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Abstract

Explicit modeling of discrete fractures at the field scale is computationally intensive, and therefore, upscaling the fractures in the context of an equivalent continuum model is indispensable for field-scale simulations. This work introduces a new upscaling technique based on the local-global multiple-boundary (LG-MB) upscaling method for fluid flow in naturally fractured reservoirs.

We extend the commonly-used local-global (LG) upscaling method by introducing a new approach to compute the fluid fluxes within the fractures crossing the grid-block boundaries. This method is based on the multiple-boundary (MB) approach applied within a local-global upscaling procedure. The global coarse-scale simulations are implemented to determine the boundary conditions for the calculations of local upscaled permeability. The procedure allows repeating the process to assure consistency between the global and local calculations. We implement the multi-point flux approximation (MPFA) finite volume method coupled with embedded discrete-fracture model within the MRST framework. We then verify the proposed LG-MB upscaling technique by comparing it with the reference fine-scale solutions and other existing local and local-global upscaling techniques. Results show that the proposed approach provides pronounced accuracy compared to local-based upscaled models with minor computational overhead. Compared to traditional local-global upscaling techniques, it provides more computational accuracy yet with the same efficiency. The superiority of the proposed LG-MB upscaling technique is attributed to two factors related to 1) the multiple-boundary technique to capture the flow behavior with high anisotropy from fractures non-alignment with the grid; 2) the use of local-global approach to accurately obtain the real flow boundary conditions. This work introduces a new upscaling method for flow simulation in naturally fractured reservoirs. We demonstrate its applicability in field applications and its superiority to existing upscaling methods. This method is accurate and straightforward and can be implemented within existing upscaling workflows.

Introduction

Accurate modeling of fluid flow in fractured formations is crucial for various environmental and energy applications, such as the oil recovery process in fractured reservoirs, geological CO₂ sequestration, geothermal energy extraction, and water management (Cardona et al., 2021; Geiger et al., 2010; Hoteit et al., 2019; Hoteit & Firoozabadi, 2008a, 2008b; Vahrenkamp et al., 2020; Younes et al., 2021). Detailed

geological characterization of fractured reservoirs has been commonly described by the discrete-fracture model (DFM), in which matrix and fractures are explicitly represented using unstructured grids. Explicit modeling of discrete fractures at the field scale is computationally intensive, and therefore, upscaling the fractures in the context of an equivalent continuum model (ECM) is indispensable for field-scale simulations. Accurate calculation of the upscaled properties of fractured media is crucial for obtaining a successful ECM such that it could capture the key flow behaviors as the fine-scale model.

Upscaling approaches used to compute upscaled properties in fractured media could be categorized into three kinds: 1) analytical methods (e.g., Oda, 1985); 2) flow-based methods (e.g., Chen et al., 2015); 3) machine learning-based methods (e.g., He, Santoso, Alsinan, et al., 2021; He, Santoso, et al., 2020). Analytical upscaling methods suffer from strong assumptions and work well only for specific idealized cases. Machine learning-based upscaling methods lack physical insight. Herein, we focus on flow-based upscaling techniques. Based on the approach to calculate flow rates and pressure gradients, flow-based upscaling methods could be divided into volume-averaged method and boundary-integrated method (Durlafsky, 2005). For example, Lang et al. (2014) computed upscaled permeability using volume-averaged method with finite element method. Boundary-integrated method could be further divided into single-boundary (SB) upscaling method developed by Long et al. (1982) and multiple-boundary (MB) upscaling method developed by Chen et al. (2015). The difference is the way to compute flow rates, in which the SB method considers flow rates on one single boundary, either parallel or orthogonal to the pressure gradient. In contrast, the MB method considers flow rates on multiple boundaries, both parallel and orthogonal to the pressure gradient. The MB upscaling method provides a more accurate calculation in terms of upscaled properties than SB one (T. Chen et al., 2015). A detailed description of the difference will be provided in the next section. From another perspective, flow-based upscaling methods could be categorized into local, extended local, global, local-global upscaling methods according to the size of the region for the calculation of the upscaled properties. Three types of boundary conditions (no-flow, periodic, and linear pressure) are commonly implemented during local or extended local upscaling procedures. Such boundary conditions are not realistic for flow in highly heterogeneous fractured media due to the existence of fractures. Real boundary conditions should be implemented instead to obtain accurate calculations of local upscaled properties. Local-global (LG) upscaling approach offers an efficient way to obtain the real boundary conditions (Y. Chen et al., 2003).

