On the Impact of User Distribution on Cooperative Spectrum Sensing and Data Transmission with Multiuser Diversity

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In this thesis, we investigate the independent but not identically distributed (i.n.i.d.) situations for spectrum sensing and data transmission. In particular, we derive the false-alarm probability and the detection probability of cooperative spectrum sensing with the scheme of energy fusion over i.n.i.d. Nakagami fading channels. Then, the performance of adaptive modulation with single-cell multiuser scheduling over i.n.i.d. Nakagami fading channels is analyzed. Closed-form expressions are derived for the average channel capacity, spectral efficiency, and bit-error-rate (BER) for both constant-power variable-rate and variable-power variable-rate uncoded M-ary quadrature amplitude modulation (M-QAM) schemes. In addition, we study the impact of time delay on the average BER of adaptive M-QAM. From the selected numerical results, we can see that cooperative spectrum sensing and multiuser diversity brings considerably better performance even over i.n.i.d. fading environments.
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Chapter I

INTRODUCTION

The dramatic growth of wireless services over the last decade explains the great demand for radio spectrum. The spectrum resource, however, is limited and most of it has already been allocated. On the other hand, according to the report of the Federal Communications Commission (FFC) [1], many portions of the allocated spectrum are not fully utilized. In order to deal with this conflict between spectrum scarcity and spectrum under-utilization, cognitive radio has been proposed as a revolutionary technology for the next generation of wireless communication networks [2].

To guarantee that the operation of the primary users is not affected, the secondary users must possess the ability of sensing the presence of active primary users, and this process is known as spectrum sensing. In some specific environment, multi-path fading and shadowing may cause the disability of secondary users to detect a primary user. In order to solve such problems, multiple secondary users can cooperate with each other to conduct the spectrum sensing to achieve an improved performance [3].

Recently, cooperative sensing has been widely studied [1] [5], and these studies suppose that the sensing channels are independent identically distributed (i.i.d.). But this is not always the case in practice. Indeed in many instances, the received signals by cooperating secondary users may experience a variety of path loss, shadowing
or fading conditions. As such, a more appropriate and practical assumption is to consider independent but not identically distributed (i.n.i.d.) channels. In this thesis, we consider cooperative sensing in such kind of environment and we assume that the reporting channels to the fusion center are error-free. In particular, instead of sending the binary local decision, each secondary user is assumed to transmit the normalized energy statistic to the base station, which makes a final decision by fusing these local energy statistics. In this context, we calculate the false-alarm probability and miss-detection probability over i.n.i.d. Nakagami fading channels with a threshold determined by the Neyman-Pearson criterion.

As long as the free spectrum, known as the white hole [3], has been detected, one of the secondary users could be chosen to perform data transmission using this free spectrum. To meet the high demand of the wireless communications services, we could adopt adaptive modulation [6, 7, 8] to increase the link spectral efficiency. When the channel side information can be estimated and this estimation sent back to the transmitter, the transmit rate and power can be adapted relative to the channel characteristics to achieve better performance. In [9], the channel capacity with channel side information was derived for both constant-power variable-rate (CPVR) and variable-power variable-rate (VPVR) transmission. In [7], a VPVR M-ary quadrature amplitude modulation (M-QAM) scheme was proposed for data transmission over fading channels. This paper showed that there is a constant power gap between the channel capacity and the spectral efficiency, and this gap is a function of the maintained bit-error-rate (BER). In [8], variable-rate adaptation for M-QAM with constant-power allocation was analyzed over Nakagami fading channels.

In a single-cell multiuser environment, multiuser diversity [10, 11, 12, 13, 14] takes advantage of the variability in fading channels to improve the overall spectral efficiency. For instance, consider that a reasonably large number of users experiencing independent time-varying fading conditions are actively transmitting or receiving
data in a given cell. By granting the channel access only to the user with the best transmission quality, we can achieve a diversity gain so that an increase in the channel capacity and spectral efficiency can be reached. Different multiuser scheduling algorithms were proposed in [10, 11] to choose the best user and reduce the feedback load. In this thesis, we simply select the user with the largest instantaneous singal-to-noise ratio (SNR) to perform data transmission at the base station. With the best user selected, we analyze the channel capacity, average spectral efficiency and average BER for both CPVR and VPVR schemes with M-QAM over an Nakagami fading channels.

The operation of the CPVR and VPVR schemes needs accurate channel estimation at the receiver, a reliable feedback path between the estimator and transmitter, and a negligible time delay. An efficient error control scheme is therefore needed to insure the transmitter’s access of the channel side information. In this thesis, we assume perfect channel estimation and a reliable feedback, and we also quantify the effect of time delay on the proposed system under consideration. In addition, it has been shown in [8] that the time delay affects the average BER due to the variability of the channel conditions. As such, the goal is to keep the time delay under a threshold to limit the impact on the BER.

The rest of this thesis is organized as follows. In chapter II, the system model, including the sensing model, the data transmission model, and the channel statistics are introduced. Next, chapter III presents the analysis of cooperative sensing by energy detection and fusion over i.n.i.d. Nakagami fading environment. Then in chapter IV, we first review the channel capacity and spectral efficiency of the CPVR and VPVR schemes, and then give closed-form expressions for them with best-user selection over i.n.i.d. Nakagami fading channels. The performance of the adaptive M-QAM is also analyzed in this chapter. In chapter V, we consider the impact of time delay on the average BER for the data transmission. Finally, conclusions are drawn in the last chapter.
Chapter II

SYSTEM MODEL

In opportunistic cognitive-radio systems, the secondary users first cooperate with each other to perform spectrum sensing. These users are allowed to transmit data only when free spectrum bands are detected. In this chapter, we present the models used in our analysis of cooperative spectrum sensing and data transmission.

II.1 Single-Cell Multiuser Model

Consider a single-cell multiuser environment with $N$ secondary users uniformly distributed in a cell of radius $R$ around the primary user and base station. In this case, the average SNR $\bar{\gamma}$ for each user is assumed to be log-normal distributed with a shadow standard deviation of $\theta$ dB and an average SNR $\bar{\gamma}_R$ at distance $R$, and a mean value following an exponentially decreasing path loss model with an exponent $\zeta$. The probability density function (PDF) for the average SNR $\bar{\gamma}$ is given by [14],[15]:

$$f_{\bar{\gamma}}(\bar{\gamma}) = \frac{2}{c} \exp \left\{ \frac{-2\theta^2 - 2c(\bar{\gamma} - \bar{\gamma}_R)}{c^2} \right\} Q \left( \frac{2\theta^2 - c(\bar{\gamma} - \bar{\gamma}_R)}{c \cdot \theta} \right), \quad (\text{II.1})$$

where $c = 10\zeta \log_{10}(e)$ is the exponential path loss parameter, $Q(\cdot)$ is the Gaussian $Q$-function defined as the tail probability of the standard normal distribution, i.e.
\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{u^2}{2}\right) du. \]

Generally, the channels between the secondary users and the base station or primary user may be subject to Nakagami fading. Nakagami fading occurs for multi-path scattering with relatively large delay-time spreads and with different clusters of reflected waves. It represents a wide range of multipath channels via the fading parameter \( m \) [16]. For instance, the Nakagami fading includes the one-sided Gaussian fading \((m = 1/2, \text{which corresponds to worst-case fading})\) and the Rayleigh fading \((m = 1)\) as special cases.

