Optimal Target Classification Using Frequency-Based Radar Waveform Design

Sultan Z. Alshirah¹, Shahzad Gishkori², *Senior Member, IEEE*, and Bernard Mulgrew³, *Fellow, IEEE*

Abstract—In this article, we introduce a frequency-based waveform design to maximize binary target classification. The optimization problems can be solved via optimal solvers available in the literature. The presented formulation is applicable to the classification scenarios where the extended target frequency responses (TFRs) are complex, random, and normally distributed with unequal mean vectors. However, their covariance matrices are either identical or different. For the former, i.e., identical covariance matrices, the optimization problem consists of a cost function that has been obtained by deriving a closed-form expression of the probability of misclassification. For the latter, i.e., different covariance matrices, we present an optimization problem where the cost function has been obtained via the Fisher separation function. For this, we also present a closed-form expression for an approximate solution to the optimization problem. We show that the proposed solution achieves performance levels comparable to the exact solution of the optimization problem obtained via state-of-the-art optimization solvers while incurring low computational complexity. We expand on the two main scenarios by introducing clutter (i.e., signal-dependent interference) in the signal model and studying possible closed-form designs in extreme waveform energy levels. Simulations are conducted using synthetically generated data in addition to the civilian vehicle data from the Moving and Stationary Target Acquisition and Recognition (MSTAR) dataset, in order to validate the proposed method.

Index Terms—Fisher separation function, probability of misclassification, radar waveform design, target classification.

I. INTRODUCTION

RADAR waveform design is a crucial part of the signal processing chain in most radar systems and a deciding factor in its performance [1]–[4]. Recently, radar hardware became more capable of arbitrary waveform and receiver design allowing for more degrees of freedom to be exploited to maximize performance [5]. One of the main goals of radar waveform design is to maximize target classification. Bell [1]

have shown the importance of prior knowledge about targets to maximize target detection and classification performance and also introduced objective functions based on the signal-to-noise ratio (SNR) or mutual information between the target response and the received signal to design waveforms for optimal detection or target response estimation. Also, he showed that acquiring more target impulse responses (TIRs) improves target classification performance. Later, in [6]–[8], an optimal waveform and receiver design has been introduced to maximize detection performance using prior knowledge about the deterministic extended TIR. The design was extended to maximize target classification in a similar way to detection, but instead of maximizing the signal-to-interference-plus-noise ratio (SINR), it maximizes the Mahalanobis distance between two classes with deterministic TIRs in the presence of noise and clutter. When more than two target classes are to be classified or when angular uncertainty is considered, the optimality of the Mahalanobis distance approach is not guaranteed and an adaptive framework where the waveform is designed on every transmission is suggested [9]–[24]. Adaptive waveform design strategies require more computations than nonadaptive strategies as waveforms are designed continuously until a decision is made. In [25], a nonadaptive waveform and classifier design strategy based on the Fisher analysis was presented. This addressed the scenarios where a target is to be assigned to one of two or multiple classes of targets based on its extended target frequency response (TFR) in the presence of noise and angular uncertainty. The methods resulted in an optimization problem that can be solved to design the optimal waveform and classifier. To solve the optimization problems, general optimization software was used. The results showed that the Fisher-based strategy outperforms the best nonadaptive waveform design techniques found in the literature. While a great body of research has expanded on other applications and design strategies to radar waveform design (e.g., target detection, estimation, or adaptive waveform design) [26]–[33], this article only focuses on nonadaptive single-illumination target classification optimization using a waveform and receiver/classifier design.

In this article, we expand on our previous work in [25] with three main contributions. First, we introduce signal-dependent interference and clutter to the signal model for the two scenarios: 1) when the target classes share the same covariance matrix and 2) when they have different covariance matrices. Second, we formulate the waveform optimization problem based on the new signal model by deriving the objective function based on the probability of misclassification and Fisher's separability function. Finally, we derive a frequency-based

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Sultan Z. Alshirah is with the School of Engineering, Institute for Digital Communications, The University of Edinburgh, Edinburgh EH8 9YL, U.K., and also with the National Center for Telecommunications and Defense Systems Technology, Radar Department, King Abdulaziz City for Science and Technology (KACST), Riyadh 12354, Saudi Arabia (e-mail: salshirah@kacst.edu.sa).

Shahzad Gishkori is with the School of Engineering, Institute for Digital Communications, The University of Edinburgh, Edinburgh EH8 9YL, U.K., and also with the Department of Computer, Electrical, Mathematical Sciences and Engineering (CEMSE), King Abdullah University of Science and Technology, Thuwal 23955, Saudi Arabia (e-mail: shahzad.gishkori@kaust.edu.sa).