This work introduces a novel upscaling method - extending the commonly-used local-global (LG) upscaling method by introducing the multiple-boundary (MB) approach to compute the fluid fluxes within the fractures. The global coarse-scale simulations are implemented to determine the boundary conditions for the calculations of local upscaled permeability. The procedure allows repeating the process to assure consistency between the global and local calculations. We implement the multi-point flux approximation (MPFA) finite volume method for embedded discrete-fracture simulation within the MRST framework. We summarize the superiority of the proposed LG-MB upscaling technique as the following two factors related to 1) the multiple-boundary technique to capture the flow behavior with high anisotropy from fractures non-alignment with the grid; 2) the use of local-global approach to accurately obtain the real flow boundary conditions. To our best knowledge, this upscaling approach is implemented for the first time in fractured reservoirs.

The paper is organized as follows. Section 2 will detail the proposed LG-MB upscaling method. We then verify the proposed upscaling method using simple cases with analytical solutions in section 3. General applicability is demonstrated by applying it to cases with complex fracture networks in section 4. Section 5 will provide conclusions and future work of this work.

Proposed Upscaling Procedure

In this section, we first present governing equations on fine-scale (i.e., discrete-fracture model) and coarse-scale models (i.e., equivalent continuum model). We then motivate the implementation of the local-global upscaling method by highlighting the significance of boundary conditions during local upscaling. Finally,

the multiple-boundary approach used for accurate flow rate calculation is introduced. This work presents a novel upscaling method by combining the commonly-used local-global (LG) upscaling method with the multiple-boundary (MB) approach, in which the LG approach is for obtaining actual boundary conditions for local upscaling and the MB for accurate calculation of flow rates crossing the grid boundaries.

1. Governing equations on fine- and coarse-scale models

Fine-scale model (see Figure 1 (a)): considering a single-phase, steady-state, and incompressible flow, the governing equation could be written:

$$\nabla \cdot \left(-\frac{k}{\mu} \nabla p \right) = 0 \quad (1)$$

Where p is pressure; μ is fluid viscosity; k is permeability of matrix or fracture. We assume the matrix to be homogeneous and isotropic, and fracture permeability is calculated based on the cubic law (He, Sinan, et al., 2021).

$$k = \frac{a_f^2}{12} \quad (2)$$

Where a_f is the hydraulic aperture of fracture.

Coarse-scale model (see Figure 1 (b)): upscaling is required to obtain the equivalent permeability tensor for each of Cartesian grids in Figure 1 (b) so that the coarse-scale model could capture key flow patterns as the fine-scale model. The governing equation for the coarse-scale model takes the form:

$$\nabla \cdot \left(-\frac{\vec{k}}{\mu} \nabla p^c \right) = 0 \quad (3)$$

Where $\vec{k}$ is the equivalent permeability is a second-rank and symmetric tensor, as shown in Figure 1 (c), and p^c is the coarse-scale pressure.

Figure 1. Schematic showing the fine-scale model (a), coarse-scale model (b), and an exaggerated grid-block with an equivalent permeability tensor $\vec{k}$.

2. Local-Global upscaling procedure: Real boundary conditions

For a given coarse-scale grid-block (see Figure 1 (c)), the boundary conditions should be imposed in order

to compute the equivalent permeability. The boundary conditions implemented during the local upscaling significantly impact the final results (Renard & De Marsily, 1997). Generally, three types of boundary conditions are widely used (see Figure 2 (a), (b), and (c)). No-flow boundary conditions are the most straightforward to implement, which results in diagonal permeability tensor with off-diagonal values of zero. Although periodic and linear boundary conditions generate full permeability tensor, such boundary conditions are not realistic for flow in highly heterogeneous fractured media due to the existence of fractures. Moreover, they assume a locally constant pressure difference along the coarse-scale grid. Global fine-scale simulations are performed to extract the actual boundary conditions over the coarse-scale grid block. All the above three assumed local boundary conditions would break down, to some extent, especially in the highly heterogeneous and anisotropic system (Y. Chen et al., 2003).

