillator are four resistors and two capacitors [13]. Op-amp sine-wave oscillators operate without an externally-applied input signal. Some combinations of positive and negative feedback are used to drive the op amp into an unstable state and this causes the output to oscillate. The derived equations for the frequency of oscillation and the condition of oscillation are demonstrated later. The schematic of the basic Wien oscillator is shown in Fig. 1.

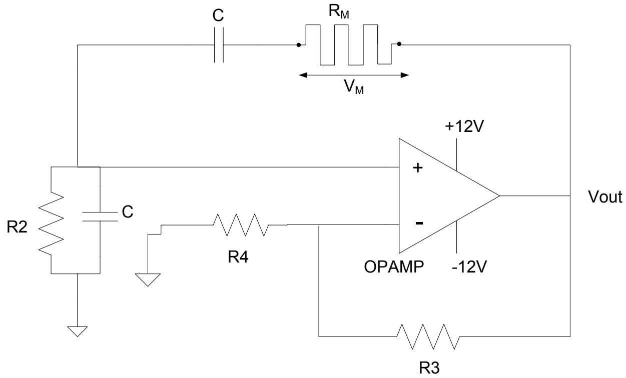

Fig. 2: Schematics of Wien oscillator with Memristor

Considering the circuit shown in Fig. 1 the characteristics equation of this system can be derived in the following form:

$$s^2 + bs + d = 0 \qquad (1)$$

$$b = \frac{1}{R_2C_2} + \frac{1}{R_1C_1} - \frac{R_3}{R_1R_4C_2}, d = \frac{1}{R_1R_2C_1C_2} \qquad (2)$$

The condition of sustained oscillation is given by

$$\frac{C_2}{C_1} + \frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad (3)$$

The frequency of oscillation is expressed as:

$$f = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} \qquad (4)$$

For sustain oscillation *b* must be equal to zero in (1). The location of the poles in this system can be found by solving the quadratic equation in (1). When b=0, there will be two conjugate poles lying on the imaginary axis. If b is a non zero value then those two poles will be located in either the right half or left half of the s-plane. For oscillations to be sustained, the poles of the system must lie on the imaginary axis. . Shifting of poles from the imaginary axis will result in a damped oscillation, which is not a sustained oscillation.

B. Wien Oscillator with Memristor

Referred to Fig. 1, R_1 is replaced with Memristor and both $C_1$and C_2 are changed to same valued capacitance C as shown in Fig. 2. The resistance of the Memristor will be designated as R_M and the voltage across the Memristor will be designated as V_M. From the simulation results R_M is found to be oscillating. So R_M will have an average value which can be denoted as R_{avg}. These changes will certainly modify equations (1)-(4). The frequency of oscillation of the new system, f_M can be written as:

$$f_M = \frac{1}{2\pi C\sqrt{R_{avg}R_2}} \qquad (5)$$

The condition of oscillation is modified as:

$$1 + \frac{R_{avg}}{R_2} = \frac{R_3}{R_4} \equiv gain \qquad (6)$$

Dmitri B. Strukov *et al*. first proposed a physical model to realize the Memristor as a two terminal device [6]. They considered a thin semiconductor film of thickness sandwiched between two metal contacts. The semiconductor film has a region with a high concentration of dopants having

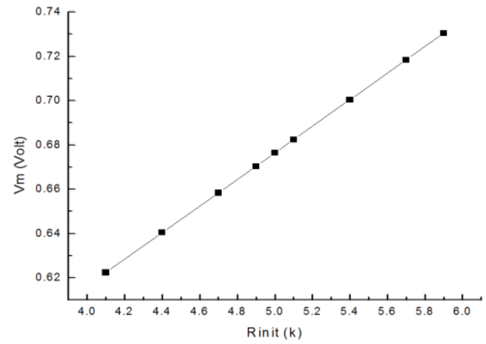

Fig. 3: Voltage across the Memristor (V_M) from simulation

low resistance R_{on}, and the remainder has a low dopant concentration and higher resistance R_{off}. The total resistance of the Memristor, R_M, is a sum of the weighted (depending on the width of the doped and undoped regions) resistances of the doped and undoped regions. The SPICE model proposed by Zdeněk Biolek *et al.* [14, 15] can be used to simulate the effect of Memristor in various circuits. The model is implemented as a SPICE subcircuit with the following parameters: the initial R_{init} resistance, the R_{off} and R_{on} resistances, the width of the thin film *D*, the dopant mobility μ_v, and the exponent *p* of the window function. To counterpart the effect of the nonlinear dopant drift they have proposed a window function with *p* as the parameter to set the nonlinearity. The differences between models with linear and nonlinear drifts decrease if *p* increases. Recent mathematical modeling of Memristor in case of periodic signals is proposed in [16, 17].

