

**Joint Subcarrier Pairing and Resource Allocation  
for Cognitive Network and Adaptive Relaying  
Strategy**

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# ABSTRACT

## Joint Subcarrier Pairing and Resource Allocation for Cognitive Network and Adaptive Relaying Strategy

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Recent measurements show that the spectrum is under-utilized by licensed users in wireless communication. Cognitive radio (CR) has been proposed as a suitable solution to manage the inefficient usage of the spectrum and increase coverage area of wireless networks. The concept is based on allowing a group of secondary users (SUs) to share the unused radio spectrum originally owned by the primary user (PUs). The operation of CR should not cause harmful interference to the PUs. In the other hand, relayed transmission increases the coverage and achievable capacity of communication systems and in particular in CR systems. In fact there are many types of cooperative communications, however the two main ones are decode-and-forward (DAF) and amplify-and-forward (AAF). Adaptive relaying scheme is a relaying technique by which the benefits of the amplifying or decode and forward techniques can be achieved by switching the forwarding technique according to the quality of the signal. In this dissertation, we investigate the power allocation for an adaptive relaying protocol (ARP) scheme in cognitive system by maximizing the end-to-end rate and searching the best carriers pairing distribution. The optimization problem is under

the interference and power budget constraints. The simulation results confirm the efficiency of the proposed adaptive relaying protocol in comparison to other relaying techniques, and the consequence of the choice of the pairing strategy.

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# LIST OF ABBREVIATIONS

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<b>Symbol</b>	<b>Meaning</b>
OFDM	Orthogonal Frequency Division Multiplexing
AAF	Amplify And Forward
DAF	Decode And Forward
ARP	Adaptive Relaying Protocol
PU	Primary User
SU	Secondary User
CR	Cognitive Radios
R	Relay
CS	Cognitive Source
D	Destination
SNR	Signal to Noise Ratio
PSD	Power Spectrum Density

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# LIST OF SYMBOLS

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Symbol	Meaning
$N$	number of subcarriers
$\Delta f$	subcarrier bandwidth
$\Phi_i, \Psi(e^{j\omega})$	power spectrum density
$P_{SR,RD,1,2}^i$	power allocated to subcarrier $i$
$T_s$	symbol duration
$I_i$	mutual interference introduced to the primary user
$d_i$	spectral distance
$G_i, Y_i, H_{SR,RD}^i$	channel gain
$\Omega_{1,2}^i$	interference factor
$\sigma^2$	noise variance
$x$	transmitted signal
$y$	received signal
$R$	rate or capacity
$\log_2(x)$	logarithm in base 2 of $x$
$\gamma_{th}$	SNR threshold
$\mathbb{E}[x]$	expected value of random variable $x$
$\alpha, t$	indicator coefficients
$I_{th}$	maximum interference tolerable by the primary user
$\mathcal{L}$	Lagrangian
$g$	dual function
$[x]^+$	$\max(0, x)$

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# Chapter I

## Introduction

The growth of the technology has affected directly modern communication systems. This expansion can be observed when a comparison is made between the earlier systems with some bits per second as a communication rate and the 300 Mbps already considered in the long term evolution (LTE) wireless communication systems. The growth of the data rate in wireless standards and services was accompanied by a rise in applications and costumers which imply an augmentation in the demand for the limited frequency spectrum. The natural spectrum resource may not be able to respond to the emerging and future technology demands.

In current systems, frequency allocation, type of service, maximum transmission powers, and duration of license are managed by governmental agencies, which apply the “command-and-control” allocation model by assigning a fixed frequency block for each communication service. This scheme is statistic and inflexible in spectrum management which leads, as shown by practical measurements, to inefficient use of the provided spectrum because the licensed users are not necessarily using the allocated portion of spectrum at all times or locations, and in the same time prevent other users from accessing the unused spectrum.

Cognitive Radio (CR) can manage the spectrum utilization by detecting the spec-

trum holes and avoiding the occupied spectrum by using the available part of spectrum. In fact, the spectrum utilization can be improved by allowing the Secondary Users (SUs) to use the vacant channels left by the licensed users (PUs) [1]. Such systems have to distribute their limited resources among the SUs in order to maximize the capacity without causing harmful interference to the PUs (see e.g. [2], [3]). Since Orthogonal Frequency Division Multiplexing (OFDM) is widely used in various wireless system, and shows a high spectral efficiency and flexibility, it is often recommended for cognitive radio systems [4].

To increase coverage and achievable capacity of the communication system, relays (R) are used to transfer the information from the cognitive source (CS) to the destination (D) when the direct link is not available [5](in some cases, even if there exists a direct link, the relays are used to improve the performance of the communication systems). The resource allocation problem for the non-cognitive OFDM based relay system has been widely studied [6], [7]. In [8], a cooperative scheme, with decode and forward technique, is combined with the cognitive radio to produce a communication system with high performance and more coverage area. Note that even in cooperative communication, both transmits, namely source and relay, have to be aware about the interference threshold tolerated by the PU.

In cooperative communication systems, the most known relaying techniques are Amplify-And-Forward (AAF) and Decode-And-Forward (DAF). In the AAF case, the relay amplifies the received signal from the source (S) by some factor to be determined, then forward it to D. However, the relay working with the DAF strategy decodes “perfectly” the received signal from S and then encodes it again (with the same code known by S and D) and forwards it to D. Note that these procedures are done at each subcarrier. The disadvantages of these two techniques of relaying come with the fact that the AAF relay can amplify the noise coming form the (S-R) link, which degrades the signal quality. On the other hand, the DAF relay causes a propagation of error

in case of uncorrect decoding of the information symbols.

The adaptive relaying or Adaptive Relaying Protocol (ARP), as named in [6], is one of the proposed solutions that benefits from the advantages of DAF and AAF and aims to minimize the disadvantages of them. In [7], the relay can execute AAF and DAF, and there is a technique based on the Signal-to-Noise-Ratio (SNR) which trigger the switching between the AAF and DAF strategies. It assumes that at high SNR (for an SNR above some SNR threshold), the relay can decode perfectly so it works with DAF, and for low SNR (below the threshold), when it is harder to decode correctly it works with AAF to avoid propagation error.

In this dissertation, we integrate this technique of adaptive relaying in a CR based environment and we look at the power allocation at the relaying and the distribution of the sub-carriers to the relay. The problem now is how to allocate optimally the power at the transmitters (S and R) to reach high capacity using ARP and without causing harmful interference to the primary user from the cognitive transmitters. We propose a solution to this problem, an algorithm based on the dual problem and sub-gradient method [9–11]. For simplicity, we begin by selecting the subcarrier and assumed that the relay uses the same sub-carrier for receiving (from S) and for transmission (to D), besides we used other type of pairing selection, like the random selection and optimal selection carrier from S-R to R-D and compared the performances of different schemes.

The reminder of the thesis is organized as follows. In chapter II, we present the system model and the matching subcarrier problem with a proposed algorithm and illustrated by some numerical results to compare the performance of the different type of relaying (AAF, DAF and ARP). In chapter III, we investigate the pairing problem by including the pairing parameters to the optimization problem studied in chapter II. More specifically, the same algorithm is used with some modification to find the best subcarrier distribution and we end the chapter with some simulation results showing



the difference between the pairing techniques used in this work. Finally, we conclude this thesis in chapter IV, where we summarize the main results and briefly discuss potential extensions of the work carried-out in the framework of this thesis.

# Chapter II

## Near Optimal Algorithm for Matching Pairing Scheme

In this chapter, we focus on the simple case in which the power allocation of a cognitive system with one relay system using a matching pairing strategy is adopted. In particular, we assume that the relay forwards the signal over the same received subcarrier. This simple case will show us the difference in performance between the three types of cooperative schemes as presented in Chapter I.

### II.1 System Model

In this work, an OFDM-based relay CR system is considered. The CR relay system coexists with the primary system in the same geographical location. We assume that there is no direct link between the cognitive source (CS) and the destination D, so that S tries to communicate with D through the relay (see Fig.II.1). The frequency spectrum of the CR system is divided into  $N$  subcarriers each having a  $\Delta f$  bandwidth. We assume that the CR system can transmit through the unused PU band without exceeding the maximum interference power  $I_{th}$ , that can be tolerated by PU. The

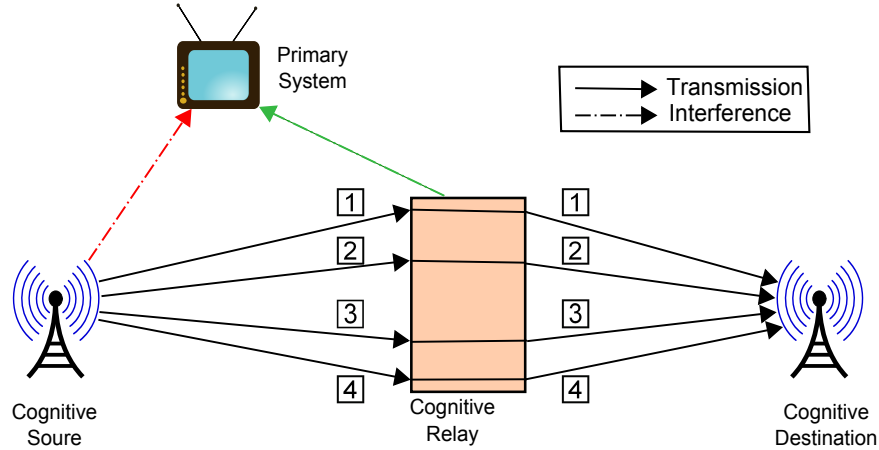


Figure II.1: Illustration of a cooperative relay cognitive radio network with  $N=4$ .

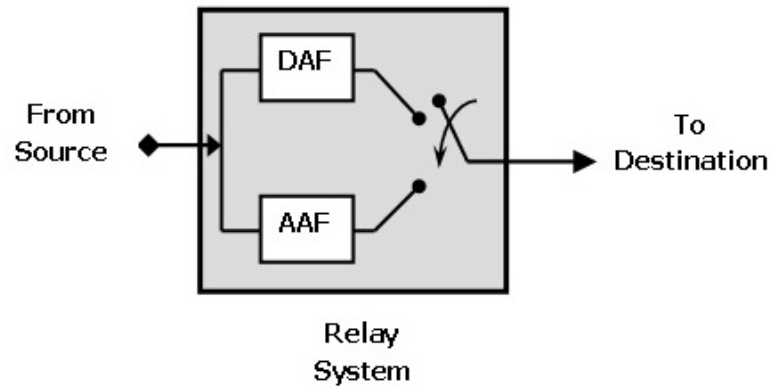


Figure II.2: Block diagram for the structure of an adaptive relay.

relay is assumed to be half-duplex, so receiving and forwarding at two different time slots. In the first time slot,  $S$  transmits to the Relay ( $R$ ), while in the second time slot,  $R$  forwards the signal to  $D$  with the ARP technique. It has been assumed that we have one relay which can work with different channels. Note that the relay can forward the data using two techniques (DAF and AAF) by switching between them as presented in Fig.II.2. The calculation of the mutual interference between PU, SU and the relay is presented in the next section.