Figure 2. Four types of boundary conditions during the calculation of local upscaled properties.

The coupled local-global method developed by Y. Chen et al. (2003) offers an efficient approach to extracting the actual pressure boundary conditions over the local domain without solving the global fine-scale simulations. We extend the commonly-used local-global (LG) upscaling method by introducing a new approach called multiple-boundary (MB) to compute the fluid fluxes crossing the grid-block boundaries. Figure 3 illustrates the flowchart of the proposed LG-MB upscaling method. We detail the whole upscaling procedure as follows. We first perform standard (extended) local multiple-boundary upscaling with constant pressure difference over the grid and linear pressure boundary conditions to obtain an initial upscaled $\vec{k}$. Based on the initial $\vec{k}$, we conduct global coarse-scale simulations to obtain the coarse-scale pressure (denoted by p^c). These coarse-scale pressures are then interpolated to provide local

boundary conditions for the (extended) local region. The local flow problems are solved again with these interpolated pressure boundary conditions, and the updated $\vec{k}$ are recalculated based on the flow rates using the MB approach. We again conduct the global coarse-scale simulations to update the coarse-scale pressures. The procedure is repeated to assure consistency between the global and local calculations. A detailed description of the MB approach for accurate flow rate computation is provided in the following subsection.

Figure 3. Flowchart of the proposed LG-MB upscaling method.

3. Flow rate computation: Single-boundary versus Multiple-boundary

Accurate calculation of flow rates is crucial for obtaining an accurate equivalent permeability tensor. This section will deliver two methods for the flow rates computation. The commonly used single-boundary method computes the flow rates only based on one single-boundary, either parallel (for calculating q_x) or orthogonal (for calculating q_y) to the pressure gradient. The method neglects the effect of fracture orientation for cases that are not aligned with the grid boundaries. The expressions for computing q_x and q_y take the form:

$$\begin{aligned} q_x &= \int_0^{l_y} (q_r \bar{n}_x) dy \\ q_y &= \int_0^{l_x} (q_u \bar{n}_y) dx \end{aligned} \quad (4)$$

Where q_x and q_y are flow rates in X and Y directions, respectively. q_r and q_u are flow rates through right and upper boundaries, respectively, in which flow rates come from the matrix (denoted by q^m) and fracture (q^f); $\bar{n}_x$ and $\bar{n}_y$ are the unit vectors in X and Y directions, respectively (see Figure 4); l_x and l_y are the lengths of the domain in X and Y directions, respectively (see Figure 4).

In contrast, the multiple-boundary method computes flow rates based on multiple boundaries, both parallel

and orthogonal to the pressure gradient by considering the fracture orientations (see red dashed line in Figure 4). The method is based on the fact that flow rates within fractures are vectors, which could be divided into two components in X and Y directions (see Figure 4). Thus, the new expressions for computing q_x and q_y could be written based on multiple boundaries as follow:

$$\begin{aligned} q_x &= \int_0^{l_y} (q_r^f \overline{n_x}) dy + \int_0^{l_x} (q_u^f \overline{n_x}) dx + \int_0^{l_y} (q_l^f \overline{n_x}) dy \\ q_y &= \int_0^{l_x} (q_u^f \overline{n_y}) dx + \int_0^{l_x} (q_r^f \overline{n_y}) dx + \int_0^{l_y} (q_l^f \overline{n_y}) dx \end{aligned} \quad (5)$$

The parts in blue highlight the difference between the single-boundary and multiple-boundary methods, indicating the necessity of considering the fracture orientation influence in the accurate computation of flow rates. The accuracy of the multi-boundary method in flow rate computation has been demonstrated by T. Chen et al. (2015) and will be implemented in this work.

Figure 4. Fracture flow rates (denoted by q^f) cross the boundaries with subscripts u , r and l representing upper, right and lower boundaries, respectively. The red dashed lines represent possible fractures that are non-alignment with the grid boundaries.

The proposed LG-MB upscaling procedure, to our best knowledge, honors the most advantages compared to other existing upscaling methods, in which the LG approach is implemented for extracting the actual boundary conditions for local upscaling and the MB method is for accurate calculation of flow rates crossing the grid boundaries. We implement the multi-point flux approximation (MPFA) finite volume method for embedded discrete-fracture simulation within the MRST framework (Lie et al., 2012).

