Performance of Large Intelligent Surface-enabled Cooperative Networks Over Nakagami- m Channels

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Abstract—In this paper, we analyze the system performance of a large intelligent surface (LIS) enabled wireless system over Nakagami- m channels. We derive closed-form expressions of the outage probability, ergodic capacity, and average bit error rate using different approximation methods, namely, central limit theorem (CLT), Gamma and generalized- K (K_G). The effects of a number of passive LIS elements (N) and fading parameters on the system performance are examined. Results show that the Gamma and K_G approximations are precise given different values of N , while the CLT approximation's accuracy depends on the number of LIS elements implemented. Finally, analytical findings are validated by thorough Monte Carlo simulations.

Index Terms—Bit error rate (BER), cooperative communications, ergodic capacity, large intelligent surface (LIS), outage probability (OP).

I. INTRODUCTION

Large intelligent surfaces (LISs) have been proposed as a promising technology for the next generation of wireless networks. The LIS is a device consisting of a large number of reflective elements, with every element having a capability of controlling the phase and amplitude of incident signals [1]. So, with the proper phase-shift adjustment and signal alignment technique, the system performance at a receiver side can be improved significantly [2]. Due to its low energy consumption, cost-effectiveness, and high spectrum efficiency, LIS can become one of the solutions to overcome the fundamental challenges of fifth-generation (5G) and beyond wireless networks. With its increasing popularity, different LIS configurations have been studied, e.g., large intelligent meta-surfaces (LIM), reconfigurable intelligent surface (RIS), etc. [3], [4].

The performance of relay-aided and LIS-aided systems was compared in [5], where the results demonstrated that the latter obtains better performance than the former due to the large number of reflective elements deployed. The LIS/antenna (LISA) technology was studied in [6] while taking into account the basic characteristics and highlighting a variety of potential LISA-based applications. The authors in [7] investigated the beamforming optimization in RIS-aided systems under discrete phase-shift constraints and showed that one-bit shifters can achieve equal asymptotic power gain as continuous shifters do, although with constant power loss. Another application of the RIS/LIS-aided communications in non-orthogonal multiple access (NOMA) networks was studied in [8], [9]. It was noted that the direct channel between the source and destination

nodes does not affect the system performance and thus can be neglected while providing a sufficient number of reflective elements at a relay node.

Most of the works mentioned above approximated the statistical distributions of channel links in the wireless networks with the help of a central limit theorem (CLT) using Gaussian distribution. Its accuracy depends on the number of reflective elements integrated; for example, a small number of RIS elements leads to in a considerable gap between the analytical and simulated results. Moreover, using the CLT, analytical expressions for the performance metrics, such as average symbol error probability, outage probability and achievable rate, of a RIS system were derived in [10]. The authors confirmed that the derived analytical expressions based on the CLT approximation become tighter to the simulated ones given a large number of reflective elements; therefore, it can be expected that these expressions would perform better for LIS-aided systems. Another type of approximation found to be more precise than the latter for RIS scenarios is a Gamma approximation [11]. To address the CLT limitation, the authors in [12] proposed an approach based on the sum of generalized K -distribution (K_G) random variables (RVs) to ensure the results tightness for the systems with a few reflective elements deployed. Their results showed that the K_G distribution can tightly approximate the RIS-aided system and perform better than the Gaussian approximation under Rayleigh fading.

Considering the aforementioned works, this letter examines the performance of a LIS-aided communication system over Nakagami- m fading channels under three different approximations, i.e., Gaussian, Gamma and K_G . First, closed-form expressions for outage performance (OP), ergodic capacity (EC), and average bit error rate (BER) are analytically derived for given approximations. Second, the accuracy and tightness of the obtained expressions are verified via Monte Carlo simulations. We show that the CLT approximation is not that accurate, even considering a large number of reflective elements, while Gamma and K_G methods are more attractive in terms of their accurateness for the considered LIS system. Through our analysis, we verify that LIS has a real potential of enhancing the system performance, and, by controlling the propagation environment, boosting the signal quality at the receiver.