Bernard Mulgrew is with the School of Engineering, Institute for Digital Communications, The University of Edinburgh, Edinburgh EH8 9YL, U.K. (e-mail: bernie.mulgrew@ed.ac.uk).

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closed-form solution to the optimization problem to maximize Fisher's separability function. The closed-form solution can be used for the scenario where the covariance matrices are identical or when they are not. We test the closed-form solution against flat spectrum wideband waveform, optimized waveforms obtained from the optimization algorithms and closed-form solutions optioned at low and high waveform energy levels. The initial results in this article were preliminary published and presented in [34].

The rest of this article is sectioned as follows. In Section II, we present the signal model and the statistical properties of all signals. In Section III, we derive the optimization problem for the two main scenarios and derive the closed-form solution of the optimization problems. In Section IV, we present the synthetic and real data simulations, and in Section V, we share our conclusion.

Notations: x is a scalar, $\mathbf{x}$ (small case bold) is a column vector, X is the Fourier transform of $\mathbf{x}$, $\mathbf{X}$ (capital case bold) is a matrix, $(\cdot)^H$ is the Hermitian transpose, $\text{trace}(\cdot)$ is the trace function, $|\cdot|$ is the absolute value operator, $\mathcal{CN}(M, \Sigma)$ is a vectorized complex Gaussian distribution with mean vector M and covariance matrix Σ , and $\Re x$ and $\Im x$ are the real and imaginary parts of x , respectively.

II. SIGNAL MODEL

A. Signal Model

Assuming that a radar is transmitting the continuous time-domain radar pulse $x(t)$ where a target with extended impulse response $r(t)$ is detected and to be classified, the continuous radar return signal $y(t)$ is expected to be given by

$$y(t) = x(t) \otimes r(t) + x(t) \otimes c(t) + n(t) \quad (1)$$

where $\otimes$ is the linear convolution operator, $n(t)$ is the additive noise signal at the receiver, and $c(t)$ is the clutter response signal.

Assuming that the frequency snapshot model is used (where $y(t)$ is Fourier-transformed into the frequency domain where the time-bandwidth product $w\Delta t$ satisfies $w\Delta t \geq 16$ [2], [34] where Δt is the length of the received signal in the time domain and 16 is the minimum time-bandwidth product in the frequency snapshot model), the vectorized radar return in the frequency domain Y can be calculated from (1) as in the following:

$$Y = \Omega_X R + \Omega_X C + N \quad (2)$$

where R is the extended frequency response of the target; N is the frequency transformed noise vector; C is the frequency transformed clutter vector, and $\Omega_X = \text{diag}(X)$ is the diagonal matrix with the frequency transform of the radar waveform on its diagonal, while Y, X, R, C , and N are all $m \times 1$ vectors.

B. Statistical Properties

In this article, we assume that the TFR, the clutter, and the noise are all random with known statistical properties. The vectors R_i , C , and N are realizations from complex-valued

Gaussian random fields with known means and covariance matrices where R_i is the TFR from the class indexed with i for $i = 1$ and 2.

Given that the vectors are distributed as follows:

$$R_i \sim \mathcal{CN}(M_{R_i}, \Sigma_{R_i}) \text{ for } i = 1, 2 \quad (3)$$

$$C \sim \mathcal{CN}(M_0, \Sigma_C) \quad (4)$$

$$N \sim \mathcal{CN}(M_0, \Sigma_N) \quad (5)$$

where M_0 is an all-zero vector and assuming that R_i , C , and N are all uncorrelated, then, the statistical distribution of Y for the i th class is given by

$$Y_i \sim \mathcal{CN}(M_{Y_i}, \Sigma_{Y_i}) \quad (6)$$

where

$$M_{Y_i} = \Omega_X M_{R_i} \quad (7)$$

and

$$\Sigma_{Y_i} = \Omega_X (\Sigma_{R_i} + \Sigma_C) \Omega_X^H + \Sigma_N. \quad (8)$$

III. PROPOSED METHOD

A. Objective Function

Given the statistical distributions of Y , the classifier is to be designed to assign the radar return Y_i (also known as the data point) to one of two classes with different mean vectors and identical or nonidentical covariance matrices. The classifier design depends on the objective function and whether the covariance matrices, Σ_{R_1} and Σ_{R_2} , are identical or not.

1) *Target Classes With Different Means and Different Covariance Matrices:* We start with the more general scenario where the two distributions have different mean vectors and covariance matrices.

In this scenario, the following conditions hold.

