Performance Analysis of Switching Based Techniques for Wireless Applications

Thesis by
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ABSTRACT

Performance Analysis of Switching Based Techniques for Wireless Applications

Switching techniques have been first proposed as a spacial diversity techniques. These techniques have been shown to reduce considerably the processing load while letting multi-antenna systems achieve a specific target performance. In this thesis, we take a different look at the switching schemes by implementing them for different other wireless applications. More specifically, this thesis consists of three main parts, where the first part considers a multiuser environment and an adaptive scheduling algorithm based on the switching with post-selection scheme for statistically independent but non-identically distributed channel conditions. The performance of this switched based scheduler is investigated and a multitude of performance metrics are presented. In a second part, we propose and analyze the performance of three switched-based algorithms for interference reduction in the downlink of over-loaded femtocells. For instance, performance metrics are derived in closed-form and these metrics are used to compare these three proposed schemes. Finally in a third part, a switch based opportunistic channel access scheme is proposed for a cognitive radio system and its performance is analyzed in terms of two new proposed metrics namely the average cognitive radio access and the waiting time duration.
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Chapter I

Introduction

It is well-known that diversity techniques can mitigate the effect of multipath fading. Many diversity combining techniques have been proposed and their performance over fading channel have been analyzed. Among them, maximum ratio combining (MRC), equal gain combining (EGC), and selection combining (SC), have received considerable attention. However, these techniques require relatively high complex receiver structures, which may not be suitable for mobile stations where the need for a simple receiver design is of primary concern. Thus, alternative techniques, known as switched combining techniques, have been proposed [1]–[4]. These techniques monitor and process only one diversity branch at a given time from all available receive branches [3], and provide a low-complexity solution to multipath fading [2]. Switch and examine combining (SEC) is a known switched-based diversity technique [3], [4] that reduces the processing power consumption at the receiver as switching between branches is only necessary when the instantaneous signal-to-noise ratio (SNR) of the current branch falls below a specific threshold.

In this work, we take a different look at switching-based diversity techniques and we seek to implement them in different wireless communications applications. Specifically, these techniques can be applied in a multiuser environment, where the users
channel fluctuation is exploited to select a suitable user. Thus, extensive research have been devoted to the performance of user scheduling algorithms and many improvements of these algorithms have been proposed. Among these improvements, the one proposed in the first part in this work, in which we investigate an adaptive scheduling algorithm for multiuser environments with statistically independent but non-identically distributed (i.n.d.) channel conditions. The algorithm aims to reduce the feedback load by examining the users channels sequentially based on a switching algorithm. It also provides improved performance by realizing a post-examining best user selection. For the considered scheme we present new formulations for the statistics of the signal-to-noise ratio (SNR) of the scheduled user under i.n.d. channel conditions. We capitalize on the findings and present various performance and processing complexity measures for adaptive discrete-time transmission. The results are then extended to investigate the effect of outdated channel estimates on the statistics of the scheduled user SNR as well as some other performance measures. Numerical results are provided to clarify the usefulness of the scheduling algorithm under perfect or outdated channel estimates.

In a second part of this work, we propose an adequate method to improve the interference mitigation capability of a recently investigated switched-based interference reduction scheme for single downlink channel assignment in over-loaded femtocells. It is assumed that the available orthogonal channels for femtocell network are distributed among access points in close vicinity, where each of which knows its allocated channels a priori. Each femtocell has a single antenna, employs the open access strategy, and can reuse its allocated channels simultaneously while scheduling concurrent new requests. Moreover, the access points can not coordinate their transmissions, and can receive limited feedback from active users. In this context, we present low-complexity schemes to identify a suitable channel to serve the desired scheduled user by maintaining the interference power level within a tolerable range. They attempt
to either complement the switched-based scheme by minimum interference channel selection or adopt different interference thresholds on available channels, while aiming to reduce the channels examination load. The optimal thresholds for interference minimization at the desired receive station are quantified for different performance criteria. The performance and processing load measures of the proposed schemes are obtained analytically, and then compared with those of the single-threshold switched-based scheme via numerical and simulation results.

In the third and last part of this thesis, switching algorithms are implemented to opportunistically access the spectrum in a cognitive radio situation. In particular, we investigate the performance of a cognitive radio transceiver that can monitor multiple channels and opportunistically use any one of them should it be available. In our work, we propose and compare two different opportunistic channel access schemes. The first scheme applies when the secondary user (SU) has access to only one channel. The second scheme applies when the SU has access to multiple channels but can at a given time monitor and access only one channel. Two switching strategies, namely the switch and examine and the switch and stay strategies, are proposed. For these proposed access schemes, we investigate their performance by deriving the analytical expression of two novel metrics: the average access duration and the average waiting time and based on these two metrics a time average SU throughput formula is proposed to predict the performance of the secondary cognitive system.
Chapter II

Adaptive Scheduling with Post-Examining User Selection Under Non-Identical Fading

II.1 Introduction

Multiuser diversity exploits channel variations to improve the capacity of wireless systems by allowing the user having the best channel conditions to access available downlink resources [5]–[7]. Several opportunistic scheduling schemes have been suggested for independent and identically distributed (i.i.d.) channels of active users [8], [9]. Specifically, in the best selection algorithm, the serving base station (BS) probes all users and selects the one with the best instantaneous signal-to-noise ratio (SNR), an equivalent approach to the selection combining (SC) scheme with spatial diversity [1, sec. 9.7]. However, this algorithm requires relatively significant overhead in order to keep the main BS aware of channel state information (CSI) of all active users, and it may result in unfair scheduling of the system resources among different users. The issue of fairness has indeed gained considerable attention, and corresponding
scheduling schemes have been proposed in [10], [40].

To reduce the feedback load while achieving a specific target performance, a switched-based algorithm has been investigated in [4], and then applied in the context of multiuser transmission in [11]. In this case, the downlink service is provided to an active user whose instantaneous SNR can satisfy target performance. It has been shown that the algorithm in [11] can reduce the overhead load.

A noticeable drawback of the algorithm investigated in [11] is that the main BS does not take advantage of the available CSI of active users when all examined users undergo relatively poor channel conditions. To this end, a post-examining selection for the switched-based algorithm has been proposed for space receive diversity systems in [12]. In this case, the switched-based algorithm is complemented by ordering of the instantaneous SNRs obtained from different users and then selecting the user that has the best channel conditions to receive downlink information. However, the results presented in [12] are limited to Rayleigh fading model. Moreover, they can not be used for the general and practical case when active users experience independent and non-identically distributed (i.n.d.) channel conditions. The findings in [12] are also applicable for non-adaptive transmission scenario. They are valid only for the case when perfect knowledge of the CSI of all diversity channels is available at the receive station.

This chapter aims to investigate an adaptive scheduling algorithm with post-examining selection when active users undergo i.n.d. channel conditions. The adaptation to channel conditions is proposed based on the active users instantaneous SNRs to choose from different quadrature-amplitude constellations over discrete-time intervals, while reducing the feedback load by using the proposed algorithm. The first part of the chapter presents new general formulations for the statistics of the instantaneous SNR of the selected user under i.n.d. channel conditions. These formulations are then used to study the case when active users undergo i.n.d. Nakagami-$m$ fading, from
which results for Rayleigh fading model can be deduced as limiting cases. The second part of the chapter capitalizes on the results in the first part, and investigates various performance and processing complexity measures for the scheduling algorithm in combination with discrete-rate adaptation. Specifically, results for the average spectral efficiency (ASE), average bit error rate (BER), and average feedback load (AFL) are quantified. Moreover, the effect of outdated CSI estimates on the efficiency of the algorithm is thoroughly investigated, in which new analytical formulations are presented to characterize the statistics of the outdated resulting SNR as well as the average BER of the adaptive system. Numerical results are provided to clarify the usefulness of the algorithm, and to compare the performance and complexity for the cases of perfect and outdated channels estimates.

The remainder of the chapter is organized as follows. Section II.2 outlines the system model, and presents the statistics of the resulting SNR of the scheduled user for non-adaptive algorithm with post-examining selection under i.n.d. channel conditions. Section II.3 provides analytical results for the ASE, average BER, and AFL for the SNR-based adaptive system. Section II.4 quantifies the effect of outdated CSIs on the statistics of the resulting SNR as well as the average BER, and section II.5 discusses selected numerical results. Finally, concluding remarks are given in section II.6.

II.2 System Model and Statistics of Resulting SNR

This section outlines the system model, and presents the general formulations for the resulting SNR of the scheduled user per the scheduling algorithm under consideration. The results are then used to study the case when the examined users undergo i.n.d. Nakagami-\(m\) fading.
II.2.1 System Model and Introductory Discussion

It is assumed that a total of $L$ users exist in the coverage area of interest. The users can receive downlink information from a known BS, which is equipped with a single antenna and can use a single channel to serve one of the active users at a time. The mobile stations are assumed to be space-limited, and each of which has only one antenna to receive and transmit in time division duplex (TDD) fashion, thereby the channel reciprocity can be exploited to extract the CSI of each examined user.

The multiuser access scheme serves different users using time division multiplexing (TDM). A guard period is inserted per each packet interval to select a suitable user and assign a suitable transmission mode. During each guard period, the BS listens to the requests from active users, and aims to serve a user whose channel quality is good enough to support reliable reception of the projected signaling constellation. The selection of the user is carried out per the scheduling algorithm under consideration, where the user having the best channel quality among all examined users is chosen only if all examined users fail to satisfy the projected performance. The long-term fairness among different users can be achieved by rearranging the users requests over successive packet durations.

Apparently, the probing period of the users requests is random and varies with the number of active users. However, it is always upper bounded by the time spent in the full feedback system that aims to select the user with the best channel conditions. It is worth mentioning herein that the allocated guard period does not carry useful information, and therefore it has nothing to do with the achieved system ASE. Moreover, the guard period can then be designed a priori to handle limited number of users each time, independently of how many users will be active. The scheduling algorithm considered herein aims to reduce the processing and feedback loads during the allocated guard period, while satisfying the projected performance level for one of the active users at a time.
Knowing that the projected performance can be mapped into a specific threshold on the resulting instantaneous SNR, the BS starts probing the users sequentially, where the estimated instantaneous SNR of the firstly probed user is compared against the SNR threshold. If it is found that the first user instantaneous SNR is above the threshold, the first user is allocated the available packet interval, and no further processing is required. Otherwise, the BS probes another user and examines its estimated instantaneous SNR against the threshold. The BS continues examining the instantaneous SNRs of the probed users until either one user having instantaneous SNR above the threshold is found or all active users have been examined. In the latter case, the BS selects the user having the best SNR, and allocate the constellation size that this user can decode reliably with the specified target error rate.

The proposed scheduling algorithm in combination with the adaptive transmission can provide many benefits. Specifically, it can be useful for wireless applications in which the active users are treated equally by the service provider. It can be applied to serve active users who demand data applications, for which the sensitivity to the time delay required to establish the downlink service can be tolerated. The proposed algorithm can be potentially attractive to resolve the failure in uplinks as the feedback information is needed only if the previously tested users can not meet the transmission requirements. However, it gains full advantage of knowing the CSI of all active users within the allocated guard period to achieve the performance of the best user selection. The adaptation in the transmission mode can support the usefulness of the proposed scheme by exploiting the channel variations among different active users to further reduce the processing and feedback loads.

II.2.2 Statistics of Resulting SNR

Define $\gamma_l$, for $l = 1, 2, \ldots, L$, as the instantaneous SNR associated with the $l$th user. Also, define $\Gamma$ as the resulting SNR of the scheduled user. Based on the discussion
above, it can be written that $\Gamma = \gamma_1$ if it is found that $\gamma_1 > \gamma_T$, where $\gamma_T$ is the SNR threshold. Moreover, it is seen that $\Gamma = \gamma_j$, for $j = 2, \ldots, L$, if it is found that $\gamma_j > \gamma_T$ and $\gamma_p < \gamma_T$, for all $p < j$. Finally, $\Gamma = \max\{\gamma_1, \ldots, \gamma_L\}$ if it is found that $\gamma_p < \gamma_T$, for $p = 1, 2, \ldots, L$. Using the aforementioned results, the cumulative distribution function (CDF) of the resulting SNR, which is defined as

$$F_\Gamma(x) = \sum_{l=1}^{L} \Pr\{\Gamma < x, \Gamma = \gamma_l\},$$

where the symbol $\Pr\{\cdot\}$ refers to the probability of the quantity between brackets, can be given as

$$F_\Gamma(x) = \prod_{l=1}^{L} F_{\gamma_l}(x) (U(x) - U(x - \gamma_T))$$

$$+ \left( \sum_{l=1}^{L} \prod_{i=1}^{l-1} F_{\gamma_i}(\gamma_T) [F_{\gamma_l}(x) - F_{\gamma_l}(\gamma_T)] + \prod_{l=1}^{L} F_{\gamma_l}(\gamma_T) \right) U(x - \gamma_T) \quad (II.1)$$

where $F_{\gamma_l}(x)$, for $l = 1, 2, \ldots, L$ refers to the CDF of the instantaneous SNR associated with the $l$th user and $U(\cdot)$ is unit step function. The probability density function (PDF) of the resulting SNR, which is defined as $f_\Gamma(x) = dF_\Gamma(x)/dx$, can be deduced from (II.1), and the result is given by

$$f_\Gamma(x) = \sum_{l=1}^{L} \left( f_{\gamma_l}(x) \prod_{i=1, i \neq l}^{L} F_{\gamma_i}(x) (U(x) - U(x - \gamma_T)) + \left( \prod_{i=1}^{l-1} F_{\gamma_i}(\gamma_T) \right) f_{\gamma_l}(x) U(x - \gamma_T) \right),$$

(II.2)

where $f_{\gamma_l}(x) = dF_{\gamma_l}(x)/dx$. Note that the results in (II.1) and (II.2) are applicable for any channel fading models. The following subsection considers the case of Nakagami-$m$ fading as a specific example.

**II.2.3 Results for the Case of Nakagami-$m$ Fading**

For the case when the channel fading envelopes of active users follow Nakagami-$m$ distribution, the statistics of the instantaneous SNRs of individual users are $f_{\gamma_l}(x) = (m_l/\bar{\gamma}_l)^{m_l} x^{m_l-1}/\Gamma(m_l) e^{-(m_l x)/\bar{\gamma}_l}$ and $F_{\gamma_l}(x) = 1 - \Gamma(m_l, (m_l/\bar{\gamma}_l)x)/\Gamma(m_l); x \geq 0$, where $\Gamma(m_l, (m_l/\bar{\gamma}_l)x)$ is the incomplete gamma function and $\Gamma(m_l)$ is the gamma function.
where \( \bar{\gamma}_l \) and \( m_l \) are the average SNR and fading parameter associated with the \( l \)th user, respectively, \( \Gamma(x) \) is the Gamma function, and \( \Gamma(x, y) \) is the upper incomplete Gamma function. To obtain (II.1) and (II.2) in tractable forms, for the case when \{\( m_l \)\} take integer values, the terms \( F_0(x) = \prod_{l=1}^{L} F_{\gamma_l}(x) \) and \( f_0(x) = \sum_{i=1}^{L} f_{\gamma_i}(x) \prod_{j=1, j \neq l}^{L} F_{\gamma_j}(x) \) that appear in (II.1) and (II.2) can be expressed, using the identity \( \Gamma(n+1, x) = e^{-x} n! \sum_{i=0}^{n} \frac{x^i}{i!}, \) for \( n = 0, 1, \ldots \), as

\[
F_0(x) = \sum_{i=0}^{L} (-1)^i \sum_{\tau(i, L)} e^{-x \sum_{k=1}^{i} \frac{i_k m_k}{\bar{\gamma}_k}} \sum_{l=0}^{U} C_{i, L} x^l, \tag{II.3}
\]

\[
f_0(x) = \sum_{j=1}^{L} \left( \frac{m_j}{\bar{\gamma}_j} \right)^{m_j} \frac{x^{m_j-1}}{(m_j - 1)!} e^{-m_j x / \bar{\gamma}_j} \tag{II.4}
\]

\[
\sum_{j=1}^{U} (-1)^i \sum_{\bar{\tau}(i, L-1)} e^{-x \sum_{k=1}^{i} \frac{i_k m_k}{\bar{\gamma}_k}} \sum_{l=0}^{\bar{U}} \bar{C}_{i, L-1} x^l, \tag{II.5}
\]

where \( \tau(i, L) \) is the set of \( L \)-tuples such that \( \tau(i, L) = \{(i_1, i_2, \ldots, i_L) : i_k \in \{0, 1\}, \sum_{k=1}^{L} i_k = i\}, U = \sum_{k=1}^{L} L_k, L_k = i_k (m_k - 1), C_{i, L} = \sum_{\omega(l, L)} \prod_{k=1}^{L} \frac{1}{\gamma_k} \gamma_k^{l_k}, \omega(l, L) \) is the set of \( L \)-tuples such that \( \omega(l, L) = \{(l_1, l_2, \ldots, l_L) : l_k \in \{0, 1, \ldots, L_k\}, \sum_{k=1}^{L} l_k = l\}. \) The symbol \( \sim \) over a given term in (II.5) indicates that the argument for \( k = j \) (the index of the outer summation) is not included in the definition of that term [13 Appendix]. For example, for a given value of \( j \), it follows that \( \bar{U} = U - L_j = \sum_{k=1, k \neq j}^{L} L_k. \)

