Modeling of MEMS piezoelectric energy harvesters using electromagnetic and power system theories

Mahmoud Al Ahmad[1], Amro M. Elshurafa[2], Khaled N. Salama[2] and H. N. Alshareef[1,*]

[1] Material Sciences and Engineering, Physical Sciences and Engineering Division, King Abdullah University of Science & Technology (KAUST), Thuwal, Saudi Arabia, 23955-6900.

[2] Electrical Engineering, Physical Sciences and Engineering Division, King Abdullah University of Science & Technology (KAUST), Thuwal, Saudi Arabia, 23955-6900.

*Corresponding author: husam.alshareef@kaust.edu.sa

Abstract

This work proposes a novel methodology for estimating the power output of piezoelectric generators. An analytical model that estimates, for the first time, the loss ratio and output power of piezoelectric generators, based on the direct mechanical-to-electrical analogy, electromagnetic theory, and power system theory, is developed and takes into account the dimensions and material properties of the generator. The mechanical-to-electrical analogy and power system theory allow deriving an equivalent input impedance expression for the network. Further, electromagnetic theory allows deducing the equivalent electromechanical loss of the piezoelectric generator. By knowing the mechanical input power and the loss of the network, calculating the output power of the piezoelectric device becomes a straightforward procedure. Experimental results based on published data are also presented to validate the analytical solution. Moreover, and to fully benefit from the well established electromagnetic and electric circuit theories, further analysis efforts on the resonant frequency, bandwidth, and sensitivity are presented. Compared to conventional modeling methods currently being adopted in the literature, the proposed method provides, relatively easily, significant additional information that is crucial for enhanced device operation. Finally, and with the study provided in this paper, optimizing piezoelectric harvesters is simplified.

Nomenclature

α Constant relating the current density to strain rate
β Constant relating the strain to output voltage specifically at resonance
δ The real part of the complex frequency variable
ε Dielectric constant
Γ Reflection coefficient
σ_{in} Input stress
ω The imaginary part of the complex frequency variable
ζ Unitless damping ratio

a	A constant being either 1 or 2 depending on the wiring of the harvester
b_m	Damping coefficient
c_p	Elastic Modulus of the piezoelectric material
c_{sh}	Elastic Modulus of the center shim
C_b	The capacitance of the piezoelectric bender
C_k	An equivalent capacitance representing mechanical stiffness
d_{31}	Piezoelectric strain coefficient
g	Acceleration of gravity
i	Electric current
i_{in}	Input current
j	Complex number (i.e. the square root of –1)
k_1	A constant relating stress to force
k_2	A constant relating the strain to the deflection
k_{31}	Piezoelectric coupling coefficient
l_e	The length of the electrode in the piezoelectric harvester
L_m	Equivalent inductance representing mass
m	Mass
n	Turn ratio
p	Normalized complex frequency variable
P_{in}	RMS incident power
P_L	RMS power delivered to the load
P_{LR}	Power loss ratio
P_R	RMS reflected power
R	Load resistance
R_b	A resistor representing mechanical damping
S	Strain
t_c	The thickness of the ceramic/piezoelectric material
t_{sh}	The thickness of the shim
V	Voltage
w	The width of the ceramic/piezoelectric material
Z_{in}	Input impedance

1. Introduction

Energy harvesting from vibrations can provide reasonable amounts of electric energy for self-powered sensor applications [1], and can be realized by various devices such as piezoelectric cantilevers. These cantilevers have demonstrated high power densities (200 μmW/cm^3), which is second highest after solar [2]. The basic structure for a piezoelectric energy harvester includes a piezoelectric layer attached to a vibrating mechanical structure. The vibration causes bending (stress) in this piezoelectric layer and subsequently the bending induces electric charges. In many cases, a proof mass is attached to the end of the piezoelectric beam to covert the vibration, or equivalently acceleration, into an effective inertial force to further increase the bending of the beam. The mass also can be varied to tune the effective resonant frequency of the structure to the frequency of vibrations, thereby maximizing the output power as much as possible.

Modeling piezoelectric materials is being actively pursued in the literature to meet the requirements of design and development engineers. Despite the many articles published in the literature [3-7], a comprehensive model that effectively relates the power output to the structure of the piezoelectric harvester and materials involved in a quick and easy manner is still lacking. As such, this work proposes an innovative model for the quick estimation of the output power of piezoelectric generators; the model is based on a newly proposed impedance analogy and utilizes electromagnetic and power system theories.