## II.2 Interference Analysis

The mutual interference introduced to PU by the  $i^{th}$  subcarrier in OFDM systems is presented in [12]. Assume that  $\Phi_i$  is the power spectrum density (PSD) of the  $i^{th}$  subcarrier. The form of the PSD depends directly on the multicarrier wave form technique. In our case, when an OFDM based is used, the PSD at the  $i^{th}$  subcarrier band can be written as

$$\Phi_i(f) = P_i T_s \left( \frac{\sin \pi f T_s}{\pi f T_s} \right)^2, \quad (\text{II.1})$$

where  $P_i$  is the total transmit power emitted by the  $i^{th}$  subcarrier, and  $T_s$  is the symbol duration. Hence, the mutual interference introduced by the  $i^{th}$  subcarrier to PU,  $I_i(d_i, P_i)$ , can be found by integrating the PSD of the  $i^{th}$  subcarrier over the PU band,  $B$ , and can be obtained using the following expression [8]

$$I_i(d_i, P_i) = \int_{d_i - B/2}^{d_i + B/2} G_i \Phi_i(f) df \triangleq P_i \Omega^i, \quad (\text{II.2})$$

where  $d_i$  and  $G_i$  denote the spectral distance and the channel gain respectively, between the  $i^{th}$  subcarrier and the PU band, while  $\Omega^i$  is the interference factor of the  $i^{th}$  subcarrier to the PU band. Note that (II.2) express the interference in terms of the total transmit power  $P_i$  of the  $i^{th}$  subcarrier linearly, which will be used to solve the optimization problem in the next sections.

By the same analysis, the interference power introduced by PU signal into the band of the  $i^{th}$  subcarrier is expressed as [8]

$$J_i = \int_{d_i - \Delta f/2}^{d_i + \Delta f/2} Y_i \Psi(e^{j\omega}) d\omega, \quad (\text{II.3})$$

where  $\Psi(e^{j\omega})$  is the PSD of PU signal and  $Y_i$  is the channel gain between the  $i^{th}$

subcarrier and the PU signal. By completing the interference analysis of the different agent of the cognitive system, we can formulate the optimization problem before proceeding to the solution.

## II.3 Capacity Analysis and Problem Formulation

Let us first define the variables of the problem. Let  $(P_{SR}^i; P_{RD}^i)$  be the power transmitted over the  $i^{th}$  subcarrier in the (S-R;R-D) link. The  $i^{th}$  subcarrier channel gain over the (S-R;R-D) link is given by  $(H_{SR}^i; H_{RD}^i)$ . Finally, the noise variance is assigned by  $\sigma_i^2 = \sigma_{AWGN}^2 + J_i$ , where  $\sigma_{AWGN}^2$  is the variance of the additive white Gaussian noise (AWGN) and  $J_i$  is the interference introduced by the PU signal into the  $i^{th}$  subcarrier which is evaluated using (II.3). This interference can be modeled as an AWGN as described in [2]. To make the analysis more clear, the noise variance  $\sigma^2$  is assumed to be the same for all subcarriers and both time slots.

### II.3.1 Processing During the First Time Slot

Let  $x_{s,i}$  be the transmitted signal from S over the  $i^{th}$  channel. The received signal at the relay R over the  $i^{th}$  subcarrier in the first time slot is given by

$$y_{SR}^i = \sqrt{H_{SR}^i P_{SR}^i} x_{s,i} + n_{SR}^i, \quad (\text{II.4})$$

where  $n_{SR}^i$  is the noise between S and R with a variance  $\sigma_{SR,i}^2 = \sigma^2$ , and  $i = \{1, 2, \dots, N\}$  denote the  $i^{th}$  subcarrier.

According to the Shannon capacity formula, the transmission rate of the  $i^{th}$  subcarrier between the source and the relay  $R_{1,i}$  can be calculated as

$$R_{1,i} = \frac{1}{2} \log_2 \left( 1 + \frac{P_{SR}^i H_{SR}^i}{\sigma^2} \right). \quad (\text{II.5})$$

As it has been mentioned above, we should limit the interference caused by the CS to the PU, which give us the following interference constraint [8]

$$\sum_{i=1}^N P_{SR}^i \Omega_{SP}^i \leq I_{th}, \quad (\text{II.6})$$

where  $\Omega_{SP}^i$  denotes the interference factor of the  $i^{th}$  subcarrier to the PU band.

### II.3.2 Capacity in the Second Time Slot

In the second time slot, the relay decodes and re-encodes or amplifies the signal over the  $i^{th}$  channel, depending on the received SNR, then forwards it to the destination.

This means that the transmit signal from the relay over the  $i^{th}$  channel is

$$x_{RD}^i = \begin{cases} \sqrt{P_{RD}^i} \cdot x_s & \text{for } P_{SR}^i \gamma_{SR}^i \geq \gamma_{th} \\ \beta_i \sqrt{P_{RD}^i} \cdot y_{SR} & \text{for } P_{SR}^i \gamma_{SR}^i < \gamma_{th}, \end{cases} \quad (\text{II.7})$$

where  $P_{SR}^i \gamma_{SR}^i = \frac{P_{SR}^i H_{SR}^i}{\sigma^2}$  is the received SNR via the source-relay link, and  $\gamma_{th}$  is the threshold SNR to ensure successful decoding. We assume that we have successful decoding when  $P_{SR}^i \gamma_{SR}^i$  is above  $\gamma_{th}$ . In (II.7),  $\beta_i$  is an amplification factor used by the relay to amplify the signal using the AAF mode. The choice of  $\beta_i$  should assure the normalization of the total transmit power to the same value with all the AAF channels. It is defined in [13, Eq. (9)] as

$$\beta[i] = \frac{1}{\sigma \sqrt{P_{SR}^i \gamma_{SR}^i + 1}}. \quad (\text{II.8})$$

At the destination the received signal over the  $i^{th}$  channel can be written as

$$y_{RD}^i = \sqrt{H_{RD}^i} x_{RD}^i + n_{RD}^i. \quad (\text{II.9})$$

Let us define two variables:  $\gamma_{SR}^i = \frac{H_{SR}^i}{\sigma^2}$  and  $\gamma_{RD}^i = \frac{H_{RD}^i}{\sigma^2}$ . Using (II.7) and (II.9), we derive the expression of the total SNR delivered via the  $i^{th}$  channel as

$$\gamma_{ARP}^i = \begin{cases} P_{RD}^i \gamma_{RD}^i & \text{if } P_{SR}^i \gamma_{SR}^i \geq \gamma_{th} \\ \gamma_{AF}^i & \text{otherwise,} \end{cases} \quad (\text{II.10})$$

where  $\gamma_{AF}^i$  is the SNR for the set of channels that work on amplify-and-forward, and it is given by

$$\gamma_{AF}^i = \frac{\mathbb{E}[y_{RD,AAF}^i]^2}{\mathbb{E}[(y_{RD,AAF}^i)^2] - \mathbb{E}[y_{RD,AAF}^i]^2} = \frac{P_{SR}^i \gamma_{SR}^i P_{RD}^i \gamma_{RD}^i}{P_{RD}^i \gamma_{RD}^i + P_{SR}^i \gamma_{SR}^i + 1}, \quad (\text{II.11})$$

where  $\mathbb{E}[y]$  denotes the expected value of the random variable  $y$ .

Back to the Shannon capacity formula, we calculate the rate of the channel in the second time slot for the two cases as

$$\begin{cases} R_{2,DAF,i} = \frac{1}{2} \log_2 \left( 1 + \frac{P_{RD}^i H_{RD}^i}{\sigma^2} \right) & \text{for the DAF case} \\ R_{2,AAF,i} = \frac{1}{2} \log_2 \left( 1 + \frac{P_{RD}^i \gamma_{RD}^i P_{SR}^i \gamma_{SR}^i}{P_{RD}^i \gamma_{RD}^i + P_{SR}^i \gamma_{SR}^i + 1} \right) & \text{for the AAF case.} \end{cases} \quad (\text{II.12})$$

Note that  $R_{2,AAF,i}$  is not jointly concave in  $P_{RD}^i$  and  $P_{SR}^i$ . To make the analysis simpler, we adopt the following approximation

$$R_{2,AAF,i} \approx \frac{1}{2} \log_2 \left( 1 + \frac{P_{RD}^i \gamma_{RD}^i P_{SR}^i \gamma_{SR}^i}{P_{RD}^i \gamma_{RD}^i + P_{SR}^i \gamma_{SR}^i} \right). \quad (\text{II.13})$$

This approximation is used in [14] and is based on the assumption that the system has a high SNR for the amplified signal between the relay and the destination. It is proved in [15] that this approximation is also accurate even in the moderate-low SNR regime.