Another important quantity that will be used in the following sections is the incomplete moment generating function (MGF) of \( \Gamma \). This quantity, which is defined as \( M_{\Gamma}(s, \gamma) = \int_{\gamma}^{+\infty} e^{-sx} f_{\Gamma}(x) dx \), includes the MGF of \( \Gamma \) as a special case (i.e. \( M_{\Gamma}(s) = M_{\Gamma}(s, 0) \)), and it can be written as per the result in (II.2) as

\[
M_{\Gamma}(s, \gamma) = \sum_{i=1}^{L} \int_{\gamma}^{+\infty} e^{-sx} f_{\Gamma_i}(x) dx = \sum_{i=1}^{L} M_{\Gamma_i}(s, \gamma), \tag{II.6}
\]

where \( M_{\Gamma_i}(s, \gamma) \) refers to the incomplete MGF of \( \Gamma \) when the \( l \)th user is chosen, which
can be obtained, after substituting the results in (II.3) and (II.5) into (II.2), as

\[ M_{\Gamma_l}(s, \gamma) = A_l(s, \gamma) \ U(\gamma - \gamma_T) + \left[ A_l(s, \gamma_T) + \frac{1}{(m_l - 1)!} \left( \frac{m_l}{\gamma_l} \right)^{m_l - 1} \sum_{i=0}^{L-1} \sum_{\gamma_{(i, L-1)}}^{\gamma_l} \sum_{j=0}^{\hat{\gamma}} \hat{C}_{j, L-1} \right] \times \Phi \left( m_l + j, \left( s + \frac{m_l}{\gamma_l} + \sum_{k=1, k \neq j}^{i} i_k m_k \frac{\gamma_l}{\gamma_k} \right), \gamma, \gamma_T \right) \ (U(\gamma) - U(\gamma - \gamma_T)), \]

(II.7)

where \( \Phi(m, x, y, z) = x^{-m} \left[ \Gamma(m, x, y) - \Gamma(m, x, z) \right] \), and \( A_l(s, \gamma) \) is given by

\[ A_l(s, \gamma) = \prod_{i=1}^{l-1} F_{\gamma_l}(\gamma_T) \frac{1}{(m_l - 1)!} \left( \frac{m_l}{\gamma_l} \right)^{m_l} \left( \frac{m_l}{\gamma_l} + s \right)^{-m_l} \Gamma \left( m_l, \frac{m_l}{\gamma_l} + s \gamma \right). \]  

(II.8)

II.3 Adaptive Scheduling with Post-Examining Selection for i.n.d. Users

This section considers the use of the scheduling algorithm with post-examining selection described in the previous section in combination with discrete-time variable-rate quadrature-amplitude modulation. Specifically, the range of the resulting instantaneous SNR is divided into \( N + 1 \) fading regions, for which the fixed set of adaptation thresholds \( \{ \gamma^*_1, \gamma^*_2, \ldots, \gamma^*_N, \gamma^*_{N+1} \} \) are used to define the boundary points, where \( \gamma^*_{N+1} = +\infty \). In this case, the constellation of size \( c_n = 2^{R_n} \) (where \( R_n \) represents the spectral efficiency in bits/sec/Hz based on Nyquist criterion) is chosen when it is found that \( \Gamma \in [\gamma^*_n, \gamma^*_{n+1}] \) such that a target BER is satisfied.

For the adaptive post-selection scheme, the SNR threshold \( \gamma_T \) that has been used in the preceding two subsections can be selected, according to the projected performance level, to be equal to \( \gamma^*_n \), which is one of the adaptation thresholds \( \{ \gamma^*_1, \gamma^*_2, \ldots, \gamma^*_N, \gamma^*_{N+1} \} \). Specifically, for the system that aims to maximize the bandwidth efficiency regardless of the processing load requirements, the suitable selection
of $\gamma_T$ will be $\gamma_T = \gamma_N^*$. On the other hand, the processing and feedback loads needed to handle the search for an appropriate user reduce with the decrease in $\gamma_T$, despite of the achieved bandwidth efficiency. The following parts discusses the analytical formulations of performance and processing complexity measures, respectively.

### II.3.1 Performance Measures

The expression for the ASE, which counts for the average number of bits from different constellations that can communicated per second per Hertz while satisfying a target BER [21], can be obtained as a weighted sum of the spectral efficiencies of different constellations, which are denoted by $\{R_n\}_{n=1}^N$ above. The weights, defined as $\{P_n\}_{n=1}^N$, are the likelihoods that $\Gamma$ fails within the corresponding adaptation regions ($P_n = \Pr\{\Gamma \in [\gamma_n^*, \gamma_{n+1}^*)\}$, for $n = 1, 2, \ldots, N$). Therefore, the expression for the ASE can be written as $\text{ASE} = \sum_{n=1}^N R_n P_n$, where $P_n = F_\Gamma(\gamma_{n+1}^*) - F_\Gamma(\gamma_n^*)$, and $F_\Gamma(x)$ is given in a general form in (II.1).

The average BER, which is defined as the ratio of the average number of bits detected in error from different constellations to the total average number of transmitted bits per second per Hertz, can be obtained as

$$\text{BER} = \frac{\sum_{n=1}^N R_n \text{BER}_n}{\text{ASE}},$$

(II.9)

where $\text{BER}_n = \mathbb{E}\{\text{BER}_n | \Gamma \in [\gamma_n^*, \gamma_{n+1}^*)\}$, and $\text{BER}_n$ for the possible signaling schemes under consideration can be generally approximated by $\text{BER}_n \simeq a_n e^{-b_n \gamma/c_n}$, in which $a_n$, $b_n$ and $c_n$ are region-dependent constants that are related to the constellation size chosen for the $n$th region, and $\gamma$ is used to refer to the received SNR. Note that some signaling schemes for which this approximate result can be applied can be found in [19], [20]. Specifically, in [19], it has been reported that this approximate result is very accurate, especially at high values of the received SNR and when $M \neq 2$ (which
is the case herein when $\gamma_T > \gamma_1^*$. Otherwise, it represents a tight lower bound for the exact result. Using this approximate result for $BER_n$, it can be written that

$$BER_n \simeq \int_{\gamma_n^n}^{\gamma_n^{n+1}} a_n e^{-b_n \gamma/c_n} f_\Gamma(\gamma) d\gamma \simeq a_n \left[ M_\Gamma\left( \frac{b_n}{c_n}, \gamma_n^* \right) - M_\Gamma\left( \frac{b_n}{c_n}, \gamma_n^{n+1} \right) \right], \quad (II.10)$$

where $M_\Gamma(s, \gamma)$ is given in (II.6) for the case of i.n.d. Nakagami-$m$ fadings.

**II.3.2 Processing Load Measure**

Define $N_e$ as a discrete-valued random variable that takes values from the set $\{1, 2, \ldots, L\}$, and it is used to refer to the number of users needing to be examined per each packet transmission. If $T$ is used to refer to the time duration that takes the BS to examine one user, the probing time delay, which is proportional to the processing load, can be written as $T_D = N_e T$. This probing time is always less than or equal to the probing time of the full feedback scheme that chooses the user with the best channel conditions after receiving requests from all active users, which is given by $T_D, \text{Full} = LT$.

For a given value of the SNR threshold $\gamma_n^*$, it can be shown, for the case i.n.d. users, that $\Pr\{N_e = K\} = \prod_{k=1}^{K-1} F_{\gamma_k}(\gamma_n^*)[1 - F_{\gamma_k}(\gamma_n^*)]$, for $K = 1, \ldots, L - 1$, and $\Pr\{N_e = L\} = \prod_{k=1}^{L-1} F_{\gamma_k}(\gamma_n^*)$. The expression for the AFL can be then obtained as

$$N_e = \sum_{K=1}^{L-1} K \prod_{k=1}^{K-1} F_{\gamma_k}(\gamma_n^*)[1 - F_{\gamma_k}(\gamma_T)] + L \prod_{k=1}^{L-1} F_{\gamma_k}(\gamma_n^*), \quad (II.11)$$

which is generally applicable for any channel fading models.

**II.4 Effect of Outdated Channel Estimates**

The analytical development in the previous two sections is applicable for the case when the scheduler has perfect knowledge of the examined users instantaneous SNRs. However, in practice, the CSI estimates may be outdated. Specifically, the scheduler
can have perfect knowledge of the CSI of examined users at time $t$. However, the CSI may be imprecise at the time the scheduler selects the suitable user, which is denoted by $t + \tau$, where $\tau$ represents the time delay that is equal to the scheduling time duration. Consequently, the adaptation to channel conditions will be performed based on outdated CSI. This imperfectness can degrade the projected performance of the adaptive scheduler.

This section investigates the effect of outdated CSI available at the transmitter on the performance of the adaptive scheduling algorithm when the examined users undergo i.n.d. channel conditions.

II.4.1 Statistics of the Resulting SNR under Outdateness

This subsection presents general analytical formulations for the statistics of the resulting SNR, considering the effect of outdated CSI estimates. The analysis is then specialized for the case of i.n.d. Nakagami-$m$ fading conditions.

**General Case**

Define $\Gamma^{(\tau)}$ as the resulting SNR of the scheduled user when the transmission takes place, which is a delayed version of the true value, $\Gamma$, by a time duration of $\tau$. Also, define $\Gamma^{(\tau)}_l$ as a delayed version of $\Gamma_l$ when the $l$th user is selected, and let $f_{\Gamma_l,\Gamma_l^{(\tau)}}(\cdot,\cdot)$ be the joint PDF of $\Gamma^{(\tau)}_l$ and $\Gamma_l$. The marginal PDF of $\Gamma^{(\tau)}_l$, for $l = 1, 2, \ldots, L$, which is given by $f_{\Gamma^{(\tau)}_l} = \int_0^\infty f_{\Gamma_l,\Gamma_l^{(\tau)}}(u, x)du$, can be obtained as [22, eq. (4)]

$$f_{\Gamma^{(\tau)}_l}(x) = \int_0^{+\infty} f_{\Gamma_l}(u) \frac{f_{\gamma_l,\gamma_l^{(\tau)}}(u, x)}{f_{\gamma_l}(x)}du, \quad (II.12)$$

where $f_{\gamma_l}(x)$ is the PDF of the instantaneous SNR associated with the $l$th user, $f_{\Gamma^{(\tau)}_l}(x)$ is the joint PDF of the true SNR and its delayed version for each individual user, and $f_{\Gamma_l}(x)$ is the PDF of the resulting SNR, $\Gamma_l$, when the $l$th user is selected,
which can be read from (II.2) as

$$f_{\Gamma_l}(x) = f_{\gamma_l}(x) \prod_{i=1, i \neq l}^L F_{\gamma_i}(x) (U(x) - U(x - \gamma_T)) + \left( \prod_{i=1}^{l-1} F_{\gamma_i}(\gamma_T) \right) f_{\gamma_l}(x) U(x - \gamma_T).$$

(II.13)

Substituting the result in (II.13) into (II.12), the PDF of $\Gamma^{(\tau)}$, which is given by

$$f_{\Gamma^{(\tau)}}(x) = \sum_{l=1}^L f_{\Gamma_l^{(\tau)}}(x),$$

can be written as

$$f_{\Gamma^{(\tau)}}(x) = \sum_{l=1}^L \left[ f_{\Gamma_l^{(\tau)}}^{(1)}(x) + \left( \prod_{i=1}^{l-1} F_{\gamma_i}(\gamma_T) \right) f_{\Gamma_l^{(\tau)}}^{(2)}(x) \right],$$

(II.14)

where $f_{\Gamma_l^{(\tau)}}^{(1)}(x) = \int_{\gamma_T}^{\gamma_l} \prod_{i=1, i \neq l}^L F_{\gamma_i}(u) f_{\gamma_l, \gamma_i^{(\tau)}}(u, x) du$ and $f_{\Gamma_l^{(\tau)}}^{(2)}(x) = \int_{\gamma_T}^{+\infty} f_{\gamma_l, \gamma_i^{(\tau)}}(u, x) du$.

Results for Nakagami-$m$ Fading

In this part, results for the PDF and conditional MGF of $\Gamma^{(\tau)}$ are shown for the case when the channels fading envelopes undergo Nakagami-$m$ distribution.

PDF of the Resulting Outdated SNR

It is seen from (II.14) that the terms $f_{\Gamma_l^{(\tau)}}^{(1)}(x)$ and $f_{\Gamma_l^{(\tau)}}^{(2)}(x)$ depend on the joint PDF of $\gamma_l$ and $\gamma_l^{(\tau)}$, which, for the case under consideration, is given by [14, eq. (126)]

$$f_{\gamma_l, \gamma_l^{(\tau)}}(x_1, x_2) = \frac{m_l^{m_l+1}(x_1 x_2)^{m_l-1}}{\Gamma(m_l)(1 - \rho_l) \rho_l^{m_l} \gamma_l^{m_l+1}} e^{-\frac{m_l}{(1 - \rho_l) \gamma_l^{m_l+1}}} I_{m_l-1} \left( 2m_l \sqrt{\frac{\rho_l x_1 x_2}{(1 - \rho_l) \gamma_l^{m_l+1}}} \right),$$

(II.15)

where $I_{\mu}(\cdot)$ is the modified Bessel function of the first kind of order $\mu$, and $0 \leq \rho_l \leq 1$, for $l = 1, \ldots, L$, denotes the correlation coefficient between $\gamma_l$ and $\gamma_l^{(\tau)}$. The value of $\rho_l$ may be expressed as a function of the delay $\tau$ as $\rho = |J_0(2\pi f_D \tau)|^2$, where $J_0(x)$ is the zero-order Bessel function of the first kind and $f_D$ is the maximum Doppler frequency shift. The best scenario takes place when $\tau \to 0$, at which $\rho \to 1$, whereas
the worst scenario happens when \( \tau \to +\infty \), at which \( \gamma_i \) and \( \gamma_i^{(r)} \) will be uncorrelated. The terms \( f^{(1)}_{\Gamma_j} (x) \) and \( f^{(2)}_{\Gamma_j} (x) \) that appear in (II.14) can now be calculated, using the results in (II.3), (II.5), and (II.15), as

\[
f^{(1)}_{\Gamma_j} (x) = \sum_{l=0}^{L-1} (-1)^{l} \sum_{\tilde{\tau}(l,L-1)} \sum_{l=0}^{U} \tilde{C}_{l,L-1} \tilde{F}_j \left( l, \sum_{k=1, k \neq j}^{L} i_k m_k, x \right), \tag{II.16}
\]

\[
f^{(2)}_{\Gamma_j} (x) = \frac{m_j (1 - \rho_j) m_j - 1}{(m_j - 1)!} \left( \frac{\Sigma_j}{2} \right)^{m_j - 1} e^{-\frac{m_j}{\tau_j} x} \times _1 F_1 \left( m_j, m_j, \frac{(1 - \rho_j) \Sigma_j^2}{4 m_j}, x \right) - \tilde{F}_j (0, 0, x), \tag{II.17}
\]

where the function \( \tilde{F}_j (\cdot, \cdot, \cdot) \) in the preceding equations is evaluated as

\[
\tilde{F}_j (l, A, x) = \int_{0}^{\tau_T} u^l e^{-Au} \tilde{f}_{\gamma_j^{(r)}} (u, x) \, du
\]

\[
= \frac{m_j^{m_j+1} \left( \frac{\Sigma_j}{\gamma_j} \right)^{m_j-1} \left( \frac{1}{\Delta_j} \right)^{l+m_j/2+1/2} x^{-m_j-1} e^{-\frac{m_j}{\gamma_j} x} \left[ \frac{\Sigma_j}{2 \sqrt{\Delta_j}} \right]^{m_j-1} \Gamma(l+m_j) \Gamma(m_j)}{(m_j - 1)! \rho_j \Sigma_j^2 \gamma_j^{m_j+1}} \times x^{-m_j-1} _1 F_1 \left( l + m_j, m + j, \frac{\Sigma_j^2}{4 \Delta_j}, x \right) - \left( \frac{1}{2} \right)^{l+m_j-1} e^{\frac{\Sigma_j^2}{2 \Delta_j}} Q_{2l+m_j,m_j-1} \left( \frac{\Sigma_j}{\sqrt{2 \Delta_j} \sqrt{x}}, \sqrt{2 \gamma_j \Delta_j} \right), \tag{II.18}
\]

and \( \Delta_j = A + \frac{m_j}{(1 - \rho_j) \gamma_j}, \ \Sigma_j = 2 m_j \sqrt{\rho_j \gamma_j}, \ _1 F_1 (\cdot, \cdot, \cdot) \) is the confluent hypergeometric function, and \( Q_{m,n} (a, b) = \int_{b}^{\infty} x^m e^{-\frac{a^2 + b^2}{2}} I_n (ax) \, dx \) is the Nuttall Q function [15], [16], which is a generalization of the Marcum Q function, \( Q_n (a) = \frac{1}{a^n} \int_{b}^{\infty} x^n e^{-\frac{a^2 + b^2}{2}} I_n (ax) \, dx \) [17].