The model, initially, relies on the equivalent circuit representation of electromechanical and piezoelectric transducers [8]. The equivalent circuit representation can model electromechanical transducers (i.e. the electrical and mechanical domains), using simple circuit elements, given the analogy in the differential equations describing both domains, and couple both domains by an ideal electromechanical transformer [8, 9]. Then, to derive the loss ratio between the input power and output power of the resultant network, electromagnetic theory (in the form of the reflection coefficient) is utilized after calculating the equivalent electromechanical input impedance. With the loss ratio calculated, it is possible to estimate the output power if the input power is known. Hence, the input power was also analyzed and a closed form expression was deduced.

In order to verify the analytical model proposed, experimental results based on published data are shown and indeed agree very well with the proposed theoretical analysis. Given the presented method in this paper, optimizing piezoelectric harvesters becomes a quick and easy procedure, and reduces the time cycle required to obtain a working prototype significantly.

2. Power and loss ratio definition

One possible way to describe a two–port network is by deriving the input impedance, $Z_{in}(p)$, where $p = \delta + j\omega$ is the normalized complex frequency variable. For our purposes, it is desirable to derive an equivalent expression for the input impedance of the electromechanical network and as such, definitions of gain (or loss) in a general two–port network should be provided first.

Consider figure 1, where P_{in} is the root mean square (RMS) incident power, P_R is the RMS power reflected back to the generator, and P_L is the RMS power delivered to the load. In other words, $P_{in} = P_L + P_R$ assuming no other losses exist in the system. Relying on electromagnetic theory, the power loss ratio, P_{LR}, of the network is defined as the ratio of the power available from the source to the power delivered to the load. Explicitly [10],

$$P_{LR} = \frac{P_{in}}{P_L} = \frac{1}{1 - |\Gamma(p)|^2} \tag{1}$$

In which,

$$\Gamma(p) = \frac{Z_{in}(p) - 1}{Z_{in}(p) + 1} \tag{2}$$

It is important for the reader to conceptualize the terminology within the context herein based on electromechanical analogy. Explicitly, P_{in} represents the input mechanical power which corresponds to the vibration source and $Z_{in}(p)$ denotes the electromechanical input impedance for the piezoelectric cantilever. Thus, knowing the electromechanical loss and the corresponding mechanical input power will determine the output electrical power delivered to the electrical load.

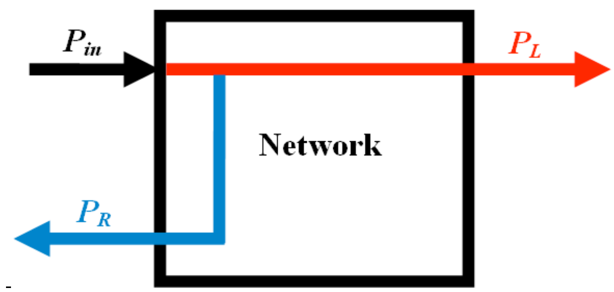

Figure 1. A conceptual schematic for an electrical network describing the incident, reflected, and delivered powers.

3. Input impedance derivation

Describing piezoelectric bimorph harvesters using an electric circuit based on the electromechanical analogy is shown in figure 2 as introduced by Roundy and Wright [9], and was obtained by lumping the distributed energy stored and energy dissipated in the system into circuit elements. When the composite beam is subject to a mechanical load, the strain induced in the piezoelectric material generates a voltage, which represents a conversion from the mechanical to the electrical domain; this conversion is accounted for by the ideal transformer and its turn ratio. Note also that the power output of a piezoelectric generator is a function of the magnitude of the mechanical input.

