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Abstract

Modeling fractured reservoirs requires an accurate assessment of the hydraulic properties of discrete fractures. High-resolution Navier-Stokes simulations provide an accurate description of the flow within the fractures, including fracture upscaling. However, its excessive computational requirements make it impractical. The traditionally used cubic law for fracture upscaling is known for its poor accuracy. This work introduces a new analytical model based on cubic law to estimate hydraulic properties of 3D rock fractures with the irregular contact-area distribution.

We first briefly review the applicability of the existing models in the literature. Most of the current models exhibit major limitations related to prediction accuracy and practicality. For instance, most models neglect the effect of contact-area distribution. As a result, they fail to capture properly the heterogeneous and anisotropic flow behaviors caused by irregular contact-area distribution. In addition, some models work well only for idealized cases, such as parallel-smooth fractures with specific shapes of contact areas. Others are only valid for cases with fractional contact area and relative roughness less than certain values. Alternatively, we propose a dynamic indicator using a particle-percolation approach to quantify the effect of the contact-area distribution. We then introduce a more general modified cubic law, which accounts for different factors, including variable aperture, roughness, flow tortuosity, and contact-area distribution.

The proposed model predictions are then compared with the solutions of the 3D high-resolution Navier-Stokes simulations and experimental measurements collected from public resources. We assess the predictability of our model and benchmark it with other existing models from the literature with dozens of fracture cases. We show significant superiority of our model compared with other existing models, especially for cases with pronounced fracture anisotropy where the impact of the fracture contact area is dominant. Moreover, results show that the hydraulic aperture decreases with the increase of frictional contact area almost linearly. The contact-area distribution shows a significant impact on hydraulic properties even with the same size of contact regions.

We introduce a new analytical model to estimate the hydraulic properties of 3D partially-open rock fractures. Based on our benchmark that included dozens of fractures, our model is superior to commonly used models in the literature. The proposed model also retains the simplicity and efficiency of the cubic law, and can be easily implemented in workflows for reservoir characterization and modeling.

Introduction

Modeling fluid flow in fractured rocks requires an accurate assessment of the hydraulic properties of discrete fractures. High-resolution Navier-Stokes (NS) equations provide an accurate description of flow within fractures, including fracture upscaling. Its high computational costs, however, make it infeasible for practical applications. Subsurface rock fractures are subjected to stress, resulting in partially-open fractures (i.e., some parts with zero apertures) (He et al., 2020; He, Santoso, et al., 2021; J B Walsh, 1981). This work introduces a new analytical model based on cubic law to estimate hydraulic properties of 3D partially-open rock fractures with the irregular contact-area distribution.

The cubic law, assuming rock fractures as two smooth and parallel surfaces separated by a constant distance, has been widely implemented in many disciplines due to its simplicity and efficiency. Natural rock fractures, however, are formed by two roughed surfaces with variable aperture fields and irregular contact areas, which leads to significant deviations of hydraulic properties from the actual ones (Yeo, 2001; Zimmerman et al., 1992; Zimmerman & Bodvarsson, 1996).

Many efforts, including machine learning, have been proposed in the literature to improve the accuracy and applicability of cubic law. Examples include 1) modification of the definition of the aperture used in cubic law, such as arithmetic average (Brown, 1987), geometric average (Renshaw, 1995), averages proposed by others (e.g., Neuzil & Tracy, 1981; Zimmerman et al., 1991); 2) inclusion of correction factors that account for roughness influence (e.g., Louis, 1969; Patir & Cheng, 1978), tortuosity influence (e.g., Brown, 1987; Waite et al., 1999), combined roughness-tortuosity influence (e.g., He, Sinan, et al., 2021; Xiao et al., 2013); 3) implementation of machine learning techniques (He, Zhu, et al., 2021; Sun et al., 2020). All above modifications, however, are designed for 2D rock fractures or 3D fully-open fracture cases (e.g., no contact areas). In other words, these above models will break down when dealing with partially open fractures. Only a few researchers incorporate the contact-area influence by including a contact correction term (Mofakham et al., 2018; Walsh, 1981; Wang et al., 2014; Yeo, 2001; Zimmerman et al., 1992). All these models, however, neglect the effect of contact-area distribution. As a result, they fail to capture properly the heterogeneous and anisotropic flow behaviors caused by irregular contact-area distribution. Moreover, some models show limited applicability. For example, some work well only for parallel-smooth fractures with specific shapes of contact areas (e.g., Mofakham et al., 2018; Walsh, 1981; Zimmerman et al., 1992). Others are only valid for fracture cases with fractional contact area and relative roughness less than certain values (e.g., Wang et al., 2014; Yeo, 2001; Zimmerman & Bodvarsson, 1996). A detailed description of these models and their limitations will be presented in the next section.