- 1) An already detected target with extended impulse/frequency response is to be classified into one of two classes (binary classification problem).
- 2) i is the class identifier for $i = 1$ and 2.
- 3) All statistical properties of all classes are known *a priori*.
- 4) $M_{Y_1} = \Omega_X M_{R_1} \neq M_{Y_2}$.
- 5) $\Sigma_{Y_1} = \Omega_X (\Sigma_{R_1} + \Sigma_C) \Omega_X^H + \Sigma_N \neq \Sigma_{Y_2} = \Omega_X (\Sigma_{R_2} + \Sigma_C) \Omega_X^H + \Sigma_N$.

For this kind of classification problem, the performance can be enhanced by employing the concepts of Fisher discriminant analysis [35].

Basically, the analysis aims at finding the projection vector $\mathbf{w}$ that maximizes the distance between classes' means while minimizing the variance of each class [35]. This is achieved by maximizing the classes' separation, which is defined by the Fisher separation function as follows:

$$f(\mathbf{w}) = \frac{\mathbf{w}^H (M_{Y_1} - M_{Y_2}) (M_{Y_1} - M_{Y_2})^H \mathbf{w}}{\mathbf{w}^H (\Sigma_{Y_1} + \Sigma_{Y_2}) \mathbf{w}}. \quad (9)$$

The optimal projection vector $\mathbf{w}_{opt}$ that maximizes Fisher's separation function $f(\mathbf{w})$ is well defined and is given by

$$\mathbf{w}_{opt} = \alpha (\Sigma_{Y_1} + \Sigma_{Y_2})^{-1} (M_{Y_1} - M_{Y_2}) \quad (10)$$

where α is a constant [35].

By substituting (7), (8), and (10) into (9), the objective function becomes variable only in $\mathbf{\Omega}_X$ that is dependent on X (the Fourier transform of the radar probing signal $\mathbf{x}$).

If $g_2(\mathbf{\Omega}_X)$ is the objective function resulting from the substitutions, then (9) can be written as

$$g_2(\mathbf{\Omega}_X) = \frac{\Delta M_R^H \mathbf{\Omega}_X^H \mathbf{S}^{-1} \mathbf{\Omega}_X \Delta M_R (\Delta M_R^H \mathbf{\Omega}_X^H \mathbf{S}^{-1} \mathbf{\Omega}_X \Delta M_R)}{(\Delta M_R^H \mathbf{\Omega}_X^H \mathbf{S}^{-1} \mathbf{\Omega}_X \Delta M_R)} \\ = \Delta M_R^H \mathbf{\Omega}_X^H (\mathbf{\Omega}_X (\mathbf{\Psi}) \mathbf{\Omega}_X^H + 2\mathbf{\Sigma}_N)^{-1} \mathbf{\Omega}_X \Delta M_R \quad (11)$$

where $\mathbf{S} = \mathbf{\Omega}_X (\mathbf{\Psi}) \mathbf{\Omega}_X^H + 2\mathbf{\Sigma}_N$, $\mathbf{\Psi} = \mathbf{\Sigma}_{R_1} + \mathbf{\Sigma}_{R_2} + 2\mathbf{\Sigma}_C$, and $\Delta M_R = M_{R_1} - M_{R_2}$.

The new objective function $g_2(\mathbf{\Omega}_X)$ then becomes dependent only on the radar waveform X , which can be designed so that the classification is set where the maximum Fisher's separation can be achieved. We use a constant energy constraint to limit the energy of the waveform to the constant value ϵ_x . The energy constraint is used here because it is tractable. The optimization problem can then be written as follows:

$$\arg \max_{\mathbf{\Omega}_X} g_2(\mathbf{\Omega}_X) \\ \text{s.t. } \text{trace}(\mathbf{\Omega}_X \mathbf{\Omega}_X^H) = m\epsilon_x. \quad (12)$$

2) *Target Classes With Different Means and Identical Covariance Matrices:* In this scenario where two distributions share the same covariance matrix with different mean vectors, it is possible to derive a closed-form direct measure of the classification performance, i.e., the probability of misclassification. This allows us to calculate the theoretical performance of the waveform if we are employing the same classifier defined in the derivation.

The classification problem is defined as follows.