We also suppose that the channels between the users and the base station or primary user are slowly-varying flat-fading such that the channel condition changes at a rate much slower than the symbol data rate. In this case, the channel remains roughly constant over hundreds of symbols. For Nakagami fading channels, the instantaneous SNR \( \gamma \) of each user is Gamma distributed with the parameters \((\bar{\gamma}, m)\) and the PDF of \( \gamma \) is thus given by [16]

\[
f_{\text{Nak}}(\gamma) = \left(\frac{m}{\bar{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left(-\frac{m}{\bar{\gamma}} \gamma\right), \quad \gamma \geq 0, \tag{II.2}
\]

where \( \Gamma(\cdot) \) is the Gamma function.

### II.2 Cooperative Sensing Model

In the sensing period, each secondary user performs local sensing by energy detection independently, and then reports these energy values to the base station over error-free reporting channels. These sensing channels for the secondary users are assumed to be independent to each other, but not identically distributed and subject to the same kind of fading.

With such an i.n.i.d. assumption, the SNR of the received signal from the primary user for \( k \)-th secondary user can be denoted as \( \gamma_k \), with its mean \( \bar{\gamma}_k \), the distribution
of which is subject to (II.1), and PDF $f_{\gamma_k}(\gamma_k)$ as shown in (II.2). The $k$-th secondary user measures the normalized energy statistic (with respect to its own energy) of the signal it has received as $E_k$, which has a central chi-square distribution under the hypothesis $H_0$ (inactive primary user) and non-central chi-square distribution with a center parameter $2\gamma_k$ under the hypothesis $H_1$ (active primary user) \cite{17}:

$$E_k \sim \begin{cases} \chi^2_{2u} & H_0 \\ \chi^2_{2u}(2\gamma_k) & H_1 \end{cases},$$

where $u = TW$ is the time bandwidth product with $T$ the observation time interval and $W$ the one-sided bandwidth.

The base station collects the normalized energy statistic from each secondary user using the energy fusion scheme \cite{4} to make a decision. Under perfect reporting channel conditions, the fusion energy statistic yields

$$E = \sum_{k=1}^{N} E_k.$$  \hspace{1cm} (II.4)

Based on (II.4), it is not difficult to show that the fusion energy statistic $E$ is also subject to a central chi-square distribution under the hypothesis $H_0$ and a non-central chi-square distribution with a center parameter $2\gamma_c$ under the hypothesis $H_1$:

$$E \sim \begin{cases} \chi^2_{2Nu} & H_0 \\ \chi^2_{2Nu}(2\gamma_c) & H_1 \end{cases},$$

where $\gamma_c$ is the combined SNR as

$$\gamma_c = \sum_{k=1}^{N} \gamma_k.$$  \hspace{1cm} (II.6)
As such, the PDF of $E$ conditioned on $\gamma$ is given by

$$f_{E}(x) = \begin{cases} \frac{1}{2} \frac{1}{N\Gamma(Nu)} x^{N u - 1} e^{-\frac{x}{2}} & \text{H}_0 \\ \frac{1}{2} \left( \frac{x}{2 \gamma_c} \right)^{\frac{N u - 1}{2}} e^{-\frac{2 \gamma_c + x}{2}} I_{N u - 1} \left( \sqrt{2 \gamma_c x} \right) & \text{H}_1 \end{cases},$$

where $I_M(\cdot)$ is the $M$-th order modified Bessel function of the first kind.

### II.3 Data Transmission Model

After the spectrum sensing, the secondary users can use the free spectrum that was previously allocated to the primary user, if the spectrum is detected to be free. At the beginning of each time slot, the user with the largest instantaneous SNR is selected to perform data transmission in this time slot. Let $\gamma_i$ denote the instantaneous SNR for the $i$-th user\(^1\) and user $b$ as the selected best user such that $b = \arg \max_{i \in \mathcal{R}} \{\gamma_i\}$, where $\mathcal{R} = \{1, 2, \cdots, N\}$. We can then write\(^2\)

$$\gamma_b = \max \{\gamma_1, \gamma_2, \cdots, \gamma_N\}. \quad (\text{II.7})$$

For Nakagami fading channels, it is easy to show that the cumulative distribution function (CDF) of the instantaneous SNR of the best user can be written as

$$F_{\gamma_b}(\gamma) = \prod_{i=1}^{N} F_{\gamma_i}(\gamma) = \prod_{i=1}^{N} \left( 1 - \frac{\Gamma(m_i, \frac{m_i}{\bar{\gamma}_i} \gamma)}{\Gamma(m_i)} \right), \quad \gamma \geq 0, \quad (\text{II.8})$$

where $\Gamma(\cdot, \cdot)$ is the upper incomplete gamma function. Using the relation $\Gamma(n, x) =$

---

\(^1\)In this thesis, we use $\gamma_k$ to denote the received SNR from the primary user for the $k$-th secondary user, and the mean and PDF of $\gamma_k$ is $\bar{\gamma}_k$ and $f_{\gamma_k}(\gamma)$. To clarify, we use $\gamma_i$ to denote the received SNR from the channel for data transmission for the $i$-th user.

\(^2\)In the cooperative sensing model, $\gamma_c$ is used to denote the combined SNR in (II.6), the PDF of which is $f_{\gamma_c}(\gamma)$ given in (III.3). While in the data transmission model, $\gamma_b$ in (II.7) denotes the SNR of the best user, and its PDF is given in (II.10).
\[ \Gamma(n)e^{-x}\sum_{i=0}^{n-1}\frac{x^i}{i!}, \] the CDF of \( \gamma_b \) can then be expressed for integer \( m_i \)’s as

\[ F_{\gamma_b}(\gamma) = \sum_{S,L} (-1)^K C_{S,L} \gamma^L \cdot e^{-E_S \gamma}, \quad \gamma \geq 0, \tag{II.9} \]

where \( S = \{s_1, s_2, \ldots, s_K\} \) is a subset of \( \mathcal{R} \), \( \sum_S \) is the sum over all possible \( S \), and \( \sum_{S,L} \) is the sum defined by

\[
\sum_{S,L} = \sum_S \sum_{l_{s_1}=0}^{m_{s_1}-1} \sum_{l_{s_2}=0}^{m_{s_2}-1} \cdots \sum_{l_{s_K}=0}^{m_{s_K}-1}.
\]