Fig. 1. A LIS-enabled wireless system.

II. SYSTEM AND CHANNEL MODELING

A. System Topology

In this paper, we consider the signal transmission arising from the single-antenna source (S) and designated to the user (U) via LIS deployed with N passive elements (see Fig. 1). It is assumed that, due to the dense urban environment, S and U do not have a direct line-of-sight (LoS) channel link. Moreover, the changes in an incident signal are considered to be in-phase, that is, the phases are controlled perfectly, as in [13], using a constructive signal accumulation at U . All channel links (g_n and h_n) are subject to a Nakagami- m fading model and the channel envelopes are assumed to be independently but not necessarily identically distributed (i.n.i.d.).

B. Transmission Protocol

The signal received at U can be written as

$$y = \sqrt{P_S} \left(\sum_{n=1}^N g_n h_n d_{sr}^{-\frac{\tau}{2}} d_{ru}^{-\frac{\tau}{2}} \alpha_n e^{j\theta_n} \right) x + n, \quad (1)$$

where P_S , x and n are the transmit power, message of S and additive white Gaussian noise (AWGN) at a receiver node, with variance N_0 , respectively. d_{sr} and d_{ru} are the distances¹ related to the S -LIS and LIS- U links; τ stands for the path-loss exponent. θ_n and α_n are the adjustable phase shift and reflection coefficient of the n th LIS element. Thus, the corresponding SNR at U can be expressed as

$$\gamma = P_S \frac{\left| \sum_{n=1}^N g_n h_n d_{sr}^{-\frac{\tau}{2}} d_{ru}^{-\frac{\tau}{2}} \alpha_n e^{j\theta_n} \right|^2}{N_0}. \quad (2)$$

¹Considering the fact that S and U are located at much more longer distances than the distances between the LIS elements, we assume that all the distances related to the individual elements are identical.

Recalling the perfect phase adjustment, in order to maximize the received SNR, the phase shifts can be optimized as [10]

$$(\theta_1^{opt}, \dots, \theta_N^{opt}) = (-(\arg[g_1] + \arg[h_1]), \dots, -(\arg[g_N] + \arg[h_N])), \quad (3)$$

so that eventually each term of the summation will be co-phased. Therefore, the optimal received SNR is found as

$$\gamma = \frac{P_S}{N_0} \left(\sum_{n=1}^N |g_n| |h_n| \tilde{\gamma}_n \right)^2, \quad (4)$$

where $\tilde{\gamma}_n = \alpha_n d_{sr}^{-\frac{\tau}{2}} d_{ru}^{-\frac{\tau}{2}}$.

III. PERFORMANCE ANALYSIS

In this section, we study the OP, EC and BER using the abovementioned approximations for the considered LIS-assisted wireless system.

A. Outage Performance

Generally, the OP is defined as the probability that the achievable received SNR at the destination node falls below the predefined SNR-associated rate threshold γ_{th} . To investigate the outage performance of the proposed system model, we rewrite $\gamma = \frac{P_S X^2}{N_0}$, where $X = \sum_{n=1}^N Y_n \tilde{\gamma}_n$, $Y_n = |g_n| |h_n|$, and Y_n is subject to the double Nakagami- m distribution.

1) *CLT approximation*: The CLT states that with a given sufficiently large number of passive LIS elements, the probability density function (PDF) and cumulative density function (CDF) of the sum of N i.n.i.d. double Nakagami- m RVs can be approximated by a Gaussian distribution as [14]

$$f_X^{CLT}(x) = \frac{\phi_X}{\sqrt{2\pi\sigma_X^2}} \exp\left(-\frac{(x - \mu_X)^2}{2\sigma_X^2}\right), \quad (5)$$

$$F_X^{CLT}(x) = 1 - \phi_X Q\left(\frac{x - \mu_X}{\sigma_X}\right), \quad (6)$$