The function \( Q_{m,n} \) in this case has a special form where \( m + n \) takes odd integers, which, according to [16], can be obtained in terms of the Marcum Q function and the modified Bessel function of the first kind, both of which are implemented in common mathematical packages, as

\[
Q_{2l+m_j,m_j-1} (a, b) = \sum_{k=1}^{l+1} c_k (l) a^{m_j+2k-3} Q_{m_j+k-1} (a, b) + e^{-\frac{a^2+b^2}{2}} \sum_{k=1}^{l} P_{l,k} (b^2) a^{k-1} b^{m_j+k} I_{m_j+k-2} (ab), \tag{II.19}
\]
where \( c_k(l) = 2^{l-k+1} \frac{\rho^l}{(l-1)!} \left( \frac{l+m_j-1}{l-k+1} \right)^{l-k+1} \rho \), \( P_{l,k}(b^2) = \sum_{r=0}^{l-k} \frac{d_r(l,k)b^{2r}}{r!} \), and
\[
d_r(l,k) = 2^{l-k-r} \frac{(l-r-1)!}{(l-k-r)!} \left( \frac{l+m_j-1}{l-k-r} \right)^{l-k-r}.
\]

**MGF of the Resulting Outdated SNR** Based on the representation in (II.14), the MGF of \( \Gamma^{(r)} \) can be written as
\[
M_{\Gamma^{(r)}}(s) = \sum_{j=1}^{L} \left[ M_{\Gamma^{(r)}}^{(1)}(s) + \left( \prod_{i=1}^{j-1} \hat{F}_{\gamma_i}(\gamma_T) \right) M_{\Gamma^{(r)}}^{(2)}(s) \right],
\]
where \( M_{\Gamma^{(r)}}^{(1)}(s) \) and \( M_{\Gamma^{(r)}}^{(2)}(s) \) are evaluated as
\[
M_{\Gamma^{(r)}}^{(1)}(s) = \int_{0}^{\infty} e^{-sx} f_{\Gamma^{(r)}}^{(1)}(x) \, dx = \frac{\sum_{j=1}^{L} \hat{C}_{l,L-1} \, \tilde{M}_j \left( l, \sum_{k=1,k \neq j}^{L} \frac{i_km_k}{\gamma_k}, s \right)}{\sum_{i=1}^{j-1} \hat{F}_{\gamma_i}(\gamma_T)},
\]
\[
M_{\Gamma^{(r)}}^{(2)}(s) = \int_{0}^{\infty} e^{-sx} f_{\Gamma^{(r)}}^{(2)}(x) \, dx = \Theta_{\gamma}^{(2)}(0,0) \, \tilde{M}_j^{(2)}(0,0,s),
\]
in which the term \( \tilde{M}_j \) is given by
\[
\tilde{M}_j(l,A,s) = \int_{0}^{\infty} e^{-sx} F_j(l,A,x) \, dx = \Theta_{\gamma}^{(1)}(l,A) \, \tilde{M}_j^{(1)}(l,A,s) - \Theta_{\gamma}^{(2)}(l,A) \, \tilde{M}_j^{(2)}(l,A,s). \]

The functions in the previous equation can be obtained as
\[
\Theta_{\gamma}^{(1)}(l,A) = \frac{m_j^{m_j+1}(l+m_j)!}{((m_j - 1)!)^2(1 - \rho_j)\rho_j^{m_j-1} \gamma_j^{m_j+1}} \left( \frac{1}{\Delta_j} \right)^{l+m_j/2+1/2} \left( \frac{\Sigma_j}{2\sqrt{\Delta_j}} \right)^{m_j-1},
\]
\[
\Theta_{\gamma}^{(2)}(l,A) = \frac{m_j^{m_j+1}}{(m_j - 1)!(1 - \rho_j)\rho_j^{m_j-1} \gamma_j^{m_j+1}} \left( \frac{1}{2} \right)^{l+m_j/2-1/2} \left( \frac{1}{\Delta_j} \right)^{l+m_j/2+1/2},
\]
\[
\tilde{M}_j^{(1)}(l,A,s) = \int_{0}^{\infty} x^{m_j-1} e^{-\left( \frac{m_j}{(1-\rho_j)^{\gamma_j}} \right)x} \frac{\Sigma_j^2}{4\Delta_j} \, dx,
\]
\[
\tilde{M}_j^{(2)}(l,A,s) = \int_{0}^{\infty} \frac{x^{m_j-1}}{\frac{\Sigma_j^2}{4\Delta_j}} e^{-\left( \frac{m_j}{(1-\rho_j)^{\gamma_j}} \right)x} \frac{\Sigma_j}{2\Delta_j} \, dx. \]
The integrals associated with the terms $\tilde{M}_j^{(1)}(l, A, s)$ and $\tilde{M}_j^{(2)}(l, A, s)$ above can be evaluated with the help of the identities in [18, Eq. (7.621.4)] and [15, Eqs. (7) and (11)] to give

\[
\tilde{M}_j^{(1)}(l, A, s) = (m_j - 1)! \left( \frac{m_j}{(1 - \rho_j)\bar{\gamma}_j} + s \right)^{-m_j} {}_2F_1\left( l + m_j, m_j; m_j; \frac{\Sigma_j^2}{4\Delta_j} \left( \frac{m_j}{(1 - \rho_j)\bar{\gamma}_j} + s \right)^{-1} \right),
\]

\[
\tilde{M}_j^{(2)}(l, A, s) = \sum_{k=1}^{l+1} c_k(l) \left( \frac{\Sigma_j}{\sqrt{2\Delta_j}} \right)^{m_j+2k-2} I_{1,k}(A, s) + e^{-\gamma_T\Delta_j} \sum_{k=1}^{l} P_{l,k}(2\gamma_T\Delta_j) \left( \frac{\Sigma_j}{\sqrt{2\Delta_j}} \right)^{m_j+k} \left( \frac{\Sigma_j}{\sqrt{2\Delta_j}} \right)^{k-1} I_{2,k}(A, s),
\]

where the terms $I_{1,k}(\cdot, \cdot)$ and $I_{2,k}(\cdot, \cdot)$ are computed as

\[
I_{1,k}(A, s) = (m_j + k - 2)! \left( \frac{2}{\kappa_j(s)} \right)^{m_j+k-1} e^{-\frac{2\gamma_T\kappa_j(s)}{\Sigma_j^2}} \sum_{u=0}^{m_j+k-2} \frac{1}{u!} \left( \frac{2\gamma_T\kappa_j(s)}{\Sigma_j^2} \right)^{u}, \tag{II.28}
\]

\[
I_{2,k}(A, s) = \left( \frac{\sqrt{\gamma_T\Sigma_j}}{\sqrt{2}} \right)^{m_j+k-2} \left( \frac{m_j}{(1 - \rho_j)\bar{\gamma}_j} + s \right)^{-\frac{m_j+k}{2}} e^{-\frac{\gamma_T(1-\rho_j)\bar{\gamma}_j\Sigma_j^2}{4(m_j+s(1-\rho_j)\bar{\gamma}_j)}}, \tag{II.29}
\]

and $\kappa_j(s) = \frac{m_j}{(1-\rho_j)\bar{\gamma}_j} - \frac{\Sigma_j^2}{4\Delta_j} + s$.

### II.4.2 Effect of Outdated Estimation on the Average BER

The expression for the average BER, as per the definition in (II.9), for the case when the resulting SNR is outdated can be written as

\[
\overline{\text{BER}} = \frac{\sum_{n=1}^{N} R_n\text{BER}'_n}{\text{ASE}}, \tag{II.31}
\]
where \( \text{BER}'_n = \sum_{l=1}^{L} \text{BER}'_{n,l} \), and

\[
\text{BER}'_{n,l} = \int_{\gamma_n}^{\gamma_n+1} \int_{0}^{\infty} \text{BER}(a_n, b_n, c_n, \gamma_n^r) f_{\gamma_n^r | \gamma}(\gamma^r | \gamma) d\gamma^r f_{\Gamma_1}(\gamma) \ d\gamma \\
= \int_{\gamma_n}^{\gamma_n+1} \text{BER}_{\gamma_n^r | \gamma}(a_n, b_n, c_n, \gamma) f_{\Gamma_1}(\gamma) \ d\gamma
\]

(II.32)

where \( f_{\Gamma_1}(\gamma) \) is given in (II.13) and \( f_{\gamma_n^r | \gamma}(\gamma^r | \gamma) \) represents the conditioned PDF which can be obtained using the joint PDF \( f_{\gamma_n^r | \gamma}(\gamma^r, \gamma) \) in (II.15) as

\[
f_{\gamma_n^r | \gamma}(\gamma^r | \gamma) = \frac{m_l}{(1 - \rho_l)\gamma_l} \left( \frac{\gamma^r}{\rho_l \gamma} \right)^{(m_l-1)/2} e^{-\frac{m_l(\rho_l \gamma + \gamma r)}{(1 - \rho_l)\gamma_l}} I_{m_l-1} \left( 2m_l \frac{\sqrt{\rho_l \gamma r}}{(1 - \rho_l)\gamma_l} \right). \tag{II.33}
\]

Noting that \( \text{BER}(a_n, b_n, c_n, \gamma) = a_n e^{-\frac{b_n}{c_n} \gamma} \) as defined previously, the expression of \( \text{BER}_{\gamma_n^r | \gamma}(a_n, b_n, c_n, \gamma) \) in (II.32) can be written as \( \text{BER}_{\gamma_n^r | \gamma}(a_n, b_n, c_n, \gamma) = a_n M_{\gamma_n^r | \gamma}(\frac{b_n}{c_n}, \gamma) \), where \( M_{\gamma_n^r | \gamma}(s, \gamma) \) refers to the conditional MGF of \( \gamma_n^r \), which can be obtained, using the result of \( f_{\gamma_n^r | \gamma}(\gamma^r | \gamma) \) in (II.33) and with the help of (II.15), as

\[
M_{\gamma_n^r | \gamma}(s, \gamma) = \int_{0}^{\infty} e^{-s\gamma} f_{\gamma_n^r | \gamma}(\gamma^r | \gamma) \ d\gamma^r = \left( \frac{m_l}{m_l + s\gamma_l(1 - \rho_l)} \right)^{m_l} e^{-\frac{m_l \rho_l s}{m_l + s\gamma_l(1 - \rho_l)}}. \tag{II.34}
\]

Substituting the result in (II.34) into (II.32), and then using the result for the incomplete MGF of \( \Gamma_1 \) presented in (II.7), the final expressions for the average BER can be written as

\[
\text{BER} = \frac{1}{\text{ASE}} \sum_{n=1}^{N} \sum_{l=1}^{L} R_n \sum_{l=1}^{L} \alpha_{n,l} \left[ M_{\Gamma_1}(\beta_{n,l}, \gamma_n^*) - M_{\Gamma_1}(\beta_{n,l}, \gamma_{n+1}^*) \right], \tag{II.35}
\]

where \( M_{\Gamma_1}(\cdot, \cdot) \) is given by (II.7) for the case of i.n.d. Nakagami-\( m \) fadings, \( \alpha_{n,l} = a_n (\frac{m_l c_n}{c_n m_l + b_n \gamma_l(1 - \rho_l)})^{m_l} \) and \( \beta_{n,l} = \frac{m_l \rho_l b_n}{c_n m_l + b_n \gamma_l(1 - \rho_l)} \), for \( l = 1, \ldots, L \) and \( n = 1, \ldots, N \), which depend on the correlation coefficients \( \rho_l \), the average SNR \( \gamma_l \), and the adaptive transmission scheme constants \( a_n, b_n \), and \( c_n \).
II.5 Numerical Results and Discussions

This section shows some numerical results obtained based on the analytical development in the preceding sections to study the performance and processing load requirement of the scheduling algorithm under i.n.d. channel conditions for both non-adaptive and adaptive transmission modes. Due to the significant time needed to execute the simulation studies, the analytical results presented above have been supported by Monte-Carlo simulations for only specific cases (e.g., case of fixed average SNRs).

In the following results, it is assumed that the users are uniformly distributed in the cell of interest having a radius of $R$. Each user has a different average SNR, denoted by $\bar{\gamma}_l$, for $l = 1, \ldots, L$, which is assumed to be log-normal distributed with shadowing standard deviation of $\theta = 3$ dB. The PDF of the average SNR $\bar{\gamma}_l$, for $l = 1, \ldots, L$, has a form of

$$f_{\bar{\gamma}}(\bar{\gamma}) = \frac{2}{c} e^{\frac{2\theta^2 - 2c(\bar{\gamma} - \bar{\gamma}_R)}{c^2}} Q\left(\frac{2\theta^2 - c(\bar{\gamma} - \bar{\gamma}_R)}{c \theta}\right),$$

(II.36)

where $\bar{\gamma}_R$ is the average SNR at distance $R$, $\alpha$ is the path loss exponent ($\alpha = 3.5$ in the numerical results), $c = 10\alpha \log(e)$ and $Q(\cdot)$ is the Gaussian $Q$-function. For the adaptive system, an eight-region (i.e. $N = 8$) adaptive discrete-rate quadrature-amplitude modulation scheme is assumed with a target BER of $BER_0 = 10^{-3}$ and $\{R_n\}_{n=1}^8 = [1, 2, 3, 4, 5, 6, 7, 8]$. For this considered scheme, the constants $a_n$, $b_n$ and $c_n$ defined previously for the BER are given by: $a_n = 0.2$, $b_n = 3/2$ and $c_n = c'_n - 1$, where $c'_n \in \{2, 4, 8, 16, 32, 64, 128, 256\}$ represents the constellation size for each region. The regions thresholds [$\gamma^*_1, \gamma^*_2, \ldots, \gamma^*_N$] are calculated using the target BER$_0$, and according to [19], they are given by [6.78, 10.25, 13.93, 17.24, 20.39, 23.47, 26.51, 29.54] dB, respectively.

Figs. III.6 show the ASE as a function of $\bar{\gamma}_R$ for different values of switching
Figure II.1: Average spectral efficiency as a function of the average SNR at a distance $R$ from the BS. The results are shown for a target BER of $10^{-3}$, $L = 50$, different values of $\gamma_T$, and $m = 1$ under perfect knowledge of CSI at the serving BS and adaptive transmission.

threshold $\gamma_T$ and assuming a target BER of $10^{-3}$ and $L = 50$. It is clear that the increase in $\bar{\gamma}_R$ increases the ASE as this increase in $\bar{\gamma}_R$ is equivalent to decreasing the radius of the cell. Moreover, the increase in $\gamma_T$ improves the ASE, where the best behavior is observed when the best selection algorithm is used (i.e. case of $\gamma_T \to +\infty$). This can be explained by noting that, with the increase in $\gamma_T$, the BS selects a user with better channel and transmits at a higher rate leading to improved ASE.

Fig. IV.3 depicts the AFL as a function of $\bar{\gamma}_R$ for different values of $\gamma_T$ when $L = 50$. From Fig. III.6 and Fig. IV.3 the tradeoff between the performance (i.e. the ASE) and processing and feedback loads (i.e. through AFL) is observed. Specifically, while the increase in $\gamma_T$ improves the ASE when $\gamma_T \to +\infty$, the BS will need to probe all users, regardless of $\gamma_T$ in order to achieve the best possible ASE. On the other hand, the proposed algorithm clarifies that the ASE can be still improved
Figure II.2: Average feedback load as a function of the average SNR at a distance $R$ from the BS. The results are shown for the cases studied in Fig. III.6.

for any value of $\gamma_T$ as $\bar{\gamma}_R$ increases, but the AFL can be reduced significantly. An interesting observation takes place when $\bar{\gamma}_R > 40$ dB, where examining a single user can be sufficient to achieve the best possible ASE with the proposed algorithm, but all users should be probed when the best user selection algorithm is used.

Figs. IV.2 and II.4 depict average BER as a function of $\bar{\gamma}_R$ for a target BER of $10^{-3}$ and different values of $\gamma_T$ and $m$, respectively, when $L = 5$. From the two figures, it is clear that the average BER is always less than the targeted one. Also, it is seen that the average BER of the considered algorithm improves as $\gamma_T$ increases, where the best performance is observed with the use of the best selection algorithm. On the other hand, it is clear from Fig. II.4 that the average BER varies with the value of $m$ only for relatively high values of $\bar{\gamma}_R$.

Fig. IV.4 shows the effect of outdated CSIs estimates of scheduled users on the
Figure II.3: Average BER as a function of the average SNR at a distance $R$ from the BS. The results are shown for a target BER of $10^{-3}$, $L = 5$, different values of $\gamma_T$, and $m = 1$ under perfect knowledge of CSI at the serving BS and adaptive transmission.
Figure II.4: Average BER as a function of the average SNR at a distance $R$ from the BS. The results are shown for a target BER of $10^{-3}$, $L = 5$, $\gamma_T = \gamma_0^* \text{ dB}$, and different values of fading parameters of active users $m = [m_1, \ldots, m_5]$ under perfect knowledge of CSI at the serving BS and adaptive transmission. The figure clarifies the effect of the fading parameters on the average BER performance.
average BER as a function of $\bar{\gamma}_R$ for different values of the delay $\tau$ when $L = 5$ and $\gamma_T = \gamma_5^* \text{ dB}$. It is seen that the outdated estimations of CSIs can severely degrade the average BER performance where the target BER can not be met for all considered cases of $\tau > 0 \ (\rho < 1)$. However, with the increase in $\bar{\gamma}_R$, it is seen that the target BER can be satisfied for the cases of $\tau > 0 \ (\rho < 1)$, but full compensation for the effect of $\tau > 0 \ (\rho < 1)$ is no longer possible. An important conclusion that can be drawn is the relation between the average SNR at the edge of the cell and the time delay $\tau$, from which the transmit power can be adjusted to guarantee the target BER.

Fig. IV.5 shows the ASE as a function of $L$ for different values of $\bar{\gamma}_R$. Apparently, the increase in the number of users can be useful to improve the system performance. The slope of improvement varies with the specified value of $\bar{\gamma}_R$. The increase in $\bar{\gamma}_R$
increases the slope of improvement and reduces the required processing and feedback loads. However, this improvement decreases gradually with \( L \) when \( \bar{\gamma}_R \) is relatively low. These observations support the discussion presented in subsection II.A, which suggests prior adjustment of the guard period to handle limited number of users each time.