To make things more clear, a new binary variable  $\alpha_i$  is defined in a way that it

takes the values “0” or “1” to indicate if the relay uses the DAF case ( $\alpha_i = 0$ ) or the AAF case ( $\alpha_i = 1$ ). We denote also by  $\mathcal{A}$  the set of index of channels that work on AAF, and  $\mathcal{D}$  as the set of index of channels that work on DAF

$$\mathcal{A} = \{i, \alpha_i = 1\}; \mathcal{D} = \{i, \alpha_i = 0\}. \quad (\text{II.14})$$

As in the DAF case, we compute the interference caused by the relay to the PU for the AAF case. Using the interference analysis done above and the expression given in [16, Eq. (17-18)], we get the following interference constraint in the  $2^{nd}$  time slot ( $R \rightarrow D$ )

$$\sum_{i \in \mathcal{A}} P_{RD}^i \Omega_{RP}^i + \sum_{i \in \mathcal{D}} P_{RD}^i \Omega_{RP}^i \leq I_{th}. \quad (\text{II.15})$$

### II.3.3 Total Capacity

It is clear that the capacity has different expression for both time slots and forwarding techniques. Therefore we need to find a unified expression that will be used as the objective function of the upcoming optimization problem. In fact, the achievable rate in each subcarrier is the minimum rate between both time slots. Thus the transmission rate is given by

$$R_i = \alpha_i \min \{R_{1,i}, R_{2,DAF,i}\} + (1 - \alpha_i) \min \{R_{1,i}, R_{2,AAF,i}\}. \quad (\text{II.16})$$

The maximum capacity is achievable when the rate in the first time slot is equal to the rate in the second time slot for every subcarrier if it is possible. Thus from (II.16), we should have the following capacity relation for the  $i^{th}$  subcarrier to achieve maximum rate

$$R_{1,i} = \begin{cases} R_{2,DAF,i} & \text{for } i \in \mathcal{D} \\ R_{2,AAF,i} & \text{for } i \in \mathcal{A}. \end{cases} \quad (\text{II.17})$$



Hence, we get two cases. For the DAF case, the equality is achievable by assembling (II.17), (II.5), and (II.12) to derive the following relation between the transmission powers

$$P_{RD}^i = \frac{P_{SR}^i H_{SR}^i}{H_{RD}^i} \quad \text{for } i \in \mathcal{D}. \quad (\text{II.18})$$

However, if we look to the formula of the rate in the AAF case (II.12), we can see that the rate in the second time slot is always less than the rate in the first time slot and cannot reach it. In fact, we have

$$\frac{P_{RD}^i \gamma_{RD}^i P_{SR}^i \gamma_{SR}^i}{P_{RD}^i \gamma_{RD}^i + P_{SR}^i \gamma_{SR}^i} < \frac{P_{SR}^i H_{SR}^i}{\sigma^2} \quad \text{for } i \in \mathcal{A}. \quad (\text{II.19})$$

This means that the achievable rate of these channels is equal to the rate in the second time slot. According to these derivations, we find the total expression of the rate in our model as

$$R = \sum_{i \in \mathcal{A}} \frac{1}{2} \log_2 \left( 1 + \frac{P_{RD}^i \gamma_{RD}^i P_{SR}^i \gamma_{SR}^i}{P_{RD}^i \gamma_{RD}^i + P_{SR}^i \gamma_{SR}^i} \right) + \sum_{i \in \mathcal{D}} \frac{1}{2} \log_2 (1 + P_{SR}^i \gamma_{SR}^i). \quad (\text{II.20})$$

### II.3.4 Optimization Problem of the Subcarrier Matching Technique

Our objective is to maximize the total capacity of the CR system while the interference introduced to the primary user is below the tolerated threshold. Therefore, the

optimization problem can be formulated as follows

$$\begin{aligned}
& \max_{P_{SR}^i, P_{RD}^i} \sum_{i=1}^N R_i \\
& \text{Subject to} \\
& \text{- (Interference at first time slot)} \\
& \sum_{i=1}^N P_{SR}^i \Omega_{SP}^i \leq I_{th} \tag{II.21} \\
& \text{- (Interference at second time slot)} \\
& \sum_{i \in \mathcal{A}} P_{RD}^i \Omega_{RP}^i + \sum_{i \in \mathcal{D}} P_{RD}^i \Omega_{RP}^i \leq I_{th} \\
& P_{SR}^i \geq 0; \quad P_{RD}^i \geq 0.
\end{aligned}$$

In this problem, the power constraints at each transmitter (S and R) are missing. However, when we take a look at the interference constraints, we note that the power constraint is defined indirectly. Moreover, we use the identity  $\Omega_{SP}^i \geq \min_j \Omega_{SP}^j$  to ensure the following inequality  $\sum_{i=1}^N P_{SR}^i \leq \frac{I_{th}}{\min_j \Omega_{SP}^j}$ . Thus the interference constraint implies, indirectly, the power constraint in the two time slots. This analysis can help us in this chapter because the problem is relatively simple and it is not a mixed integer programming problem so the power constraint can be omitted. However, in the next chapter, the problem is more complex and we should define the power constraints at the transmitters to avoid divergency of the algorithm.

We assume that all fading gains are perfectly known. The channel gains between the CR system parts (S, R and D) can be obtained by channel estimation techniques, the channel gains between the CR system and the PU can be obtained by estimating the received signal power from the primary terminal when it transmits [17].

At the end of this section and by assembling the previous equations and relations,

we can re-write the optimization problem given in (II.21) as

$$\begin{aligned}
& \max_{P_{SR}^i, P_{RD}^i} \frac{1}{2} \sum_{i \in \mathcal{A}} \log_2 \left( 1 + \frac{P_{RD}^i \gamma_{RD}^i P_{SR}^i \gamma_{SR}^i}{P_{RD}^i \gamma_{RD}^i + P_{SR}^i \gamma_{SR}^i} \right) + \frac{1}{2} \sum_{i \in \mathcal{D}} \log_2 (1 + P_{SR}^i \gamma_{SR}^i) \\
& \quad \text{s.t.} \\
& \quad \sum_{i=1}^N P_{SR}^i \Omega_{SP}^{*i} \leq I_{th}; \\
& \quad \sum_{i \in \mathcal{A}} P_{RD}^i \Omega_{RP}^i + \sum_{i \in \mathcal{D}} \frac{P_{SR}^i H_{SR}^i}{H_{RD}^i} \Omega_{RP}^i \leq I_{th}; \\
& \quad P_{SR}^i \geq 0; \\
& \quad P_{RD}^i \geq 0.
\end{aligned} \tag{II.22}$$

Under the previous assumption of perfect knowledge of the channel coefficient and the noise variance, the problem is a convex optimization problem with the parameter  $P_{RD}^i$  and  $P_{SR}^i$ . In the next section, we solve this problem using the Lagrangian method and the Karush Kuhn Tucker (KKT) conditions. Moreover, using the fact that the problem is convex, the dual solution and the primal solution are the same, so the problem can be solved using the dual formulation.

## II.4 Problem Solution

For simplicity reasons, and for making the mathematical notation easy to follow, we denote the following,  $P_{SR}^i$  by  $P_1^i$ ,  $P_{RD}^i$  by  $P_2^i$ ,  $\gamma_{SR}^i$  by  $\gamma_1^i$ ,  $\gamma_{RD}^i$  by  $\gamma_2^i$ ,  $\Omega_{SP}^i$  by  $\Omega_1^i$ , and  $\Omega_{RP}^i$  by  $\Omega_2^i$ .

### II.4.1 Dual Problem

The Lagrangian function with Lagrangian multipliers  $\lambda$ ,  $\mu$  can be written as

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \sum_{i=1}^N \left[ \alpha_i \log_2 \left( 1 + \frac{P_1^i \gamma_1^i P_2^i \gamma_2^i}{P_1^i \gamma_1^i + P_2^i \gamma_2^i} \right) + (1 - \alpha_i) \log_2 (1 + P_1^i \gamma_1^i) \right] \\ & + \mu \left( I_{th} - \sum_{i=1}^N P_1^i \Omega_1^i \right) + \lambda \left( I_{th} - \sum_{i=1}^N \alpha_i P_2^i \Omega_2^i + (1 - \alpha_i) \frac{P_1^i H_{SR}^i}{H_{RD}^i} \Omega_2^i \right). \end{aligned} \quad (\text{II.23})$$

Note that we substitute  $\mathcal{A}$  and  $\mathcal{D}$  by their definition and we include  $\alpha_i$  in the Lagrangian to simplify the computation.

We develop the Lagrangian to get the following expression

$$\begin{aligned} \mathcal{L} = & \sum_{i=1}^N \alpha_i \left[ \frac{1}{2} \log_2 \left( 1 + \frac{P_1^i \gamma_1^i P_2^i \gamma_2^i}{P_1^i \gamma_1^i + P_2^i \gamma_2^i} \right) - \lambda P_2^i \Omega_2^i - \mu P_1^i \Omega_1^i \right] \\ & + (1 - \alpha_i) \left[ \frac{1}{2} \log_2 (1 + P_1^i \gamma_1^i) - \lambda \frac{P_1^i H_{SR}^i}{H_{RD}^i} \Omega_2^i - \mu P_1^i \Omega_1^i \right] + (\lambda + \mu) I_{th}. \end{aligned} \quad (\text{II.24})$$

Now we solve this problem by using the dual approach. But first, let us first define the dual problem and the dual function as

$$\min_{\mu \geq 0, \lambda \geq 0} g(\mu, \lambda) \quad (\text{II.25})$$

where

$$g(\mu, \lambda) \triangleq \max_{P_1^i \geq 0, P_2^i \geq 0, \alpha_i} \mathcal{L}. \quad (\text{II.26})$$

From (II.24), and for a given set of  $\alpha_i$ , the problem can be divided into  $N$  independent problems. Thus we divide the dual function (the Lagrangian) into  $N$  dual function (Lagrangian), such that  $g_i(\mathcal{L}_i)$ . For each subcarrier  $i$ , for given  $\lambda$  and  $\mu$ , and according to the value of  $\alpha_i$  (which can take two values, 0 or 1), it appears that there are two cases:

- Case  $\alpha_i = 1$

In this case the relay is working on AAF for the  $i^{th}$  subcarrier, we will have in (II.23) only the terms related to the AAF approach. Hence the dual function can be simplified as follows

$$\begin{aligned} g_i(\mu, \lambda) &= \max_{P_1^i, P_2^i \geq 0} \mathcal{L}_i \\ &= \max_{P_1^i, P_2^i \geq 0} \frac{1}{2} \log_2 \left( 1 + \frac{P_1^i \gamma_1^i P_2^i \gamma_2^i}{P_1^i \gamma_1^i + P_2^i \gamma_2^i} \right) - \lambda P_2^i \Omega_2^i - \mu P_1^i \Omega_1^i. \end{aligned}$$

The maximum of  $\mathcal{L}_i$  can be found by searching the partial derivative of  $\mathcal{L}_i$  subject to  $P_1^i$  and  $P_2^i$  which leads to

$$\frac{\partial \mathcal{L}_1}{\partial P_1^i} = \frac{(P_2^i)^2 (\gamma_2^i)^2 \gamma_1^i}{(P_2^i \gamma_2^i + P_1^i \gamma_1^i)(P_2^i \gamma_2^i + P_1^i \gamma_1^i + P_2^i \gamma_2^i P_1^i \gamma_1^i)} - \mu \Omega_1^i \quad (\text{II.27})$$

$$\frac{\partial \mathcal{L}_1}{\partial P_2^i} = \frac{(P_1^i)^2 (\gamma_1^i)^2 \gamma_2^i}{(P_2^i \gamma_2^i + P_1^i \gamma_1^i)(P_2^i \gamma_2^i + P_1^i \gamma_1^i + P_2^i \gamma_2^i P_1^i \gamma_1^i)} - \lambda \Omega_2^i. \quad (\text{II.28})$$

We then equal both (II.27) and (II.28) to zero. The solution of these equations leads to  $P_1^{i*} = c_i P_2^{i*}$ , where  $c_i = \sqrt{\frac{\gamma_2^i \lambda \Omega_2^i}{\gamma_1^i \mu \Omega_1^i}}$ . Thus the new value of  $P_2^i$  is

$$P_2^{i*} = \left[ \frac{\gamma_2^i}{\mu c_i \Omega_1^i (\gamma_2^i + c_i \gamma_1^i)} - \frac{1}{c_i \gamma_1^i} - \frac{1}{\gamma_2^i} \right]^+, \quad (\text{II.29})$$

where  $[x]^+ = \max(0, x)$ .