where $\phi_X = 1/Q\left(-\frac{\mu_X}{\sigma_X}\right)$ is the normalization coefficient, and $Q(\cdot)$ is the Gaussian Q function, defined by [15, Eq. (6.287.3)]. The parameters μ_X and σ_X^2 are calculated as

$$\mu_X = N \mathbb{E}[Y_n^2] \tilde{\gamma}_n, \quad (7)$$

$$\sigma_X^2 = N \sigma_{Y_n}^2 \tilde{\gamma}_n, \quad (8)$$

where the h th moment of Y_n can be defined as [16]

$$\mathbb{E}[Y_n^h] = \prod_{i=1}^2 \frac{\Gamma(m_i + h/2)}{\Gamma(m_i)} \left(\frac{\Omega_i}{m_i}\right)^{h/2}, \quad (9)$$

where m denotes the Nakagami- m fading parameter, $\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx$ is the Gamma function.

The outage performance of the system can be defined as

$$\begin{aligned} F_\gamma(z) &= \Pr(\gamma < z) \\ &= \Pr\left(X < \sqrt{\frac{z}{\gamma}}\right) = F_X\left(\sqrt{\frac{z}{\gamma}}\right). \end{aligned} \quad (10)$$

or equivalently, for the CLT approximation,

$$F_{\gamma}^{\text{CLT}}(z) = 1 - \phi_X Q \left(\frac{\sqrt{\frac{z}{\bar{\gamma}}} - \mu_X}{\sigma_X} \right), \quad (11)$$

where $\bar{\gamma} = \frac{P_S}{N_0}$ is the transmit SNR. Thus, the OP can be obtained using its mathematical formulation as

$$P_{\text{out}}^{\text{CLT}}(\gamma_{\text{th}}) = \Pr(\gamma < \gamma_{\text{th}}) = F_{\gamma}^{\text{CLT}}(\gamma_{\text{th}}). \quad (12)$$

2) *Gamma approximation:* Another approach is to apply the Gamma approximation, whose PDF and CDF are given by

$$f_X^{\Gamma}(x) = \frac{1}{\Gamma(l)\theta^l} x^{l-1} \exp\left(-\frac{x}{\theta}\right), \quad (13)$$

$$F_X^{\Gamma}(x) = \frac{1}{\Gamma(l)} \gamma\left(l, \frac{x}{\theta}\right), \quad (14)$$

where $\gamma(s, x) = \int_0^x t^{s-1} \exp(-t) dt$ is the lower incomplete Gamma function; l and θ are the shape and scale parameters calculated after equating the mean and variance of the Gamma distribution ($l\theta$ and $l\theta^2$) to μ_X and σ_X^2 , respectively. Similar to (10), the CDF of γ is then written accordingly

$$F_{\gamma}^{\Gamma}(z) = \frac{1}{\Gamma(l)} \gamma\left(l, \frac{\sqrt{z}}{\sqrt{\bar{\gamma}}\theta}\right). \quad (15)$$

Finally, the OP is defined as

$$P_{\text{out}}^{\Gamma}(\gamma_{\text{th}}) = F_{\gamma}^{\Gamma}(\gamma_{\text{th}}). \quad (16)$$

3) *K_G approximation:* In [12], the authors applied K_G distribution to approximate the sum of double-Rayleigh RVs by using the approach [17]. Due to the demonstrated tightness, we exploit to further extend the proposed method for the considered system model over double Nakagami- m channels.