Alternatively, we propose a dynamic indicator using a particle-percolation approach to quantify the effect of the contact-area distribution. We then introduce a modified cubic law designed for more general cases, which accounts for different factors, including variable fracture aperture, wall roughness, flow tortuosity, and contact-area distribution. The proposed model shows general applicability for fracture cases with any relative roughness and fractional contact area. The proposed model predictions are then compared with the solutions of the 3D high-resolution Navier-Stokes (NS) simulations and experimental measurements collected from public resources. We assess the predictability of our model and benchmark it with other existing models from the literature with several fracture cases. This work introduces a new analytical model to estimate the hydraulic properties of 3D partially-open rock fractures. Based on our benchmark that included dozens of fractures, our model is superior to commonly used models in the literature. The proposed model also retains the simplicity and efficiency of the cubic law, and can be easily implemented in workflows for reservoir characterization and modeling.

The paper is organized as follows. In section 2, we present a comprehensive review of existing models and their limitations. Section 3 provides the development of the proposed model. We then show the results and discussion in section 4, in which the proposed model is compared to the high-resolution NS solutions, public experimental measurements, and other existing models. Finally, we close the paper with the conclusions and future work.

Literature Review - Limitations

The modifications on 2D and 3D fully open fractures will break down when dealing with 3D fracture cases with contact areas. Contact areas should be treated differently from conductive void areas (Piggott & Elsworth, 1992; Walsh, 1981; Zimmerman et al., 1992). These researchers investigated the contact-area influence on flow by considering parallel-plate fracture with contact areas, where nominal aperture of the conductive void areas is kept to be constant (Figure 1). This section will provide a comprehensive review of almost all existing models that consider the contact-area influence and a summary of their limitations.

Figure 1. Flow through the parallel-plate fracture with circular contact area.

Walsh (1981) investigated the contact-area influence by considering the parallel plate fracture with circular contact area. The following equation was derived using Maxwell's effective medium approach:

$$a_h^3 = a_n^3 \frac{1-c}{1+c} \quad (1)$$

Where a_h is hydraulic aperture, a_n is nominal aperture, c is fractional contact area.

Zimmerman et al. (1992) extended Walsh (1981)'s model to more general cases with randomly oriented elliptical contact area, based on the basic solution to two-dimensional flow around an elliptical obstacle derived by (Obdam & Veling, 1987) and proposed:

$$a_h^3 = a_n^3 \frac{1-\beta c}{1+\beta c} \quad (2)$$

Where,

$$\beta = (1+\alpha)^2 / 4\alpha \quad (3)$$

Where α is aspect ratio of the elliptical contact area. Eq. (2) reduces to Eq. (1) when $\alpha = 1$. For the other cases (i.e., $\alpha > 1$), β is always greater than unity, exhibiting a monotonical increase as the ellipse

becomes more elongated. It, therefore, indicates elliptical shape shows a more significant influence than the circular one. Zimmerman et al. (1992) further demonstrated Eq. (2) could be applied to parallel-plate fracture with the irregularly shaped contact area only if the actual contact area is replaced by an elliptical shape honoring the same perimeter.