- Case  $\alpha_i = 0$

For this case, the relay switch to the DAF technique at the  $i^{th}$  subcarrier having  $i \in \mathcal{D}$ . The problem of the DAF relaying has been solved in [8]. We just have to know the value of  $P_1^{i*}$  which can be obtain from the following relation  $P_2^{i*} = \frac{P_1^{i*} H_{SR}^i}{H_{RD}^i}$ . The solution is found to be given, in this case, by the following

expression

$$P_1^{i*} = \left[ \frac{1}{\mu\Omega_1^i + \lambda \frac{H_{SR}^i}{H_{RD}^i} \Omega_2^i} - \frac{1}{\gamma_1^i} \right]^+. \quad (\text{II.30})$$

By obtaining the optimal values of the transmitted powers  $P_1^{i*}$  and  $P_2^{i*}$ , the dual function is now a function of  $\mu$  and  $\lambda$ . In the next section, we use an algorithm named subgradient algorithm [9] that proceeds to the search of the optimum values of  $\mu$  and  $\lambda$  iteratively.

## II.4.2 Sub-gradient Method to Solve the Dual Problem

With the obtained optimal values of primal variables ( $P_1^{i*}$ ,  $P_2^{i*}$ ), the dual problem can be solved using the sub-gradient method [9], [10], [11]. In fact our algorithm is based on the calculation of the Lagrangian multipliers  $\lambda$  and  $\mu$  in each iteration. The decision about the type of relaying mode over each subcarrier is made using (II.10). The implementation procedures is described in Algorithm II.1.

---

### Algorithm II.1 Power Allocation Algorithm

---

- 1: **Initialize**  $\lambda = \lambda_0$  and  $\mu = \mu_0$
  - 2: **for**  $k = 1$  to  $Iter_{max}$  **do**
  - 3:   Compute  $c_i$ ,  $P_2^i$  and  $P_1^i$  using (II.29),  $\forall i$
  - 4:   Set  $\alpha_i$  by the decision rule presented in (II.10)
  - 5:   Compute  $P_1^j$  and  $P_2^j$  using (II.30),  $\forall j$
  - 6:   Set  $\alpha_j$  by the decision rule presented in (II.10)
  - 7:   **if**  $\alpha_i = \alpha_j = 1$  **then**
  - 8:     Choose  $P_1$  and  $P_2$  according to Step-3
  - 9:   **else if**  $\alpha_i = \alpha_j = 0$  **then**
  - 10:     Choose  $P_1$  and  $P_2$  according to Step-5
  - 11:   **else**
  - 12:     Choose  $\alpha$  that maximize the capacity
  - 13:   **end if**
  - 14:    $\mu^{(k+1)} \leftarrow \mu^{(k)} - \delta^{(k)} (I_{th} - \sum_{i=1}^N P_1^i \Omega_i^*)$
  - 15:    $\lambda^{(k+1)} \leftarrow \lambda^{(k)} - \delta^{(k)} (I_{th} - \sum_{i=1}^N P_2^i \Omega_2^i)$
  - 16: **end for**
-

The parameter  $\delta^{(k)}$ , appear in lines 14 and 15 of Algorithm II.1, denote the step size of the  $k^{\text{th}}$  iteration. This algorithm is well described in [9], [10], [11], where many types of step size can be used in the sub-gradient algorithm. In our model, we try different step sizes and then use the best one in terms of best performance and less complexity.

## II.5 Simulation Results

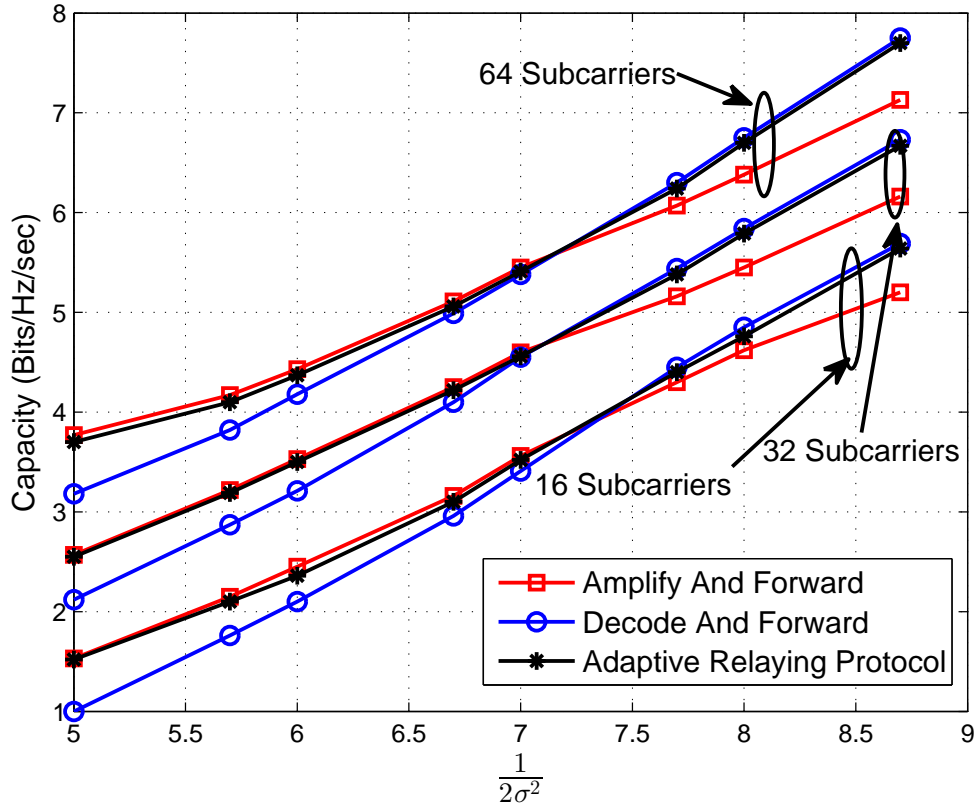


Figure II.3: Achieved capacity for different SNR, different number of sub-carriers, and  $I_{th} = 10^{-5}W$

The simulations are performed under the scenario given in Section II.1. An OFDM system of  $N$  subcarriers ( $N \in \{16, 32, 64\}$ ) at the source and destination and one relay system is assumed. The values of  $T_s$ ,  $\Delta f$  and  $I_{th}$  are assumed to be  $4\mu$  seconds, 0.3125

MHz and  $-20$  dBm respectively. The channel gains are outcomes of independent Rayleigh distributed random variables with mean equal to 1.

Fig.II.3 plots the average capacity using the different schemes (AAF, DAF,ARP) vs. the SNR and using different values of total number of subcarriers with  $N = \{16, 32, 64\}$ . It is shown that for low values of SNR ( $\frac{1}{2\sigma^2} < 7$ ) and for each value of  $N$ , the DAF relay decoding procedure is not perfect. Therefore the AAF performs better than the DAF and provides higher capacity. However, at high SNR ( $> 7$ ) values the behavior of the system become inverse to the previous situation. Here, decoding can be done “perfectly” and the propagation of errors due to the amplification in AAF process has more chances to occur. Thus, in this SNR region the performance achieved by the DAF mode is higher than that achieved by the AAF.

It can be also shown that the ARP relaying protocol achieves, for the different depicted values of SNR, the best results. This can be explained by the fact that the ARP protocol is able to switch (in an adaptive way) from one relaying mode to another (AAF or DAF) using in each moment the relaying mode that achieves the best performance. In other words, the ARP tends to use the AAF relaying protocol for low values of SNR, and use the DAF for higher SNRs. Thus, ARP is able to take advantage of each relaying mode depending on the SNR range. Fig.II.3 shows, finally, how the system capacity scales as function of the increase in the total number of carriers of the system.

Fig.II.4 depicts the average capacity using all the relaying schemes (AAF, DAF, ARP) having two interference threshold values which are  $10^{-5}W$  and  $3 \times 10^{-5}W$ . It can be shown that the crossing point between the DAF and the AAF curves occurs at different SNR values when the system has different interference constraints. Note that, for high values of interference threshold, the source and the relay will be able to transmit with more power than with small interference constraints ( $I_{th}$  is small). This result implies that the decode procedure can be done correctly at the relay phase



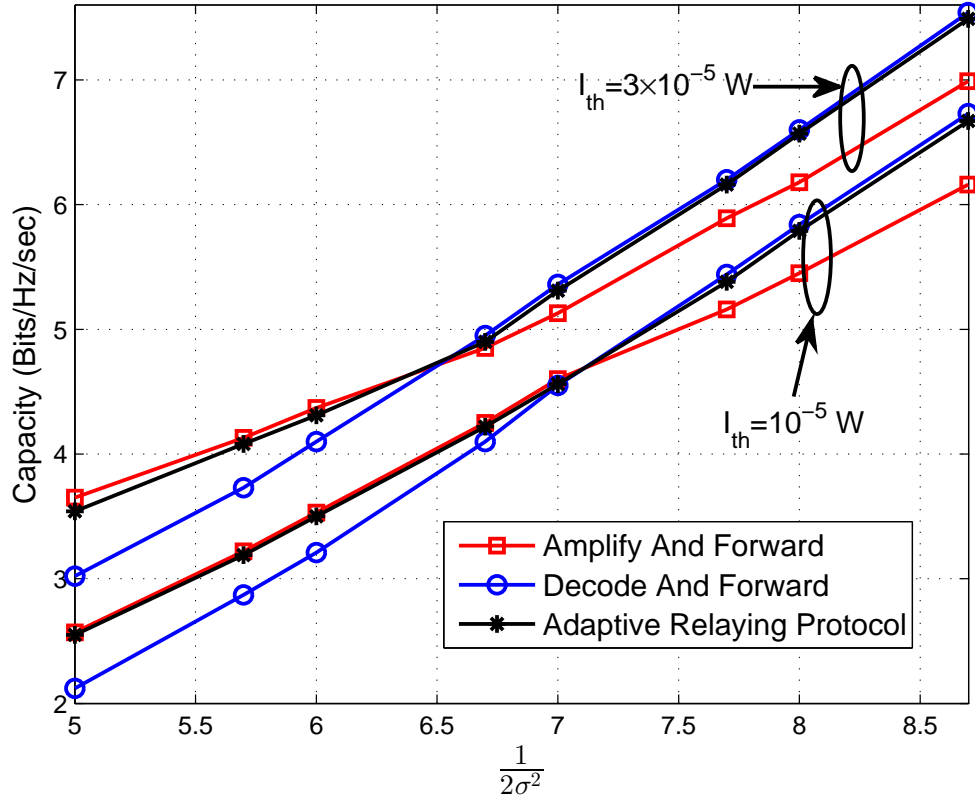


Figure II.4: Achieved capacity for different SNR, 32 subcarriers, and two values of the interference threshold:  $I_{th} = 10^{-5}W$  and  $I_{th} = 3.10^{-5}W$ .

even for high noise variance.