We first consider $G = \sum_{s=1}^S R_s^2$, where R_s is a K_G RV. The author in [17] stated that the sum of K_G RVs can be tightly approximated by the PDF of $\sqrt{G}$, approximating G with the squared K_G distribution. Then, the PDF of X is expressed as

$$f_X^{K_G}(x) = \frac{4\phi_H^{k_w+m_w} x^{k_w+m_w-1}}{\Gamma(m_w)\Gamma(k_w)} K_{k_w-m_w}(2x\phi_H), \quad (17)$$

where $K_v(\cdot)$ is the modified v th-order Bessel function of the second kind [15, Eq. (8.432)], $\phi_H = \left(\frac{k_w m_w}{\Omega_w}\right)^{\frac{1}{2}}$, and $\Omega_w = \mathbb{E}[X^2]$. The shaping parameters m_w and k_w are calculated as

$$m_w = \frac{-\beta + \sqrt{\beta^2 - 4ac}}{2a}, \quad (18)$$

$$k_w = \frac{-\beta - \sqrt{\beta^2 - 4ac}}{2a}. \quad (19)$$

If k_w and m_w are conjugate complex numbers, they are set to the magnitude of the estimated complex parameters. The values of β , a , and c are expressed as follows

$$\beta = \mathbb{E}[X^6]\mathbb{E}[X^2] - 4(\mathbb{E}[X^4])^2 + 3(\mathbb{E}[X^2])^2\mathbb{E}[X^4],$$

$$a = \mathbb{E}[X^6]\mathbb{E}[X^2] + (\mathbb{E}[X^2])^2\mathbb{E}[X^4] - 2(\mathbb{E}[X^4])^2,$$

$$c = 2(\mathbb{E}[X^2])^2\mathbb{E}[X^4].$$

Using a multinomial identity and (9), these parameters are defined by evaluating the required moments of X as

$$E[X^n] = \sum_{n_1=0}^n \sum_{n_2=0}^{n_1} \cdots \sum_{n_{N-1}=0}^{n_{N-2}} \binom{n}{n_1} \binom{n_1}{n_2} \cdots \binom{n_{N-2}}{n_{N-1}} \times \mathbb{E}[Y_1^{n-n_1}] \mathbb{E}[Y_2^{n_1-n_2}] \cdots \mathbb{E}[Y_{N-1}^{n_{N-2}}]. \quad (20)$$

Remark 1: The h th moments of X for the LIS-assisted wireless network are computationally complex, especially, for $h = \{4, 6\}$. Therefore, to further facilitate the research in this area, they are calculated and drawn in Table I.

Finally, the PDF and CDF of γ can be expressed as the squared K_G distribution as

$$f_{\gamma}^{K_G}(z) = \frac{2(\tilde{\gamma}_n \phi_H)^{k_w+m_w} z^{\frac{k_w+m_w}{2}-1}}{\tilde{\gamma}^{\frac{k_w+m_w}{2}} \Gamma(m_w) \Gamma(k_w)} \times K_{k_w-m_w}\left(2\tilde{\gamma}_n \phi_H \sqrt{\frac{z}{\tilde{\gamma}}}\right), \quad (21)$$

$$F_{\gamma}^{K_G}(z) = \frac{G_{1,3}^{2,1}\left(k_w, m_w, 0 \mid \frac{z(\tilde{\gamma}_n \phi_H)^2}{\tilde{\gamma}}\right)}{\Gamma(m_w) \Gamma(k_w)}. \quad (22)$$

where $G_{p,q}^{m,n}\left(\begin{smallmatrix} a_1, \dots, a_p \\ b_1, \dots, b_q \end{smallmatrix} \mid \eta x\right)$ is the Meijer G-function [15].

Remark 2: It is important to highlight that the direct application of a K_G distribution is characterized by insufficient accuracy for the case of LIS with large N .