Verification: Single Rotating Fracture

We first verify the proposed LG-MB upscaling approach by applying it to a set of single rotating fractures with known analytical solutions.

We consider a single rotating fracture in a Cartesian matrix with a dimension of $1\text{m} \times 1\text{m}$ (see Figure 5). The centroid of the square domain exactly overlaps with the center of fracture. The fracture aperture is assumed to be 1 mm with a permeability of 8.44×10^7 mD based on the cubic law (He, Hoteit,

et al., 2020; He, Santoso, Zhu, et al., 2021; He, Sinan, et al., 2021). We assume the matrix as homogeneous and isotropic media with a permeability of 10 mD. The solid black line represents the rotating fracture with azimuth θ ranging from 0 to π with the step size of $\pi/18$.

Figure 5. A rotating fracture within a 2D square matrix domain. The solid black line represents the fracture with an azimuth of θ .

The equivalent permeability tensor of the above fracture configuration could be derived analytically based on the coordinate rotations of the permeability tensor ellipse (Lough et al., 1998). A detailed description of analytical computation could be found in **Appendix A**. We then compare our numerical results with the known analytical solutions. Numerical stability is assured by increasing the number of grids until the two consecutive solutions are within 1 %. Figure 6 shows a perfect match between the proposed upscaling method and analytical results for different fracture azimuths. No iteration is required since only one coarse-scale grid is in this verification case.

Figure 6. Comparison between the proposed upscaling method and analytical solutions in equivalent permeability tensor components for different fracture azimuths.

Application: Complex Fracture Networks

This section will demonstrate the general applicability of the proposed upscaling approach by extending it to a more complex fracture network. We consider a 2D porous media with a dimension of $10 \text{ m} \times 10 \text{ m}$, including complex fracture networks, as shown in Figure 7 (a). This discrete-fracture characterization is upscaled to a coarse-scale model with 10×10 Cartesian grids with fracture networks superimposed (Figure 7 (b)). An exaggerated $1 \text{ m} \times 1 \text{ m}$ grid-block with one fracture is shown in Figure 7 (c).

Figure 7. (a) Discrete-fracture characterization with complex fracture networks; (b) upscaled coarse-scale model with 10×10 Cartesian grids; (c) an illustrated grid-block with one fracture.

We first solve Eq. (1) to obtain the fine-scale solutions on the fractured domain in Figure 7 (a). We implement a two-point flux approximation (TPFA) finite volume method coupled with the embedded

discrete-fracture model (EDFM) (Figure 8, Row 1, left). The EDFM avoids the complex meshing for cases with narrow intersecting angles of fracture networks and offers compatibility with the current commercial software due to the feature of Cartesian grids. For comparison purposes, we average the fine-scale solutions on the upscaled coarse-scale model (Figure 8, Row 1, right), which will be as the reference solutions compared with other flow-based upscaling methods. The other upscaling approaches, including the proposed one, are shown on the left-hand side of Row 2, 3, 4, 5 of Figure 8, respectively. We apply a multi-point flux approximation (MPFA) finite volume scheme to solve Eq. (3) because of the upscaled anisotropic permeability tensor. The corresponding absolute error between these models and average fine-scale solutions are illustrated on the right-hand side of Row 2, 3, 4, 5 of Figure 8, respectively. Among them, our proposed local-global multiple-boundary (LG-MB) upscaling method shows the best accuracy, followed by the local upscaling method coupled with the multiple-boundary flux computation method under linear boundary conditions (denoted as Local-MB-Linear), Local-SB-Linear, and Local-no flux. We summarize the superiority of the proposed LG-MB upscaling technique as the following two factors related to 1) the multiple-boundary technique to accurately capture the flow behavior with high anisotropy from fractures non-alignment with the grid (i.e., accurate calculation of the flow rates); 2) the use of local-global approach to accurately obtain the real flow boundary conditions for accurate local upscaling.