![Figure II.6: Average spectral efficiency as a function of the number of users. The results are shown for a target BER of \( 10^{-3} \), \( \gamma_T = \gamma_2^* \), \( m = 1 \), and different values of the average SNR at a distance \( R \) from the BS under perfect knowledge of CSI at the serving BS and adaptive transmission.](image)

### II.6 Concluding Remarks

This work has investigated a multiuser scheduling algorithm with post-examining selection when active users undergo i.n.d. channel conditions, and treated the cases of non-adaptive and adaptive transmission modes. The adaptation for channel cond-
tions has been proposed to choose from different signaling constellations over discrete-time intervals, while reducing the feedback load via the employed algorithm. General formulations for the statistics of the resulting SNR of the scheduled user have been presented. These formulations have been used to study various performance and processing measures in combination with discrete-rate adaptation. The effect of outdated CSI estimates on the efficiency of the algorithm have been also investigated, for which analytical formulations for the outdated resulting SNR as well as the average BER of the adaptive system have been presented. The numerical results have clarified the effect of the adaptation thresholds and fading conditions of active users on the system performance and processing and feedback loads. They also verified the advantages of the proposed adaptive algorithm for different operating conditions.
Chapter III

Performance Improvement of
Switched-Based Interference
Mitigation for Downlink Channel
Assignment in Over-Loaded
Femtocells

III.1 Introduction

The use of femtocells has been suggested to bring cellular network much closer to users equipment (UEs). This technology attempts to improve the system capacity and to extend the spatial coverage of existing cellular systems [23], [24]. Femtocells are usually installed indoors either privately by cellular users or in public places by network operators, wherein the UEs are likely to experience frequent service interruption. They are expected to cover short ranges, operate at low cost with low power consumption, while maintaining the existing cellular architecture.
The utilization of available radio resources in such two-tier networks, which contains the existing cellular network and femtocell network, has been receiving significant attention. Specifically, techniques like multiple-antenna configurations [25]–[27], cooperative communication [28]–[30], and cognitive radio [31]–[33], can potentially be attractive. However, these techniques can face many practical challenges, among which are the need for advanced management and coordination of distributed radio resources as well as the excessive need for feedback information. This work considers a single-antenna point-to-point communication scenario in which coordination between operating access points (i.e. femtocell and/or macrocell base stations) is infeasible. It is considered that the femtocells in close vicinity are allocated orthogonal channels to minimize the inter-cell interference. However, other interference sources can be still noticeable, which are the cross-tier interference and the co-tier intra-cell interference. The cross-tier interference can be observed between the macrocell and femtocell networks due to reusing the same channels in the two networks simultaneously. On the other hand, the co-tier intra-cell interference is due to the co-channel interference generated in each femtocell or macrocell separately when the available channels are reused aggressively.

The effect of cross-tier interference can be reduced when the open-access control strategy in each network is used [34]. However, the reduction of intra-cell interference in each operating femtocell demands a separate treatment, which is desired to maintain the processing load as low as possible. In [35], a single-threshold interference reduction scheme has been proposed for over-loaded open-access femtocells. The results have been shown when each UE employs single-antenna and can convey limited feedback. However, the scheme in [35] may suffer from performance loss when the interference power levels on examined channels are relatively large or the available channels are extensively reused, which can be the case in dense open-access femtocells.

\footnote{The processing load can be related to the number of operational steps required to identify a suitable channel in order to serve the desired UE.}
This chapter proposes low-complexity schemes to improve the interference mitigation capability of the single-threshold interference reduction scheme. The proposed schemes aim to find the suitable channel to serve the desired user in each cell separately while reducing the number of examined channels when all channels are overloaded. They target the case when the feedback information from active UEs is limited to just specify the index of the suitable channel and mode of transmission. Specifically, three schemes are investigated, and their modes of operation are discussed. The first scheme suggests complementing the single-threshold switched-based interference reduction scheme by selecting the channel whose interference power level is minimum when all available channels are found unsuitable. It takes advantage of knowing the interference power levels on available channels noncoherently when all channels are found unsuitable, and attempts to identify the channel whose associated interference power level is minimal. On the other hand, capitalizing on the multi-branch diversity optimization in [36], the second scheme proposes to adaptively vary the interference threshold on available channels according to the predicted interference power levels on the previously examined channels. It considers the case when ranking the interference powers on available channels is infeasible, and aims to reduce the processing load in searching for a suitable channel to serve the desired UE. The third scheme proposes a sequential processing by supporting the optimized multi-threshold switched-based scheme with post minimum interference power channel selection.

The contribution of this work is summarized as follows. Low-complexity schemes to improve the capability of the single-threshold switched-based scheme in mitigating the effect of intra-cell interference\(^2\) in over-loaded open-access femtocells are proposed. The proposed schemes are applicable for the case when the femtocells access points in close vicinity are allocated reusable orthogonal channels, employ single antennas, can not coordinate their transmissions, can schedule concurrent active requests for

\(^2\)The proposed schemes can be also applied to mitigate other sources of interference. The scenarios encountering such cases will be treated in another work.
downlink service, and receive limited feedback information. The first scheme complements the single-threshold scheme by selecting the channel whose interference power level is minimum when all channels fail to satisfy the specified interference threshold. The mode of operation of this scheme is described and some important performance metrics, such as outage probability, average bit error rate (BER), and average number of examined channels, are quantified and discussed. The second scheme proposes to vary the interference thresholds among available channels to enhance the desired user performance and/or to reduce the number of examined channels. The optimal thresholds that can either minimize the average resulting interference power, minimize the average BER, or maximize the average received signal-to-interference-plus-noise ratio (SINR) are thoroughly studied for generalized models of interference powers on available channels. The third proposed scheme investigates the benefits of using the first and second schemes jointly via sequential processing. Numerical results, which are verified by simulations, are provided to clarify the gains obtained when using the proposed schemes as compared to the single-threshold scheme with no post-processing capability.

The rest of the chapter is organized as follows. Section III.2 revisits the system model. Section III.3 presents the switched-based scheme with post channel selection, and presents the statistics of the resulting interference power as well as some important performance measures. Section III.4 investigates the adaptive variation of the interference threshold among different channels, and quantifies the optimal thresholds for various performance metrics. Section III.5 discusses the joint deployment of the two schemes presented in sections III.3 and III.4, and section III.6 shows some numerical comparisons, are supported by simulations, for the performance and processing load of the proposed schemes. Finally, concluding remarks are provided in section III.7.
III.2 System Model

The system model under consideration assumes that the macrocell of interest contains a number of femtocells, which can be either deployed individually by private users or jointly to cover a relatively small public area. The femtocells in close vicinity are allocated orthogonal channels, and can update their transmit powers periodically in order to minimize the co-channel interference between operating access points. They also place no restrictions on the identities of served UEs, thereby alleviating any possible cross-tier interference between the macrocell network and femtocell network, which are assumed to share the same available resources simultaneously.

The active UEs in the femtocell of interest can be served by a total of $K$ orthogonal channels, where each UE can access only one channel at a given time due to capacity and/or spatial coverage constraints. The case when the number of active UEs in each femtocell at a given time exceeds the number of available channels is of interest herein. In this case, and to reduce the likelihood of service interruption, the available channels can be reused to support more UEs as long as the interference experienced by the active UEs can be tolerated, while maintaining a target performance level.

Each access point is equipped with an isotropic antenna and can schedule the concurrent UEs requests per each channel via an orthogonal time division scheme. On the other hand, each UE is equipped with an isotropic antenna, can transfer limited feedback to the serving access point, and can sense the interference powers on different channels noncoherently and periodically. The access points are assumed to operate independently with no coordination between them, which is the same case considered for the active UEs in each femtocell. The channel conditions and traffic load are assumed to be static for at least one packet duration\textsuperscript{3}. However, the results in this work are generally applicable for any statistical models of interference power levels,

\textsuperscript{3}For the case of fast varying traffic or high mobility, the identification of a suitable channel can be made based on the average interference conditions. The results in this work can be applied directly to handle this situation.
and can be used when fast variation of interference levels on the available channels is observed. Each packet time duration contains a guard period that can be used by the active users to identify their modes of transmission and suitable downlink channels. Although the scheduling gain is not shown herein explicitly, the analysis focuses on the processing load and achieved performance of the scheduled (i.e., desired) UE.

III.3 Switched-Based Scheme with Post Minimum Interference Channel Selection

In this section, the improvement achieved by employing post minimum interference channel selection is quantified. The section contains four parts. The first part presents general formulations for the statistics of the resulting interference power. These statistics are then used in the second part to obtain analytical results for various performance measures. The average number of the examined channels, which is linked to the processing load, is characterized in the third part. Finally, the fourth part discusses the optimization of the interference threshold from different perspectives.

III.3.1 Resulting Interference Power

This subsection presents the statistics of the resulting interference power, which is denoted by $s_I$, when the interference power levels on available channels are tested against a single threshold, referred to as $s_T$. The results are applicable for the general case of non-identically distributed interference power levels, which can follow any statistical models.

The scheme herein starts examining interference powers on the active channels either arbitrarily, from the recently used channel, or based on the long-term average interference powers. The process is terminated when a channel whose interference power is below the specific threshold is found or the last channel is reached. When
none of available channels can satisfy the interference threshold, the channel whose interference power level is minimum is identified by ordering the interference power levels on available channels according to their power strengths. The UE then specifies the index of the selected channel to the serving access point for subsequent packet transmission.

**Mode of Operation**

The mode of operation is described as follows. During the guard period of each packet duration, the interference power on the current channel, which is denoted by $s_1$, is tested against $s_T$. If $s_1 < s_T$, then the current channel is used for subsequent transmission, and no farther operations are required. On the other hand, if $s_1 > s_T$, switching to another channel, which can be the closest channel to the previously tested one, is made, and its interference power, referred to as $s_2$, is tested against $s_T$. The second examined channel is then used if $s_1 > s_T$ and $s_2 < s_T$. Otherwise, a third channel is examined.

In general, the $k$th channel is selected if $s_k < s_T$ and each channel of the first $k-1$ examined channels has its interference level above the channel threshold. For the case when all the $K$ channels have their interference powers above the specified threshold, the receive station can take the advantage of knowing all interference powers on available channels, and then selects the one whose interference power level is the minimal.

**Statistics of Resulting Interference Power**

Based on the mode of operation presented above, the cumulative distribution function (CDF) of $s_I$, which is defined as $F_{s_I}(x) = \sum_{k=1}^{K} \Pr\{s_I < x, s_I = s_k\}$, can be expressed
as

$$F_{s_I}(x) = \left[ 1 - \prod_{k=1}^{K} (1 - F_{s_k}(x)) \right] U(x-s_T) + \left[ \sum_{k=1}^{K} \prod_{j=1}^{k-1} (1 - F_{s_j}(s_T)) F_{s_k}(x) \right] (U(x) - U(x-s_T)),$$

(III.1)

where $F_{s_k}(.)$ is the CDF of the interference power associated with the $k$th examined channel and $U(.)$ is the unit step function. The probability density function (PDF) of $s_I$ can be obtained by direct differentiation of the CDF of $s_I$ in (III.1), and can be written as

$$f_{s_I}(x) = \sum_{j=1}^{K} f_{s_j}(x) \prod_{k=1,k\neq j}^{K} (1 - F_{s_k}(x)) U(x-s_T)$$

$$+ \left[ \sum_{k=1}^{K} \prod_{j=1}^{k-1} (1 - F_{s_j}(s_T)) f_{s_k}(x) \right] (U(x) - U(x-s_T)),$$

(III.2)

where $f_{s_k}(.)$ is the PDF of the interference power associated with the $k$th examined channel. The results in (III.1) and (III.2) are new formulations that are applicable for any statistical models of interference powers on available channels, and can be used to study various performance measures.

### III.3.2 Performance Analysis

This section provides formulations for the statistics of the received SINR. These results are needed to quantify different performance measures.

**Received SINR**

The received SINR at the desired UE for the case under consideration can be written as

$$\gamma_{\text{SINR}} = \frac{s_D}{s_I + \sigma^2},$$

where $s_D$ is the received desired signal power, $s_I$ is the resulting instantaneous interference power, and $\sigma^2$ is the power of the background white noise.

This model considers the case when the desired UE receives the same power level,

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4Details of the derivations are omitted herein for brevity.
regardless of the used channel, which is a valid assumption for short-range radio links.\(^5\) However, the limitation comes from increasing the reuse percentage of the available channels, which results in varying the interference powers of the available channels.

When the desired UE undergoes similar Rayleigh fading statistics across the \(K\) channels, the PDF of \(s_D\) can be simply given by \(f_{s_D}(x) = \frac{1}{\bar{s}_D} e^{-x/\bar{s}_D}, x \geq 0\), and the CDF can be given by \(F_{s_D}(x) = 1 - e^{-x/\bar{s}_D}, x \geq 0\), where \(\bar{s}_D\) is the long-term average value of \(s_D\). Assuming that there are a total of \(q_k\) interfering UEs accessing the \(k\)th channel simultaneously, the CDF and PDF of \(s_k\) can be written, using the noncoherent superposition principle, as \(F_{s_k}(x) = F_{\sum_{i=1}^{q_k} s_{k,i}}(x)\) and \(f_{s_k}(x) = f_{\sum_{i=1}^{q_k} s_{k,i}}(x)\), respectively, where \(s_{k,i}\) is the interference power of \(i\)th interfering user on the \(k\)th channel.

For the specific case of small-scale Rayleigh fading, the PDF of \(s_{k,i}\) can be given by \(f_{s_{k,i}}(x) = \frac{1}{\bar{s}_{k,i}} e^{-x/\bar{s}_{k,i}}, x \geq 0\), where \(\bar{s}_{k,i}\) is the long-term average value of \(s_{k,i}\). Since the interfering UEs are physically separated and undergo statistically independent fading processes, the statistics of \(\sum_{i=1}^{q_k} s_{k,i}\) can be obtained as (see, e.g., [37, eq. (14)])

\[
F_{\sum_{i=1}^{q_k} s_{k,i}}(x) = 1 - \sum_{i=1}^{q_k} \xi_{k,i} e^{-x/\bar{s}_{k,i}}, \quad f_{\sum_{i=1}^{q_k} s_{k,i}}(x) = \sum_{i=1}^{q_k} \frac{\xi_{k,i}}{\bar{s}_{k,i}} e^{-x/\bar{s}_{k,i}},
\]

where \(\xi_{k,i} = \prod_{u=1, u \neq i}^{q_k} (1 - \bar{s}_{k,u}/\bar{s}_{k,i})^{-1}\).

**PDF of \(\gamma_{\text{SINR}}\)**

Using the statistics of \(s_I, s_k, s_D\), for \(k = 1, \ldots, K\), and \(s_D\) presented in the previous parts, the PDF of \(\gamma_{\text{SINR}}\), which is defined as \(f_{\gamma_{\text{SINR}}}(x) = \int_0^{+\infty} (t + \sigma^2) f_{s_D}(x(t + \sigma^2)) f_{s_I}(t) dt\),[38]

\(^5\)The analysis can also handle the case when the desired received power varies with the index of the allocated channel.
can be expressed as

\[
f_{\gamma_{\text{SINR}}}(x) = \sum_{j=1}^{K} \int_{st}^{+\infty} (t + \sigma^2)f_{s_D}(x(t + \sigma^2)) f_{s_k}(t) F_{0,j}(t) dt + \sum_{k=1}^{K} \prod_{j=1}^{k-1} (1 - F_{s_j}(st)) \int_{0}^{st} (t + \sigma^2)f_{s_D}(x(t + \sigma^2)) f_{s_k}(t) dt, \tag{III.4}
\]

where \(F_{0,j}(t) = \prod_{k=1,k\neq j}^{K} (1 - F_{s_k}(t))\). To express the result in (III.4) in a tractable form, it is seen that, under the effect of Rayleigh fading, the term \(F_{0,j,k}(t)\) is the product of sum of weighted exponentials, and hence it can be expressed as a sum of a weighted exponential terms as \(F_{0,j}(t) = \sum_{i=1}^{Q_j} A_i e^{-B_i t}\), where \(Q_j = \sum_{k=1,k\neq j}^{K} q_k\), \(A_i = \prod_{k=1}^{K-1} a_k\) such that \((a_1, a_2, ..., a_{K-1})\) is a set of all the possible elements where each of which is selected from a different set from \(\{V_1, V_2, ..., V_K\}/\{V_j\}\), which represents the set of all subsets excluding the \(j\)th one, where \(V_k = \{\xi_{k,1}, \xi_{k,2}, ..., \xi_{k,q_k}\}\), for \(k = 1, ..., K\). For the term \(B_i\) in the preceding equation, it can be expressed as \(B_i = \sum_{k=1}^{K-1} b_k\) such that \((b_1, b_2, ..., b_{K-1})\) is a set of all the possible elements where each of which is selected from a different set from \(\{W_1, W_2, ..., W_K\}/\{W_j\}\), where \(W_k = \{1/s_{k,1}, 1/s_{k,2}, ..., 1/s_{k,q_k}\}\), for \(k = 1, ..., K\). Substituting the result for \(F_{0,j}(t)\) into the general formula in (III.4), the PDF of \(\gamma_{\text{SINR}}\) becomes

\[
f_{\gamma_{\text{SINR}}}(x) = \sum_{j=1}^{K} \sum_{i=1}^{q_j} \sum_{l=1}^{Q_j} \frac{\xi_{j,i} A_i}{s_{j,i} s_D} g_1(x, st) + \sum_{k=1}^{K} \left( \prod_{j=1}^{k-1} \sum_{i=1}^{q_j} \xi_{j,i} e^{-\frac{2}{s_{j,i}}} \right) \sum_{l=1}^{q_k} \frac{\xi_{k,l}}{s_{k,l} s_D} g_2(x, st), \tag{III.5}
\]

where \(g_1(x, st)\) and \(g_2(x, st)\) can be obtained using [18, eq. (3.381.3)], and they are given by

\[
g_1(x, y) = \left( B_l + \frac{x}{s_D} \right)^{-2} e^{B_l \sigma^2} \Gamma \left( 2, (y + \sigma^2) \left( B_l + \frac{x}{s_D} \right) \right),
\]

\[
g_2(x, y) = \left( \frac{x}{s_D} + \frac{1}{s_{k,l}} \right)^{-2} e^{\frac{x^2}{s_{k,l}}} \left[ \Gamma \left( 2, \sigma^2 \left( \frac{x}{s_D} + \frac{1}{s_{k,l}} \right) \right) - \Gamma \left( 2, (y + \sigma^2) \left( \frac{x}{s_D} + \frac{1}{s_{k,l}} \right) \right) \right], \tag{III.6}
\]
in which $\Gamma(\mu, x)$ refers to the upper incomplete Gamma function [18, sec. 8.35].