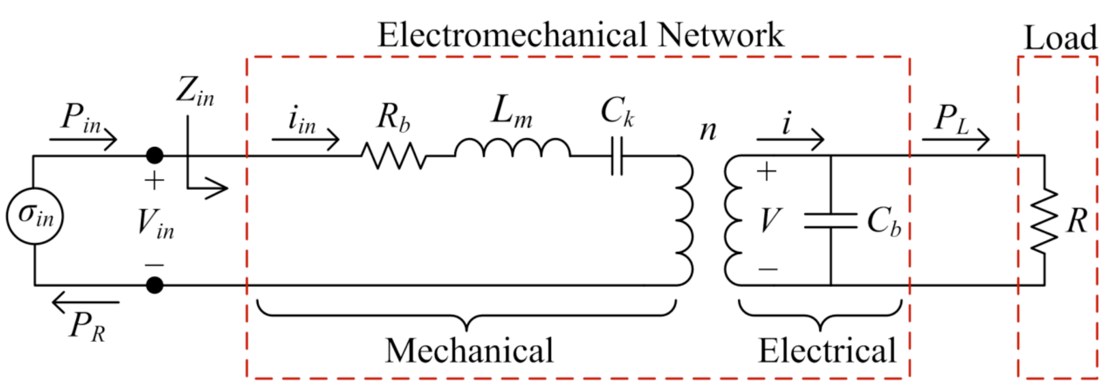

Figure 2. Equivalent circuit representation of the piezoelectric generator along with power definitions.

As with purely electrical circuits, the governing equations are then determined by using Kirchhoff's current and voltage laws. Explicitly, and as derived in [9],

$$\sigma_{in} = L_m \ddot{S} + R_b \dot{S} + \frac{S}{C_k} + nV \tag{3}$$

$$i = C_b \dot{V} + V / R \tag{4}$$

In equations 3 and 4, a dot represents the first derivative and double-dot represents the second derivative. For these expressions to be transformed into a usable system model, equivalent expressions have been derived also as [9]:

$$n = \frac{-ad_{31}c_p}{2t_c} \tag{5}$$

$$R_b = k_1 k_2 b_m \tag{6}$$

$$L_m = k_1 k_2 m \tag{7}$$

$$C_b = \frac{a^2 \varepsilon w l_e}{2t_c} \tag{8}$$

$$C_k = c_p^{-1} \tag{9}$$

The current generated as a result of the mechanical stress evaluated at zero electric field is given as

$$i = awl_e d_{31} c_p \dot{S} \tag{10}$$

The Laplace transform of the strain term is found to be[1]:

$$S = \frac{a\varepsilon}{t_c d_{31} c_p p} \left(p + \frac{1}{RC_b} \right) V \tag{11}$$

To derive the electrical input impedance for the network, we first refer to the mechanical side of the network as primary and the electrical side as secondary. Next, and to use equations (1) and (2), an expression for the input electromechanical impedance, or Z_{in}, must be derived; this is done by finding the Thevenin equivalent [11] of the network in figure 2 looking into it from the primary side, i.e.

$$Z_{in}(p) = \frac{V_{in}(p)}{i_{in}(p)} = \frac{\sigma_{in}(p)}{i(p)} \tag{12}$$

Therefore, and after some manipulation,

$$Z_{in}(p) = \left(\frac{1}{\alpha}\right)\left(pL_m + R_b + \left(\frac{1}{pC_k}\right) + n\beta\right) \tag{13}$$

[1] There is a typographical error in this equation in [9].

where $\beta = \dfrac{a\varepsilon}{t_c d_{31} c_p}$ and $\alpha = awl_e d_{31} c_p$.

Equation (13) represents the electromechanical input impedance of the piezoelectric cantilever. Note that any change in the geometries, damping, etc, will be translated to this derived input impedance shown in equation (13). In other words, this input impedance describes the dynamic behavior of the piezoelectric harvester.

4. Input and output power calculation

With the definitions and derivations above, it is possible to calculate the output power if the input power is known. Similarly, the input power can be calculated if the output power is known. For the purposes of this paper, as the case generally is in the field, the former is applicable rather than the latter.

In order to determine the input power, it is assumed that the network in figure 2 is going to be excited at resonance. Note that at resonance, the imaginary part of the impedance at the primary side becomes zero and the impedance becomes purely real (i.e. resistive). As such, the power that is delivered to the primary side of the circuit at resonance is only governed by R_b.

Because R_b is the only component that governs the power now, one may be tempted to say that the power is going to be dissipated in this resistor and no power will be transferred to the secondary side. However, remember that the power we refer to here is in the mechanical domain, and will not be dissipated literally because R_b serves purely a modeling purpose. Explicitly, the resistor R_b models the structural damping of the piezoelectric harvester and dissipates some energy in the form of heat. The remaining energy, which is of interest to us, is going to be transferred to the secondary side of the network based on the loss ratio calculated.