Zimmerman & Bodvarsson (1996) further generalized its application to rough rock fractures by incorporating the effects of roughness and contact area. The second term indicates the roughness influence, and the third term for contact-area influence. They adopted the contact correction term $(1 - 2c)$ proposed by Kirkpatrick (1973) instead of the one in Eq. (2), as it's tricky to determine the aspect ratio of irregularly shaped contact areas, especially with rough fracture cases.

$$a_h^3 = a_m^3 \left[1 - 1.5 \left(\frac{\sigma}{a_m} \right)^2 \right] (1 - 2c) \quad (4)$$

Where a_m and σ is the arithmetic mean and standard deviation of the aperture fields.

Yeo (2001) modified the contact correction in Eq. (4) to $(1 - 2.4c)$ based on numerical simulation of Reynolds equation, yielding better estimation, especially for high fractional contact area.

$$a_h^3 = a_m^3 \left[1 - 1.5 \left(\frac{\sigma}{a_m} \right)^2 \right] (1 - 2.4c) \quad (5)$$

Wang et al. (2014) modified Eq. (5) by representing the roughness correction term based on fractal characteristics of rough fracture.

$$a_h^3 = a_m^3 \exp^{-(D-2)} (1 - 2.4c) \quad (6)$$

Where D is the fractal dimension of rough fracture.

Mofakham et al. (2018) proposed a new model by combining the roughness correction term from Zimmerman & Bodvarsson (1996) and the contact correction term from Walsh (1981).

$$a_h^3 = a_n^3 \left[1 + 9.0 \left(\frac{\sigma}{a_m} \right)^2 \right]^{-1/2} \frac{1 - c}{1 + c} \quad (7)$$

We summarize these models and their limitations in Table 1. The objective of this work is to develop an accurate and robust analytical model that could overcome these limitations. We will detail the development of the proposed model in the next section.

Table 1 summarizes the above models and their limitations.

Year	Model	Limitations
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1981 Walsh	$a_h^3 = a_n^3 \frac{1-c}{1+c}$	<ul style="list-style-type: none"> Valid for parallel-plate fractures with circular contact areas Fail to capture the effect of contact-area distribution Fail to capture anisotropic flow behavior caused by contact areas
1992 Zimmerman et al	$a_h^3 = a_n^3 \frac{1-\beta c}{1+\beta c}$	<ul style="list-style-type: none"> Valid for parallel-plate fractures with elliptical contact areas Regarding irregularly-shaped contact areas, it's difficult to determine the proper aspect ratio Fail to capture the effect of contact-area distribution Cannot better capture anisotropic behavior caused by contact areas
1996 Zimmerman and Bodvarsson	$a_h^3 = a_m^3 \left[1 - 1.5 \left(\frac{\sigma}{a_m} \right)^2 \right] (1-2c)$	<ul style="list-style-type: none"> Only valid for $c \leq 0.5$ and $(\sigma/a_m) \leq 0.8165$ Fail to capture the effect of contact-area distribution Fail to capture anisotropic flow behavior caused by contact areas
2001 Yeo	$a_h^3 = a_m^3 \left[1 - 1.5 \left(\frac{\sigma}{a_m} \right)^2 \right] (1-2.4c)$	<ul style="list-style-type: none"> Only valid for $c \leq 0.4167$ and $(\sigma/a_m) \leq 0.8165$ Numerical simulations based on Reynolds equations are used to modify the contact correction term from $(1-2c)$ to $(1-2.4c)$ Fail to capture the effect of contact-area distribution Fail to capture anisotropic flow behavior caused by contact areas
2014 Wang et al	$a_h^3 = a_m^3 \exp^{-(D-2)} (1-2.4c)$	<ul style="list-style-type: none"> Only valid for $c \leq 0.4167$ Fail to capture the effect of contact-area distribution Fail to capture anisotropic flow behavior caused by contact areas Complex process to calculate D
2018 Mofakham et al	$a_h^3 = a_m^3 \left[1 + 9.0 \left(\frac{\sigma}{a_m} \right)^2 \right]^{-1/2} \frac{1-c}{1+c}$	<ul style="list-style-type: none"> No physical insight – just a simple combination of roughness and contact correction terms, in which contact correction term is adopted from the parallel-plate fracture with circular cases Fail to capture the effect of contact-area distribution Fail to capture anisotropic flow behavior caused by contact areas

Proposed Model Development

In this section, we will present a modified cubic law designed for more general cases, which accounts for different factors, including variable fracture aperture, wall roughness, flow tortuosity, and contact-area distribution. The effect of contact-area distribution and anisotropic flow behaviors are captured using a dynamic indicator based on the particle-percolation approach.