In (II.9), \( l_S, E_S \) and \( C_{S,L} \) are given by

\[
l_S = l_{s_1} + l_{s_2} + \cdots + l_{s_K}
\]

\[
E_S = \frac{m_{s_1}}{\bar{\gamma}_{s_1}} + \frac{m_{s_2}}{\bar{\gamma}_{s_2}} + \cdots + \frac{m_{s_K}}{\bar{\gamma}_{s_K}}
\]

\[
C_{S,L} = \prod_{i=1}^{K} \frac{1}{l_{s_i}!} \left( \frac{m_{s_i}}{\bar{\gamma}_{s_i}} \right)^{l_{s_i}}.
\]

When \( S \) is an empty set (\( S = \phi \)), we have \( l_S = 0, E_S = 0 \) and \( C_{S,L} = 1 \). By simply differentiating (II.9) over \( \gamma \), the PDF of \( \gamma_b \) can then be written as

\[ f_{\gamma_b}(\gamma) = \sum_{S,L} (-1)^K C_{S,L} (l_S - E_S \gamma) \gamma^{L-1} e^{-E_S \gamma}, \quad \gamma \geq 0. \tag{II.10} \]
Chapter III

COOPERATIVE SPECTRUM SENSING

In this chapter, we analyze the performance of cooperative spectrum sensing when energy detection is used over Nakagami fading channels. Each secondary user conducts sensing independently, and the base station collects the energy statistics from each secondary user, based on which the final decision on spectrum sensing is made. In addition, some numerical results are shown at the end of this chapter.

III.1 Performance Analysis

After collecting the energy values from each secondary user, the base station compares the energy statistic $E$ to a threshold $\lambda$ to make a decision. If $E < \lambda$, the base station decides on $H_0$, otherwise decides on $H_1$ if $E > \lambda$. In these conditions, the false-alarm and detection probabilities conditioned on $\gamma_c$ are given by [17]

\begin{align*}
P_f &= \frac{\Gamma(Nu, \lambda/2)}{\Gamma(Nu)} \\
P_d &= Q_{Nu}(\sqrt{2\gamma_c}, \sqrt{\lambda})
\end{align*}

(III.1)
where $\Gamma(\cdot, \cdot)$ is the upper incomplete gamma function, $u$ is the time bandwidth product, $\gamma_c$ is the combined SNR in (II.6), and $Q_M(a, b)$ is the generalized Marcum Q-function, i.e. $Q_M(a, b) = \frac{1}{a} \int_b^\infty x^M \exp \left( -\frac{x^2 + a^2}{2} \right) I_{M-1}(ax) dx$. The averaged detection probability can then be computed in different fading environments as

$$P_d = \int_0^{+\infty} Q_{Nu} \left( \sqrt{2\gamma}, \sqrt{\lambda} \right) f_{\gamma_c}(\gamma) \, d\gamma. \quad (\text{III.2})$$

Since the conditional false-alarm probability is independent of $\gamma_c$, $\bar{P}_f = P_f$.

The threshold $\lambda$ can be determined using the Neyman-Pearson criterion. In particular, given a desired false-alarm probability $P_f = \eta$, the threshold $\lambda$ can be determined and can be used in (III.2) to find the average detection probability.

For distinct $\gamma_k$’s, the PDF of the combined SNR $\gamma_c = \sum_{k=1}^N \gamma_k$ is given by

$$f_{\gamma_c}(\gamma) = \sum_{k=1}^N \sum_{j=1}^{m_k} \frac{A_{kj}}{\Gamma(j)} (\frac{m_k}{\bar{\gamma}_k})^{m_k} \gamma^{j-1} \exp \left\{ -\frac{m_k}{\bar{\gamma}_k} \gamma \right\}, \quad \gamma > 0, \quad (\text{III.3})$$

where the coefficient $A_{kj}$ is determined by

$$A_{kj} = \frac{(-1)^{m_k-j} d^{m_k-j} M_{\gamma_c} (s) \left( 1 - \frac{\bar{\gamma}_k s}{m_k} \right)^{-m_k}}{\Gamma(j)! \left( \frac{m_k}{\bar{\gamma}_k} \right)^{m_k}} \bigg|_{s = \frac{m_k}{\bar{\gamma}_k}}$$

with $M_{\gamma_c} (\cdot)$ denoting the moment generation function of $\gamma_c = \sum_{k=1}^N \gamma_k$ given by

$$M_{\gamma_c}(s) = \left\{ \prod_{k=1}^N \left( 1 - \frac{\bar{\gamma}_k s}{m_k} \right)^{-m_k} \right\},$$

since we assumed independent fading of each channel.

Inserting (III.3) in (III.2), we get the average detection probability with the help of [19, (29)] as

$$\bar{P}_{d,Nak} = \sum_{k=1}^N \sum_{j=1}^{m_k} \frac{A_{kj}}{2j-1 \Gamma(j)} \left( \frac{m_k}{\bar{\gamma}_k} \right)^{m_k} G_{Nu}, \quad (\text{III.4})$$
where $G_{Nu}$ can be determined recursively as

$$G_M = G_{M-1} + \frac{\Gamma(j)\lambda^{M-1}e^{-\frac{\lambda}{2}}}{2^{M-j}(M-1)!} \left(1 + \frac{m_i}{\bar{\gamma}_k}\right)^{-j} {}_1F_1\left(j; M; \frac{\lambda}{2}\frac{\bar{\gamma}_k}{\bar{\gamma}_k + m_k}\right)$$

where the initial $G_1$ given by

$$G_1 = \int_0^\infty x^{2j-1}\exp\left\{-\frac{m_k}{2\bar{\gamma}_k}x^2\right\} Q\left(x, \sqrt{\lambda}\right) dx,$$

and ${}_1F_1(\cdot; \cdot; \cdot)$ is the confluent hypergeometric function. $G_1$ can be written in a closed form with the help of [19, (25)] as:

$$G_1 = \frac{2^{j-1}\Gamma(j)^2\bar{\gamma}_k^{j+1}}{(m_k + \bar{\gamma}_k)m_k^j} \exp\left\{-\frac{\lambda}{2} m_k + \bar{\gamma}_k\right\} \left[\sum_{t=0}^{j-2} \left(\frac{m_k}{m_k + \bar{\gamma}_k}\right)^t \left(-\frac{\lambda}{2} \frac{\bar{\gamma}_k}{m_k + \bar{\gamma}_k}\right) + \left(1 + \frac{m_k}{\bar{\gamma}_k}\right) \left(\frac{m_k}{m_k + \bar{\gamma}_k}\right)^{j-1} L_{j-1}\left(-\frac{\lambda}{2} \frac{\bar{\gamma}_k}{m_k + \bar{\gamma}_k}\right)\right],$$

where $L_t(\cdot)$ is the Laguerre polynomials with degree $t$ [18].