Therefore, in this paper, we apply a Hankel's expansion that makes the K_G distribution highly accurate as [18]

$$f_X^H(x) = \frac{2\phi_H \exp(-2\phi_H x)}{\psi_H} \sum_{s=0}^M b_s (2\phi_H x)^{\omega_s-1}, \quad (23)$$

$$F_X^H(x) = \frac{1}{\psi_H} \sum_{s=0}^M b_s \gamma(\omega_s, 2\phi_H x), \quad (24)$$

where $\omega_s = \mu_H - s$, $b_s = \sum_{s=0}^M \frac{\Gamma(\frac{1}{2}+s)}{\Gamma(\frac{1}{2}-s)2^s}$, $\mu_H = m_w + k_w - \frac{1}{2}$, and $\psi_H = \sum_{s=0}^M b_s \Gamma(\omega_H)$. M is defined as [18]

$$M = \begin{cases} 1, & \text{if } 0 \leq k_w - m_w < 0.5, \\ \lceil k_w - m_w - 0.5 \rceil, & \text{if } k_w - m_w \geq 0.5. \end{cases} \quad (25)$$

Now, with the help of (23), (24) and (25), the PDF and CDF of γ can be respectively described as

$$f_{\gamma}^H(z) = \frac{\phi_H \exp\left(-2\tilde{\gamma}_n \phi_H \sqrt{\frac{z}{\tilde{\gamma}}}\right)}{\psi_H} \times \sum_{s=0}^M b_s (2\phi_H)^{\omega_s-1} \tilde{\gamma}_n^{\omega_s} \left(\frac{\sqrt{z}^{\omega_s-2}}{\sqrt{\tilde{\gamma}}^{\omega_s}}\right), \quad (26)$$

$$F_{\gamma}^H(z) = \frac{1}{\psi_H} \sum_{s=0}^M b_s \gamma\left(\omega_s, 2\tilde{\gamma}_n \phi_H \sqrt{\frac{z}{\tilde{\gamma}}}\right). \quad (27)$$

Finally, the OP can be obtained as

$$P_{\text{out}}^H(\gamma_{\text{th}}) = F_{\gamma}^H(\gamma_{\text{th}}). \quad (28)$$

B. Ergodic Capacity

1) *CLT approximation*: EC can be defined as [19]

$$C = \mathbb{E} [\log_2(1 + \gamma)]. \quad (29)$$

Then, substituting $\gamma = \tilde{\gamma}x^2$, it can be rewritten as

$$C = \frac{1}{\ln(2)} \int_0^\infty \ln(1 + x^2\tilde{\gamma}) f_X(x) dx. \quad (30)$$

By inserting (5) into (30), we obtain

$$C = \frac{1}{\ln(2)} \frac{\phi_X}{\sqrt{2\pi\sigma_X^2}} \int_0^\infty \ln(1 + x^2\tilde{\gamma}) \exp\left(-\frac{(x - \mu_X)^2}{2\sigma_X^2}\right) dx. \quad (31)$$

Now, by using the curve fitting [20], $\ln(1 + x^2\tilde{\gamma})$ can be well-approximated as $\sum_{i=1}^4 \zeta_i \exp(\psi_i \tilde{\gamma} x^2)$, where $\zeta = [6.4678, -1.8264, -2.7948, -1.6552]$ and $\psi = [0, 0.037, 0.004, 0.274]$. Then, (31) can be rewritten as

$$C \approx \frac{1}{\ln(2)} \frac{\phi_X}{\sqrt{2\pi\sigma_X^2}} \sum_{i=1}^4 \zeta_i \exp(-(\eta_i x^2 + 2\delta x + \rho)) dx, \quad (32)$$

where $\eta_i = \frac{2\psi_i\sigma_X^2+1}{2\sigma_X^2}$, $\delta = -\frac{\mu_X}{2\sigma_X^2}$, and $\rho = \frac{\mu_X^2}{2\sigma_X^2}$. With the help of [15, Eq. (2.33.1)], the capacity can be written as

$$C^{\text{CLT}} \approx \sum_{i=1}^4 \frac{\zeta_i \phi_X}{2 \ln(2) \sqrt{2\eta_i \sigma_X^2}} \exp\left(\frac{\delta^2 - \eta_i \rho}{\eta_i}\right) \times \left(1 - \operatorname{erf}\left(\frac{\delta}{\sqrt{\eta_i}}\right)\right), \quad (33)$$

where $\operatorname{erf}(\cdot)$ indicates the error function [15, Eq. (8.250.1)].