The equivalent channel model developed by Walsh & Brace (1984) states the expression of computing rock permeability (see Figure 2) and takes the form:

$$k = \phi^3 \frac{V_b^2}{\beta \tau^2 A_s^2} \quad (8)$$

Where V_b is the total volume, including pore and solid volume; β is the shape factor, for circular tubes $\beta = 2$ and parallel plates $\beta = 3$; A_s is the surface area of pore spaces; τ is the porosity.

Figure 2. Equivalent channel model for permeability analysis of rock (modified from Walsh & Brace (1984)) left: tortuous flow path represented by the streamline; right: chosen representative network with A_x/A equal to the porosity of the rock in left and the tortuosity of dl/dx equal to the one (denoted by L/X) in left.

We apply the concept of the equivalent channel by extending it from porous media to rock fractures. Figure 3 shows the theoretical analysis for the development of the proposed model. We consider two cases: (1) fracture without contact areas (see Figure 3 (b)) ; (2) fracture with contact areas (see Figure 3 (c)). We make the following assumptions:

- 1) The solid volume is negligible compared to the void volume constrained by the top and bottom fracture surfaces. It implies the total volume is equal to the fracture void volume.
- 2) We assume the total volumes of these two cases are the same.
- 3) The matrix permeability is negligible compared to the fracture permeability, which shows the rock permeability is equal to the fracture permeability.
- 4) For case 1, the tortuosity is assumed to be unity (i.e., $\tau = 1$). This assumption is supported by the streamlines that are essentially parallel to the direction of the pressure gradient in fully-open fractures.

Based on these four assumptions, the permeability of fracture cases with contact areas could be derived theoretically. We will detail the procedure as follows:

Regarding case 1 (see Figure 3 (b)), the fracture permeability could be written based on Eq. (8):

$$k_I = \phi^3 \frac{V_b^2}{\beta \tau^2 A_s^2} = \frac{V_b^2}{\beta A_s^2} \quad (9)$$

Where V_b is the volume constrained by the top and bottom fracture surfaces and has the following form:

$$V_b = A_p a_m \quad (10)$$

Where A_p is the projected surface area of A_s and a_m is the average aperture.

Regarding case 2 (see Figure 3 (c)), the porosity of fracture decreases because of the existence of contact areas and could be written as:

$$\phi_{contact} = 1 - c \quad (11)$$

Where c is the fractional contact area. The corresponding fracture permeability takes the form:

$$\frac{k_{II}}{k_I} = \frac{(1-c)^3}{\tau_{contact}^2} \quad (12)$$

The ratio of Eq. (12) to Eq. (9) yields:

$$\frac{k_{II}}{k_I} = \frac{(1-c)^3}{\tau_{contact}^2} \quad (13)$$

The theoretical model proposed by Zimmerman & Bodvarsson (1996) describes the relationship between hydraulic aperture and mechanical aperture for 3D fully-open fractures and takes the form:

$$a_h^3 = a_m^3 \left[1 + 9.0 \left(\frac{\sigma}{a_m} \right)^2 \right]^{-1/2} \quad (14)$$

Combining Eq. (14) and Eq. (13) yields the proposed new model for 3D rough fracture cases with contact areas.

$$a_h^3 = a_m^3 \left[1 + 9.0 \left(\frac{\sigma}{a_m} \right)^2 \right]^{-1/2} \frac{(1-c)^3}{\tau_{contact}^2} \quad (15)$$

Our proposed model shows general applicability compared to other existing models and works for fracture cases with any ranges of relative roughness (i.e., σ/a_m could be any value) and fractional contact area (i.e., $0 \leq c \leq 1$). Eq. (15) will reduce to Eq. (14) when $c = 0$. The fracture is full of solid with zero permeability when the extreme case of $c = 1$. The physical insight of each term in the right-hand side of Eq. (15) could be understood as the 1st term to account for the effect of aperture distribution, 2nd term as the effect of roughness, 3rd term as the effect of contact areas, and the 4th term reflect the effect of flow tortuosity.