For our simulations we have used R_2=5k, C_1=C_2=C=3.2μF, R_4=100k, R_1 (in Fig. 1) is replaced with Memristor (Fig. 2). In the SPICE model only R_{init} is changed and the value of *p* is given 10. *p=10* is good enough for linear approximation. The other parameters were kept as it is in the model file [15]. R_{init} was changed from 4.1k to 5.9k for simulation and also for mathematical modeling.

III. Results and Discussion

Sustained oscillations are found for each R_{init} value of R_M. In Fig. 3 the simulated values of V_M (voltage across the Memristor) is shown for every value of R_{init}. From this figure it is easily observed that V_M has linear relation with R_{init}. So V_M (the peak voltage of V_M) can be modeled with straight line approximation. V_M can be expressed as a function of R_{init}:

$$V_M = 0.3764 + 0.06R_{init} \qquad (7)$$

The calculated values of V_M using (7) give similar results as simulation and the error in this case is less than 0.22%. Thus V_M can be approximated by two methods: the first method is by using (7) where V_M can be calculated for every R_{init}. The second method would be to use conventional Wien oscillator (Fig. 1). As the average values of V_M are almost constant whether a Memristor is used or not if similar values of R_1 and R_{init} are used so V_1 can be taken from conventional Wien oscillator to be used as V_M in the proposed formula described later. In Fig. 4 the simulation result is shown for R_M and V_{out}. V_{out} is the final output voltage. For this simulation, R_{init}=5k. Similar results are found for other values of R_{init}. The sustained oscillation is clearly observed in both the final output V_{out} andR_M (Fig. 4). R_3 has to be changed to accomplish this oscillation. The values of R_3 are found equal to the calculated values derived from (6). In this case R_{init} is taken as R_{avg}. This approximation does not alter the result as R_{avg} (from the simulation) is very close to R_{init} with an error less than 0.02%. The most interesting observation is that the output oscillation is sustained as shown in Fig. 4. Though from the SPICE model [15] R_M can take values like R_{on}=0.1k, R_{off}=16k, and R_{init} but from the simulation it is found that R_M is oscillating in a different range across R_{avg}. So Memristor resistance, R_M can be modeled as:

$$R_M = R_{avg} + \Delta R_M \sin(\omega t \pm \varphi) \quad (8)$$

Where, ΔR_M is the sinusoidal amplitude. If only the magnitude peaks of R_M is considered then (8) can be written as:

$$R_{max\,,min} = R_{avg} \pm \Delta R_M \quad (9)$$

As R_M is oscillating, It will have a maximum (R_{max}) and a minimum (R_{min}) values. Using the concept described in [16] and reusing (5) and (6) the maximum and minimum values of R_M can be modeled as:

$$R_{max}{}^2 = R_{min}{}^2 \pm \left[\frac{2V_M k\left(R_{off} - R_{on}\right)}{\pi f_M}\right] \quad (10)$$

Where, R_{on} and R_{off} are described previously. k is the same factor as described in [3] and it is given as $k = (\mu_v R_{on})/D^2$

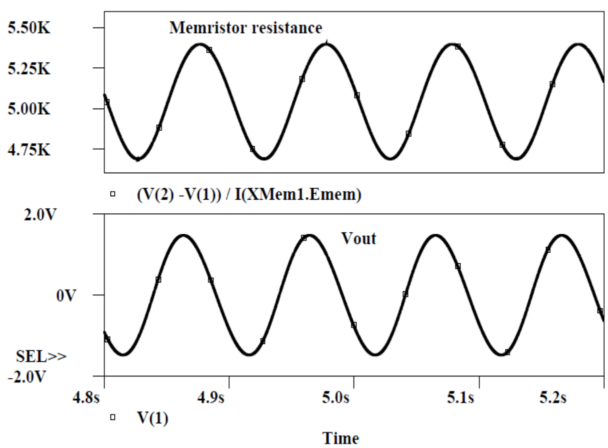

Fig. 4: Simulation result for R_M and V_{out} of the final output oscillation.