Figure 8. Pressure maps for the reference fine-scale solutions (Row 1, left), the averaged fine-scale solutions (Row 1, right), other flow-based upscaling methods, such as Local-no flux (Row 2, left), Local-SB-Linear (Row 3, left), Local-MB-Linear (Row 4, left), and the proposed LG-MB (Row 5, left). The corresponding absolute error maps of the pressure are shown on the right-hand side of Row 2, 3, 4, 5, respectively.

Conclusions

This work introduces a new upscaling technique based on the local-global multiple-boundary (LG-MB) upscaling method for fluid flow in naturally fractured reservoirs. The main conclusions are summarized as follows:

- We extend the commonly-used local-global (LG) upscaling method by introducing the multiple-boundary (MB) approach to compute the fluid fluxes within the fractures crossing the grid-block boundaries.
- Real boundary conditions should be implemented in highly heterogeneous fractured reservoirs due to the existence of fractures.
- We should consider the fracture orientation effect when computing the flow rates. The flow rates crossing non-alignment grid blocks are vectors, which could be divided into two components, parallel and orthogonal to the pressure gradient.
- The proposed LG-MB upscaling approach is demonstrated to a set of single rotating fractures with known analytical solutions, which shows a perfect match between them.
- General applicability is verified by extending it to a more complex fracture network. The proposed upscaling approach shows the best accuracy compared to other existing local, traditional local-global upscaling methods.
- We summarize the superiority of the proposed LG-MB upscaling technique as the following two factors related to 1) the multiple-boundary technique to accurately capture the flow behavior with high anisotropy from fractures non-alignment with the grid (i.e., accurate calculation of the flow rates); 2) the use of local-global approach to accurately obtain the real flow boundary conditions for accurate local upscaling.
- We implement the multi-point flux approximation (MPFA) coupled with the embedded discrete-fracture model (EDFM) to solve the fine-scale and coarse-scale models. The MPFA numerical method could adequately handle fully anisotropic permeability tensor cases, yet the two-point flux approximation (TPFA) would fail to capture the anisotropic behavior. The implementation of EDFM avoids complex meshing issues, especially for fracture networks with small intersecting angles.
- To our best knowledge, this upscaling approach is implemented for the first time in fractured reservoirs. This method is accurate and straightforward and can be implemented within existing upscaling workflows.

Future work will focus on more complex cases, such as realistic cases taken from natural outcrop and an extension to 3D cases, and include more physics, such as two-phase flow considering capillarity and gravity effects.

Appendix A: Permeability Tensor Calculation

For a fracture with $\theta = 0$, the equivalent permeability tensor could be computed as:

$$K(0) = \begin{bmatrix} K_{xx}^0 & 0 \\ 0 & K_{yy}^0 \end{bmatrix} \quad (A1)$$

Where K_{xx}^0 and K_{yy}^0 are the X-direction and Y-direction components of the equivalent permeability tensor, respectively. Taking weighted arithmetic average yields:

$$K_{xx}^0 = \frac{k_m(L - a_f) + k_f a_f}{L} \quad (A2)$$

Taking weighted harmonic average yields:

$$K_{yy}^0 = \frac{L}{(L - a_f)/k_m + a_f/k_f} \quad (\text{A3})$$

Where k_m and k_f are permeabilities of matrix and fracture, respectively; a_f is fracture hydraulic aperture. L is length of the square domain.

For a fracture with random θ , the corresponding equivalent permeability tensor could be computed as:

$$K(\theta) = \begin{bmatrix} K_{xx}^\theta & K_{xy}^\theta \\ K_{yx}^\theta & K_{yy}^\theta \end{bmatrix} = \begin{bmatrix} K_{xx}^0 \cos^2 \theta + K_{yy}^0 \sin^2 \theta & (K_{xx}^0 - K_{yy}^0) \cos \theta \sin \theta \\ (K_{xx}^0 - K_{yy}^0) \cos \theta \sin \theta & K_{xx}^0 \sin^2 \theta + K_{yy}^0 \cos^2 \theta \end{bmatrix} \quad (\text{A4})$$

Plugging Eq. (A2) and Eq. (A3) into Eq. (A4) yields the equivalent permeability tensor for cases with any random θ .

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Additional Information

Any discussion on this paper is highly encouraged. All codes are developed in a MATLAB environment and are available upon request (xupeng.he@kaust.edu.sa). Citing the paper is required once implementing the original or modified codes.

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