Because of the existence of contact areas, the significant tortuous flow will occur around contact areas. The tortuosity, defined as the ratio of flow-path to nominal distances, not only depends on the sizes of contact areas also their locations. Thus, we use the indicator $\tau_{contact}$ to capture the contact-area distribution

and anisotropic flow behaviors. In this work, we present an efficient approach to estimate the streamline within fractures based on the particle-percolation method without solving any simulations. We detail the workflow as follows.

Figure 3. Analysis model for 3D contacted, rough fracture. (a) Illustration for a 3D partially-open fracture; (b) fracture without contact areas; (c) fracture with contact areas and its simplified parallel-plate model with equal size of contact areas. Note: for illustration purposes, 2D fractures are used in (b) and (c).

Figure 4 illustrates the algorithm implemented for estimating the potential flow streamlines. This algorithm is based on the concept the flow is always along the direction, where its aperture honors the biggest values. Provided with the given aperture fields, the potential next point is determined based on the the aperture values. If the potential next point is touching the no-flow boundaries, it fails to form a complete flow path and stops. If it is within the inside of the domain, it will continue until it reaches the outlet boundary to create a complete flow path. The contact areas are assigned with apertures of zero. When all flows end, we calculate each streamline length and further its tortuosity. Finally, the overall tortuosity is computed by averaging all these individual tortuosities. Regarding different contact-area distributions, the resulting potential streamlines are different. We can differentiate different cases by using the dynamic indicator, i.e., the overall tortuosity.

In summary, we provide a modified cubic law, which accounts for different factors, including variable fracture aperture, wall roughness, flow tortuosity, and contact-area distribution. The proposed model could describe the flow behaviors within 3D partially-open fractures and demonstrate more general applicability designed for fracture cases with any relative roughness and any fractional contact areas. To our best knowledge, our model is the first to capture the effect of contact-area distribution and anisotropic flow behaviors. The proposed model also retains the simplicity and efficiency of the cubic law, and can be easily implemented in workflows for reservoir characterization and modeling. We demonstrate the superiority of the proposed model by comparing it with other existing models, high-resolution NS solutions, experimental measurements in the next section.

Figure 4. Schematic of the algorithm used for estimating potential streamlines.

Results and Discussion

In this section, we will demonstrate the proposed model by comparing it with experimental results, high-resolution NS solutions, and further other existing analytical models.

1. Comparison with experimental data

We test our proposed model with experimental measurements by (Xiao et al., 2013), in which the coupled flow-normal-shear tests are conducted. We choose the cases with a shear of 1 mm under the normal stress of 3 MPa. All the required parameters values are available in Table 1-5 in the reference (Xiao et al., 2013) except for Wang et al. (2014)'s model since no information regarding fractal dimension is provided. The proposed model shows the best accuracy among all available analytical models, as shown in Figure 5.

Figure 5. Comparison between different models with experimental measures by Xiao et al. (2013). They are sorted in descending order in terms of accuracy.

2. Comparison with high-resolution NS solutions

High-resolution Navier-Stokes (NS) equations accurately describe flow within fractures, including fracture upscaling, and its solutions will be considered as the ground truth. (Brush & Thomson, 2003; He, Sinan, et al., 2021; Robert W Zimmerman & Yeo, 2000). The superiority of the proposed model will be illustrated in the following two aspects.