As a special case, the average detection probability for Rayleigh fading channels is given by making all $m_i = 1$ in (III.4) resulting in

$$P_{d_{Ray}} = \sum_{k=1}^{N} A_k \left\{\frac{\Gamma(Nu - 1, \frac{\lambda}{2})}{\Gamma(Nu - 1)} + e^{-\frac{\lambda}{2(Nu - 1)}} P\left(Nu - 1, \frac{\lambda}{2}\frac{\bar{\gamma}_k}{1 + \bar{\gamma}_k}\right) \left(1 + \frac{1}{\bar{\gamma}_k}\right)^{Nu-1}\right\},$$

(III.5)

where $P(M, x) = \gamma(M, x)/\Gamma(M)$ with $\gamma(M, x) = \Gamma(M) - \Gamma(M, x)$ as the lower incomplete gamma function, and $A_k$ is determined by

$$A_k = M_{\gamma_c}(s)(1 - \bar{\gamma}_k s)|_{s=1/\bar{\gamma}_k}$$

where $M_{\gamma_c}(s) = \left\{\prod_{n=1}^{N} (1 - \bar{\gamma}_k s)\right\}^{-1}$. 
III.2 Numerical Results

Performance of the network is often described through its receive operating characteristic (ROC) curves \((P_d \text{ versus } P_f)\) or complementary ROC curves \((P_m = 1 - P_d \text{ versus } P_f)\) for different situations of interests. In the following figures, we sample the set of average SNR \(\{\bar{\gamma}_k, n = 1, 2, \cdots, N\}\) from (II.1)\(^1\), with the standard deviation \(\theta = 3 \text{ dB}\) and the path-loss exponent \(\zeta = 3\), and plot the performance curves by a Monte Carlo method using the Neyman-Pearson criterion to calculate the threshold for a decision.

![Figure III.1: Complementary ROC curves for different values of \(N\) and a Nakagami parameter \(m = 3\) (\(\bar{\gamma}_R = 8 \text{ dB and } u = 5\)).](image)

Ignoring the fading and noise of the reporting channels between the secondary users and the fusion center, we examine the performance of energy fusion over i.n.i.d. sensing channels with Nakagami fading. In Fig[III.1] we generate the complementary ROC curves for Nakagami fading for different numbers \(N\) of cooperative sec-

\(^1\)In this thesis, we generate the average SNR \(\{\bar{\gamma}_k\}'s \text{ or } \{\bar{\gamma}_i\}'s \text{ as per (II.1)}\) with the assumption that the users are uniformly distributed in the cell, and that the \(\{\bar{\gamma}_k\}'s \text{ or } \{\bar{\gamma}_i\}'s \text{ are independent. In reality, the } \{\bar{\gamma}_k\}'s \text{ or } \{\bar{\gamma}_i\}'s \text{ may be dependent because of correlated shadowing. In this case, the } \{\bar{\gamma}_k\}'s \text{ or } \{\bar{\gamma}_i\}'s \text{ should be sampled from any available joint distribution of } \{\bar{\gamma}_k\}'s \text{ or } \{\bar{\gamma}_i\}'s \text{ as } f_{\bar{\gamma}_1, \bar{\gamma}_2, \cdots, \bar{\gamma}_N(\gamma_1, \gamma_2, \cdots, \gamma_N)}, \text{ which will be largely determined by the location of each user.}
Figure III.2: Complementary ROC curves for different values of $\bar{\gamma}_R$ and a Nakagami parameter $m = 3$ ($N = 5$ dB and $u = 5$).

Secondary users. Clearly the greater is the number of cooperative secondary users, the higher performance can the network achieve. From the figure, we can see that the miss-detection probability decreases greatly with the number of secondary users $N$ increasing slightly, which shows that cooperation improves the performance of sensing.

In Fig. III.2, we generate complementary ROC curves for Nakagami conditions for different average SNR $\bar{\gamma}_R$ at distance $R$. From the figure, we notice that there is a great improvement in the performance with several dB’s increment in $\bar{\gamma}_R$. The cooperating secondary users are distributed in a cell centered around the primary user. A higher value of $\bar{\gamma}_R$ means better conditions of the sensing channels between the secondary users and the primary user, or means that these secondary users are distributed in a closer proximity to the primary user.

In Fig. III.3, we plot the complementary ROC curves for different values of Nakagami parameters over Nakagami fading channels. As we can see in the figure, there is an improvement of roughly one order of magnitude for the miss-detection probability from $m = 1$ to $m = 2$. This improvement diminishes when $m$ is reduced by half and
Figure III.3: Complementary ROC curves for different values of Nakagami parameter $m$ ($N = 5$, $\bar{\gamma}_R = 8$ dB and $u = 5$).

...increases when $m$ is doubled.
Chapter IV

DATA TRANSMISSION

When some spectrum bands are detected to be free, one of the secondary user can be selected for data transmission. By choosing the best secondary user, we can adopt adaptive modulation to improve the performance. In this chapter, we analyze the performance of the data transmission when the CPVR and VPVR schemes are adopted. We conclude our chapter with some numerical results to illustrate our schemes.

IV.1 Channel Capacity and Spectral Efficiency

Assuming perfect channel estimation and reliable feedback, the time-varying channel capacity of a fading channel $< C >$ [Bits/Sec] with variable power allocation and rate adaptation is given by [9]

$$< C > = \max_{P(\gamma)} \left\{ \int_0^\infty W \log_2 \left( 1 + \gamma \frac{P(\gamma)}{P} \right) f_\gamma(\gamma) d\gamma \right\}, \quad \text{(IV.1)}$$

where $W$ [Hz] is the channel bandwidth, $f_\gamma(\gamma)^\text{1}$ is the distribution of the received SNR, $P$ is the average transmit power, and $P(\gamma)$ is the instantaneous transmit power.

---

1In this thesis, $f_\gamma(\gamma)$ represents the distribution of the received SNR from arbitrary fading channels. For example, it is given by (II.3) in the case of Nakagami fading.
chosen relative to $\gamma$, which is subject to the power constraint

$$\int_0^\infty P(\gamma) f_\gamma(\gamma)d\gamma \leq \overline{P}, \quad (IV.2)$$

If we choose uncoded M-QAM for the adaptive modulation scheme of choice, the BER with variable power allocation can be well approximated by \[7\]:

$$\text{BER}(M, \gamma) \simeq 0.2 \exp\left(-\frac{3\gamma}{2(M-1)} \frac{P(\gamma)}{\overline{P}}\right). \quad (IV.3)$$

To maintain a fixed BER of $\text{BER}_0$, the constellation size $M$ is adjusted as

$$M = \left(1 + \frac{\gamma}{K_0}\right) \frac{P(\gamma)}{\overline{P}}, \quad (IV.4)$$

where $K_0 = -\frac{2}{3}\ln(5\text{BER}_0)$.