### Outage Probability

The outage probability is defined as the probability that $\gamma_{\text{SINR}}$ is below a certain outage threshold, which gives $P_{\text{out}}(x) = F_{\gamma_{\text{SINR}}}(x)$, where $x$ represents the outage threshold. Following similar procedure to that described above to find the PDF of $\gamma_{\text{SINR}}$, the outage probability expression can be found as

$$P_{\text{out}}(x) = \int_0^{+\infty} F_{s_D}(x(t + \sigma^2)) f_{s_1}(t) dt$$

$$= \sum_{j=1}^{K} \sum_{q_j}^{Q_j} \xi_{j,i} A_l I_1(x, s_T) + \sum_{k=1}^{K} \left( \prod_{j=1}^{k-1} \xi_{j,i} e^{-\frac{x}{s_j,i}} \right) \sum_{l=1}^{q_k} \xi_{k,l} I_2(x, s_T),$$

(III.7)

where the terms $I_1(x, s_T)$ and $I_2(x, s_T)$ in the preceding equation can be obtained as

$$I_1(x, y) = e^{-\left( B_l + \frac{1}{s_{j,i}} \right)} y \left[ \left( B_l + \frac{1}{s_{j,i}} \right)^{-1} \left( B_l + \frac{1}{s_{j,i}} + \frac{x}{s_D} \right)^{-1} e^{-\frac{y + \sigma^2}{s_D} x} \right],$$

(III.8a)

$$I_2(x, y) = s_{k,l} \left( 1 - e^{-\frac{x}{s_{k,l}}} \right) - e^{-\frac{x}{s_D}} \left( \frac{1}{s_{k,l}} + \frac{x}{s_D} \right)^{-\frac{1}{y}} \left( 1 - e^{-\left( \frac{1}{s_{k,l}} + \frac{x}{s_D} \right)y} \right).$$

(III.8b)

### Performance Measures

It is known that various performance measures, such as the average bit error rate (BER) of digital signaling schemes, can be computed using the moment-generating function (MGF) of $\gamma_{\text{SINR}}$, which is defined as $M_{\gamma_{\text{SINR}}}(e) = E_{\gamma_{\text{SINR}} \{ e^{-e \gamma_{\text{SINR}}} \}}$. It is proposed to use the result in (III.7) in order to obtain the analytical expression for
\[ M_{\gamma \text{SINR}}(e) = s \int_{0}^{+\infty} e^{-ex} F_{\gamma \text{SINR}}(x) dx \]
\[ = \sum_{j=1}^{K} \sum_{i=1}^{Q_j} \sum_{l=1}^{q_i} \frac{\xi_{j,i} A_l}{\bar{s}_{j,i}} M_1(e, s_T) + \sum_{k=1}^{K-1} \left( \prod_{j=1}^{q_i} \sum_{i=1}^{Q_j} \frac{\xi_{j,i} e^{-e x}}{s_{j,i}} \right) \sum_{l=1}^{q_k} \frac{\xi_{k,l}}{\bar{s}_{k,l}} M_2(e, s_T), \]

(III.9)

where the terms \( M_1(e, s_T) \) and \( M_2(e, s_T) \) in (III.9) can be obtained using [18, Eqs. (3.381.3)] and [18, Eqs. (8.359.1)], and the results are given by

\[ M_1(e, y) = \int_{0}^{+\infty} e^{-ex} I_1(x, y) dx \]
\[ = \left( B_l + \frac{1}{\bar{s}_{j,i}} \right)^{-1} \left( B_l + \frac{1}{\bar{s}_{j,i}} \right)^y e^{s_D e^{(s_D + \sigma^2)}} \left( B_l + \frac{1}{\bar{s}_{j,i}} \right) E_i \left( -\left( B_l + \frac{1}{\bar{s}_{j,i}} \right) \left( y + \sigma^2 + e \bar{s}_D \right) \right), \]

\[ M_2(e, y) = \int_{0}^{+\infty} e^{-ex} I_2(x, y) dx \]
\[ = \bar{s}_{k,l} \left( 1 - e^{-\bar{s}_{k,l}} \right) e^{s_D e^{(s_D + \sigma^2)}} \left[ E_i \left( -\frac{e}{\bar{s}_D} + \frac{y + \sigma^2}{\bar{s}_{k,l}} \right) - E_i \left( -\frac{e}{\bar{s}_D + \sigma^2} \right) \right], \]

(III.10)

where \( E_i(x) \) represents the exponential integral function, which is defined as \( E_i(x) = \int_{-\infty}^{x} \frac{1}{t} e^t dt, x > 0 \) [18]. As an example, the average BER of the binary differential phase-shift keying (B-DPSK) signaling scheme can be obtained as \( \bar{P}_b = 1/2 M_{\gamma \text{SINR}}(1) \).

### III.3.3 Average Number of Examined Channels

The average number of examined channels is related to the processing load required by the proposed scheme to allocate a suitable channel to the desired UE. The number of examined channels can be used to quantify the number of channel switchings by simply reducing the number of examined channels by one. Define \( N \) as a discrete random variable that takes on values from the set \( \{1, 2, \ldots, K\} \). The value of \( N \) refers to the number of examined channels at a given time. The probability mass function
(PMF) of $N$ can be quantified analytical based on the mode of operation described in subsection III.3.1. Specifically, it can be noted that $\Pr \{ N = 1 \} = \Pr \{ s_1 < s_T \} = F_{s_1}(s_T)$. Moreover, for $k = 2, \ldots, K - 1$, $\Pr \{ N = k \} = F_{s_k}(s_T) \prod_{n=1}^{k-1} (1 - F_{s_n}(s_T))$. Finally, $\Pr \{ N = K \} = \prod_{n=1}^{K-1} (1 - F_{s_n}(s_T))$, which refers the probability that the $K - 1$ previously examined channels have their interference power levels above the specified threshold.

Using the results above for the PMF of $N$, the average number of examined channels during the search for a suitable channel to serve the desired UE using the scheme under consideration, which is denoted by $\bar{N} = \sum_{k=1}^{K} k \Pr \{ N = k \}$, is given by

$$\bar{N} = \sum_{k=1}^{K-1} k F_{s_k}(s_T) \prod_{n=1}^{k-1} (1 - F_{s_n}(s_T)) + K \prod_{n=1}^{K-1} (1 - F_{s_n}(s_T)),$$

(III.11)

which is applicable for any statistical models of interference power levels. It can be seen from (III.11) that, when $s_T \to 0$, the average number of examined channels based on their interference power levels approaches the total number of available channels. On the other hand, when $s_T$ approaches infinity, it is clear that the first channel will be used directly and no examination of other channels is needed.

### III.3.4 Optimization of Interference Threshold

It is noted that the value of $s_T$ has significant impact on the processing load and achieved performance of the proposed scheme. Specifically, for the extreme case when $s_T$ is set to zero, the proposed scheme acts as the minimum interference channel selection scheme, whereas when $s_T$ is set very high, the proposed scheme does not take advantage of the available channels to reduce the interference power experienced by the desired UE as it will keep using the first examined channel all the time.

From the processing load perspective, the use of $s_T \to +\infty$ is desired to avoid
examining and testing the occupancy of other channels. However, this choice of the value of $s_T$ results in large performance loss. On the other hand, and from the system performance perspective, setting the value of $s_T = 0$ provides the best possible performance gain, but it requires the largest processing load. Therefore, minimizing the processing load requires different optimal interference threshold than that required to maximize the system performance. In between these values of $s_T$, the proposed scheme allows for a balance between the processing load reduction and performance enhancement. A relatively small value of $s_T$ is expected to give priority for improving the achieved performance, whereas a relatively large interference margin can be satisfied with a relatively low processing load at the receive station.

III.4 Multi-Threshold Switched-Based Interference Reduction Scheme

This section presents a different approach that can improve the performance and/or reduce the processing load of the switched-based interference reduction scheme when ranking the predicted interference power levels is infeasible. The approach proposes to vary the interference threshold on different channels according to the interference power levels on previously examined channels.

III.4.1 Resulting Interference Power

This subsection presents the statistics of $s_I$ when the interference power levels on available channels are tested against multiple thresholds, are non-identically distributed, and can follow any statistical models. This scheme terminates the search for the suitable channel when a channel whose interference power is below the specific channel threshold is found or the last channel is reached. In the later case, and when ranking the interference power levels is infeasible, the last channel is used.
Mode of Operation

When a total of \( K \) orthogonal channels are available to serve the desired UE, a total of \( K - 1 \) interference thresholds, which are referred to as \( \{ s_{T,1}, \ldots, s_{T,K-1} \} \), are used and updated sequentially. During the guard period, the interference power on the current channel, which is denoted by \( s_1 \), is tested against \( s_{T,1} \). If \( s_1 < s_{T,1} \), then the current channel is used for subsequent transmission. On the other hand, if \( s_1 > s_{T,1} \), switching to another channel, which can be the closest channel to the previously tested one, is made, and its interference power, referred to as \( s_2 \), is tested against \( s_{T,2} \). The second examined channel is then used if \( s_1 > s_{T,1} \) and \( s_2 < s_{T,2} \).

In general, the \( k \)th channel is selected if \( s_k < s_{T,k} \) and each channel of the first \( k-1 \) examined channels has its interference level above the channel threshold. For the case when the first \( K - 1 \) channels have their interference powers above the specified thresholds, the UE does not need to examine the last channel, which is directly used.

Statistics of Resulting Interference Power

Based on the mode of operation described above, the PDF of \( s_I \) can be expressed as

\[
f_{s_1}(x) = \prod_{j=1}^{K-1} \left(1 - F_{s_j}(s_{T,j})\right) f_{s_K}(x) + \sum_{k=1}^{K-1} \prod_{j=1}^{k-1} \left(1 - F_{s_j}(s_{T,j})\right) f_{s_k}(x) \left( U(x) - U(x - s_{T,k}) \right),
\]

where \( F_{s_k}(.) \) is the CDF of the instantaneous interference power associated with the \( k \)th examined channel and \( f_{s_k}(.) \) is its PDF, as defined previously. The important issue to explore herein is to link the interference thresholds with a specific performance measure. The following subsection treats this case, and presents analytical formulations for the optimization of the interference thresholds for various performance measures. The results are generally valid for any statistical models of interference power levels on available channels.
III.4.2 Optimization of Interference Thresholds

This subsection presents the optimal thresholds that guarantee average resulting interference power minimization, average received SINR maximization, or average BER minimization, respectively.

Average Interference Power Minimization

The average value of the resulting interference power can be obtained as
\[ \bar{I} = \int_{0}^{+\infty} x f_s(x) \, dx. \]

Using the result in (III.12), it can be written as
\[ \bar{I} = K - 1 \prod_{j=1}^{K-1} (1 - F_{s_j}(s_{T,j})) \int_{0}^{+\infty} x f_{s_{K}}(x) \, dx + \sum_{k=1}^{K-1} \prod_{j=1}^{k-1} (1 - F_{s_j}(s_{T,j})) \int_{0}^{s_{T,k}} x f_{s_k}(x) \, dx. \]

(III.13)

The objective is to find the sequence of optimal switching thresholds \( \{ s_{T,1}^*, s_{T,2}^*, \ldots, s_{T,K-1}^* \} \) that minimizes the value of \( \bar{I} \). Notice that there is no threshold for the last channel because the last channel is selected directly if the first \( K - 1 \) channels are found unsuitable.

Since each channel undergoes an independent interference, the last switching threshold in the sequence can be found by setting \( \frac{\partial \bar{s}_I}{\partial s_{T,1} \ldots \partial s_{T,K-1}} = 0 \). It can be expressed as
\[ \frac{\partial \bar{s}_I}{\partial s_{T,1} \ldots \partial s_{T,K-1}} = \prod_{j=1}^{K-1} \left( -f_{s_j}(s_{T,j}) \right) \int_{0}^{+\infty} x f_{s_{K}}(x) \, dx + s_{T,K-1} f_{s_{K-1}}(s_{T,K-1}) \prod_{j=1}^{K-2} \left( -f_{s_j}(s_{T,j}) \right) . \]

(III.14)

Setting the result in (III.14) to zero results in
\[ s_{T,K-1}^* = \int_{0}^{+\infty} x f_{s_{K}}(x) \, dx = \bar{s}_K, \]

(III.15)

where \( \bar{s}_K \) is the average interference power in the \( K \)th channel, as defined previ-
ously. Substituting \( s_{T,K-1} \) by \( s_{T,K-1}^* \) in (III.13) and taking another partial derivative \( \frac{\partial \bar{s}_I}{\partial s_{T,1} \cdots \partial s_{T,K-2}} \), we obtain

\[
\frac{\partial \bar{s}_I}{\partial s_{T,1} \cdots \partial s_{T,K-2}} = \prod_{j=1}^{K-2} (-f_{s_j}(s_{T,j})) \int_0^{s_{T,K-1}} x f_{s_{K-1}}(x) dx + s_{T,K-2} f_{s_{K-2}}(s_{T,K-2}) \prod_{j=1}^{K-3} (-f_{s_j}(s_{T,j})) \\
+ (1 - F_{s_{K-1}}(s_{T,K-1})) \prod_{j=1}^{K-2} (-f_{s_j}(s_{T,j})).
\]

(III.16)

Setting the result in (III.16) to zero leads to

\[
s_{T,K-2}^* = \bar{s}_K \left( 1 - F_{s_{K-1}}(s_{T,K-1}^*) \right) + \int_0^{s_{T,K-1}} x f_{s_{K-1}}(x) dx.
\]

(III.17)

Following the same procedure described above, a recursive relation for the optimal thresholds \( \{s_{T,n}^*\}_{n=1}^{K-1} \) can be defined, and the results are summarized as

\[
\begin{align*}
\left\{ \begin{array}{l}
s_{T,K-1}^* = \bar{s}_K, \\
s_{T,n}^* = s_{T,n+1}^* \left( 1 - F_{s_{n+1}}(s_{T,n+1}^*) \right) + \int_0^{s_{T,n+1}} x f_{s_{n+1}}(x) dx, n = 1, 2, \ldots, K-2.
\end{array} \right.
\end{align*}
\]

(III.18)

For the case of Rayleigh faded interfering users on available channels, and using the statistics of the interference powers on different channels given in (III.3), the results in (III.18) can be obtained with the help of the identity in [18, eq. (3.381.1)] to give

\[
\begin{align*}
\left\{ \begin{array}{l}
s_{T,K-1}^* = \sum_{i=1}^{q_K} \xi_{K,i} \bar{s}_{K,i}, \\
s_{T,n}^* = \sum_{i=1}^{q_{n+1}} \xi_{n+1,i} \left[ s_{T,n+1}^* e^{-\frac{s_{T,n+1}^*}{\bar{s}_{n+1,i}}} + \bar{s}_{n+1,i} \gamma \left( 2, \frac{s_{T,n+1}^*}{\bar{s}_{n+1,i}} \right) \right], n = 1, 2, \ldots, K-2,
\end{array} \right.
\end{align*}
\]

(III.19)

where \( \gamma(a,x) \) is the lower incomplete Gamma function [18, Sec. 8.35].
Average SINR Maximization

Based on the development in subsection [III.3.2] and using the result in (III.12), the PDF of $\gamma_{\text{SINR}}$ for the case under consideration can be obtained as

$$f_{\gamma_{\text{SINR}}}(x) = \prod_{j=1}^{K-1} (1 - F_{s_j}(s_{T,j})) \int_0^{+\infty} (t + \sigma^2) f_{s_D}(x(t + \sigma^2)) f_{s_K}(t) dt$$

$$+ \sum_{k=1}^{K-1} \prod_{j=1}^{k-1} (1 - F_{s_j}(s_{T,j})) \int_0^{s_{T,k}} (t + \sigma^2) f_{s_D}(x(t + \sigma^2)) f_{s_k}(t) dt,$$