- 1) An already detected target with extended impulse/frequency response is to be classified into one of two classes (binary classification problem).
- 2) i is the class identifier for $i = 1$ and 2.
- 3) ω_i is the state of nature of the target [35].
- 4) All statistical properties of all classes are known *a priori*.
- 5) $M_{Y_1} = \mathbf{\Omega}_X M_{R_1} \neq M_{Y_2}$
- 6) $\mathbf{\Sigma}_{Y_1} = \mathbf{\Omega}_X (\mathbf{\Sigma}_{R_1} + \mathbf{\Sigma}_C) \mathbf{\Omega}_X^H + \mathbf{\Sigma}_N = \mathbf{\Sigma}_{Y_2} = \mathbf{\Omega}_X (\mathbf{\Sigma}_{R_2} + \mathbf{\Sigma}_C) \mathbf{\Omega}_X^H + \mathbf{\Sigma}_N = \mathbf{\Sigma}_Y$.

It can be shown [34] that the probability of misclassification P_{mc} is given by

$$P_{mc} = Q\left(\sqrt{(M_{Y_1} - M_{Y_2})^H \mathbf{\Sigma}_Y^{-1} (M_{Y_1} - M_{Y_2})/2}\right). \quad (13)$$

While P_{mc} can be used as the objective function that should be minimized to maximize the classification performance, we can also make use of the properties of the Q -function and the square root function to further reduce the optimization problem objective to only maximize the term inside the square root in (13), which is $(M_{Y_1} - M_{Y_2})^H \mathbf{\Sigma}_Y^{-1} (M_{Y_1} - M_{Y_2})$.

The optimization problem is then

$$\arg \max_{\mathbf{\Omega}_X} g_1(\mathbf{\Omega}_X) = (M_{Y_1} - M_{Y_2})^H \mathbf{\Sigma}_Y^{-1} (M_{Y_1} - M_{Y_2}) \\ \text{s.t. } \text{trace}(\mathbf{\Omega}_X \mathbf{\Omega}_X^H) = m\epsilon_x \quad (14)$$

where ϵ_x is the energy of the time-domain waveform in $\mathbf{x}$.

B. Waveform Design

The waveform design section is divided into Sections III-B1 and III-B2. Section III-B1 covers the optimal and closed-form solution for the waveform design problem where classes have different covariance matrices, which is the more general case where the covariance matrices of the classes can be different or identical. Section III-B2 presents optimal waveform design methods for the scenario where the covariance matrices for classes distributions are identical.

1) *Optimal Waveform and Closed-Form Design for Two Classes With Different Covariance Matrices:* A similar waveform design strategy can be adopted for the scenario where target classes have different means and difference covariance matrices. In this scenario, minimizing P_{mc} is replaced with maximizing Fisher separation and the optimization problem in (14) is replaced with (12). This is because deriving P_{mc} for classes with different means vectors and different covariance matrices is a very challenging problem. The Fisher discriminant analysis on the other hand is used regardless of whether the matrices are identical or not.

The problem in (12) and (14) can be solved using the optimization toolbox of MATLAB. The solution to the optimization problem is used to design the optimal X and ultimately obtain the optimal radar waveform $x(t)$ that maximizes the performance of target classification. However, in the following, we propose a closed-form solution to the problem, which is derived using Lagrange multipliers, given that $\mathbf{\Sigma}_{R_1}$, $\mathbf{\Sigma}_{R_2}$, $\mathbf{\Sigma}_C$, and $\mathbf{\Sigma}_N$ can be approximated to be diagonal matrices (off-diagonal elements are negligible) [36].

Let $\mathbf{\Psi} = \mathbf{\Sigma}_{R_1} + \mathbf{\Sigma}_{R_2} + 2\mathbf{\Sigma}_C = \text{diag}(P)$ and $\mathbf{\Sigma}_N = \text{diag}(\mathbf{\Xi})$, where $P = [p_1^2, p_2^2, \dots, p_m^2]^T$, $\mathbf{\Xi} = [\sigma_1^2, \sigma_2^2, \dots, \sigma_m^2]^T$, and $\Delta M_R = [\Delta\mu_1, \Delta\mu_2, \dots, \Delta\mu_m]^T$, and then, $g_2(\mathbf{\Omega}_X)$ in (11) can be expressed as

$$g_2(\mathbf{\Omega}_X) = \Delta M_R^H \mathbf{\Omega}_X^H (\mathbf{\Omega}_X (\mathbf{\Psi}) \mathbf{\Omega}_X^H + 2\mathbf{\Sigma}_N)^{-1} \mathbf{\Omega}_X \Delta M_R \\ = \sum_{i=1}^m \left(\frac{|\Delta\mu_i|^2 |x_i|^2}{|x_i|^2 (p_i^2) + 2\sigma_i^2} \right) \\ = \sum_{i=1}^m \frac{\frac{|\Delta\mu_i|^2}{(p_i^2)} |x_i|^2}{|x_i|^2 + \frac{2\sigma_i^2}{(p_i^2)}} = \sum_{i=1}^m \frac{\alpha_i |x_i|^2}{|x_i|^2 + \beta_i} \quad (15)$$

assuming that $p_i^2 \neq 0 \forall i$.