**IV.2 Constant-Power Variable-Rate Adaptation**

With a constant transmit power $P(\gamma) = \overline{P}$, the channel capacity $< C >_{\text{cpvr}}$ can be easily derived from (IV.1) as \[20\]

$$< C >_{\text{cpvr}} = W \int_0^\infty \log_2(1 + \gamma)f_\gamma(\gamma)d\gamma. \quad (IV.5)$$

Substituting (II.10) in (IV.5) yields the channel capacity per unit bandwidth over i.n.i.d. Nakagami fading channels with the best user selection

$$< \frac{C}{W} >_{\text{cpvr}} = \log_2(e) \sum_{S,L} (-1)^K C_{S,L} (l_S \mathcal{I}_{l_S}(E_S) - E_S \mathcal{I}_{l_S+1}(E_S)), \quad (IV.6)$$
where $\mathcal{I}_n(\mu)$ is defined in [20 (32)], and its closed-form evaluation for a positive integer $n$ is also derived in [20 (78)]. $\sum_{S,L}, l_S, E_S$ and $C_{S,L}$ are defined in (II.3) as

$$\sum_{S,L} = \sum_S l_{s_1} = 0 \sum_{l_{s_2} = 0} m_{s_2} - 1 \cdots \sum_{l_{s_K} = 0} m_{s_K} - 1 ,$$

$$l_S = l_{s_1} + l_{s_2} + \cdots + l_{s_K} ,$$

$$E_S = \frac{m_{s_1}}{\bar{\gamma}_{s_1}} + \frac{m_{s_2}}{\bar{\gamma}_{s_2}} + \cdots + \frac{m_{s_K}}{\bar{\gamma}_{s_K}} ,$$

$$C_{S,L} = \prod_{i=1}^K \frac{1}{l_{s_i}!} (\frac{m_{s_i}}{\bar{\gamma}_{s_i}})^{l_{s_i}} ,$$

where $S = \{s_1, s_2, \cdots, s_K\}$ is a subset of $\mathcal{R}$, and $\sum_S$ is the sum over all possible $S$.

With the constant power scheme and $M$ chosen in (IV.4), the spectral efficiency for continuous-rate M-QAM can be approximated by [8]

$$< \frac{R}{W} >_{cpvr} = \int_0^\infty \log \left( 1 + \frac{\gamma}{K_0} \right) f_{\gamma}(\gamma) d\gamma , \quad (IV.7)$$

Substituting (II.10) in (IV.7), the above spectral efficiency per unit bandwidth can be written as

$$< \frac{R}{W} >_{cpvr} = \log_2(e) \sum_{S,L} (-1)^K C_{S,L} K_0^{l_S} (l_S I_{l_S}(E_S K_0) - E_S K_0 I_{l_S+1}(E_S K_0)) .$$

(IV.8)

For practical use, the constellation size $M$ is often restricted to $2^n$ for a positive integer $n$. We divide the whole SNR range into $T + 1$ fading regions, and assign $M_n$ as the constellation size to the $n$-th region. When the estimated SNR falls in the $n$-th region, $M_n$ is chosen for M-QAM. With the constellation size $M_n$ and the boundary SNRs $\gamma_n$ chosen as the same as in [8 (30)], we have the spectral efficiency for constant-power discrete-rate (CPDR) scheme as the sum of the data rates ($\log_2(M_n)$) associated with each region multiplied by the probability of the instantaneous SNR.
\( \gamma \) falling in that region
\[
< \frac{R}{W} >_{cpdr} = \sum_{n=1}^{T} r_n p_n,
\] (IV.9)
where \( r_n = \log_2(M_n) = n, p_n = \int_{\gamma_n}^{\gamma_n+1} f_{\gamma_n}(\gamma) d\gamma = F_{\gamma_n}(\gamma_{n+1}) - F_{\gamma_n}(\gamma_n) \), which can be easily computed with [II.8].

For continuous-rate M-QAM, it always operates at the target BER \( 0 \). When the constellation size is restricted to some integer values, however, the discrete M-QAM operates at an average \(< BER >_{cpdr} \) lower than BER \( 0 \). Similar to the calculation of the spectral efficiency, the average BER with constellation restriction is the ratio of the average number of bits in error over the total average number of transmitted bits [8]:
\[
< BER >_{cpdr} = \frac{\sum_{n=1}^{T} r_n \overline{BER}_n}{\sum_{n=1}^{T} r_n p_n},
\] (IV.10)
where \( \overline{BER}_n \) is calculated by
\[
\overline{BER}_n = \int_{\gamma_n}^{\gamma_{n+1}} BER(M_n, \gamma) f_{\gamma_n}(\gamma) d\gamma.
\] (IV.11)

Substituting \( BER(M_n, \gamma) = \frac{1}{5}(5BER_0)^{\frac{\gamma}{\gamma_n}} \) into (IV.11), we have the \( \overline{BER}_n \) written as
\[
\overline{BER}_n = \sum_{S,L} (-1)^K \frac{C_{S,L}}{5^{\beta_{S_n}}} \left( I_S \Gamma(l_S, \beta_{S_n} \gamma_n, \beta_{S_n} \gamma_{n+1}) - \frac{E_S}{\beta_{S_n}} \Gamma(l_S + 1, \beta_{S_n} \gamma_n, \beta_{S_n} \gamma_{n+1}) \right),
\] (IV.12)
where \( \beta_{S_n} = E_S + 1.5K_0/\gamma_n \), and \( \Gamma(n, a, b) = \Gamma(n, a) - \Gamma(n, b) \).

### IV.3 Variable-Power Variable-Rate Adaptation

By optimizing the channel capacity [IV.1] with the power constraint [IV.2], we have the channel capacity with optimal power allocation as [9]
\[
< C >_{vpvr} = W \int_{\gamma_0}^{\infty} \log_2 \left( \frac{\gamma}{\gamma_0} \right) f_{\gamma}(\gamma) d\gamma,
\] (IV.13)
where $\gamma_0$ is the optimal cutoff SNR below which data transmission is suspended and can be determined by solving

$$
\int_{\gamma_0}^{+\infty} \left( \frac{1}{\gamma_0} - \frac{1}{\gamma} \right) f_\gamma(\gamma) \, d\gamma = 1.
$$

(IV.14)

Rewrite it in terms of the CDF of $\gamma_b$ as

$$
\frac{1 - F_{\gamma_b}(\gamma_0)}{\gamma_0} = 1 + \sum_{S,L} (-1)^K C_{S,L} \gamma_0^{l_S} \left( l_S \Gamma(l_S - 1, E_S \gamma_0) - \Gamma(l_S, E_S \gamma_0) \right).
$$

(IV.15)

When $l_S = 0$ or $1$, we have $\Gamma(0, E_S \gamma_0) = E_1(E_S \gamma_0)$, where $E_1(\cdot)$ is the exponential integral of the first order [18], and $F_{\gamma_b}(\gamma_0)$ can be easily evaluated using (II.8).