Fig.II.5 shows the capacity of the system versus the SNR having the relay system at different distances from the source. A general observation is that the ARP achieves higher capacity when relay is near the destination, then the performances decrease as soon as we have the relay at middle distance between the S and D, and near to the source respectively. It can be shown that the crossing point between the use of the AAF or the DAF appears at lower values of SNR if the relay system is located near the source. This can be explained by the fact that the relay receives data at high SNR in this case, which means that decoding can be done correctly and as such the relay switches to DAF mode. However, when the relay is near the destination the intersection point appears at high SNR. In this case, the received signal at the relay

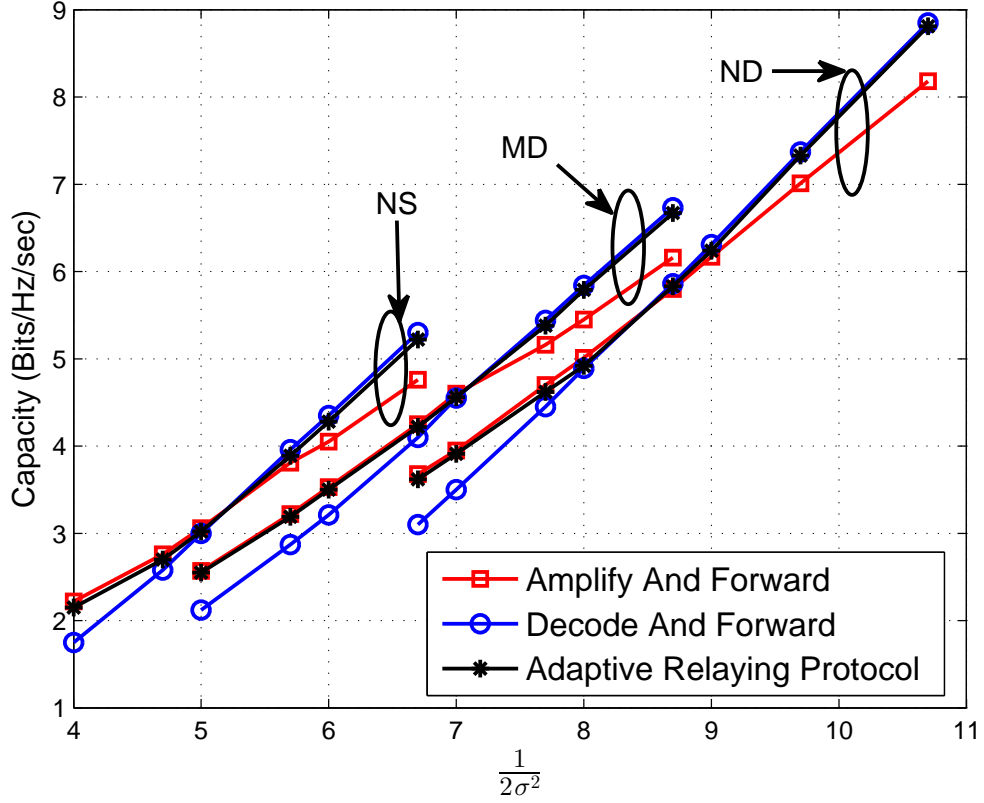


Figure II.5: Achieved capacity for different SNR, different position of the relays,  $I_{th} = 10^{-5}W$  and 32 subcarriers: NS: Near Source, MD: Middle Distance, ND: Near Destination

has a low SNR, which favors AAF since AAF performs better than DAF for low and moderate values of SNR.

As a general observation from Figures II.3, II.4, and II.5, is that it can be shown that the ARP scheme behavior always reaches the optimal scheme for different SNR values. However the major limitation of the proposed scheme is its complexity. Thus, a new algorithm with much less complexity is required to make a step towards possible real implementation. Further work should focus on the development of suboptimal algorithm that achieves a near optimal performance with affordable complexity of implementation.

Fig.II.6 shows capacity performance comparison using the matching and random pairing techniques for different values of SNR. In matching pairing technique, the

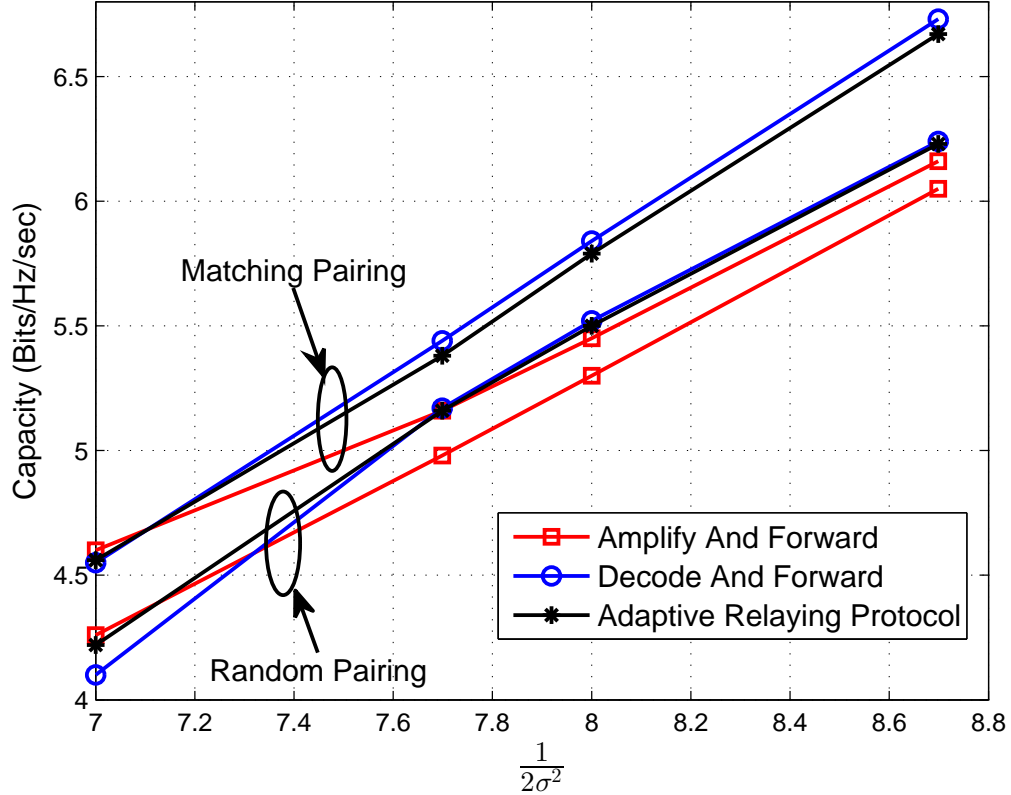


Figure II.6: Achieved capacity for different SNR,  $I_{th} = 10^{-5}W$ , 32 subcarriers and two types of subcarrier pairing: subcarrier matching and random pairing

same carrier  $k$  is used in both time slots ( in S-R and R-D links). However, with random pairing technique assigned carrier in the second time slot will be chosen randomly. It can be shown in this figure that higher capacity is achieved by matching carriers pairing than using the random assignment process of the carriers from S-R to R-D. It can be also observed that the ARP relaying technique achieves best performances in both cases; matching, and random pairing for different values of SNRs. We can conclude that using the matching pairing technique with ARP relaying strategy, higher capacity performance could be achieved for a wide range of SNR values.

## II.6 Concluding Remarks

In this chapter, a near optimal power allocation algorithm in cognitive radio with cooperative communication agent (R) works on adaptive technique of signal forwarding by switching between the DAF and AAF. The aim of this strategy is to avoid error propagation and noise amplification. The simulation results confirm the best choice of this scheme comparing to DAF and AAF. However, the results show that the choosing of the subcarrier selection is very important to improve the performance of the proposed relaying scheme. This matter forms the subject of the next chapter in which we focus on choosing the best pairing strategy to improve the system capacity.

# Chapter III

## Subcarrier Pairing for Adaptive Relaying Protocol

As it was mentioned in the previous Chapter, there are many types of pairing techniques to switch the sub-carriers from the first link to the second link. It has been shown that the pairing strategy has an important impact on the resulting capacity. Therefore, in order to reach maximum capacity, with limited resources, we should carefully choose the pairing technique. One solution is to introduce the subcarrier pairing in the final optimization problem in order to find the optimum pairing distribution that maximizes the capacity without losing too much in terms of complexity.