2) *Gamma approximation*: Using the Gamma approximation, EC can be expressed as

$$C = \frac{1}{\ln(2)\Gamma(l)} \int_0^\infty x^{l-1} \ln(1 + x^2\tilde{\gamma}) \exp\left(-\frac{x}{\theta}\right) dx. \quad (34)$$

According to [21], the natural logarithm and exponential function can be respectively transformed as

$$\ln(1 + x^2\tilde{\gamma}) = G_{2,2}^{1,2} \left(\begin{matrix} 1, 1 \\ 1, 0 \end{matrix} \middle| x^2\tilde{\gamma} \right), \quad (35)$$

$$\exp\left(-\frac{x}{\theta}\right) = G_{0,1}^{1,0} \left(\begin{matrix} - \\ 0 \end{matrix} \middle| \frac{x}{\theta} \right). \quad (36)$$

Further, by substituting (35) and (36) into (34) and using [21, Eq. (21)], we can rewrite the final closed-form EC for the Gamma distribution as

$$\begin{aligned} C^\Gamma &= \frac{1}{\ln(2)\Gamma(l)\theta^l} \int_0^\infty x^{l-1} G_{0,1}^{1,0} \left(\begin{matrix} - \\ 0 \end{matrix} \middle| \frac{x}{\theta} \right) \\ &\quad \times G_{2,2}^{1,2} \left(\begin{matrix} 1, 1 \\ 1, 0 \end{matrix} \middle| x^2\tilde{\gamma} \right) dx \\ &= \frac{2^{l-1}}{\ln(2)\Gamma(l)\sqrt{\pi}} G_{4,2}^{1,4} \left(\begin{matrix} 1, \frac{1-l}{2}, \frac{2-l}{2}, 1 \\ 1, 0 \end{matrix} \middle| 4\theta^2\tilde{\gamma} \right). \end{aligned} \quad (37)$$

3) *K_G approximation*: Following (30), EC can be written as

$$C = \frac{\phi_H \exp(-2\phi_H x)}{\ln(2)\psi_H} \sum_{s=0}^M \int_0^\infty \ln(1 + \tilde{\gamma}_n^4 x^2 \tilde{\gamma}) b_s (2\phi_H x)^{\omega_s-1} dx. \quad (38)$$

Using (35) and (36), (38) can be expressed as

$$C = \frac{\phi_H}{\ln(2)\psi_H} \sum_{s=0}^M b_s (2\phi_H)^{\omega_s-1} \int_0^\infty x^{\omega_s-1} \times G_{2,2}^{1,2} \left(\begin{matrix} 1, 1 \\ 1, 0 \end{matrix} \middle| \tilde{\gamma}_n^4 x^2 \tilde{\gamma} \right) G_{0,1}^{1,0} \left(\begin{matrix} - \\ 0 \end{matrix} \middle| 2\phi_H x \right) dx. \quad (39)$$

Finally, using [21, Eq. (21)], EC can be derived in closed-form as follows

$$C^H = \frac{\phi_H}{\ln(2)\psi_H\sqrt{\pi}} \sum_{s=0}^M b_s 2^{\omega_s-1} \times G_{4,2}^{1,4} \left(\begin{matrix} 1, \frac{1-\omega_s}{2}, \frac{2-\omega_s}{2}, 1 \\ 1, 0 \end{matrix} \middle| \frac{\tilde{\gamma}_n^4 \tilde{\gamma}}{\phi_H^2} \right). \quad (40)$$

C. Average BER

The BER is one of the valuable performance metrics whose expression, considering different types of binary modulation schemes, is expressed as [16]

$$P_e = \int_0^\infty p \operatorname{erfc}(\sqrt{qx}) f_\gamma(x) dx, \quad (41)$$

where $\operatorname{erfc}(\cdot)$ is complementary error function [15, Eq. (8.250.4)]. p and q are the parameters standing for various modulating schemes.