By substituting (II.10) in (IV.13), the channel capacity per unit bandwidth with optimal power allocation can be written as

$$
< \frac{C}{W} >_{vpvr} = \log_2(e) \sum_{S,L} (-1)^K C_{S,L} \gamma_0^{l_S} \left( l_S J_{l_S}(E_S \gamma_0) - E_S \gamma_0 J_{l_S + 1}(E_S \gamma_0) \right),
$$

(IV.16)

where the definition of $J_n(\mu)$ is given in [20, (14)], and its closed-form evaluation for a positive integer $n$ is also derived in [20, 64].

With the optimal power allocation and $M$ chosen in (IV.4), the spectral efficiency is given by [7]

$$
< \frac{R}{W} >_{vpvr} = \int_{\gamma_K}^{\infty} \log_2 \left( \frac{\gamma}{\gamma_K} \right) f_\gamma(\gamma) \, d\gamma,
$$

(IV.17)

where $\gamma_K$ is the cutoff fade depth under which no power is allocated to transmit data, which is determined by

$$
K_0 \int_{\gamma_K}^{+\infty} \left( \frac{1}{\gamma_K} - \frac{1}{\gamma} \right) f_\gamma(\gamma) \, d\gamma = 1.
$$

(IV.18)

The spectral efficiency with optimal power allocation over i.n.i.d. Nakagami fading
channels can be derived by simply replacing $\gamma_0$ in (IV.16) with $\gamma_K$

$$<\frac{R}{W}>_{\text{vpvr}}= \log_2(e) \sum_{S,L} (-1)^K C_{S,L} \gamma_K^S (l_S \mathcal{J}_{l_S}(E_S \gamma_K) - E_S \gamma_K \mathcal{J}_{l_S+1}(E_S \gamma_K)).$$

(IV.19)

Similar to the constellation restriction in the CPVR scheme, we also divide the whole SNR range into $T+1$ fading regions, but restrict our constellation size as $M_0 = 0, M_1 = 2, \text{ and } M_n = 2^{(n-1)}, n = 2, \cdots, T$. The boundary SNR $\gamma_n$ is determined by $\gamma_n = \gamma^* M_n$ with a parameter $\gamma^* > 0$ which satisfies the power constraint (IV.20) if we adopt the same power allocation scheme as in [7]

$$\sum_{n=1}^{T} \frac{M_n-1}{K_0} \int_{\gamma^* M_n}^{\gamma^* M_{n+1}} \frac{f_{\gamma_b}(\gamma)}{\gamma} d\gamma = 1.$$

(IV.20)

The spectral efficiency can also be calculated by (IV.9) with $p_n = F_{\gamma_b}(\gamma^* M_{n+1}) - F_{\gamma_b}(\gamma^* M_n)$, where $F_{\gamma_b}(\cdot)$ is given by (II.8), and $r_1 = 1, r_n = 2(n-1), n = 2, 3, \cdots, T$. Note that the the BER is always equal to BER$_0$, i.e. $<\text{BER}>_{\text{vpdr}} = \text{BER}_0$.

### IV.4 Numerical Results

In the following figures, we consider a single-cell multiuser environment with a shadow standard deviation of $\theta = 3$ dB and an exponent $\zeta = 3$ for the path loss. The average SNR $\tilde{\gamma}_i$’s for each user are supposed to be i.n.i.d. following the distribution given in (II.1) with a cell-range SNR $\tilde{\gamma}_R$, and generated in our simulation based on the procedure described in the Appendix. In our results, the Nakagami parameter for each user is fixed as $m = 3$.

In Fig. [IV.1] we plot the channel capacity and spectral efficiency for different numbers of users for both the CPVR and VPVR schemes. From this figure, we can see that the channel capacity and spectral efficiency for CPVR and VPVR schemes are almost the same. With the constellation restriction, the spectral efficiency for CPVR
scheme degrades considerably, while for the VPVR scheme, the effect of constellation restriction is offset by the continuous power allocation. Note that all the users are distributed in a single cell with the cell-range SNR $\bar{\gamma}_R = 0$ dB. When the number of users increases, we maintain the $\bar{\gamma}_R$ unchanged so that the cell radius remains the same. We can see from the figure that both the channel capacity and spectral efficiency are marginally decreasing. So the number of users in the cell should be moderate in order to achieve a diversity gain without causing too much complexity.

In Fig. IV.2, we generate the spectral efficiency for different target BER$_0$ for both the continuous-rate and the discrete-rate schemes. In this figure, we constrain the number of the users to 5 with $\gamma_R = 0$ dB. As expected, the larger the BER$_0$ is, the greater efficiency we can achieve. In addition, the efficiency for discrete-rate scheme will approach the continuous-rate efficiency as the number of regions increases.

In Fig. IV.3 and Fig. IV.4, the channel capacity and spectral efficiency for different cell-range SNR $\bar{\gamma}_R$ are shown for the CPVR and VPVR schemes, respectively. In Fig.

Figure IV.1: Channel capacity and spectral efficiency for different numbers of users ($\bar{\gamma}_R = 0$ dB and BER$_0 = 10^{-6}$).
Figure IV.2: Spectral efficiency for different values of the target BER \( \bar{\gamma}_R = 0 \) dB and \( N = 5 \).

In IV.3, we see a saturation of the spectral efficiency at large SNR for the discrete-rate CPVR scheme. For the large SNR range, the current constellation size is not enough for transmission with a higher efficiency, and larger constellation size is frequently used to achieve better performance. While in Fig. IV.4, the power allocation for discrete-rate VPVR scheme will be highly suboptimal for large SNR. In these circumstances, the power is far from being fully utilized and no \( \gamma^* \) is found as the solution for (IV.20), so that no results for large SNR are shown in the figure. In order to utilize the power fully, larger constellation size is also needed to achieve a better performance.

In Fig. IV.5 we compare the BER for discrete-rate CPVR schemes with different regions. From the curves in this figure we see that the discrete-rate CPVR scheme operates at a BER smaller than the target BER\( \bar{\gamma}_R \), and the more regions the whole SNR is divided into, the higher BER we have. As the number of regions goes to infinity, the scheme will converge to the continuous-rate scheme and operate at the target BER\( \bar{\gamma}_R \).
Figure IV.3: Channel capacity and spectral efficiency for different values of cell-range SNR $\bar{\gamma}_R$ for the CPVR scheme ($N = 5$ and BER$_0 = 10^{-6}$).

Figure IV.4: Channel capacity and spectral efficiency for different values of cell-range SNR $\bar{\gamma}_R$ for the VPVR scheme ($N = 5$ and BER$_0 = 10^{-6}$).
Figure IV.5: BER for different values of the cell-range SNR $\bar{\gamma}_R$ for the CPVR scheme ($N = 5$).
Chapter V

TIME DELAY EFFECT

V.1 Introduction

Recall that the choice of the constellation size $M$ and power allocation $P(\gamma)$ are based on a channel estimation at time $t$, while the data are sent over the channel at time $t + \tau$, where $\tau$ is the time delay for the channel estimation and feedback. If the time delay $\tau$ degrades the BER significantly, then the adaptive technique will not work. Assuming perfect channel estimation and reliable feedback, we analyze in this section the impact of time delay on BER with the best user selection in an i.n.i.d. Nakagami fading environment.