### III.1 System Architecture

The same OFDM cooperative system described in section II.1 is used in this part with some modification. In fact, a SU, present in the same coverage area of the PU and can communicate through the PU spectrum without causing harmful interference to the adjacent PUs. We assume the absence of a direct link between CS and D. Thus the SU is reaching the destination using the ARP technique of relaying through one relay

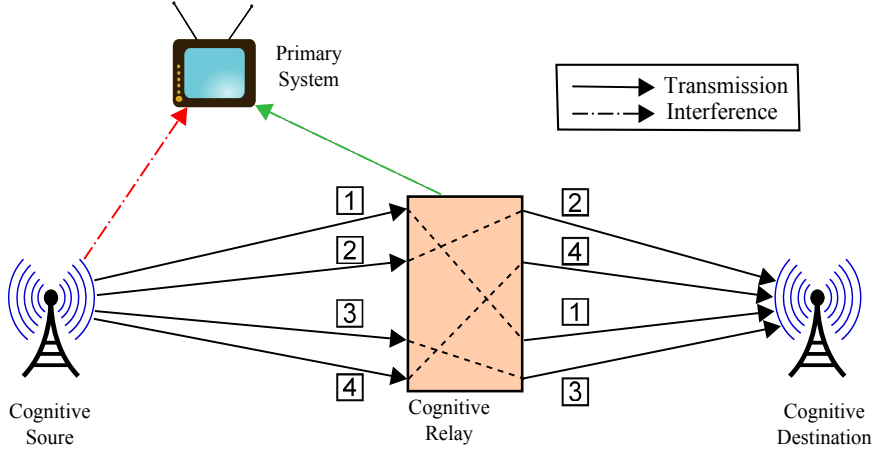


Figure III.1: Cooperative relay cognitive radio network with  $N=4$ . (Dashed lines show the pairing over relay and center lines represent the interference)

R. It is assumed that the data is multiplexed into OFDM with several sub-carriers whose total number is equal to  $N$ . Thus, the used spectrum by the CS is divided into  $N$  sub-carriers each having a  $\Delta f$  bandwidth. Both S and R can transmit over the PU spectrum and interfere with its signal without exceeding the maximum interference power tolerated by PU,  $I_{th}$ . As mentioned before, the source and the relay transmit in two different time slots in a way that the link (S-R) is active at the first time slot while the link (R-D) is active in the second time slot.

The main change in this model is at the relay side which has to distribute the subcarriers to maximize the total rate. In fact, it creates different pairs of subcarrier  $(k, l)$  that assure the transmission in both time slots, the  $k^{th}$  subcarrier in the first time slot is paired with the  $l^{th}$  in the second time slot. Fig.III.1 illustrates the analyzed system.

The interference calculations are the same as the ones done in section II.2, and the interference introduced by the  $k^{th}$  subcarrier to PU,  $I_k(d_k, P_k)$ , can as such be expressed as

$$I_k(d_k, P_k) \triangleq P_k \Omega_k. \quad (\text{III.1})$$

Also the interference introduced by the PU is modeled as AWGN with variance

$J_i$ . In what follows the noise variance is denoted by  $\sigma_i^2 = \sigma^2$  and it is assumed to be the same for all subcarriers and in both time slots.

## III.2 Problem Formulation

### III.2.1 Total Capacity

The variables of the problem are defined as follows.  $P_1^k$  ( $P_2^l$ ) is the power transmitted over the  $k^{\text{th}}$  ( $l^{\text{th}}$ ) subcarrier in the S-R (R-D) link, respectively. The  $k^{\text{th}}$  ( $l^{\text{th}}$ ) subcarrier channel gain over the S-R (R-D) link is given by  $H_1^k$  ( $H_2^l$ ) respectively. Using the notation used in the previous chapter for the transmitted and received signals in the different system hops, we can show that the capacity in the first time slot can be written as follows

$$\mathcal{C}_{1,k} = \frac{1}{2} \log_2(1 + P_1^k \gamma_1^k). \quad (\text{III.2})$$

However in the second time slot, we get two possible formula depend on the type of forwarding technique used at the relay. Hence if the sub-carriers pair  $(k, l)$  is used for transmission, the achieved capacity in the second time slot is given by

$$\mathcal{C}_2[k, l] = \begin{cases} \mathcal{C}_{DF}[k, l] = \frac{1}{2} \log_2(1 + P_2^l \gamma_2^l) & \text{if DAF is used} \\ \mathcal{C}_{AF}[k, l] = \frac{1}{2} \log_2\left(1 + \frac{P_1^k P_2^l \gamma_1^k \gamma_2^l}{P_2^l \gamma_2^l + P_1^k \gamma_1^k + 1}\right) & \text{if AAF is used} \end{cases} \quad (\text{III.3})$$

Note that the same amplification factor defined in (II.8) is used by the relay in the AAF case. The variable  $\gamma$  was given in (II.3.2) and is defined as

$$\begin{cases} \gamma_1^k = \frac{H_1^k}{\sigma^2} \\ \gamma_2^l = \frac{H_2^l}{\sigma^2}. \end{cases} \quad (\text{III.4})$$

For each channel couple  $(k, l)$  the total rate is the minimum rate between both

time slots. If the couple  $(k, l)$  is working on DAF, the rate can be obtained by simply equalizing  $\mathcal{C}_{1,k}$  to  $\mathcal{C}_{DF}[k, l]$ . This equality reduces the number of variables by making a relation between the power allocation in the first and second link as  $P_2^l = \frac{\gamma_1^k}{\gamma_2^l} P_1^k$  for the channel couple  $(k, l)$ . However, if  $(k, l)$  is operating under the AAF protocol, the following inequality solves the problem but does not reduce the number of variables

$$P_1^k \gamma_1^k > \frac{P_1^k P_2^l \gamma_1^k \gamma_2^l}{P_2^l \gamma_2^l + P_1^k \gamma_1^k}. \quad (\text{III.5})$$

As defined in (II.14), we can use the binary variable  $\alpha$  to combined the two values of the capacity according to the forwarding technique. In fact we know that  $\alpha_l = 1$  indicate that the  $l^{\text{th}}$  subcarrier is operating on AAF, and  $\alpha_l = 0$  means that the  $l^{\text{th}}$  subcarrier forwards by DAF. Furthermore, we introduce a new parameter  $t_{k,l}$  to obtain the optimal possible combination  $(k, l)$  in pairing at R. In fact  $t_{k,l}$  takes the values "0" and "1". The "1" means that the couple  $(k, l)$  exists and "0" in the case the couple  $(k, l)$  does not exist. If we model  $t_{k,l}$  by an  $(N \times N)$  matrix  $T$ , the constraints on  $t_{k,l}$  imply that the sum over each column of  $T$  equals to 1, and the sum over each row of  $T$  equals also to 1. The simplest choice of  $T$  is the identity. In this case, R forwards the signal over the same received subcarrier (see Chapter II). To get a closed form of the total rate of the cognitive system, we introduce the previous changes to get the following expression

$$\mathcal{C} = \sum_{k=1}^N \sum_{l=1}^N t_{k,l} (\alpha_l \mathcal{C}_{AF}[k, l] + (1 - \alpha_l) \mathcal{C}_{DF}[k, l]). \quad (\text{III.6})$$

By writing the final expression of the total system capacity, a new constraint on the indicators appears. The first relates to the subcarrier pairing constraint about



$t_{k,l}$  and the second is about  $\alpha_l$

$$\sum_{k=1}^N t_{k,l} = 1, \forall l; \quad \sum_{l=1}^N t_{k,l} = 1, \forall k$$

$$\alpha_l = 0 \text{ or } 1, \forall l.$$

### III.2.2 Optimization Problem

Our objective is to maximize the CR system throughput by optimizing the subcarrier pairing and allocating the best power budgets in the source and the relay among assigned subcarrier pairs, with taking care of the instantaneous interference introduced to PU that should be below the maximum limit using ARP. The interference constraints can be defined using the interference analysis developed in section II.2, to get the following equations

$$\sum_{k=1}^N P_1^k \Omega_1^k \leq I_{th}, \quad (\text{III.7})$$

$$\sum_{l=1}^N P_2^l \Omega_2^l \leq I_{th}, \quad (\text{III.8})$$

where  $\Omega_1^k$  and  $\Omega_2^l$  are the interference factors in each slot.

Therefore, we can formulate the optimization problem as follows

$$\begin{aligned}
& \max_{P_1^k, P_2^l, \alpha_l, t_{k,l}} \mathcal{C} \\
& s.t. \\
& \text{--(C1: Source power constraint)} \\
& \quad \sum_{k=1}^N P_1^k \leq P_S \\
& \text{--(C2: Relay power constraint)} \\
& \quad \sum_{l=1}^N P_2^l \leq P_R \\
& \text{--(C3: First time slot interference)} \\
& \quad \sum_{k=1}^N P_1^k \Omega_1^k \leq I_{th} \\
& \text{--(C4: Second time slot interference)} \\
& \quad \sum_{l=1}^N P_2^l \Omega_2^l \leq I_{th} \\
& \text{--(C5: Forwarding technique constraint)} \\
& \quad \alpha_l \in \{0, 1\}, \quad \forall l \\
& \text{--(C6: Subcarrier pairing constraint)} \\
& \quad \sum_{k=1}^N t_{k,l} = 1, \quad \forall l; \quad \sum_{l=1}^N t_{k,l} = 1, \quad \forall k
\end{aligned} \tag{III.9}$$

$P_S$  and  $P_R$  are the available power budgets in the source and the relay, respectively. The instantaneous fading gains are assumed to be perfectly known. The channel gains can be estimated using classical channel estimation techniques.

### III.3 Optimal Power Allocation

Solving problem (III.9) with respect to the optimization variables  $P_1^k, P_2^l, t_{k,l}$  is a mixed binary integer programming problem. The problem in (III.9) is satisfying the time sharing condition presented in [18] for larger  $N$ . By consequence the duality gap of the problem is negligible as the number of subcarrier is sufficiently large (i.e.

$N > 8$ ) regardless of the convexity of the problem. By solving the dual problem, we get an asymptotically optimal solution [18].