The optimization problem becomes

$$\arg \max_{x_i} \sum_{i=1}^m \frac{\alpha_i |x_i|^2}{|x_i|^2 + \beta_i} \quad \text{s.t. } \sum_{i=1}^m |x_i|^2 = m\epsilon_x \quad (16)$$

where $\alpha_i = (|\Delta\mu_i|^2 / (p_i^2))$ and $\beta_i = (2\sigma_i^2 / (p_i^2))$.

To solve this problem, we use a Lagrange multipliers approach. It is important to note that using Lagrange multipliers does not necessarily find the global maxima of the optimization problem if the strong duality does not hold and the duality gap is not zero. This means that there is always the possibility that there is a better waveform than the diagonal waveform, which can be found by other methods. We define

the Lagrangian of our primal problem $\mathcal{L}(X, \lambda)$ as follows:

$$\mathcal{L}(X, \lambda) = \sum_{i=1}^m \frac{\alpha_i |x_i|^2}{|x_i|^2 + \beta_i} - \lambda \left(\sum_{i=1}^m |x_i|^2 - m\epsilon_x \right) \quad (17)$$

and we define the dual function $L_D(\lambda) = \max_{x_i} \mathcal{L}(X, \lambda)$.

Then, maximizing $\mathcal{L}(X, \lambda)$ w.r.t absolute of each element of X amounts to

$$\begin{aligned} \frac{\partial \mathcal{L}(X, \lambda)}{\partial x_i} &= \frac{2\alpha_i \beta_i |x_i|}{(|x_i|^2 + \beta_i)^2} - 2\lambda |x_i| = 0 \\ \Rightarrow [-2\lambda(|x_i|^2)^2 - 4\lambda\beta_i|x_i|^2 \\ &\quad + (2\alpha_i\beta_i - 2\lambda\beta_i^2)]|x_i| = 0 \end{aligned} \quad (18)$$

and thus,

$$|x_i| = 0 \quad (19)$$

is a possible solution where zero energy of the radar waveform is allocated at the i th frequency bin.

The other four possible solutions come from

$$\begin{aligned} |x_i|^2 &= \frac{4\lambda\beta_i \pm \sqrt{(-4\lambda\beta_i)^2 - 4(-2\lambda)(2\alpha_i\beta_i - 2\lambda\beta_i^2)}}{-4\lambda} \\ &= \frac{4\lambda\beta_i \pm \sqrt{4^2\lambda^2\beta_i^2 + 4^2\alpha_i\beta_i\lambda - 4^2\lambda^2\beta_i^2}}{-4\lambda} \\ &= \frac{4\lambda\beta_i \pm \sqrt{4^2\alpha_i\beta_i\lambda}}{-4\lambda} \\ &= \frac{\lambda\beta_i \pm \sqrt{\lambda\alpha_i\beta_i}}{-\lambda} \\ &= -\beta_i \mp \frac{\sqrt{\alpha_i\beta_i}}{\sqrt{\lambda}}. \end{aligned} \quad (20)$$

Please note that λ is not restricted by a sign because the multipliers of equality constraints are not restricted to be positive or negative but can be either.

Now, substituting (20) into (18), we can write

$$\begin{aligned} L_D(\lambda) &= \sum_{i=1}^m \frac{\alpha_i \frac{\lambda\beta_i \pm \sqrt{\lambda\alpha_i\beta_i}}{-\lambda}}{\frac{\lambda\beta_i \pm \sqrt{\lambda\alpha_i\beta_i}}{-\lambda} + \beta_i} - \lambda \left(\sum_{i=1}^m \frac{\lambda\beta_i \pm \sqrt{\lambda\alpha_i\beta_i}}{-\lambda} - m\epsilon_x \right) \\ &= \sum_{i=1}^m \frac{\alpha_i \lambda\beta_i \pm \alpha_i \sqrt{\lambda\alpha_i\beta_i}}{\lambda\beta_i \pm \sqrt{\lambda\alpha_i\beta_i} - \lambda\beta_i} + \sum_{i=1}^m \left(\lambda\beta_i \pm \sqrt{\lambda\alpha_i\beta_i} \right) + \lambda m\epsilon_x \\ &= \sum_{i=1}^m \frac{\alpha_i \lambda\beta_i \pm \alpha_i \sqrt{\lambda\alpha_i\beta_i}}{\pm \sqrt{\lambda\alpha_i\beta_i}} + \sum_{i=1}^m \left(\lambda\beta_i \pm \sqrt{\lambda\alpha_i\beta_i} \right) + \lambda m\epsilon_x \\ &= \sum_{i=1}^m \left(\pm \sqrt{\lambda\alpha_i\beta_i} + \alpha_i \right) + \sum_{i=1}^m \left(\lambda\beta_i \pm \sqrt{\lambda\alpha_i\beta_i} \right) + \lambda m\epsilon_x \\ &= \pm \sum_{i=1}^m \sqrt{\lambda\alpha_i\beta_i} + \sum_{i=1}^m \alpha_i + \sum_{i=1}^m \lambda\beta_i \pm \sum_{i=1}^m \sqrt{\lambda\alpha_i\beta_i} + \lambda m\epsilon_x \\ &= \left(m\epsilon_x + \sum_{i=1}^m \beta_i \right) (\sqrt{\lambda})^2 \pm 2 \sum_{i=1}^m \sqrt{\alpha_i\beta_i} (\sqrt{\lambda}) + \sum_{i=1}^m \alpha_i \end{aligned} \quad (21)$$