V.2 Performance Analysis

We denote $\rho$ as the correlation factor between the SNR $\gamma$ at time $t$ and the SNR $\gamma_\tau$ at time $t + \tau$. $\rho$ can be expressed in terms of time delay $\tau$, the mobile speed $v$, and the wavelength of the carrier frequency $\lambda_c$ by $\rho = J_0^2(2\pi f_D \tau)$ [21], where $J_0(\cdot)$ is the zero-order Bessel function of the first kind, and $f_D = v/\lambda_c$ is the maximum Doppler
frequency shift. With $\rho$, the conditional PDF of $\gamma_\tau$ over $\gamma_b$ is given by [16]

$$
f_{\gamma | \gamma_b}(\gamma | \gamma) = \frac{m_b}{(1 - \rho)\gamma_b} \left( \frac{\gamma}{\rho \gamma} \right)^{m_b - 1} \exp \left( -\frac{m_b \rho \gamma + \gamma}{\gamma_b (1 - \rho)} \right) I_{m_b - 1} \left( \frac{2m_b \sqrt{\rho \gamma \gamma_\tau}}{(1 - \rho)\gamma_b} \right),
$$

(V.1)

where $m_b$ and $\bar{\gamma}_b$ are the Nakagami parameter and average SNR for the best user $b$, respectively, and $I_n(\cdot)$ is the modified Bessel function of the first kind.

With constant power allocation, the instantaneous BER is given by $\text{BER}(\gamma, \gamma_\tau) = \frac{1}{5}(5\text{BER}_0)^{\frac{2\gamma}{\gamma}}$. Averaging it over (V.1) yields the conditional average BER on $\gamma$ as

$$
\text{BER}(\gamma) = \frac{1}{5} \left( \frac{\gamma}{\gamma + \alpha} \right)^{m_b} \exp \left( -\frac{\beta \gamma}{\gamma + \alpha} \right),
$$

where $\alpha = \frac{3(1 - \rho)K_0\bar{\gamma}_b}{2m_b}$ and $\beta = \frac{3}{2}K_0\rho$. Averaging $\text{BER}(\gamma)$ over the PDF of $\gamma_b$ given in (II.10) yields the average BER for CPVR scheme accounting for the impact of the time delay given by

$$
< \text{BER}_{\tau} >_{\text{cpvr}} = \frac{1}{5} \sum_{S,L} (-1)^K C_{S,L} \left( l_S \mathcal{F}_{m_b}^{0,\infty}(\alpha, \beta, l_S - 1, E_S) - E_S \mathcal{F}_{m_b}^{0,\infty}(\alpha, \beta, l_S, E_S) \right),
$$

(V.2)

where $\mathcal{F}_{m}^{x,y}(\alpha, \beta, a, b)$ is defined as

$$
\mathcal{F}_{m}^{x,y}(\alpha, \beta, a, b) = \int_x^y \left( \frac{\gamma}{\gamma + \alpha} \right)^{m} \exp \left( -\frac{\beta \gamma}{\gamma + \alpha} \right) \gamma^a e^{-b\gamma} d\gamma.
$$

Considering the restriction of the constellation, the instantaneous BER yields $\text{BER}(M_n, \gamma_\tau) = \frac{1}{5}(5\text{BER}_0)^{\frac{2\gamma_n}{\gamma_n}}$. Averaging this expression over the conditional PDF (V.1) yields the conditional average BER on $\gamma$ for the discrete rate as

$$
\text{BER}_n(\gamma) = \frac{1}{5} \left( \frac{\gamma_n}{\gamma_n + \alpha} \right)^{m_b} \exp \left( -\frac{\beta \gamma}{\gamma_n + \alpha} \right).
$$

The average BER is calculated by (IV.10) with the $\text{BER}_n$ calculated by (IV.11), the
result of which is given by

\[
\text{BER}_n = \frac{1}{5} \left( \frac{\gamma_n}{\alpha + \gamma_n} \right)^{\text{mb}} \sum_{S,L} (-1)^K C_{S,L} \left( l_S \Gamma(l_S, \alpha_{S_n} \gamma_n, \alpha_{S_n} \gamma_{n+1}) \right. \\
\left. - \frac{E_{S}}{\alpha_{S_n}} \Gamma(l_S + 1, \alpha_{S_n} \gamma_n, \alpha_{S_n} \gamma_{n+1}) \right), \\
(V.3)
\]

where \(\alpha_{S_n} = E_S + \frac{\beta}{\alpha + \gamma_n}\) with \(\alpha\) and \(\beta\) given as above.

For the variable power scheme, the power allocation \(P(\gamma)/\overline{P}\) is chosen so that it always operates at the target BER. Taking time delay into consideration, the instantaneous BER, for both the continuous rate and discrete rate, yields \(\text{BER}(\gamma; \gamma_T) = \frac{1}{5}(5\text{BER}_0)^{\frac{\gamma_T}{\gamma}}\).

For the continuous-rate VPVR scheme, the data is transmitted only when the instantaneous SNR is above the cutoff SNR, or \(\gamma > \gamma_K\). So the average BER for continuous-rate VPVR scheme with time delay effect is given by

\[
< \text{BER}_{\tau} >_{\text{vpvr}} = \frac{1}{5} \sum_{S,L} (-1)^K C_{S,L} \left( l_S \mathcal{F}_{\gamma, \infty}^{\nu} (\alpha, \beta, l_S - 1, E_S) - E_S \mathcal{F}_{\gamma, \infty}^{\nu} (\alpha, \beta, l_S, E_S) \right). \\
(V.4)
\]

With the constellation restriction, the average BER is calculated by \([V.10]\), where \(r_1 = 1, r_n = 2(n - 1), n = 2, 3, \cdots, T, p_n = F_{\gamma}(\gamma^* M_{n+1}) - F_{\gamma}(\gamma^* M_n)\), and \(\text{BER}_n\) is given in this case by

\[
\text{BER}_n = \frac{1}{5} \sum_{S,L} (-1)^K C_{S,L} \left( l_S \mathcal{F}_{mb}^{\gamma_n, \gamma_{n+1}} (\alpha, \beta, l_S - 1, E_S) - E_S \mathcal{F}_{mb}^{\gamma_n, \gamma_{n+1}} (\alpha, \beta, l_S, E_S) \right). \\
(V.5)
\]

**V.3 Numerical Result**

In Fig. \([V.1]\) the impact of time delay on BER is studied. From the figure we can see that, if the normalized delay is maintained under a value of \(10^{-2}\), the impact of delay is tolerable and the systems will be able to operate satisfactorily. Beyond that, the
Figure V.1: BER for different values of normalized time delay ($\gamma_R = 0$ dB and $N = 5$).

time delay will degrade the BER significantly. Comparing the curves of $\text{BER}_0 = 10^{-3}$ and $\text{BER}_0 = 10^{-6}$, we can find that the smaller the $\text{BER}_0$ we want to maintain, the more sensitive the BER is to the time delay.
Chapter VI

Conclusions

In this thesis, we analyzed the performance of cooperative spectrum sensing over the independent but not identical Nakagami-fading environments. We considered an energy-fusion based scheme and derived the closed-form expressions of false-alarm probability and detection probability. It was shown by selected numerical results that cooperative spectrum sensing still works considerably well in an i.n.i.d. environment.