1) *CLT approximation*: By taking a derivative of (11), we derive the PDF of γ as

$$f_\gamma^{\text{CLT}}(z) = \frac{\phi_X}{2\sqrt{2z\tilde{\gamma}\pi\sigma_X^2}} \exp\left(-\frac{\left(\sqrt{\frac{z}{\tilde{\gamma}}} - \mu_X\right)^2}{2\sigma_X^2}\right). \quad (42)$$

Next, using (41) and (42), we evaluate the average BER as

$$P_e^{\text{CLT}} = \int_0^\infty \frac{p\phi_X}{2\sqrt{2x\tilde{\gamma}\pi\sigma_X^2}} \exp\left(-\frac{\left(\sqrt{\frac{x}{\tilde{\gamma}}} - \mu_X\right)^2}{2\sigma_X^2}\right) \times \operatorname{erfc}(\sqrt{qx}) dx. \quad (43)$$

For the best of authors' knowledge, the integral in (43) does not have a closed-form solution. Therefore, in this paper, the BER performance for the CLT approximation is evaluated numerically.

2) *Gamma approximation*: We first obtain the PDF of γ by taking a derivative of (15) as

$$f_\gamma^\Gamma(z) = \frac{z^{\frac{l}{2}-1}}{2\Gamma(l)\sqrt{\tilde{\gamma}^l}} \exp\left(-\frac{\sqrt{z}}{\sqrt{\tilde{\gamma}\theta}}\right). \quad (44)$$

Table I. The h th moments of X for different N and m using (20).

N	h	$m = 1$	$m = 2$	$m = 3$
32	4	460, 212.89	683, 097.37	785, 686.33
	6	347, 089, 602.76	593, 232, 386.45	719, 172, 506.96
64	4	6, 869, 200.92	10, 578, 401.84	12, 306, 711.07
	6	19, 029, 952, 117.34	35, 294, 940, 460.47	43, 890, 709, 549.17
96	4	33, 951, 087.97	52, 958, 674.96	61, 853, 300.08
	6	205, 395, 599, 531.19	392, 076, 603, 724.93	491, 879, 380, 153.50
128	4	106, 005, 701.76	166, 435, 103.37	194, 774, 768.18
	6	1, 122, 862, 585, 680.33	2, 175, 182, 544, 825.52	2, 741, 103, 413, 827.63

The term $\text{erfc}(\sqrt{qx})$ can be represented using [21] as

$$\text{erfc}(\sqrt{qx}) = \frac{1}{\sqrt{\pi}} G_{1,2}^{2,0} \left(\frac{1}{0, \frac{1}{2}} \middle| qx \right). \quad (45)$$

Now, using (36), (41), (44) and (45), we evaluate the average BER for the Gamma approximation as

$$\begin{aligned} P_e^\Gamma &= \frac{p}{2\Gamma(l)\sqrt{\gamma}^l} \int_0^\infty x^{\frac{l}{2}-1} \text{erfc}(\sqrt{qx}) \exp\left(-\frac{\sqrt{x}}{\sqrt{\gamma}\theta}\right) dx \\ &= \frac{p}{2\Gamma(l)\sqrt{\gamma}^l} \int_0^\infty x^{\frac{l}{2}-1} G_{0,1}^{1,0} \left(- \middle| \frac{\sqrt{x}}{\theta\sqrt{\gamma}} \right) G_{1,2}^{2,0} \left(\frac{1}{0, \frac{1}{2}} \middle| qx \right) dx \\ &= \frac{p}{2\pi\Gamma(l)} \left(\frac{q}{\gamma\theta^2} \right)^{\frac{l}{2}} G_{2,3}^{2,2} \left(\frac{\frac{1}{2} - \frac{l}{2}, 1 - \frac{l}{2}}{0, \frac{1}{2}, -\frac{l}{2}} \middle| \frac{1}{4q\theta^2\sqrt{\gamma}} \right). \end{aligned} \quad (46)$$