which is quadratic in $(\sqrt{\lambda})$ and convex.

In order to minimize the dual function $L_D(\lambda)$, now that we know that it is convex and quadratic, we just need to derive $\sqrt{\lambda}$, which satisfies $(\partial L_D(\lambda))/(\partial \sqrt{\lambda}) = 0$

$$\begin{aligned} \frac{\partial L_D(\lambda)}{\partial \sqrt{\lambda}} &= 2 \left(m\epsilon_x + \sum_{i=1}^m \beta_i \right) \sqrt{\lambda} \pm 2 \sum_{i=1}^m \sqrt{\alpha_i\beta_i} = 0 \\ \Rightarrow \sqrt{\lambda} &= \mp \frac{\sum_{i=1}^m \sqrt{\alpha_i\beta_i}}{m\epsilon_x + \sum_{i=1}^m \beta_i}. \end{aligned} \quad (22)$$

We then substitute the value of λ to obtain the optimal value of $|x_i|$ into (20) as follows:

$$\begin{aligned} |x_i|^2 &= -\beta_i \mp \frac{\sqrt{\alpha_i\beta_i}}{\sqrt{\lambda}} \\ &= -\beta_i \mp \frac{\sqrt{\alpha_i\beta_i}}{\frac{\sum_{l=1}^m \sqrt{\alpha_l\beta_l}}{m\epsilon_x + \sum_{l=1}^m \beta_l}} \\ &= -\beta_i \mp \left(\mp \frac{\sqrt{\alpha_i\beta_i}}{\frac{\sum_{l=1}^m \sqrt{\alpha_l\beta_l}}{m\epsilon_x + \sum_{l=1}^m \beta_l}} \right) \\ &= \frac{(m\epsilon_x + \sum_{l=1}^m \beta_l) \sqrt{\alpha_i\beta_i}}{\sum_{l=1}^m \sqrt{\alpha_l\beta_l}} - \beta_i \end{aligned} \quad (23)$$

by substituting the expressions of α_i and β_i

$$|x_i|^2 = \frac{\left(m\epsilon_x + \sum_{l=1}^m \frac{2\sigma_l^2}{(p_l^2)} \right) \sqrt{\frac{|\Delta\mu_i|^2}{(p_i^2)} \frac{2\sigma_i^2}{(p_i^2)}}}{\sum_{l=1}^m \sqrt{\frac{|\Delta\mu_l|^2}{(p_l^2)} \frac{2\sigma_l^2}{(p_l^2)}}} - \frac{2\sigma_i^2}{(p_i^2)}. \quad (24)$$

Because $|x_i|$ in (24) has to be positive, we conclude that the amplitude of each element of the solution waveform $X_{solution}$ is given by

$$|x_i| = \begin{cases} \sqrt{|x_i|^2}, & \text{if } |x_i|^2 \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (25)$$

and the phase can be arbitrary. Finally, x_i can be scaled to satisfy the energy constraint.

2) *Optimal Waveform Design for Two Classes With Identical Covariance Matrices:* Maximizing the classification performance is directly influenced by reducing classification errors, which translates into minimizing the probability of misclassification. Minimizing P_{mc} can be achieved, as derived in Section III-A2, by maximizing $(M_{Y_1} - M_{Y_2})^H \Sigma_Y^{-1} (M_{Y_1} - M_{Y_2})$. By expanding this term and substituting the expressions of Σ_Y , M_{Y_1} , and M_{Y_2} , we can express $g_1(\Omega_X)$ that is variable only in Ω_X as follows:

$$g_1(\Omega_X) = \Delta M_R^H \Omega_X^H (\Omega_X (\Sigma_R + \Sigma_C) \Omega_X^H + \Sigma_N)^{-1} \Omega_X \Delta M_R \quad (26)$$

where $\Delta M_R = M_{R_1} - M_{R_2}$.