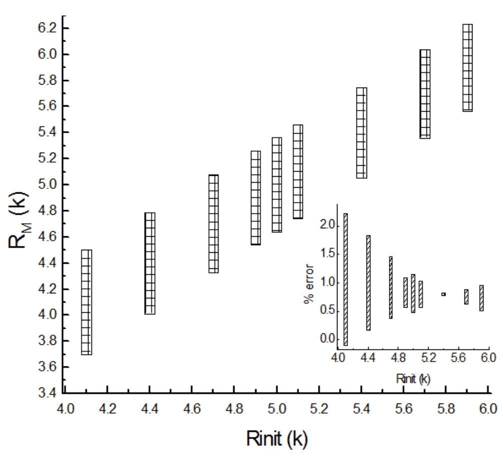

Fig. 5: Range of calculated values of R_M; Range of percentage errors between simulated and calculated R_M (inset).

Using the bionomial expansion, equation (10) can be approximated as following:

$$|R_{max} - R_{init}| \cong \left[\frac{\{V_M k\left(R_{off} - R_{on}\right)\}}{\{2\pi R_{init} f_{avg}\}}\right] \equiv \Delta R_M \quad (11)$$

For any value of R_{init}, the peaks R_{max} and R_{min} can be easily calculated using (11). In Fig. 5 the calculated range of the Memristor resistance R_M using (11) is shown for different values of R_{init}. The inset figure in Fig. 5 shows the percentage error between simulation and calculated values for every range of R_M. From Fig. 5 it is seen that the calculated and the simulated values of R_M are quite close with a maximum error of 2.2% which is negligible.

A. Poles as a function of time: "Oscillating poles"

It is well known that for any oscillation to be stable, the poles of that system must lie on imaginary axis. In other words, there should not be any real part in the value of the pole. If the poles of the system fall in right half of the *s* plane, then the system will have a over damped oscillation, which will be increasing by time, and as a result, the oscillation will saturate. If the poles of the system fall in the left half of *s* plane, the system will have a damped oscillation, which dies out eventually. Using (2), and (6) the expression for *b* and *d* can be written as:

$$b = \frac{1}{CR_M R_2}\left[R_M - R_{avg}\right] \cong \frac{\Delta R_M}{CR_M R_2}, d = \frac{1}{C^2 R_M R_2} \quad (12)$$

The solution for *s* can be written as:

$$s = \sigma \pm j\omega = \frac{b}{2} \pm j\left[\frac{\sqrt{4R_M R_2 - (\Delta R_M)^2}}{2CR_M R_2}\right] \quad (13)$$

Conventional stable oscillation can only be achieved if *b*=0 as in case of the Wien oscillator presented in Fig. 1 where the oscillation is sustained by adjusting the values of R_1, R_2, C_1, C_2, R_3, and R_4. But in the case where a Memristor is used (refer to (12)), *b* is found directly proportional to ΔR_M. So a small value of ΔR_M can give a significant value of *b*. If ΔR_M=0, poles will be on $\pm j\omega$ axis. These poles can be named as "fixed poles". In Fig. 4 we can see the resistance of Memristor, R_M is oscillating and so definitely $\Delta R_M \neq 0$ and so *b* cannot be zero. As a result the poles will not be fixed. But still sustained oscillation is found and it is clear in the simulation result of V_{out} (in Fig. 4). These poles can be named as "oscillating poles". In Fig. 6, the oscillating nature of poles is shown. In Fig. 6(a), R_M is plotted using (8) and (11). This result is almost similar to the simulation result of R_M shown in Fig. 4. As R_M is oscillating so R_M can be replaced using (8) in (12) to plot the real part of pole in (13). The plot is shown for R_{init}=5k and R_{avg}=5.045k. In the same way the imaginary part of pole is plotted in Fig. 6(c). For both cases the oscillating behavior of pole is found. Fig. 6(d) shows the complex pole of the system when R_M is changing from its minimum to maximum value. In this figure $+j\omega$ axis is shown only. The mirror image is found in $-j\omega$ axis. When R_M is at its maximum value the poleswill be at the lowest point of the straight line (in Fig. 6(d)). When R_M is at its minimum value the poles will be at the highest point of the line (in Fig. 6(d)). The pole will oscillate between these two points along the line for the whole period of oscillation. The pole for which this line crosses +jω axis corresponds to R_{avg}. Thus the poles of this system will continue to oscillate with time.