- Anisotropic behavior: flow in X- and Y-direction

We consider a parallel-plate fracture with a rectangular contact area with its centroid located at the center of the fracture, as shown in Figure 6. The related dimensions of fracture, contact area, and nominal aperture are shown in Figure 6. We investigate the anisotropic behaviors (flow in X and Y directions, respectively) caused by the rectangular contact area using this case. Obviously, all existing analytical models fail to capture the anisotropic characteristics except for the model by Zimmerman et al. (1992), yet challenging to determine the proper aspect ratio. These models lead to the same hydraulic properties in X- and Y-direction, respectively.

Figure 6. Anisotropic characteristics (flow in X and Y directions, respectively) caused by the irregular contact area.

As discussed above, we implement a dynamic tortuosity to capture the anisotropic characteristics based on the particle-percolation approach. In other words, the dynamic flow tortuosity is to differentiate the flow in X and Y directions. We compare the proposed model predictions with the high-resolution NS solutions, referred to as the ground-truth solutions. The NS equations are solved using the mixed finite element formulation within the COMSOL framework. We assure numerical stability by increasing the number of grid cells until the two consecutive solutions are within 0.1 %. A detailed description of the mixed finite element formulation of NS equations and imposed boundary conditions could be found in Appendix A. Observations from high-resolution NS simulations (see Figure 7) show different flow behaviors in X and Y directions, resulting in different hydraulic properties. Figure 8 shows the comparison between the high-resolution NS and analytical solutions. Results show that only the proposed model could

capture the anisotropic characteristics and provide the most accurate hydraulic aperture estimation among all available analytical models. These two cases are differentiated using the proposed dynamic tortuosity.

Figure 7. Streamline distributions obtained from high-fidelity NS simulations show significantly different flow behaviors from X and Y directions, respectively.

Figure 8. The superiority of the proposed model in capturing anisotropic characteristics compared to other models.

- Heterogeneous behavior: contact-area distribution

We further superiority of the proposed model in describing the heterogeneous flow behavior caused by different contact-area distributions. Figure 9 illustrates the fracture cases with the same contact area yet with different contact-area distributions. When it comes to heterogeneous characteristics, we only consider in one direction – we take flow behaviors in the X direction as an example for these two fracture cases. Streamlines from NS solutions exhibit significantly different flow behaviors for these two cases, which

would result in different hydraulic properties, as shown in Figure 10. Figure 11 shows the comparison between the high-resolution NS and analytical solutions. Results show that only the proposed model could capture the heterogeneous characteristics and provide the most accurate estimation of hydraulic aperture among all analytical models.

Figure 9. Heterogeneous characteristics caused by different contact-area distributions, although with the same contact areas.

Figure 10. Streamline distributions obtained from high-fidelity NS simulations show significantly different flow behaviors for these two cases.

Figure 11. The superiority of the proposed model in capturing heterogeneous characteristics compared to other analytical models.

Conclusions

This work provides an accurate analytical model based on cubic law for estimating hydraulic properties of 3D partially-open fractures. We summarize the main findings as follows:

- Comprehensive literature of existing analytical models is presented. Their applicability and corresponding limitations are analyzed. Most of these models show limited applicability – only work for cases with certain ranges of relative roughness and fractional contact area. In addition, these models failed to capture appropriately the heterogeneous and anisotropic flow behaviors caused by irregular contact-area distribution.
- We proposed a theoretical model based on the concept of the equivalent channel model, which honors the physical insight. This model shows general applicability by incorporating the effects of aperture distribution, roughness, contact area, and tortuosity. The effect of contact-area distribution and anisotropic flow behaviors are captured using a dynamic indicator based on the particle-percolation approach.
- We demonstrate the accuracy of the proposed model with experimental data collected from the public resources, which provides the best accuracy among all other existing analytical models.
- We illustrate the shortcomings of existing analytical models by running the high-resolution NS simulations and demonstrate the superiority of our model, which could capture the heterogeneous and anisotropic behaviors caused by the irregular contact-area distribution.

Future work will focus on more rigorous validation work: 1) collect more experimental data 2) run more high-resolution NS simulations on fracture cases with different aperture distributions and extensive ranges of relative roughness and fractional contact areas.