The design of the optimal radar waveform is then achieved by solving the optimization problem in (14). Solving (14) to find a closed-form solution is a challenging problem. However, a numerical solution for the optimization problem in (14) can be computed using MATLAB optimization toolbox (e.g., [34]).

In some special situations, it is possible to derive a closed-form solution to the optimization problems. Designing the optimal waveform in these situations will require significantly less computations. The two main special situations explored here are dependent on the relationship of the two terms in (26).

- 1) *Signal and Clutter Term*: $\mathbf{\Omega}_X(\mathbf{\Sigma}_R + \mathbf{\Sigma}_C)\mathbf{\Omega}_X^H$.
- 2) *Noise Term*: $\mathbf{\Sigma}_N$.

The first situation is defined when the waveform energy is very high such that the following approximation is accurate:

$$(\mathbf{\Omega}_X(\mathbf{\Sigma}_R + \mathbf{\Sigma}_C)\mathbf{\Omega}_X^H + \mathbf{\Sigma}_N)^{-1} \approx (\mathbf{\Omega}_X^H)^{-1}(\mathbf{\Sigma}_R + \mathbf{\Sigma}_C)^{-1}(\mathbf{\Omega}_X)^{-1}.$$

The approximation becomes more accurate as the magnitude of each element in the signal and clutter term becomes much higher than that of the corresponding element in the noise term such that the noise term becomes negligible in (26).

In this situation, we can see that the waveform design no longer matters as (26) becomes independent of $\mathbf{\Omega}_X$. This means that, as long as no element in X equals zero [see (13)], the probability of misclassification will always be given by

$$P_{mc} = Q\left(\sqrt{\Delta M_R^H(\mathbf{\Sigma}_R + \mathbf{\Sigma}_C)^{-1}\Delta M_R/2}\right). \quad (27)$$

The second situation is the opposite situation where the waveform energy is too low such that the approximation below is valid:

$$(\mathbf{\Omega}_X(\mathbf{\Sigma}_R + \mathbf{\Sigma}_C)\mathbf{\Omega}_X^H + \mathbf{\Sigma}_N)^{-1} \approx (\mathbf{\Sigma}_N)^{-1}.$$

This means that (26), in this situation, can be expressed as

$$g_1(\mathbf{\Omega}_X) = X^H \mathbf{\Omega}_M^H (\mathbf{\Sigma}_N)^{-1} \mathbf{\Omega}_M X \quad (28)$$

where $\mathbf{\Omega}_M = \text{diag}(\Delta M)$.

The values of X that maximizes the new objective function is obtained using the eigenvector $\gamma_{\lambda_{\max}}$ of the matrix $\mathbf{\Omega}_M^H (\mathbf{\Sigma}_N)^{-1} \mathbf{\Omega}_M$, which corresponds to its maximum eigenvalue $\lambda_{\max}$. Finally, the optimal X equals $\gamma_{\lambda_{\max}}$ scaled by $\sqrt{m\epsilon_x}/(\gamma_{\lambda_{\max}}^H \gamma_{\lambda_{\max}})^{1/2}$ to satisfy the energy constraint.

The probability of misclassification in this situation can be calculated using

$$P_{mc} = Q\left(\sqrt{m\epsilon_x \lambda_{\max}/2}\right). \quad (29)$$

C. Classifier Design

Here, as in [25], the classifier assigns the target to the class closest to the target return Y in terms of the Mahalanobis distance after projecting all the received data and the means by $\mathbf{w}$. The projection vector $\mathbf{w}$, depending on the scenario, is given by the following.

- 1) $\mathbf{w} = (\mathbf{\Sigma}_Y)^{-1}(M_{Y_1} - M_{Y_2})$ when the covariance matrices are identical
- 2) $\mathbf{w} = (\mathbf{\Sigma}_{Y_1} + \mathbf{\Sigma}_{Y_2})^{-1}(M_{Y_1} - M_{Y_2})$ when they are not identical.

The Mahalanobis distance for the i th class is then given by

$$d_i = \sqrt{\frac{|\mathbf{w}^H(Y - M_{Y_i})|^2}{\mathbf{w}^H \mathbf{\Sigma}_{Y_i} \mathbf{w}}}. \quad (30)$$

The target is assigned to the classes with the minimum d_i .