(III.20)

where $f_{s_k}(x)$, $k = 1,..,K$, and $f_{s_D}(x)$ are the PDFs of the interference power in the $k$th channel and the received desired power, respectively. Using the PDF of $\gamma_{\text{SINR}}$ given in (III.20), the average value of the received SINR, which is defined as

$$\bar{\gamma}_{\text{SINR}} = \int_0^{+\infty} x f_{\gamma_{\text{SINR}}}(x) dx,$$

can be written as

$$\bar{\gamma}_{\text{SINR}} = \prod_{j=1}^{K-1} (1 - F_{s_j}(s_{T,j})) \bar{s}_D \int_0^{+\infty} \frac{1}{t + \sigma^2} f_{s_K}(t) dt$$

$$+ \sum_{k=1}^{K-1} \prod_{j=1}^{k-1} (1 - F_{s_j}(s_{T,j})) \bar{s}_D \int_0^{s_{T,k}} \frac{1}{t + \sigma^2} f_{s_k}(t) dt.$$

(III.21)

The objective is then to find the sequence of optimal thresholds $\{s_{T,1}^*, s_{T,2}^*, ..., s_{T,K-1}^*\}$ that maximizes the average SINR for any statistical models of interference powers on available channels. Following an iterative procedure that is similar to the one described in the preceding subsection, the results for $s_{T,K-1}^*$ can be obtained from the partial differentiation of $\bar{\gamma}_{\text{SINR}}$ with respect to associated thresholds,

$$\frac{\partial \bar{\gamma}_{\text{SINR}}}{\partial s_{T,1}...\partial s_{T,K-1}},$$

which can be expressed as

$$\frac{\partial \bar{\gamma}_{\text{SINR}}}{\partial s_{T,1}...\partial s_{T,K-1}} = \prod_{j=1}^{K-1} (-f_{s_j}(s_{T,j})) \bar{s}_D \int_0^{+\infty} \frac{1}{t + \sigma^2} f_{s_K}(t) dt$$

$$- \prod_{j=1}^{K-1} (-f_{s_j}(s_{T,j})) \frac{\bar{s}_D}{s_{T,K-1} + \sigma^2}.$$

(III.22)
Setting (III.22) to zero, a result that contains \( s_{T,K-1} \) can be found, and therefore it can be shown that

\[
 s_{T,K-1}^* = \left[ \int_0^{+\infty} \frac{1}{t + \sigma^2} f_{s_K}(t) dt \right]^{-1} - \sigma^2. \tag{III.23}
\]

Substituting \( s_{T,K-1} \) by \( s_{T,K-1}^* \) in (III.21), and then taking another partial derivative \( \frac{\partial \text{SINR}}{\partial s_{T,1}...\partial s_{T,K-2}} \) gives, after setting the result to zero, the following formula for \( s_{T,K-2}^* \):

\[
 s_{T,K-2}^* = \left[ (1 - F_{s_{K-1}}(s_{T,K-1}^*)) \frac{1}{s_{T,K-1}^* + \sigma^2} + \int_0^{s_{T,K-1}} \frac{1}{t + \sigma^2} f_{s_{K-1}}(t) dt \right]^{-1} - \sigma^2. \tag{III.24}
\]

After adopting this procedure, it can be shown that \( \{s_{T,n}\}_{n=1}^{K-1} \) can be obtained recursively as

\[
 \begin{align*}
 s_{T,K-1}^* &= \left[ \int_0^{\infty} \frac{1}{t + \sigma^2} f_{s_K}(t) dt \right]^{-1} - \sigma^2, \\
 s_{T,n}^* &= \left[ (1 - F_{s_{n+1}}(s_{T,n+1}^*)) \frac{1}{s_{T,n+1}^* + \sigma^2} + \int_0^{s_{T,n+1}} \frac{1}{t + \sigma^2} f_{s_{n+1}}(t) dt \right]^{-1} - \sigma^2, \\
 n &= 1, 2, \ldots, K - 2.
\end{align*} \tag{III.25}
\]

For the special case when the interfering users on available channels are under Rayleigh fading processes, and using the statistics of the interference powers on different channels given in (III.3), the results in (III.25) can be expressed, using the
identity in [8, Eq. (3.381.3)], as

\[
\begin{align*}
\mathcal{S}_{T,K-1}^* & = \left[ \sum_{i=1}^{q_K} \xi_{K,i} \frac{s^{2}_{K,i}}{s^{2}_{K,i}} \right]^{-1} - \sigma^2, \\
\mathcal{S}_{T,n}^* & = \left[ \frac{1}{s_{T,n+1}^* + \sigma^2} \sum_{i=1}^{q_{n+1}} \xi_{n+1,i} e^{-\frac{s_{T,n+1}^*}{s_{n+1,i}}} + \sum_{i=1}^{q_{n+1}} \xi_{n+1,i} \frac{s_{n+1,i}^2}{s_{n+1,i}} \right]^{-1} - \sigma^2, \\
& \times \left( \Gamma \left( 0, \frac{\sigma^2}{s_{n+1,i}} \right) - \Gamma \left( 0, \frac{s_{T,n+1}^* + \sigma^2}{s_{n+1,i}} \right) \right)^{-1} - \sigma^2, n = 1, 2, \ldots K - 2.
\end{align*}
\] (III.26)

Note that the upper incomplete Gamma function when the first parameter is zero can be represented using the exponential integral function as \( \Gamma(0, x) = -Ei(-x), x > 0 \) [8, Eq. (8.359.1)].

**Average BER Minimization**

The BER of various signaling schemes can be expressed in terms of the one-dimensional Gaussian Q-function [11 ch. 3]. For the specific case of binary phase-shift keying (BPSK), the average BER can be given by

\[
\bar{P}_b = \int_0^{+\infty} Q \left( \sqrt{2x} \right) f_{\gamma_{\text{SINR}}} (x) dx
\]

\[
= \prod_{j=1}^{K-1} \left( 1 - F_{s_j}(s_{T,j}) \right) \int_0^{+\infty} (t + \sigma^2) I(t) f_{s_k}(t) dt
\]

\[
+ \sum_{k=1}^{K-1} \prod_{j=1}^{k-1} \left( 1 - F_{s_j}(s_{T,j}) \right) \int_0^{s_{T,k}} (t + \sigma^2) I(t) f_{s_k}(t) dt,
\] (III.27)

where \( I(t) = \int_0^{+\infty} Q \left( \sqrt{2x} \right) f_{s_D}(x(t + \sigma^2)) dx \), which for the case of Rayleigh fading for the desired UE, using the help of [39 App. B], can be expressed as

\[
I(t) = \frac{1}{2(t + \sigma^2)} \left( 1 - \sqrt{\frac{s_D}{s_D + \sigma^2 + t}} \right),
\] (III.28)
It is required to quantify the sequence of the optimal interference thresholds on available channels that can be jointly used to minimize the average BER. Specifically, the result for $\frac{\partial \bar{P}_b}{\partial s_{T,1} \ldots \partial s_{T,K-1}}$ can be obtained from (III.27) as

$$
\frac{\partial \bar{P}_b}{\partial s_{T,1} \ldots \partial s_{T,K-1}} = \prod_{j=1}^{K-1} (-f_{s_j}(s_{T,j})) \int_{0}^{+\infty} (t + \sigma^2)I(t)f_{s_K}(t)dt
$$

(III.29)

Setting (III.29) to zero leads to

$$
s_{T,K-1}^* = \left[ \int_{0}^{+\infty} \sqrt{1 \over \tilde{s}_D + \sigma^2 + t}f_{s_K}(t)dt \right]^{-2} - \sigma^2 - \tilde{s}_D. \quad (III.30)
$$

Inserting (III.30) in (III.27) and taking another partial differentiation $\frac{\partial \bar{P}_b}{\partial s_{T,1} \ldots \partial s_{T,K-2}} = 0$ yields

$$
(s_{T,K-2}^* + \sigma^2)I(s_{T,K-2}^*) = \int_{0}^{s_{T,K-1}^*} (t + \sigma^2)I(t)f_{s_{K-1}}(t)dt
$$

$$
+ \left(1 - F_{s_{K-1}}(s_{T,K-1}^*)\right) \left(s_{T,K-1}^* + \sigma^2\right)I(s_{T,K-1}^*). \quad (III.31)
$$

The sequence of $s_{T,n}^*$, for $n = 1, \ldots, K-1$, can be then obtained, and summarized as

$$
\begin{align*}
\begin{cases}
s_{T,K-1}^* = \left[ \int_{0}^{+\infty} \sqrt{1 \over \tilde{s}_D + \sigma^2 + t}f_{s_K}(t)dt \right]^{-2} - \tilde{s}_D - \sigma^2, \\
s_{T,n}^* = \sqrt{1 \over \tilde{s}_D + \sigma^2 + s_{T,n+1}^*} \left(1 - F_{s_{n+1}}(s_{T,n+1}^*)\right) \\
+ \int_{0}^{s_{T,n+1}^*} \sqrt{1 \over \tilde{s}_D + \sigma^2 + t}f_{s_{n+1}}(t)dt \right]^{-2} - \tilde{s}_D - \sigma^2,
\end{cases}
\end{align*}
$$

(III.32)

As an example, for the case of Rayleigh fading, the results in (III.30), using [18]...
Eq. (3.381.3), become

$$
\begin{align*}
\begin{cases}
    s^*_{T,K-1} = & \left[ \frac{q_K}{\sqrt{\bar{s}_{K,i}}} \sum_{i=1}^{q_K} \xi_{K,i} e^{\frac{\bar{s}_{D} + \sigma^2}{\bar{s}_{K,i}}} \Gamma \left( 1/2, \frac{\bar{s}_{D} + \sigma^2}{\bar{s}_{K,i}} \right) \right]^{-2} \bar{s}_D - \sigma^2, \\
    s^*_{T,n} = & \left[ \frac{1}{\bar{s}_D + s^*_{T,n+1} + \sigma^2} \sum_{i=1}^{q_{n+1}} \xi_{n+1,i} e^{\frac{s^*_{T,n+1} + \sigma^2}{\bar{s}_{n+1,i}}} + \sum_{i=1}^{q_{n+1}} \frac{\xi_{n+1,i}}{\sqrt{\bar{s}_{n+1,i}}} e^{\frac{s^*_{D} + \sigma^2}{\bar{s}_{n+1,i}}} \Gamma \left( 1/2, \frac{\bar{s}_D + \sigma^2}{\bar{s}_{n+1,i}} \right) \right]^{-2} \bar{s}_D - \sigma^2, \quad n = 1, 2, \ldots, K - 2.
\end{cases}
\end{align*}
$$

(III.33)

Note that the upper Gamma incomplete function when the first parameter is equal to 1/2 can be written in terms of the error function as $\Gamma(1/2, \alpha) = \sqrt{\pi} \text{erfc}(\sqrt{\alpha})$, [18, eq. (3.359.3)] where $\text{erfc}(\cdot)$ represents the complementary error function.

### III.4.3 Average Number of Examined Channels

Define $N$ as a discrete random variable that takes on values from the set $\{1, 2, \ldots, K - 1\}$. The value of $N$ is used to refer to the number of examined channels during the search for a suitable channel to support the desired UE using the proposed multi-threshold interference reduction scheme. Based on the mode of operation described in subsection [III.4.1] it follows that $\Pr\{N = 1\} = \Pr\{s_1 < s^*_{T,1}\}$. The probability that $N = 2$ is exactly $\Pr\{s_1 > s^*_{T,1}\} \Pr\{s_2 < s^*_{T,2}\}$. In general, $\Pr\{N = k\} = \prod_{j=1}^{k-1} \Pr\{s_j > s^*_{T,j}\} \Pr\{s_k < s^*_{T,k}\}$, for $k = 2, 3, \ldots, K - 2$. Knowing that the last channel will be used only when the $(K-1)$st channel is found unacceptable, it follows that $\Pr\{N = K - 1\} = \prod_{j=1}^{K-2} \Pr\{s_j > s^*_{T,j}\} \Pr\{s_{K-1} < s^*_{T,K-1}\} + \prod_{j=1}^{K-1} \Pr\{s_j > s^*_{T,j}\}$, which leads to $\prod_{j=1}^{K-2} \Pr\{s_j > s^*_{T,j}\}$. Combining the results above for the PMF of $N$, it can be shown that the average value of $N$, which is denoted by $\bar{N}$, in this case is given by

$$
\bar{N} = \sum_{k=1}^{K-2} \frac{k-1}{k} \prod_{j=1}^{k-1} \left( 1 - F_{s_j}(s^*_{T,j}) \right) F_{s_k}(s^*_{T,k}) + (K - 1) \prod_{j=1}^{K-2} \left( 1 - F_{s_j}(s^*_{T,j}) \right). \quad (III.34)
$$
The values of the optimal interference thresholds \( \{s^*_T,n\}_{n=1}^{K-1} \) can be read from the results in the previous subsection for different performance metrics. Due to the differences of the optimal interference thresholds between the considered performance measures, the average number of examined channels is sensitive to which performance metric needs to be optimized.

Apart from the use of the optimal interference thresholds in (III.34), two limiting cases are worth to explain. The first case is when the interference thresholds are set to very high values, which results in \( \lim_{s^*_T,j \to +\infty} (1 - F_{s,j}(s^*_T,j)) = 0 \), and therefore \( \bar{N} = 1 \). The second case is when the interference thresholds is set to very low values, which gives \( \lim_{s^*_T,j \to 0^+} F_{s,j}(s^*_T,j) = 0 \), and therefore \( \bar{N} = K - 1 \). For the first case, the proposed scheme will suggest using the first examined channel all the time, regardless of the interference power level on that channel, which leads to

\[
F_{s_1|s^*_T,1,...,s^*_T,K-2 \to +\infty}(x) = F_{s_1}(x) \quad \text{and} \quad f_{s_1|s^*_T,1,...,s^*_T,K-2 \to +\infty}(x) = f_{s_1}(x).
\]

On the other hand, the second case requires the largest processing load requirement, but can not achieve the optimal performance as the last available channel will be used all the time, which results in

\[
F_{s_1|s^*_T,1,...,s^*_T,K-2 \to 0^+}(x) = F_{s_K}(x) \quad \text{and} \quad f_{s_1|s^*_T,1,...,s^*_T,K-2 \to 0^+}(x) = f_{s_K}(x).
\]

### III.5 Multi-Threshold Interference Reduction with Post Channel Selection

This section presents a joint scheme to gain advantages of the two schemes discussed in the previous two sections in order to provide further performance improvement. It can be viewed as a generalization of the single-threshold switched-based scheme, multi-threshold interference reduction scheme, randomized channel selection, and minimum interference channel selection.

The mode of operation of the scheme under consideration is summarized as follows. A total of \( K \) orthogonal channels are available to serve the desired UE and
the optimized interference thresholds \( \{s^*_{T,1}, s^*_{T,2}, \ldots, s^*_{T,K-1}\} \) obtained according to the development in the previous section are used. In general, when \( k = 1, \ldots, K - 1 \), the \( k \)th channel is selected if \( s_k < s^*_{T,k} \) and each channel of the first \( k - 1 \) examined channels has its interference level above the channel threshold. For the case when the \( K - 1 \)th channel is found unsuitable, the UE will take the advantage of knowing all the channels interference power and select the channel with the minimum interference.

This scheme complements the multi-threshold interference reduction scheme by selecting the channel whose interference power level is minimum when all examined channels are found unsuitable. The scheme includes the minimum interference channel selection as a limiting case when the interference thresholds on different channels are set to very low values (i.e. approaching zero). It also performs similar to the multi-threshold interference reduction scheme when at least one of the examined channels is found suitable to meet the assigned channel interference threshold.

### III.6 Numerical Results and Discussions

This section presents numerical results to compare the performance of the proposed schemes with the single-threshold scheme. The numerical results, which have been obtained using the analytical findings above, are verified by Monte Carlo simulations. In generating the results, it is assumed that the users are uniformly distributed in the femtocell of interest having a radius of \( R \), and centered around the access point. The interfering users have different average powers, which are assumed to be log-normal distributed with shadowing standard deviation of \( \theta \). According to [40], the PDF of the average power associated with the \( i \)th interfering user on the \( k \)th available channel, \( \bar{s}_{k,i} \), can be expressed as

\[
f_{\bar{s}_{k,i}}(\bar{s}) = \frac{1}{c} e^{-\frac{2\theta^2 - \alpha(\bar{s} - \bar{\bar{s}}_R)}{c^2}} Q\left(\frac{2\theta^2 - \alpha(\bar{s} - \bar{\bar{s}}_R)}{c \theta}\right),
\]

where \( \bar{\bar{s}}_R \) is the average interference power at distance \( R \), \( \alpha \) is the path loss exponent (\( \alpha = 3.5 \) herein), and \( c = 10\alpha \log(e) \). Through the results, multiple non-identical realizations
for each value of $s_{k,i}$ have been generated, and then averaged to account for the randomness of interfering users locations.