To formulate and solve the dual problem, we need to find the Lagrangian of the primal problem which is given by

$$\begin{aligned} \mathcal{L} = & \sum_{k=1}^N \sum_{l=1}^N \frac{1}{2} t_{k,l} \left[ \alpha_l \log_2 \left( 1 + \frac{P_1^k \gamma_1^k P_2^l \gamma_2^l}{P_1^k \gamma_1^k + P_2^l \gamma_2^l} \right) + (1 - \alpha_l) \log_2 \left( 1 + P_1^k \gamma_1^k \right) \right] \\ & + \beta \left( P_S - \sum_{k=1}^N P_1^k \right) + \theta \left( P_R - \sum_{l=1}^N P_2^l \right) + \lambda \left( I_{th} - \sum_{k=1}^N P_1^k \Omega_1^k \right) + \mu \left( I_{th} - \sum_{l=1}^N P_2^l \Omega_2^l \right) \end{aligned} \quad (\text{III.10})$$

The dual problem associated to the primal problem is given by

$$\min_{\beta \geq 0; \theta \geq 0; \lambda \geq 0; \mu \geq 0} g(\beta, \theta, \lambda, \mu), \quad (\text{III.11})$$

where  $\beta$  and  $\theta$  are the lagrangian multipliers (dual variables) related to the power constraints at the source and the relay, and  $\lambda$  and  $\mu$  represent the dual variables associated to the interference constraints. The dual function  $g$  is defined as

$$\begin{aligned} g(\beta, \theta, \lambda, \mu) = & \max_{P_1^k > 0, P_2^l > 0, \alpha_l, t_{k,l}} \mathcal{L} \\ & s.t. \quad (\text{C5}), (\text{C6}) \end{aligned} \quad (\text{III.12})$$

We can rewrite the dual function from (III.12) as follows

$$\begin{aligned} g(\beta, \theta, \lambda, \mu) = & \max_{P_1^k > 0, P_2^l > 0, \alpha_l, t_{k,l}} \sum_{k=1}^N \sum_{l=1}^N t_{k,l} (\alpha_l \mathcal{D}_{AF}(P_1^k, P_2^l) + (1 - \alpha_l) \mathcal{D}_{DF}(P_1^k)) \\ & + \beta P_S + \theta P_R + I_{th}(\lambda + \mu) \\ & s.t. \quad (\text{C5}), (\text{C6}), \end{aligned} \quad (\text{III.13})$$

As shown in (III.13) and in order, we introduce two new functions in the dual function to simplify the computation, given by

$$\mathcal{D}_{AF}(P_1^k, P_2^l) = \frac{1}{2} \log_2 \left( 1 + \frac{P_1^k \gamma_1^k P_2^l \gamma_2^l}{P_1^k \gamma_1^k + P_2^l \gamma_2^l} \right) - \beta P_1^k - \theta P_2^l - \lambda P_1^k \Omega_1^k - \mu P_2^l \Omega_2^l, \quad (\text{III.14})$$

and

$$\mathcal{D}_{DF}(P_1^k, l) = \frac{1}{2} \log_2 \left( 1 + P_1^k \gamma_1^k \right) - \beta P_1^k - \theta \frac{P_1^k \gamma_1^k}{\gamma_2^l} - \lambda P_1^k \Omega_1^k - \mu \frac{P_1^k \gamma_1^k}{\gamma_2^l} \Omega_2^l. \quad (\text{III.15})$$

Moreover, for a given values of the different dual variables, we get two cases depending on the value of the variable  $\alpha_l$ , which are

- Case 1 : When the pair  $(k, l)$  is used for amplify and forward, i.e.  $\alpha_l = 1$ .

Assume  $(k, l)$  to be a valid subcarrier pair, the optimal power allocation can be evaluated by solving the following sub-problem for every  $(k, l)$  assignment

$$\max_{P_1^k, P_2^l} \mathcal{D}_{AF}(P_1^k, P_2^l) \quad s.t. \quad P_1^k \geq 0, P_2^l \geq 0. \quad (\text{III.16})$$

Hence, we obtain the optimal power by equating

$$\frac{\partial \mathcal{D}_{AF}(P_1^k, P_2^l)}{\partial P_1^k} = \frac{\partial \mathcal{D}_{AF}(P_1^k, P_2^l)}{\partial P_2^l} = 0. \quad (\text{III.17})$$

The optimal power in (III.16) can be expressed as follows

$$\begin{cases} P_1^{k*} = \left[ \frac{\gamma_1^k}{(\gamma_1^k + c_{k,l} \gamma_2^l) c_{k,l} (\theta + \mu \Omega_2^l)} - \frac{1}{\gamma_1^k} - \frac{1}{c_{k,l} \gamma_2^l} \right]^+ \\ P_2^{l*} = c_{k,l} P_1^{k*}, \end{cases} \quad (\text{III.18})$$

where  $c_{k,l} = \sqrt{\frac{\gamma_1^k (\beta + \lambda \Omega_1^k)}{\gamma_2^l (\theta + \mu \Omega_2^l)}}$ . Hence, the power variable in (III.13) can be eliminated by substituting the optimal power allocation found in (III.18). Then the dual

function can be easily found by searching the optimal pair  $(k, l)$  that maximizes the dual function.

- Case 2 : When the pair  $(k, l)$  is used for decode and forward, i.e.  $\alpha_l = 0$ .

In this case we assume that the pair  $(k, l)$  is a valid pair that forwards by DAF technique. The following problem should be solved for each valid pair

$$\max_{P_1^k, P_2^l} \mathcal{D}_{DF}(P_1^k, l) \quad s.t. \quad P_1^k \geq 0. \quad (\text{III.19})$$

By differentiating the previous function over  $P_1$ , we obtain the optimal power allocation in this case

$$\begin{cases} P_1^{k*} = \left[ \frac{1}{\beta + \theta \frac{\gamma_1^k}{\gamma_2^l} + \lambda \Omega_1^k + \mu \frac{\gamma_1^k}{\gamma_2^l} \Omega_2^l} - \frac{1}{\gamma_1^k} \right]^+ \\ P_2^{l*} = \frac{\gamma_1^k}{\gamma_2^l} P_1^{k*}. \end{cases} \quad (\text{III.20})$$

Like Case 1, we substitute the power variable by its optimal value to get a new problem without power parameter. Therefore, the best pair  $(k, l)$  is chosen so it maximizes the dual function.

At this stage, we get the power allocation and sub-carriers pairing in function of the dual variables so that the dual function can be written as follows

$$\begin{aligned} g(\beta, \theta, \lambda, \mu) = \max_{t_{k,l}} \sum_{k=1}^N \sum_{l=1}^N t_{k,l} (\alpha_l \mathcal{D}_{AF}(P_1^{k*}, P_2^{l*}) + (1 - \alpha_l) \mathcal{D}_{DF}(P_1^{k*})) \\ + \beta P_S + \theta P_R + I_{th}(\lambda + \mu) \\ s.t. \quad (\text{C6}). \end{aligned} \quad (\text{III.21})$$

The problem in (III.21) is a linear optimization problem which can be simply solved. The subgradient method can be used to solve the dual problem with guaranteed convergence. At this state we get all the optimal solution, i.e.  $P_{1,2}^*$ ,  $t_{k,l}^*$  and  $\alpha_l$  of

the dual function for a given dual points  $\beta$ ,  $\theta$ ,  $\lambda$  and  $\mu$ . The dual variables at the  $(i + 1)^{th}$  iteration are then updated as

$$\begin{aligned}
 \beta^{(i+1)} &= \beta^{(i)} - \delta^{(i)} \left( P_S - \sum_{k=1}^N P_1^{k*} \right) \\
 \theta^{(i+1)} &= \theta^{(i)} - \delta^{(i)} \left( P_R - \sum_{l=1}^N P_2^{l*} \right) \\
 \lambda^{(i+1)} &= \lambda^{(i)} - \delta^{(i)} \left( I_{th} - \sum_{k=1}^N P_1^{k*} \Omega_1^k \right) \\
 \mu^{(i+1)} &= \mu^{(i)} - \delta^{(i)} \left( I_{th} - \sum_{l=1}^N P_2^{l*} \Omega_2^l \right)
 \end{aligned} \tag{III.22}$$

where  $\delta^{(i)}$  is the step size that can be updated according to the nonsummable diminishing step size policy [9–11].

### III.4 Simulation Results

According to the scenario given in Section III.1, a multicarrier system of  $N = 32$  subcarrier and one relay is assumed. The values of the symbol duration  $T_s$ ,  $\Delta f$  and  $\sigma^2$  are assumed to be  $4\mu$  seconds, 0.3125 MHz and  $10^{-7}$ , respectively, as it was the case in section II.5. The channel gains are outcomes of independent Rayleigh distributed random variables with a mean equal to 1.

Fig.III.2 shows the achieved capacity for different interference and power constraints by using the adaptive relaying protocol. From the figure, it can be observed that the capacity increases as the other constraints increase. However, by fixing one of the constraints, the capacity does not change and becomes constant when some value of the other constraint is reached, this can be justified by the fact that the power allocation reaches its maximum value allowed by the changeable constraint and can not move beyond this constraint. An example of this case is clearly shown in Fig.III.3, where the evolution of the average capacity versus the interference threshold for two values of the power constraint is drawn. Note that for a fixed power constraint, the

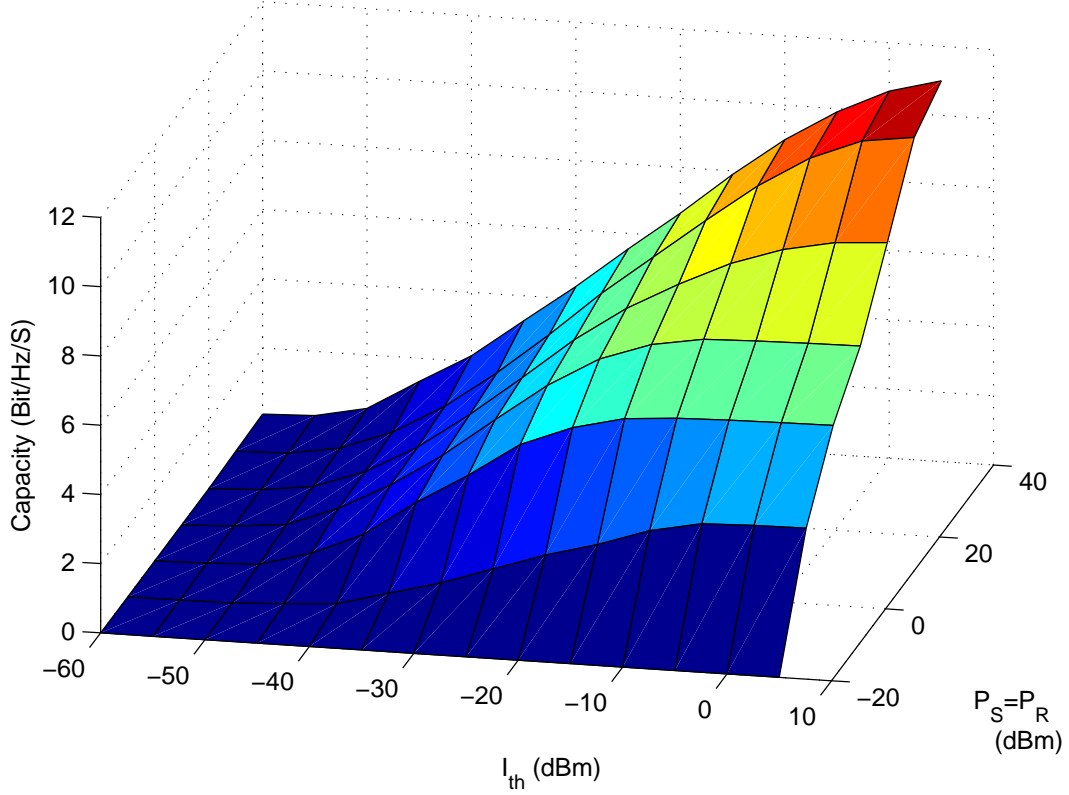


Figure III.2: Achieved capacity as function of the allowed interference threshold and power budget constraints.

system capacity becomes constant because the interference introduced to the PU using the fixed power budget is less than the interference threshold. In the other hand, the figure shows us also that the optimal pairing strategy has the best performance compared to the matched pairing strategy studied in the previous chapter or any random pairing scheme. In fact, in the optimal pairing strategy, the algorithm chooses the best combination of subcarrier pairs to be used while relaying which is, as expected, better than using the same subcarrier for relaying or using a randomly picked set of subcarrier couples.