3) K_G approximation: Using (26) and (41), the BER for the Hankel-based K_G approximation can be written as

$$\begin{aligned} P_e^H &= \frac{p(2\phi_H)^{\omega_H}}{2\psi_H} \sum_{s=0}^M \frac{b_s}{\sqrt{\gamma}^{\omega_s}} \\ &\quad \times \int_0^\infty \text{erfc}(\sqrt{qx}) x^{\frac{\omega_s}{2}-1} \exp\left(-\frac{2\phi_H}{\tilde{\gamma}_n^2} \sqrt{\frac{x}{\gamma}}\right) dx \\ &= \frac{p(2\phi_H)^{\omega_H}}{2\pi\psi_H} \sum_{s=0}^M b_s \left(\frac{\tilde{\gamma}_n^2}{q\gamma} \right)^{\frac{\omega_s}{2}} \\ &\quad \times G_{2,3}^{2,2} \left(1 - \frac{\omega_s}{2}, \frac{1}{2} - \frac{\omega_s}{2} \middle| \frac{\phi_H^2}{q\tilde{\gamma}_n^4\gamma} \right). \end{aligned} \quad (47)$$

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we present some analytical and simulation results on the performance of a LIS-enabled wireless system. Unless stated otherwise, for the simulation purposes, the following parameters are considered: $m = 2$, $\alpha_n = 1$, $d_{sr} = d_{ru} = 10$ m, $\tau = 2$, $\gamma_{th} = 5$ dB, and $N_0 = 1$. Also, we use quadrature phase shift keying (QPSK) with $p = 0.5$ and $q = 0.5$.

Fig. 2 illustrates the system OP for a different number of LIS elements. Both Gamma and K_G approximations are fairly accurate for the whole range of transmit SNR values and different N , while the CLT one fits well with the simulation results only given considerably large N . One can notice outage performance improvement as the number of N increases due to achievable high passive beamforming gains. From this perspective, it can be concluded that the increase in N supports the system's energy efficiency performance.

Fig. 3 depicts the EC results plotted versus the transmit SNR

Fig. 2. OP vs. transmit SNR for different N .

Fig. 3. EC vs. transmit SNR for different N .

for different values of N . The analytical expressions derived for the Gamma and K_G approximations are highly consistent with the simulation results compared to the CLT one. Moreover, one can notice that given linear increase in the number of N does not result in a pro-rata gain in the capacity performance. For example, at $\bar{\gamma} = 20$ dB, as N increases from 32 to 64, the EC

Fig. 4. Average BER vs. transmit SNR for different N .

improves by about 76.3%, while, when N goes up from 64 to 96, the capacity rises by approximately 47.3%.

Finally, the average BER performance is plotted in Fig. 4, considering the QPSK modulation for $N = 32, 64, 96, 128$ and $m = 1, 2, 3$. When $m = 1$, the considered fading model becomes double Rayleigh. It can be seen that bigger m corresponds to the better system performance as m stands for the number of multi-path components of the channel links. Moreover, the increase in N significantly enhances the BER performance, i.e., for $m = 2$, $N = 32, 64, 96, 128$ result in the transmit SNR of 21, 15, 11.2 and 8.5 dB required to achieve 10^{-3} . However, this improvement has a non-linear nature that brings us to the conclusion that, for the considered system model, it is not always required to have a complicated LIS-based relay as more densely deployed LIS leads to a higher amount of phase-shift errors.

V. CONCLUSION

The present work investigated the performance of the considered LIS-enabled system in terms of the OP, EC, and BER metrics considering different approximation methods. Closed-form expressions for the given performance metrics were derived. The results revealed that both Gamma and K_G perform better than the CLT. Even though both Gamma and K_G well-approximated the simulation results, the Gamma was proved to be less complicated in terms of the PDF and CDF representations, and, as a result, can be considered to be more practical. It also was shown that the fading parameter m improves the system BER performance. Finally, the system performance mainly depends on the number of elements implemented.

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