Fig. III.1 shows the outage probability for the switched-based scheme with and without post-selection as a function of $s_R$ for different values of $s_T$. In generating the plots in the figure, it is considered that the number of channels is set to $K = 4$, where each of has $q_k = 3$ interfering users. Moreover, the average power of the desired user is set $s_D = 35$ dBm, the noise variance is normalized as $\sigma^2 = 0$ dBm, the shadowing standard deviation of interfering users is chosen to be $\theta = 3$ dB, and the outage threshold is specified at $s_{th} = 0$ dB. The figure also shows results for the single-channel scenario (upper bound curve) and the noise-limited interference-free scenario (lower bound curve). It is seen that the increase in $s_R$ increases the outage probability of the two schemes as well as the single-channel scenario. This is because the increase in $s_R$ is equivalent to decreasing the radius of the femtocell, and therefore the active users become much closer to the access point and have larger powers which will increase the interference levels on the available channels. For all selected values of $s_T$, the scheme with post-selection provides improved outage performance as compared to the one without post-selection. The gap in performance between the two schemes becomes more noticeable as $s_T$ decreases. The schemes performs similarly only when $s_R$ approaches zero or infinity. Specifically, when $s_R \to 0^+$, the interference power levels on all available channels will be very low, and therefore the two schemes performs similar to the noise-limited scenario, regardless of the value of $s_T$. On the other hand, when $s_R \to +\infty$, the interference levels on available channels will be very high that diminishes the gain provided by the post minimum interference channel selection. Another advantage of the scheme with post-selection is that it can improve outage performance for a wider range of high interference powers on available channels as compared to that without post-selection capability. Finally, it is important to notice that the proposed scheme with post-selection takes full advantage of the decrease in
Figure III.1: Outage probability for the switched-based scheme with and without post-selection as a function of the average power of the UE at a distance $R$ and for different values of the interference threshold $s_T$.

The interference threshold $s_T$ by improving the performance. On the other hand, the scheme without post-selection can not guarantee better performance for all values of interference powers on available channels even when a large processing load is placed (i.e., $s_T$ is low).

Fig. III.2 shows the outage probability for the switched-based schemes with and without post-selection as a function of the average power of the desired UE, $\bar{s}_{D,R}$, at a distance $R$ for different values of $s_T$ and fixed value of $\bar{s}_R$ when $K = 4$, $q_k = 3$, $\sigma^2 = 0$ dBm, $\theta = 3$ dB, $s_{th} = 0$ dB, and $\bar{s}_R = 0$ dBm. This figure attempts to observe the effect of varying the radius of the femtocell on the desired UE average power and the achieved performance of the two schemes for different interference thresholds. It is clear that the increase in $\bar{s}_{D,R}$ improves the outage performance of the two schemes.
Figure III.2: Outage probability for the switched-based scheme with and without post-selection as a function of the average power of the desired UE at a distance $R$ for different values of the switching threshold $s_T$.

The gap in performance between the two schemes becomes wider as $\bar{s}_{D,R}$ increases. For the values of $s_T$ considered in the figure, it is seen that the performance gap between the two schemes decreases with the increase in $s_T$. The reason for this is due to the fact that the increase in $s_T$ allows the switched-based scheme to more likely find a suitable channel by examining a relatively low number of channels, and therefore, the post-selection gain diminishes. The decrease in $s_T$ improves the performance of the scheme with post-selection as the post-selection advantage becomes more feasible.

For low values of the desired UE received average power (i.e., below $-10$ dBm), due to the increase in the femtocell radius, it is seen that all cases perform similarly.

Fig. III.3 shows the average BER for B-DPSK for the switched-based scheme with and without post-selection as a function of the interference threshold $s_T$ for different
values of the average desired user power $\bar{s}_D$. The results are shown for given values of $K = 4$, $q_k = 3$, $\sigma^2 = 0$ dBm, $\theta = 3$ dB, and $\bar{s}_R = 5$ dBm. The objective of this figure to clarify the effect of $s_T$ on the achieved performance. For the two schemes, it is clear that the increase in the desired UE received power improves the average BER, regardless of the interference threshold. For any value of $\bar{s}_D$, it is seen that the two schemes perform similarly when $s_T$ is relatively large. This observation is obvious from the modes of operation of the two schemes, wherein increasing the value of $s_T$ reduces the likelihood that the post-selection will be activated, as mentioned above.

For the switched-based scheme without post-selection, it is clear that there is a unique value of $s_T$ at which the average BER is minimized. This value does not require the maximum processing load, as shown in Fig. III.4, since the minimal average BER is lower than that achieved when $s_T \to 0^+$. On the other hand, the scheme with post-selection capability can improve the average BER as long as $s_T$ decreases, but this is achieved at the expense of higher processing load as more channels has to be examined (see Fig. III.4). In any case, the post-selection advantage is seen even when the other one provides the minimal average BER.

Fig. III.5 shows the effect the shadowing standard deviation $\theta$ of the UE on the outage probability of the two schemes with and without post-selection for different values of $\bar{s}_R$. The results are shown for $K = 4$, $q_k = 3$, $\bar{s}_D = 35$ dBm, $\sigma^2 = 0$ dBm, and the outage threshold $s_{th} = 0$ dB. It is clear that the increase in $\theta$ degrades the outage performance of the two schemes. The effect of $\theta$ is more noticeable when $\bar{s}_R$ is relatively low. These observations can be explained by noting that the increase in $\theta$ widens the range over which the average interference powers on available channels can take around their mean value, which results in degraded performance. However, for all values of $\theta$ and $\bar{s}_R$, the scheme with post-selection capability maintains its performance advantage over that without post-selection.

Table III.1 shows the simulated optimal thresholds $s^*_T$ in dBm for different per-
Figure III.3: Average BER of B-DPSK of the switched-based scheme with and without post-selection as a function of the interference threshold $s_T$ for different values of the average power of the desired UE at a distance $R$. 

$K = 4$, $q_k = 3$, $s_R = 0$ dBm, $\sigma^2 = 0$ dBm, $\theta = 3$ dB
Figure III.4: Average number of examined channels for the cases in Fig. III.3. Performance criteria when the single-threshold switched scheme without post-selection is used. The results are shown for $\bar{s}_D = 20$ dBm, $\sigma^2 = 0$ dBm, $K = 8$, $q_k = 3$, for $k = 1,..,K$, and a simulation precision $\epsilon = 0.1$. It is clear that $s^*_T$ varies according to the objective function. Table III.2 shows the numerical and simulation results of the optimal interference thresholds for the multi-threshold switched scheme, $s^*_{T,n}$, for $n = 1,..,K-1$, using the same values of the parameters in Table III.1. It can be seen that the optimal thresholds for each objective function are different, and their values increase with the channel index. From Table III.1 it is seen that the minimization of the average interference power requires the largest optimal threshold (i.e. allowing for larger interference margin) among the studied performance measures. However, the situation is not the same for the multi-threshold scheme, which shows the order of the values of the optimal thresholds varies with the channel index, though the minimization of the average BER demands the largest optimal threshold for any channel.
Figure III.5: Outage probability performance of the received SINR for the post-selection switching scheme and conventional switching scheme as a function of the shadowing standard deviation $\theta$ of the UE and for different values the average power of the UE at a distance $R$. 

$K = 4, q_k = 3, s_{th} = 0 \text{ dB}, s_D = 35 \text{ dBm}, s_T = 10 \text{ dBm}, \sigma^2 = 0 \text{ dBm}$
Table III.1: Optimal interference thresholds $s_T^*$ in dB based on different performance criteria for the single threshold switching scheme. The results are shown for $\bar{s}_D = 20 \text{ dBm}$, $\sigma^2 = 0 \text{ dBm}$, $q_k = 3$, $K = 8$ and simulation precision $\epsilon = 0.1$.

<table>
<thead>
<tr>
<th>$\bar{s}_I$ minimization</th>
<th>$\bar{\gamma}_{\text{SINR}}$ maximization</th>
<th>$P_b$ minimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.8000</td>
<td>11.7000</td>
<td>12.5000</td>
</tr>
</tbody>
</table>

Table III.2: Optimal interference thresholds $s_{T,n}^*$ in dB for different performance criteria. The results are shown for $\bar{s}_D = 20 \text{ dBm}$, $\sigma^2 = 0 \text{ dBm}$, $q_k = 3$, $K = 8$ and simulation precision $\epsilon = 0.1$.

<table>
<thead>
<tr>
<th>$\bar{s}_I$ minimization</th>
<th>$\bar{\gamma}_{\text{SINR}}$ maximization</th>
<th>$P_b$ minimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>Simulation</td>
<td>Analytical</td>
</tr>
<tr>
<td>2 12.0083</td>
<td>12.0000</td>
<td>11.5108</td>
</tr>
<tr>
<td>5 13.3910</td>
<td>13.4000</td>
<td>15.3107</td>
</tr>
</tbody>
</table>

Figs. III.6 shows $\bar{s}_I$ as functions of $\bar{s}_R$ for the proposed optimal multi-threshold scheme and the optimal single-threshold scheme, considering different values of $K$. It is clear that the increase in the number of available channels improves the performance of the two schemes, as expected. It is also seen that the proposed multi-threshold scheme with optimized thresholds provides improved performance as compared to the optimized single-threshold scheme for any value of $\bar{s}_R$ and $K > 2$.

Fig. III.7 shows the average number of examined channels for the cases addressed in Fig. III.6. It is clear that the average number of examined channels at the optimal thresholds for the two schemes increase with $K$ but at different slopes. The gap between the two values of $K$ remain relatively constant when the multi-threshold scheme is used, whereas it expands substantially with the increase in $\bar{s}_R$ when the single-threshold scheme is used. This can be justified by noting that the multi-
Figure III.6: Average value of the resulting interference power for the optimal multi-threshold proposed scheme and the optimal single-threshold scheme as a function of the average signal power of the UE at a distance $R$ for different values of the number of channels $K$. 
threshold scheme adapts its thresholds according the interference power levels of different channels. However, for low values of $\bar{s}_R$, it is clear that the single-threshold scheme needs to examine less number of channels on average, but at the expense of some loss in performance as shown in Fig. III.6. Interestingly, for high values of $\bar{s}_R$, the multi-threshold scheme provides better performance than that of the single-threshold scheme, and it requires to examine less number of channels on average.

Fig. III.8 shows the outage performance for the single-threshold post-selection switched scheme, the optimized multi-threshold scheme without post-selection, and the joint scheme as functions of $\bar{s}_R$ when $K = 4$, $q_k = 3$, $s_{th} = 0$ dB, $\bar{s}_D = 35$ dBm, $\sigma^2 = 0$ dBm, and $\theta = 3$ dB. The figure compares the outage performance of the three
proposed scheme and presents the result for the single-threshold scheme without post-selection for different values of the average interference power at the femtocell edge. It is clear that the three proposed schemes provide outage performance advantage as compared to the single-threshold scheme without post-selection. The best outage performance is obtained with the use of the joint scheme. Specifically, the joint scheme performs similar to the optimized multi-threshold scheme when $\bar{s}_R$ is low, and then similar to the single-threshold post-selection scheme when $\bar{s}_R$ is large (i.e., $\bar{s}_R \geq 10 \text{ dBm}$). The optimized multi-threshold scheme outperforms the single-threshold post-selection scheme when $\bar{s}_R$ is low (i.e., $\bar{s}_R < -5 \text{ dBm}$), but the situation is reversed as $\bar{s}_R$ increases. The optimized multi-threshold scheme maintains its superior performance over the single-threshold scheme without post-selection for all values of $\bar{s}_R$.

III.7 Concluding Remarks

The chapter has proposed adequate schemes to improve the performance of the single-threshold switched-based scheme in mitigating the intra-cell interference in over-loaded open-access femtocells. They target the situation when the femtocells access points are allocated reusable orthogonal channels, employ single antennas, can not coordinate their transmissions, can schedule concurrent active requests for downlink service, and receive limited feedback information. The first scheme has suggested a post-processing minimum interference channel selection when all available channels fail to satisfy the target interference threshold. The second scheme has aimed to improve the performance without additional post-processing via varying the interference thresholds among available channels according the interference power levels on previously examined channels. The third scheme has investigated the use of the aforementioned schemes jointly via sequential processing. The modes of operation
Figure III.8: Outage probability performance of the received SINR for the post-selection switched scheme, the multi-threshold switched scheme and the joint scheme as a function of the average signal power of the UE at a distance $R$. 

$K = 4, q_k = 3, s_{th} = 0 \text{ dB}, s_D = 35 \text{ dBm}, \sigma^2 = 0 \text{ dBm}, \theta = 3 \text{ dB}$. 

Joint scheme
Post-selection scheme $s_T = 20 \text{ dBm}$
Switching scheme $s_T = 20 \text{ dBm}$
Multi-threshold scheme
Simulations
of the proposed schemes have been described, and analytical formulations for impor-
tant metrics have been thoroughly discussed. The results have showed that the
proposed schemes can improve the performance substantially as compared to that of
no post-processing capability and employs a single threshold.
Chapter IV

Opportunistic Spectrum Access for Cognitive Radio Based on Channel Switching

IV.1 Introduction

Cognitive radio has been shown to be one of the potential solutions to radio spectrum resource scarcity [41], [42]. Extensive measurements indicated that in contrast with the spectrum scarcity, at any given time and location, a large portion of licensed spectrum is unused. As a matter of fact, new opportunistic spectrum access techniques for cognitive radio emerge. These techniques, have been considered as an efficient mean to opportunistic spectrum sharing between primary users (PU), which are licensed, and secondary user (SU), which will make use of the unused spectrum at a given time and place. Opportunistic spectrum access (OSA) are considered as dynamic spectrum access techniques that allow the SU to access channels when PU are not transmitting while protecting PU from interference. The OSA techniques, present an effective strategy for the SU access by achieving a high spectrum efficiency and qual-
ity of service [43], [44]. When SU may not be able to monitor all the spectrum and have energy constrains, decentralized cognitive protocols are attractive [45]. These protocols, allow the SU to independently search for spectrum holes without a coordinator or a dedicated communication channel. Spectrum holes can be either exploited in space domain, when the SU transmit in a location where no PU are active, or in time domain, when the PU is present in that location and is found to be idle. In this chapter, the proposed OSA will focus on the temporal white spaces.

In OSA, one of the important issue is the modeling of the behavior of the PU, which depends on the application that is running in the PU. The PU activity can be modeled by a simple two-state Markov chain. This model is not always realistic, but some experimental studies have shown that this model can be a reasonable approximation of the PU behavior in some systems such as the IEEE 802.11 Wireless LAN for various traffic models [46].

In this chapter, we assume that the transmission of the PU is unslotted, which means that the PU can transmit at any time, and the traffic of the PU can be modeled by continuous-time Markov-chain. The SU is assumed to implement a decentralized cognitive protocol, to have unslotted transmission and to be able to use and sense only one channel at a time. We propose two different OSA schemes, where the first one applies when the SU have access to only one channel. Thus, the SU will periodically sense the channel and access it once the PU is sensed to be OFF. The SU during transmission will sense continuously the used channel to avoid interference with PU and immediately evacuate the band as soon as the corresponding PU appears. The second proposed OSA scheme applies when the SU is able to access multi channel but can use and sense only one channel at a time. In this case, the studied access schemes are based on two different switching schemes. The first switching scheme is the switch and examine scheme (SEC), where, once the PU appears, the SU switch sequentially to the next channel and keep switching until he find an unused channel.
The second switching scheme is the switch and stay scheme (SSC), in which the SU, when the PU is ON in a channel, will switch to the next channel and transmit if it is free or wait until it will be free. In this work, two important metrics for the SU are proposed, which are the average waiting duration and the average service time, and based on these two metrics we propose a performance measure metric denoted by time average SU throughput. With these novel performance metrics and their accurate mathematical characterization, we can predict the types of application the secondary system can support based on different PU traffic pattern.

**IV.2 System Model**

In this work, we assume that we have a PU system that have \( L \) parallel channels available for transmission. A cognitive secondary system, constituted by one transmitter and one receiver, will try to access one of the available channels opportunistically, as shown in Fig. IV.1. We assume that the occupancy of each channel by the PU system evolves independently according to a homogeneous continuous-time Markov chain with idle (OFF) and busy (ON) states, with the notion that the PU traffic is not slotted.

We denote the duration of the ON and OFF period of the PU by \( T_{p_{on}} \) and \( T_{p_{off}} \), respectively. Due to the Markovian assumption, the holding times, \( T_{p_{on}} \) and \( T_{p_{off}} \), are exponentially distributed with parameters \( \lambda \) (average ON duration) for the OFF period and \( \mu \) (average OFF duration) for the ON period, respectively.

The SU transmitter, is assumed to always have data to send, to have unslotted traffic protocol and to have the ability to sense and access only one channel at a time. Also, we assume that, when the SU is using an idle channel, the SU will sense continuously that channel and stop accessing immediately when the PU begins transmitting. Thus, the proposed OSA scheme will cause nearby zero interference to
IV.3 Performance Analysis

In this section, we study two important performance metrics for the SU which are the average SU transmission time and the average SU waiting time. These two metrics predict according to the PU activity, how much time the SU needs to wait in average to transmit and for how much time in average he can transmit. Thus, using these two metrics we can investigate when having the PU traffic statistics, what type of
applications the SU system can support.

IV.3.1 One Channel Access

In this subsection, we assume that the SU have access to only one channel. When the PU is sensed to be ON, the SU will periodically sense the channel every period of $T_s$. Once the SU find out that the PU is OFF, he starts transmitting while sensing the PU activity continuously until the PU is ON.

Average SU Service Time

Case of Poisson Traffic  We denote by $T_{on}^s$ and $T_{off}^s$ the duration of the ON and OFF periods the SU, respectively, as shown in the block diagram in Fig. IV.2. Assuming a Poisson traffic for the PU, the duration of the ON ($T_{on}^p$) and OFF ($T_{off}^p$) period are exponentially distributed and their probability density functions (PDFs) are given by

$$f_{T_{on}^p}(t) = \frac{1}{\lambda} e^{-\frac{t}{\lambda}} U(t), \quad f_{T_{off}^p}(t) = \frac{1}{\mu} e^{-\frac{t}{\mu}} U(t), \quad (IV.1)$$

respectively, where $\lambda$ and $\mu$ represent the average duration of the ON period and the average duration of the OFF period, respectively, and $U(.)$ is the unit step function.