After spectrum sensing, we also examined various schemes for the data transmission of secondary users. By choosing the best secondary user in a single-cell environment, multiuser diversity gain can be achieved. We analyzed the constant power variable rate and variable power variable rate schemes with uncoded M-QAM, and offered close-form expressions for the channel capacity, the spectral efficiency, and the BER. Numerical results confirm that by selecting the best user, we can improve the performance considerably through the diversity gain.

Finally, the impact of time delay on the BER was studied, and numerical results showed that when the normalized time delay is below $10^{-2}$, the impact on BER is tolerable. However, the BER degrades considerably when the normalized time delay exceeds $10^{-2}$.

For further works, the impact of time delay on cooperative spectrum sensing will
be studied. Just as the time delay degrades the BER of the data transmission when adaptive modulation is adopted, the time delay also degrades the performance of spectrum sensing because of the mismatch of channel conditions, which increases the probability of miss detection. In addition, the change of status of the primary user will have an impact on performance for both the spectrum sensing and data transmission. During the sensing period, the sudden come or leave of the primary user may cause trouble for the spectrum sensing, which increases the chance that miss detection or false alarm happens. For the data transmission period, the primary user traffic will also certainly degrade the BER of secondary user during the data transmission period. These two interesting problems will be investigated.
Appendix

Average SNR Random Generation

In this thesis, we describe the acceptance-rejection algorithm [22] used to generate the average SNR $\bar{\gamma}_k$’s or $\bar{\gamma}_i$’s following the PDF given in (II.1) by the acceptance-rejection algorithm. This algorithm generates sampling values from an arbitrary PDF $f_X(x)$ by using an auxiliary PDF $f_Y(x)$. The PDFs $f_X(x)$ and $f_Y(x)$ should satisfy the following assumptions:

- It is simple to sample from $f_Y(x)$;
- There exists $0 < \phi \leq 0$ such that $\phi \frac{f_X(x)}{f_Y(x)} \leq 1$ for all $x$.

As long as we find a proper $f_Y(x)$, the sampling procedure to generate samples for $f_X(x)$ consists in

- **Step 1:** Sample two independent random variables $y$ from $f_Y(x)$ and $u$ from the uniform distribution $U(0,1)$;
- **Step 2:** If $u \leq \phi \frac{f_X(y)}{f_Y(y)}$, then accept $x = y$ as a sample from $f_X(x)$, otherwise reject $y$, and go to Step 1.

\[\text{In this thesis, we use } \gamma_k \text{ to denote the received SNR from the primary user for the } k\text{-th secondary user for spectrum sensing. To clarify, we use } \gamma_i \text{ to denote the received SNR from the channel for data transmission for the } i\text{-th user.}\]
For our case, the target sampling PDF is given in (II.1). Let us denote \( x = \frac{2\theta^2-c(\bar{\gamma}-\gamma_R)}{c\theta} \) so that \( \bar{\gamma} = \bar{\gamma}_R + \frac{2\theta^2}{c} - \theta x \). The PDF of \( x \) is then given by

\[
f_X(x) = \frac{2\theta}{c} \exp \left( \frac{2\theta}{c} x - \frac{2\theta^2}{c^2} \right) Q(x).
\] (VI.1)

Note that \( Q(x) < 1 \) and \( Q(x) < \frac{1}{2} \exp \left( -\frac{x^2}{2} \right) \) for \( x > 0 \). We can find the auxiliary \( f_Y(x) \) as

\[
f_Y(x) = \begin{cases} \frac{2\theta}{c} \exp \left( \frac{2\theta}{c} (x - \frac{\theta}{c}) \right) & x \in (-\infty, \theta/c] ; \\ \frac{2\theta}{c} & x \in (\theta/c, 2\theta/c) ; \\ \frac{2\theta}{c} \exp \left( -\frac{1}{2} (x - \frac{2\theta}{c})^2 \right) & x \in [2\theta/c, +\infty) ; \end{cases}
\] (VI.2)

where \( \phi = \frac{1}{1 + 2\theta^2/c^2 + 2\theta/c\sqrt{2\pi}} < 1 \) is a parameter to assure that \( \int_{-\infty}^{+\infty} f_Y(x)dx = 1 \).

With the above \( f_X(x) \) in (VI.1) and \( f_Y(x) \) in (VI.2), we can generate the samples for (II.1) through the following procedure:

**Step 1:** Sample \( u_0 \) from the uniform distribution \( U(0, 1) \). For \( u_0 \in (0, p_1) \), go to Step 2; for \( u_0 \in (p_1, p_2) \), go to Step 3; and for \( u_0 \in (p_2, 1) \), go to Step 4;

**Step 2:** Sample \( x_0 \) from the exponential distribution \( Exp(2\theta/c) \), and calculate \( \omega = Q(y_0) \) with \( y_0 = \theta/c - x_0 \); and go to Step 5;

**Step 3:** Sample \( x_0 \) from the \( U(0, 1) \), and calculate \( \omega = \exp(2y_0\theta/c - 2\theta^2/c^2)Q(y_0) \) with \( y_0 = (1 + x_0)\theta/c \); and go to Step 5;

**Step 4:** Sample \( x_0 \) from the normal distribution \( N(0, 1) \), and calculate \( \omega = \exp(2y_0\theta/c - 2\theta^2/c^2 + (y_0 - 2\theta/c)^2/2)Q(y_0) \) with \( y_0 = 2\theta/c + |x_0| \); go to Step 5;

**Step 5:** Sample \( u \) from \( U(0, 1) \). If \( u \leq \omega \), then accept \( y_0 \) as a sample from \( f_X(x) \), otherwise reject \( y_0 \) and go back to Step 1.

**Step 6:** \( \bar{\gamma} = \bar{\gamma}_R + 2\theta^2/c - \theta y_0 \) is a sample from (II.1).

In the above \( p_1 = \int_{-\infty}^{\theta/c} f_X(x)dx = 2Q(\theta/c) \) and \( p_2 = \int_{-\infty}^{2\theta/c} f_X(x)dx = 0.5 + \exp(2\theta^2/c^2)Q(2\theta/c). \)
References