From basic electric circuit theory, it is well known that the power dissipated in a resistor is calculated simply as the product of voltage and current, or the ratio of the voltage squared to resistance. In terms of the electromechanical analogy, stress in analogous to voltage and strain rate in analogous to current. Hence we can write the power delivered to the primary side of the network as:

$$P_{in} = \sigma_{in} \dot{S} = \frac{\sigma_{in}^2}{2R_b} \tag{14}$$

and the factor of '2' in equation 14 represents the RMS value [11], since for a sinusoidal voltage with a peak voltage of V, the RMS voltage is $V/\sqrt{2}$. At resonance, the primary side of the network is comprised of a pure resistor only. Therefore the stress (or voltage) across the resistance is exactly equal to the input stress (or input voltage) generated.

5. Experimental validations and discussion

5.1. Experimental validation

The reported results in [12-13] have been chosen to validate the proposed model. The harvester is a cantilevered plate consisting of two piezoelectric layers (hence the name: bimorph), and one structural layer (shim) residing between the piezoelectric layers. To form a cantilever, the three latter layers are mounted (fixed) on one end and left free to move at the other end. Often, a mass is glued atop the free end to tune the resonant frequency of the harvester and indeed it is the case here. All parameters and material properties used in the model implementation were reported in [12-13]; nonetheless, and for convenience, we include them in table 1 for quick and easy reference.

Table 1. Parameters and material properties used in [12-13] and herein for the modeling.

Parameters or material property	Value
c_p	62 GN/m^2
c_{sh}	66 GN/m^2
w	3.2 mm
ε	1800
t_c	0.134 mm
t_{sh}	0.132 mm
l_e	25.60 mm
m	0.5894 grams
ζ	0.02
g	9.8 m/s^2
d_{31}	-1.9e-10 m/V
k_{31}	0.32

To obtain the output power, the values of the circuit elements were computed using the information in table 1. Then, equation 13 was used to compute the input impedance. Finally, the impedance values were substituted in equations 2 and 1 respectively to obtain P_L.

As evident in figure 3, the presented model follows the measured behavior very well. The discrepancy shown, however, was expected due to several factors including uncertainties in the geometry of the device, tolerances in reported material properties, difficulty in accounting for the glue-ing/bonding of the bimorph, and difficulty in accurately determining the damping coefficient. Note also that the model proposed is linear because it uses linear (passive) circuit elements only, while the stress distribution within the beam during bending is not. To account for the nonlinearity of the piezoelectric layer, a higher order circuit model must be used. Considering the simplicity of the introduced method and the difficulty of the task, the model represents an effective (and promising) method to model piezoelectric bimorphs, especially when the bending of the bimorph is not big which means the nonlinearity in the stress gradients is not severe (yet present).

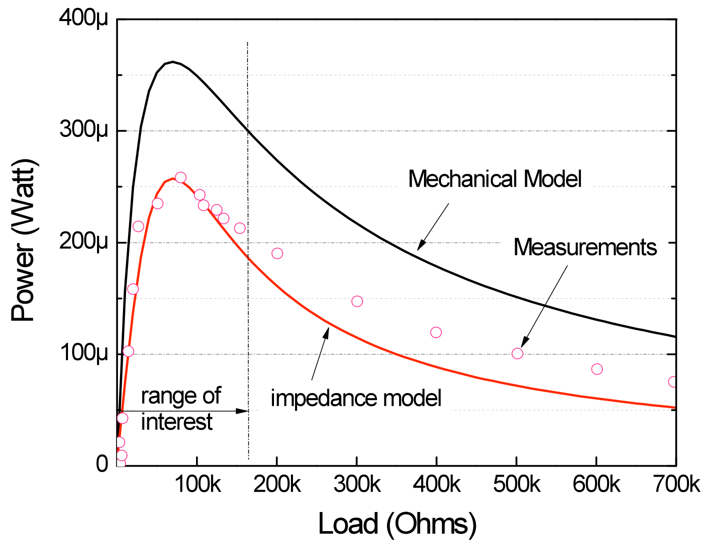

Figure 3: Simulation results and measurement results superimposed. The mechanical model has been constructed using the procedure described in [9].