The system used above assumes that the relay is located in the middle between the source and the destination because the channel gains have the same power. The position of the relay can be changed by modifying the channel power (CS to R) and

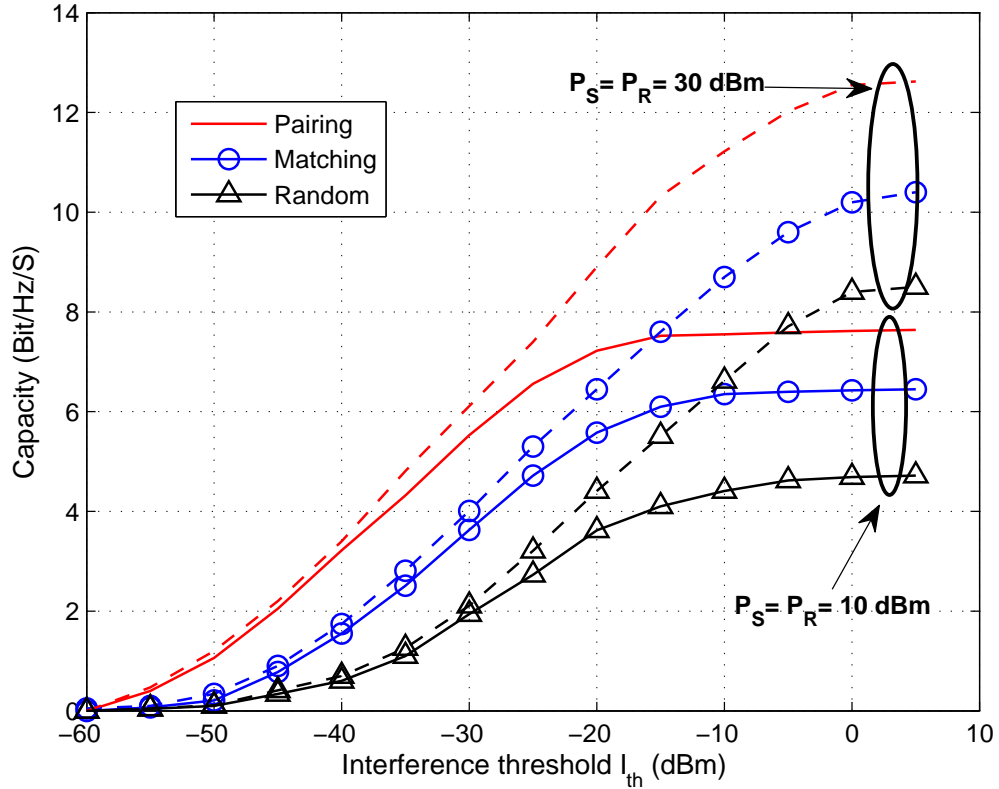


Figure III.3: Achieved capacity vs allowed interference threshold. Solid lines for  $P_S = P_R = 10$  dBm, and dashed lines for  $P_S = P_R = 30$  dBm.

(R to D). The results in Fig.III.4 confirm that the position of the relay has an impact on the system capacity. From this figure, it is shown that the best performance appears when the relay is near the source and the worst case when the relay is near the destination. Indeed, the relay receives a signal with high SNR when it is near the source. As such what ever the deployed forwarding technique (DAF or AAF), the relay transmits a signal with good quality, very near to the original signal for the DAF case and without an important noise amplification for the AAF case. However as the relay moves away from the source, the quality of the received signal at R degrades and the processing becomes more difficult which affects the capacity of the system and decreases it. Therefore the worst case appears when the relay is near the destination since the DAF will cause error propagation while AAF will cause an important noise



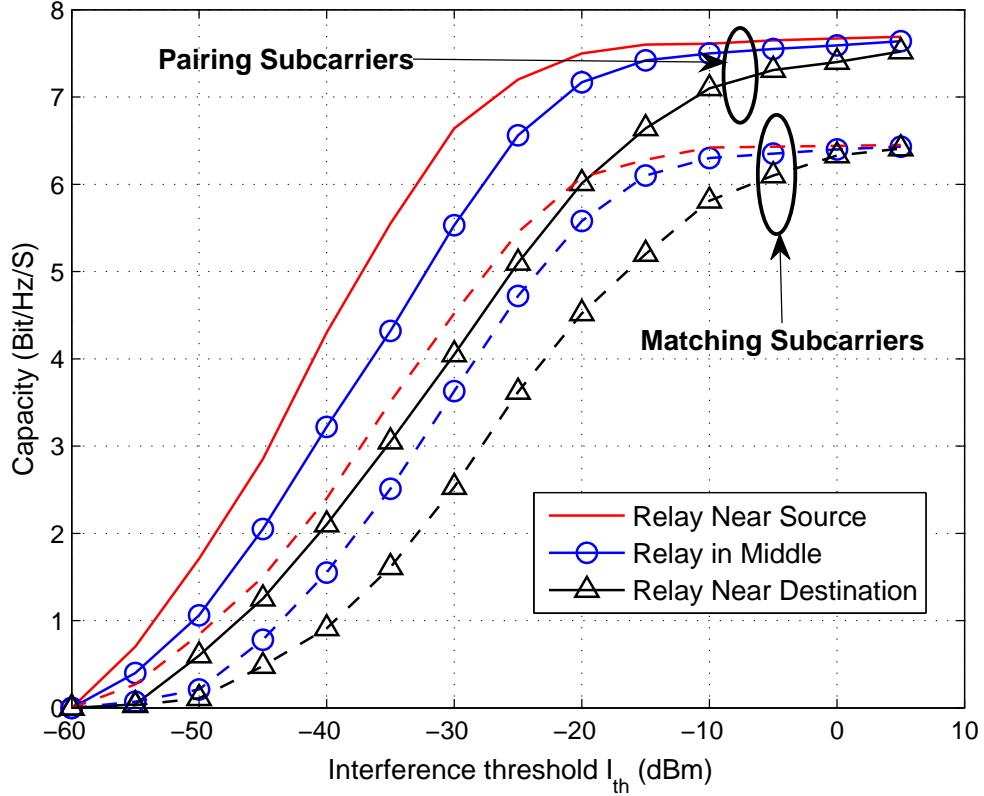


Figure III.4: System capacity for different values of interference threshold, different position of the relay and  $P_S = P_R = 10\text{dBm}$ . The solid lines present the pairing subcarrier technique and the dashed lines for the matching technique

amplification because of the low received SNR. Moreover, the curves in this figure confirm the efficiency of the proposed scheme of pairing. Note that whatever the position of the relay, the system reaches the same maximum capacity allowed by the power budget for high interference threshold. This issue can be explained by the fact that for high interference threshold the received SNR at the relay is very high even if the relay is far from the source. Therefore the capacity increases as the interference threshold increases until it saturates when the power reaches its maximum value.

## III.5 Concluding Remarks

In this chapter, we considered the problem of subcarrier pairing for the adaptive relaying protocol used in a cognitive system. The results show the efficiency of the proposed algorithm and the effect of the choice of the subcarriers pairs at the relay.

# Chapter IV

## CONCLUSION

### IV.1 Summary

In this dissertation, we considered a near optimal power allocation algorithm for an OFDM-based system with adaptive relaying protocol using one relay. In the first part, the problem is solved for the simple case of subcarrier matching to compare the performance of the ARP scheme to the classical AAF and DAF techniques. However in the second part, the goal was to maximize the capacity by jointly optimizing the subcarrier pairing, the power allocation, and the relaying technique (AAF or DAF). In our framework, we assumed a limited power budget at each transmitter, and because it is cognitive scenario, the introduced interference to the primary user was required not to exceed a predetermined tolerated threshold. The problem was formulated with the different constraints as a mixed integer programming problem. We used the dual method to solve the optimization problem iteratively using the sub-gradient algorithm.

Some selected simulation results confirmed the efficiency of the proposed relaying scenario (ARP), which offers better performance in comparison to the AAF and DAF techniques. These results showed also that the performance has a considerable

dependence on the adopted subcarrier pairing techniques at the relay. The simulation results showed finally the effects of the interference threshold tolerated by the PU and the impact of the power budget set at the transmitters.

## **IV.2 Future Research Work**

The work presented in this thesis can be extended in different ways. Firstly, another near optimal algorithm, that is less complex than the sub-gradient algorithm used in this work, can be developed. Secondly, the multi-relay and/or multi-hop scenarios can be considered. For instance, a multi-relay system can be adopted to improve the capacity, and using the pairing techniques along the lines of what was used in this thesis. Finally, investigate the impact of imperfect channel estimation on the development of the optimized schemes can be of interest.

# PUBLICATIONS

- Hamza Soury, Faouzi Bader, Musbah Shaat, and Mohamed-Slim Alouini, “Near Optimal Power Allocation Algorithm for OFDM-Based Cognitive Using Adaptive Relaying Strategy”, *To appear in the Proceedings of the International Conference on Cognitive Radio Oriented Wireless Networks (CROWNCOM 2012), Stockholm, Sweden, June 2012.*
- Hamza Soury, Ferkan Yilmaz, and Mohamed-Slim Alouini, “Average Bit Error Probability of Binary Coherent Signaling over Generalized Fading Channels Subject to Additive Generalized Gaussian Noise”, *To appear in IEEE Communications Letters.*
- Hamza Soury, Faouzi Bader, Musbah Shaat, and Mohamed-Slim Alouini, “Joint Subcarrier Pairing and Power Allocation for Adaptive Relayed OFDM Cognitive Systems”, *Submitted to IEEE Transactions on Wireless Communications.*

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