B. Frequency response

As R_M is oscillating, the frequency of oscillation f_M can be calculated by replacing R_M with $R_{avg} \pm \Delta R_M$ in (13). It is found that the frequency of oscillation will have a maximum and a minimum value. As the poles of this system are oscillating so the oscillation cannot have a single frequency component in Fourier spectra like in the case of the conventional Wien oscillator. This logical interpretation is well supported by the simulation result shown in Fig. 7. Here the Fourier spectrum is shown for the same transient response of V_{out} shown in Fig. 4. The center frequency, $f_{M,avg}$ = 9.90Hz corresponds to the purely imaginary pole and the corresponding value of R_M is R_{init}. The calculated values of the maximum and minimum value of f_M are found 9.98Hz and 9.78 Hz correspondingly and these values have very close match with $f_{M,max}$ and $f_{M,min}$ in Fig. 7. Other frequencies are due to the shifting of poles from the imaginary axis. This triangular shape of FFT validates the range of frequency of oscillation. It is to be mentioned that for any sustained oscillation the FFT of the sinusoidal oscillation is an impulse. But surprisingly when Memristor is used with Wien oscillator the FFT changes to triangular shape which means that the frequency of oscillation will not be single valued rather it will have a range.

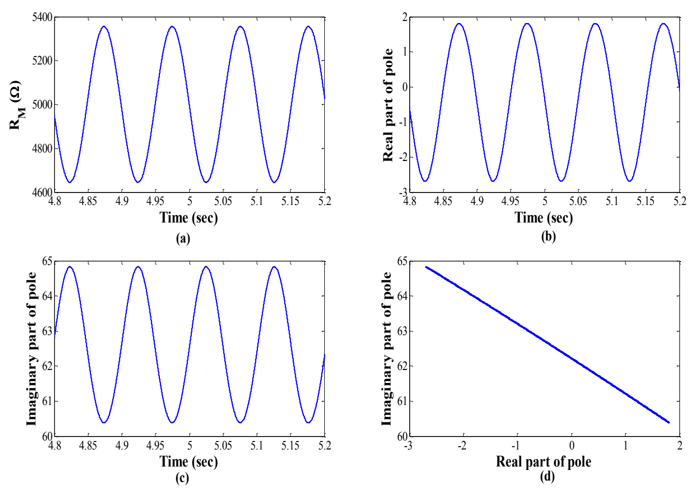

Fig. 6: (a) Oscillating resistance R_M, (b) Oscillating real part of pole, (c) Oscillating imaginary part of pole, and (d) Pole of the described system when R_M is changing from its minimum to maximum value.

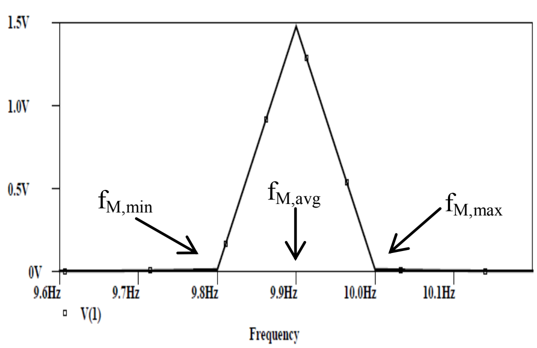

Fig. 7: FFT of output voltage (V(1) in Fig. 4)

IV. Conclusion

In this paper, we reported the effect of using a Memristor in the conventional Wien oscillator. The developed mathematical models for characterizing the oscillation in this scheme match very closely with the simulation results. The new exciting results in oscillation will definitely redefine the traditional concepts. First of all the oscillating behavior of resistance and pole of the investigated scheme can surely give sustained oscillation. Despite of oscillating pole, the stability of the system is not at all diminished. The oscillating resistance, periodic oscillation of poles and triangular shape of FFT may have interesting effect in the field of oscillation. Challenges remain with the nonlinearity of opamp and Memristor itself. The nonlinear characteristics inhibit the complexity of the analysis. The unprecedented results can have impact in designing oscillatory circuits, sensitive control systems, signal generation, amplitude and frequency modulation, etc.

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