An important comment should be made here regarding the usage of the power loss ratio. When defining the relation between P_{in} and P_L, we have preferred using the loss ratio, i.e. P_{in}/P_L, because the efficiency of piezoelectric materials is usually low as known. It was possible, for example, to use the gain definition, i.e. P_L/P_{in}. However, the gain is more suitably used when the ratio P_L/P_{in} is greater than one, which means that amplification is taking place. When the gain is less than one, then it is a case of attenuation. For the purposes of this paper, we opted to use the loss ratio definition because it is more sensible and intuitive. Nevertheless, both definitions would have been equally effective because the loss ratio is simply the reciprocal of gain.

5.2. Frequency Response

Extending the analysis further, and to leverage the well-established electromagnetic and circuit theories, we use the method proposed in this paper to plot the frequency response of the bimorph. Figure 4 plots the frequency response of the harvester, and as can be seen, there is an obvious peak at a frequency of 97 Hz compared to a measured resonant frequency of 95 Hz [12-13], which acts as further verification to our proposed method.

Another advantage of the proposed method is its capability of finding other resonant frequencies existing, i.e. harmonics, (if any). Further, crucial information about the bandwidth of the device is also attainable from the plot. Information regarding the bandwidth provides insight in the initial design stages; if the bandwidth of the device is small, then additional care should be exercised in order to fully capitalize on the harvester. However, if the bandwidth is large, then vibration frequency matching requirements are relaxed. The significance of figure 4 is that it immediately and easily, with minimal time and effort, provides important information about the resonant frequency (or frequencies), power, and bandwidth of the device; this information was not readily available previously using conventional modeling methods with this degree of simplicity.

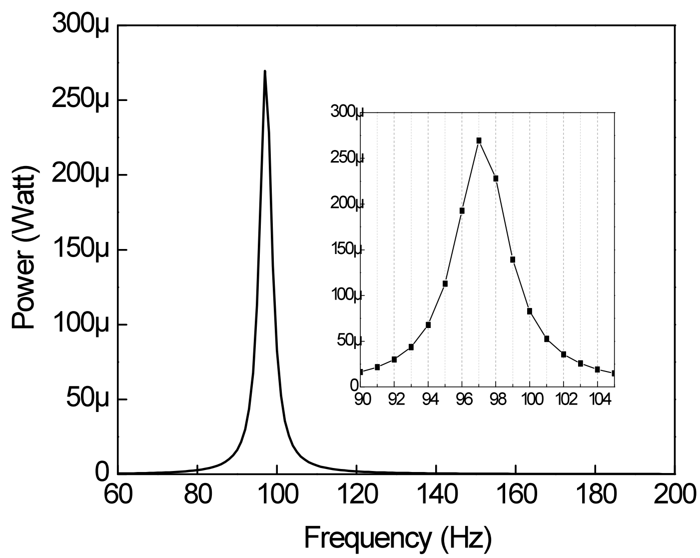

Figure 4. The frequency response of the bimorph plotted using the proposed method. The inset is an exaggerated view of the peak from 90 Hz to 105 Hz.

Before leaving this section, and in order to be accurate in describing the resonant frequency obtained in figure 4, it is important to note that the frequency obtained in this figure is the *combined effect* from both the mechanical and electrical domains. This frequency is more descriptive of the actual, measured frequency because it takes into account the capacitance C_b (see figure 2). This is another important advantage of the proposed design. However, and in fairness, the value of C_b is very small (and the value of L_m is very large compared to it). As such, the discrepancy between the resonant frequency obtain from the mechanical domain only and the resonant frequency obtained when both domain are considered are very close to each other. This is confirmed in this paper by noting the measured resonant frequency in [12-13] was reported to be 95 Hz, while the theoretical resonant frequency obtained here in 97 Hz.

5.3. Sensitivity

To study the effect of various parameters on the piezoelectric generator, we provide sensitivity analysis based, once again, on our proposed model. As mentioned in the previous section, this is easily possible given the effectiveness of the proposed model.

Although there are several parameters that could be studied (i.e. varied), we dissect the analysis to two main patterns. First, we vary the total input impedance; this way the sensitivity being analyzed is for the system as a whole. Secondly, we specifically vary the dimensionless damping ratio (ζ, where $b_m = 2\zeta\omega_n m$) and the piezoelectric stain coefficient (d_{31}) because these two parameters are particularly difficult to obtain to a high degree of accuracy as opposed to the mass or dimensions of the bimorph for example.