Appendix A: Mixed Finite Element Formulation

Consider the steady-state of incompressible, Newtonian laminar flow with no gravity effects, and the full-physics NS equations can be given as:

$$\begin{aligned}\rho(\bar{\mathbf{u}} \bullet \nabla \bar{\mathbf{u}}) &= -\nabla p + \mu \nabla^2 \bar{\mathbf{u}} \\ \nabla \bullet \bar{\mathbf{u}} &= 0\end{aligned}\tag{A1}$$

In the above equations, $\bar{\mathbf{u}}$ is velocity vector, ρ is density, p is pressure, and μ is viscosity.

We assume no-slip boundary conditions applied to the fracture walls (except for inlet and outlet surfaces) and contact areas, and pressure values are set at the fracture's outlet and inlet, as shown in Figure A1. The gradient of velocity in the direction of the pressure gradient is set to zero to guarantee the fully developed flow and to avoid the inlet effect.

Figure A1. Illustration of boundary conditions imposed on a 3D partially-open rough fracture.

The NS equations can be formulated in a mixed variational form, where the pressure, and the velocity, are simultaneously approximated. Multiplying Eq. (A1) by a test function $(\bar{\mathbf{v}}, q)$ and integrating the resulting equations over the domain Ω yields:

$$\begin{aligned}\int_{\Omega} \rho(\bar{\mathbf{u}} \bullet \nabla \bar{\mathbf{u}}) \bullet \bar{\mathbf{v}} dx &= \int_{\Omega} (-\nabla p) \bullet \bar{\mathbf{v}} dx + \int_{\Omega} (\mu \nabla^2 \bar{\mathbf{u}}) \bullet \bar{\mathbf{v}} dx \\ \int_{\Omega} (\nabla \bullet \bar{\mathbf{u}}) q dx &= 0\end{aligned}\tag{A2}$$

Applying the integration by parts technique, we have:

$$\begin{aligned}\int_{\Omega} (-\nabla p) \bullet \bar{\mathbf{v}} dx &= \int_{\Omega} p \nabla \bullet \bar{\mathbf{v}} dx - \int_{\partial\Omega} p \bar{\mathbf{v}} \bullet \bar{\mathbf{n}} ds \\ \int_{\Omega} (\mu \nabla^2 \bar{\mathbf{u}}) \bullet \bar{\mathbf{v}} dx &= -\mu \int_{\Omega} \nabla \bar{\mathbf{u}} : \nabla \bar{\mathbf{v}} dx + \mu \int_{\partial\Omega} \nabla \bar{\mathbf{u}} \bullet \bar{\mathbf{n}} \bar{\mathbf{v}} ds\end{aligned}\tag{A3}$$

The problem becomes, find $(\bar{\mathbf{u}}, p) \in W$, such that,

$$a\left(\left(\bar{u}, p\right),\left(\bar{v}, q\right)\right)=L\left(\left(\bar{v}, q\right)\right) \quad (\text{A4})$$

where $\left(\bar{v}, q\right) \in W$, and

$$a\left(\left(\bar{u}, p\right),\left(\bar{v}, q\right)\right)=\rho \int_{\Omega}\left(\bar{u} \bullet \nabla \bar{u}\right) \bullet \bar{v} d x-\int_{\Omega} p \nabla \bullet \bar{v} d x+\mu \int_{\Omega} \nabla \bar{u}: \nabla \bar{v} d x+\int_{\Omega}\left(\nabla \bullet \bar{u}\right) q d x \quad (\text{A5})$$

$$L\left(\left(\bar{v}, q\right)\right)=-\int_{\partial \Omega_N} p_{inlet} \bar{v} \bullet \bar{n} d s-\int_{\partial \Omega_N} p_{outlet} \bar{v} \bullet \bar{n} d s \quad (\text{A6})$$

In the above equation, $W=V \times Q$ is a mixed-function space such that for $q \in Q$ and $\bar{u} \in V$.

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