During the ON period of the PU, the SU performs a periodic sensing with period $T_s$, as shown in Fig. IV.2, the OFF duration of the SU can be expressed as $T_{off}^s = N T_s$, where $N$ is the number of the sensing periods before the SU switch to ON state. Note that $N$ is a random variable that depends on the ON duration of the PU.

It can be seen from Fig. IV.2 that $T_{on}^p + T_{off}^p = T_{on}^s + T_{off}^s$. Thus, the ON duration
of the SU can then be expressed as

\[ T_{on}^s = T_{on}^p + T_{off}^p - T_{off}^s \]

\[ = T_{on}^p + T_{off}^p - NT_s \]

\[ = T_{off}^p - \tau, \tag{IV.2} \]

where \( \tau = NT_s - T_{on}^p \) represents the time duration when the PU is OFF and the SU is not transmitting, \( \tau \in [0, T_s] \).

To get the average SU service duration, denoted by \( \bar{T}_{on}^s \), we first need to get the statistics of \( T_{on}^s \). Since \( T_{on}^s \) depend on the random variables \( T_{off}^p \) and \( \tau \) and \( T_{off}^p \) is exponential distributed with parameter \( \mu \), we just need to get the statistics of \( \tau \).

Conditioning on \( N \), \( T_{on}^p \mid N \) is between \((N - 1)T_s\) and \( NT_s \). Thus, knowing that \( T_{on}^p \) is exponentially distributed, \( T_{on}^p \mid N \) is a truncated exponential random variable.
and its PDF is given by

\[ f_{T_p|N}(t) = \frac{1}{\lambda} e^{-\left(\frac{(N-1)T_s}{\lambda}\right)} - e^{-\frac{NT_s}{\lambda}}, \quad (N - 1)T_s \leq t \leq NT_s. \]  

(IV.3)

The PDF of \( \tau \) conditioned on \( N \) can be determined as

\[ f_{\tau|N}(t) = f_{T_p|N}(NT_s - t) \]

\[ = \frac{1}{\lambda} \frac{e^{\frac{t-T_s}{\lambda}}}{1 - e^{-\frac{T_s}{\lambda}}}, \quad 0 \leq t \leq T_s. \]

(IV.4)

It can be seen from the previous PDF that \( f_{\tau|N}(.) \) does not depend on \( N \), thus the PDF of \( \tau \) is the same as the conditioned PDF on \( N \), i.e., \( f_\tau(t) = f_{\tau|N}(t) \). This can be explained by the fact that the Poisson model for the PU activity is memoryless.

Having the PDF of \( \tau \) and \( T_{off}^p \), we can easily get the average SU transmission time as

\[ \bar{T}_{on} = E[T_{on}] \]

\[ = E[T_{off}^p] - E[\tau] \]

\[ = \mu + \lambda - \frac{T_s}{1 - e^{-\frac{T_s}{\lambda}}}. \]

(IV.5)

It can be easily seen that when \( T_s \rightarrow 0 \) which mean that the SU continuously senses the channel, \( \frac{T_s}{1 - e^{-\frac{T_s}{\lambda}}} \rightarrow \lambda \). So \( \bar{T}_{on} \) is approaching \( \mu \) as expected, because in this case the SU will begin transmitting exactly when the PU finish and will finish exactly when the PU begin transmitting.

**Case of General PU Traffic**  This result can be generalized to any PU activity model where the truncated PDF of \( T_{on}^p \), between \( (N - 1)T_s \) and \( NT_s \), is available. If we denote by \( F_{T_{on}^p}(.) \) the cumulative distribution function (CDF) of \( T_{on}^p \), the PDF of
$T_{on}$ conditioned on $N$ is given by

$$f_{T_{on} \mid N}(t) = \frac{f_{T_{on}}(t)}{F_{T_{on}}(NT_s) - F_{T_{on}}((N - 1)T_s)},$$  \hspace{1cm} (IV.6)

$NT_s \leq t \leq (N - 1)T_s$.

Using this previous PDF, we can express the PDF of $\tau$ knowing $N$ as

$$f_{\tau \mid N}(t) = \frac{f_{T_{on}}(NT_s - t)}{F_{T_{on}}(NT_s) - F_{T_{on}}((N - 1)T_s)}, \hspace{0.5cm} 0 \leq t \leq T_s.$$  \hspace{1cm} (IV.7)

If we denote by $P_n = \Pr \{N = n\}$, we can express the probability mass function (PMF) of $N$ as a function of the CDF of $T_{on}$ as

$$P_n = \Pr \{N = n\} = \Pr \{(n - 1)T_s \leq T_{on} \leq nT_s\} = F_{T_{on}}(nT_s) - F_{T_{on}}((n - 1)T_s)$$  \hspace{1cm} (IV.8)

Having $P_n$ and $f_{\tau \mid N}(\cdot)$, we can obtain $f_\tau(\cdot)$ as

$$f_\tau(t) = \sum_{n=1}^{+\infty} P_n f_{\tau \mid n}(t) = \sum_{n=1}^{+\infty} f_{T_{on}}(nT_s - t), \hspace{0.5cm} 0 \leq t \leq T_s.$$  \hspace{1cm} (IV.9)

In this case, the average SU service duration, $\bar{T}_{on}^s$, is given by

$$\bar{T}_{on}^s = \bar{T}_{off} - \bar{\tau} = \bar{T}_{off} - \sum_{n=1}^{+\infty} \int_0^{T_s} t \ f_{T_{on}}(nT_s - t) \ dt,$$  \hspace{1cm} (IV.10)

where $\bar{T}_{off}$ is the mean value of $T_{off}$, given by the statistics of the OFF period of the PU traffic.
Average Waiting Time

The average SU waiting time, denoted by $\bar{\delta}$, is the average time for which the SU will need to wait to get access to the channel. This metric in our case can be given by $\delta = NT_s$. Since we have the PMF of $N$ which is given in general case by (IV.8), we can calculate the average value of $\delta$ as

$$
\bar{\delta} = \sum_{n=1}^{+\infty} \delta P_n
= \sum_{n=1}^{+\infty} n T_s P_n.
$$

(IV.11)

In the case of Poisson PU traffic, $P_n = e^{-(n-1)\frac{T_s}{\lambda}} \left( 1 - e^{-\frac{T_s}{\lambda}} \right)$. Thus $\bar{\delta}$ is expressed as

$$
\bar{\delta} = \sum_{n=1}^{+\infty} n T_s e^{-(n-1)\frac{T_s}{\lambda}} \left( 1 - e^{-\frac{T_s}{\lambda}} \right)
= \frac{T_s}{1 - e^{-\frac{T_s}{\lambda}}}.
$$

(IV.12)

IV.3.2 Multi-Channel Access Based on SEC Scheme

In this section, we assume that the SU can access any one of all the available channels at a time. If the sensed channel is idle, the SU will transmit in this channel and sense the channel continuously until the PU is present. Once the PU is present in the channel, the SU will switch to the next channel. If the channel is available, the SU will use it otherwise the SU switches again. We will assume that when the SU switches to a new channel, the switching duration can keep neglected and sensing duration is $T_p$.

Average SU Service Time

We again assume Poisson traffic for the PU. When the SU find an available channel after switching, the availability duration of the channel access is exponential with the
same mean $\mu$. Once the SU switches and find an available channel, it will take $T_p$ to begin transmitting. Thus, if we denote by $X$ the availability duration of the channel, the service time can be given by

$$T_{on}^s = X - T_p, \quad (IV.13)$$

and the average SU transmission time is given by

$$\bar{T}_{on}^s = \mu - T_p. \quad (IV.14)$$

**Average Waiting Time**

**Case of a Large Number of Channels**  Given the channel and sensing time $T_p$, the waiting time is a multiple of $T_p$ and can be given by

$$\delta = N \ T_p, \quad (IV.15)$$

where $N$ is a random variable that represents the number of channels that the SU has examined before finding a free channel. The discrete random variable $N$ is a Bernoulli random variable with probability $p = \frac{\lambda}{\lambda + \mu}$. If there is a large number of channels, the switched-to channels will have independent statistics. Therefore, the PMF of $N$ is

$$\Pr \{ N = n \} = (1 - p) \ p^{n-1} \left( \frac{\lambda}{\lambda + \mu} \right)^{n-1} \left( \frac{\lambda}{\lambda + \mu} \right)^n. \quad (IV.16)$$
With the PMF of $N$, we can calculate the average value of $\delta$ as

$$
\bar{\delta} = \sum_{n=1}^{+\infty} \delta P_n
$$

$$
= \sum_{n=1}^{+\infty} n T_p P_n
$$

$$
= \frac{T_p}{\mu} (\lambda + \mu). \tag{IV.17}
$$

**Case of Small Number of Channel**  The results for the waiting time obtained in the previous paragraph are for the case when we have many channels so that the SU when switching sequentially can find an available channel before going back to the first channel examined.

A much more interesting case for the SEC scheme is when the number of channels is not large enough and the SU while switching over the channels can switch back to the first channel after checking all the available channels. In this case the waiting time will be different from the one obtained previously.

We take as an example the two channel case. In this case during the waiting time the SU will switch between the two channels until he finds a free channel. The difference here with the previous case is that when the user switch back to the previous channel the probability of non access $p$ is no longer equal to $\frac{\lambda}{\lambda+\mu}$ because we know
the state of the channel in the last $T_p$ period. It can be shown that PMF of $N$ is

$$
\Pr \{ N = 1 \} = \frac{\mu}{\lambda + \mu}
$$

$$
\Pr \{ N = 2 \} = \frac{\mu}{\lambda + \mu} \Pr \{ 0 < T_{on1}^p < T_p \} = \frac{\lambda}{\lambda + \mu} \left( 1 - e^{-\frac{2T_p}{\mu}} \right)
$$

$$
\Pr \{ N = 3 \} = \frac{\mu}{\lambda + \mu} \Pr \{ T_{on1}^p > T_p \} \Pr \{ T_p < T_{on2}^p < 3T_p \}
$$

$$
= \frac{\lambda}{\lambda + \mu} e^{-\frac{T_p}{\mu}} \left( e^{-\frac{T_p}{\mu}} - e^{-\frac{3T_p}{\mu}} \right)
$$

$$
\Pr \{ N = n \} = \frac{\lambda}{\lambda + \mu} e^{-\frac{(n-2)T_p}{\mu}} \left( e^{-\frac{(n-2)T_p}{\mu}} - e^{-\frac{3T_p}{\mu}} \right),
$$

where $T_{on1}^p$ and $T_{on2}^p$ are the ON duration of the PU in the first channel and second channel, respectively.

Using the PMF of $N$ above, we can obtain the average waiting time as

$$
\bar{\delta} = \sum_{n=1}^{+\infty} \delta P_n = \frac{T_p}{\mu + \lambda} \left( \mu + \lambda \frac{2 - e^{-\frac{2T_p}{\mu}}}{1 - e^{-\frac{2T_p}{\mu}}} \right). \tag{IV.19}
$$

### IV.3.3 Multi-Channel Access Based on SSC Scheme

In this section, we assume that the SU has access to all the available channels and sense the channels sequentially. If the sensed channel is idle, the SU transmits and senses the channel continuously until the PU is present. Once the PU is present in the channel, the SU switches to the next channel. If the next channel is available the SU transmits. Else he stays on that channel and wait until it is free. During the waiting time the SU senses periodically (every $T_s$) the PU activity. In this part, we denote by $T_p$ and $T_s$ the duration of sensing and the period of sensing, respectively. Note that the switching duration is assumed to be negligible.
**Average SU Transmission Time**

To calculate the average service time for this switching scheme, we need to consider two cases, which are i) the case when the SU switches to another channel and find it idle. We denote the service duration in this case by $T_1$. ii) the case when the switch to channel is found to be busy. We denote the service duration in this case by $T_2$. The average service time is in this cases given by

$$T_{on}^{s} = p_1 T_1 + p_2 T_2, \quad (IV.20)$$

where $p_1$ and $p_2$ are the probability to find the channel idle and busy, respectively, and are given by $p_1 = \frac{\mu}{\mu + \lambda}$ and $p_2 = \frac{\lambda}{\mu + \lambda}$, respectively. Thus the average service time is given by

$$\bar{T}_{on}^{s} = \frac{\mu}{\mu + \lambda} \bar{T}_1 + \frac{\lambda}{\mu + \lambda} \bar{T}_2. \quad (IV.21)$$

When the switch to channel is found to be idle, the SU waits for $T_p$ duration the time to switch and to sense the channel and then transmit. Thus in this case knowing that we found the switch to channel idle, the average service time will be exactly the same as of the case of SEC scheme, given by $\bar{T}_1 = \mu - T_p$.

When the switch to channel is found to be busy, the SU will stay in that channel and wait until the channel is free. During the waiting time the SU will sense the channel periodically every $T_s$. Thus the service time in this case is exactly equal to the one channel case. So the average service time knowing that the switch to channel is found to be busy is given by

$$\bar{T}_2 = \mu + \lambda - \frac{T_s}{1 - e^{-\frac{T_s}{\lambda}}}. \quad (IV.22)$$
Using all the previous results we can get $T_{\text{on}}^s$ and represented as

$$T_{\text{on}}^s = \lambda + \frac{\mu (\mu - T_p)}{\mu + \lambda} - \frac{1}{\mu + \lambda} \left( \frac{T_s}{1 - e^{-\frac{T_s}{\mu}}} \right).$$

(IV.23)

**Average Waiting Time**

For this switching scheme, the average waiting time is exactly equal to the waiting time of the case when we have only one channel access if we neglect the channel switching time. Thus the average waiting time is given by

$$\bar{\delta} = \frac{T_s}{1 - e^{-\frac{T_s}{\mu}}}. $$

(IV.24)

**IV.3.4 Time Average SU Throughput**

In this part, we propose a third performance metric namely time average SU throughput based on the two previously proposed metrics, average service time and average waiting time. The time average SU throughput for all the considered access scheme is given by

$$\Gamma = \frac{T_{\text{on}}^s}{\bar{\delta} + T_{\text{on}}^s} \log_2 (1 + \gamma),$$

(IV.25)

where $\gamma$ is the signal-to-noise ratio of the SU.

**IV.4 Numerical Results and Discussions**

In this section, we present some numerical results for the average service time and average waiting time to compare the different proposed SU access schemes.

Fig. [IV.3] shows the normalized average service time of the SU for the different studied access schemes as a function of the average OFF duration of the PU $\mu$. In this figure, we assume that $T_s = 0.2$, $T_p = 0.05$ and $\lambda = 1$. It is clear that as $\mu$ increases the service duration for the SU and for all the considered schemes increases and this
is because the OFF period of the PU will be longer. On the other hand, it can be seen that the switching based schemes have better service duration compared to the one channel access scheme. Moreover, if we compare the two studied switching schemes, we can see that, in terms of service duration, the SEC is better than the SSC.

![Graph](image)

Figure IV.3: Normalized average service time of the cognitive radio for the different studied access schemes.

Fig. IV.4 shows the normalized average waiting time of the SU for the different studied access schemes as a function of the average ON duration of the PU $\lambda$. In this figure, we assume that $T_s = 0.2$, $T_p = 0.05$ and $\lambda = 1$. It is clear that as $\lambda$ increases the SU waiting duration for all the considered schemes decreases and this is because the ON period of the PU will be longer. On the other hand, it can be seen that the switching based schemes present less waiting duration for the SU compared to the one channel access scheme. Moreover, if we compare the two studied switching schemes, we can see that, in terms of waiting duration, the SEC provides lower waiting time compared to the SSC scheme.

Fig. IV.5 shows the normalized average service duration for the switched based
access schemes for different values of $T_p$ as a function of the average OFF duration $\mu$ of the PU. In this figure, we assume that $T_s = 0.2$ and $\lambda = 1$. It is clear that as $T_p$ decreases the service time decreases for the two switching considered cases, as expected. This is because when the sensing duration decreases the SU can quickly make use of the available channel.

Fig. IV.6 shows the normalized average waiting duration for the switched based access schemes for different values of $T_p$ as a function of the average ON duration $\lambda$ of the PU. In this figure, we present the results for the average waiting time of the SEC for multi-channels and two-channels case. Also in this figure, we assume that $T_s = 0.2$ and $\mu = 1$. It is clear that as $T_p$ increases the waiting time increases for the different considered access schemes. This is because the waiting time for the SEC is directly related to $T_p$. Besides, this figure shows that the multi-channels switching SEC scheme provides a smaller waiting time than the two channel case. This can be explained by the fact that when switching over multi-channels it is more likely to find
Figure IV.5: Normalized average service duration for the switched based access schemes for different values of $T_p$.

a free channel.

IV.5 Concluding Remarks

In this chapter, we proposed two different based switching spectrum access schemes to improve the performance of the SU. The first scheme named SEC and consists of switching sequentially to the next channel when the previous channel is found to be busy until an idle channel is found. The second scheme named SSC and consists of staying in the same channel if it is busy and wait until it will be idle to transmit, once the PU appears again the SU switch to the next channel. For the considered opportunistic access schemes we propose and derive two SU performance metric named average waiting and service duration. Numerical results have shown that the SEC provides lower waiting time and better service duration compared to the SSC scheme.
Figure IV.6: Normalized average waiting duration for the switched based access schemes for different values of $T_p$. 
APPENDICES
Appendix A

Papers Submitted and Under Preparation

A.1 Conference Papers


A.2 Journal Papers


Bibliography/References