To obtain the sensitivity of the harvester, Z_{in}, ζ, and d_{31} were all varied by ±5%. Figure 5 shows the effect of the variation in Z_{in} on the output power, while figure 6 shows the effect of the variation in ζ and d_{31}.

As can be seen from figures 5 and 6, a 5% change in the input impedance or the damping ratio, translates to ~20 μW change at resonance. However, a 5% change in the strain coefficient affects the output power by ~40 μW. Both values are deemed considerable if we argue that a 5% change in these parameters results a change of more than 5% in the output power. As such, it is worthwhile for a designer to spend, justifiably, extra time on carefully characterizing ζ and d_{31} to ensure that the harvester is being modeled accurately.

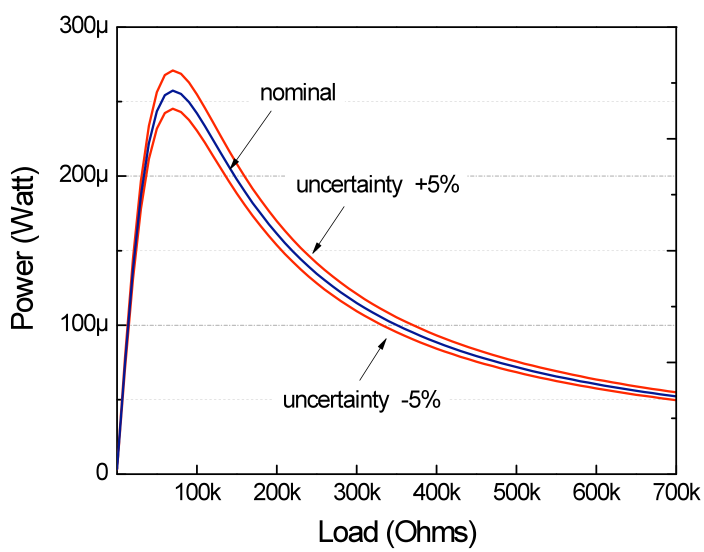

Figure 5. Sensitivity plot of the bimorph obtained after varying the input impedance by ±5%.

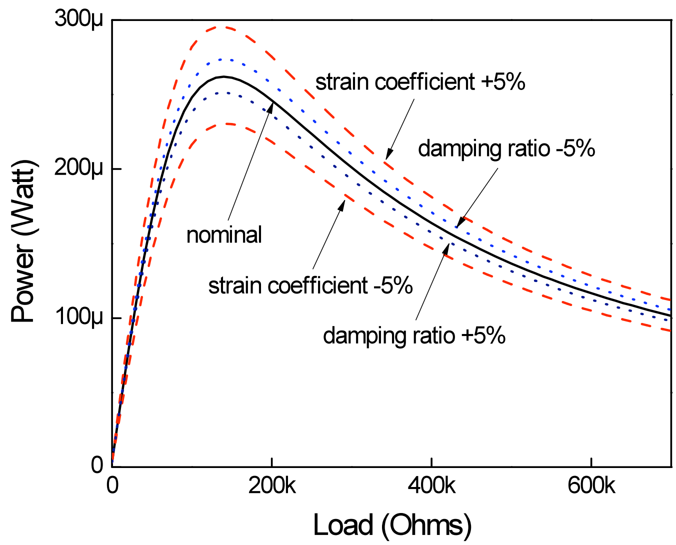

Figure 6. Sensitivity plot of the bimorph obtained after varying the damping ratio and strain coefficient by ±5% (each varied separately).

6. Conclusion

A novel method for estimating the power output of piezoelectric generators based on the electromechanical analogy incorporating, for the first time, electromagnetic theory and power system theory is presented. Among the main contributions of this paper is coupling effectively several governing equations in well-developed and understood fields to arrive ultimately at an easy way to solve the problem under study. The equations derived were verified using published experimental data, and are able to provide important information on the resonant frequency, bandwidth, and sensitivity swiftly and easily. Another important advantage of the proposed method is that it can, for the first time, describe the efficiency of the generator and hence allowing for genuine optimization.

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