Note that the choice of Mahalanobis-distance-based classifier is for simple representation, and more sophisticated classifiers can also be used.

IV. RESULTS AND DISCUSSION

In this section, we present the simulation results generated using synthetic and real targets data to study the performance of the optimized waveforms presented in Sections III-B2 and III-B1 and the closed-form solution derived in Section III-B1 (from here on referred to as “the diagonal solution” and the waveform as “the diagonal waveform”) against other radar waveforms found in the literature [25].

The four main waveform design methods to be studied in this scenario are as follows.

- 1) *Diagonal Waveform*: The waveform generated from computing the optimal waveform using the diagonal solution proposed in this article in (25)
- 2) *Chirp Waveform*: A wideband flat spectrum linearly frequency-modulated waveform.
- 3) *PS Optimal Waveform*: Obtained using MATLAB optimization toolbox to solve (12) or (14) where the algorithm “particle swarm” is used. The optimization problem in (14) is solved when the covariance matrices are identical and (12) is solved when they are not. In MATLAB, the default parameters of the particle swarm are used and only the objective function is required as input.
- 4) *Average Mahalanobis Distance (AMD) Waveform*: In the scenario, where two target classes are considered, the AMD waveform is designed in the time domain to maximize the Mahalanobis distance between classes’ mean vectors [7].

A. Results Generated Using Synthetic Data

The common simulation setup is as follows.

- 1) $m = 64$, which is the number of frequency bins for all vectors.
- 2) $n = 10\,000$, which is the number of Monte Carlo runs.
- 3) Mean vectors and covariance matrices for all target classes are generated arbitrarily at the start of the simulation.
- 4) Targets’ covariance matrices are not diagonal.
- 5) Clutter if present is assumed white with a spectral variance of 0.5.
- 6) Noise is assumed white with a spectral variance of 1.
- 7) The realizations of the target response R , noise vector N , and clutter vector C are different at each run (i.e., generated from pulse to pulse).

1) *Target Classes With Different Mean Vectors and Identical Covariance Matrices (i.e., $\mathbf{\Sigma}_{R_1} = \mathbf{\Sigma}_{R_2}$) and Clutter Is Negligible ($\sigma_c^2 = 0$)*: We start first by studying the scenario where no significant clutter response is received in the radar system (e.g., radar looking above ground) and the two classes of targets share the same covariance matrix. The standard measure of the classification performance that we will be using from here on is the probability of correct classification P_{cc} , which is the complement probability of P_{mc} (i.e., $P_{cc} = 1 - P_{mc}$).

Fig. 1. Theoretical P_{cc} versus waveform energy ϵ_x for PS optimal, diagonal, chirp, and low ϵ_x waveform design methods. Measured P_{cc} versus waveform energy ϵ_x for AMD waveform and the high ϵ_x performance limit for the scenario where the two classes have identical covariance matrices and no clutter is present.

In this scenario, we study the performance of the diagonal solution and the three other waveforms calculated directly from the theoretical probability of misclassification as derived and expressed in (13).

Fig. 1 shows the theoretical performance of the four aforementioned waveforms in terms of the probability of correct classification P_{cc} versus the time-domain waveform energy ϵ_x . The figure also shows the expected high waveform energy ϵ_x performance limit calculated using (27) as well as the theoretical performance of the low ϵ_x waveform at each energy level. Only the measured performance of AMD waveform is shown in Fig. 1. This is because the derived theoretical performance of AMD in [7] does not account for classes' covariance matrices resulting in difference between the theoretical and measured performances. As the noise variance is unchanged along the x -axis of the plot, we expect every point on the x -axis to correspond to different SNRs, which is also expected to be monotonically increasing with ϵ_x .

The optimal waveform designed using the particle swarm algorithm is shown to outperform all other waveforms with slight advantage over the diagonal solution. We can see that the diagonal waveform achieved a suboptimal performance with approximately negligible difference between its performance and the performance of the optimized waveform designed with the optimization algorithms. This, of course, comes at the expense of the additional computational complexity required to obtain the optimal waveform in comparison to the diagonal solution [37].

The high ϵ_x performance limit is calculated using (27). We can see that the performance of no waveform can surpass that of the high ϵ_x limit and the waveforms with the best performance start saturating as they get closer to the high ϵ_x limit value. The figure also shows the performance of the low ϵ_x waveform, which performs worse in comparison to other waveforms in the current energy levels where ϵ_x is not very low.

Fig. 2. Theoretical P_{cc} versus waveform energy ϵ_x for PS optimal, diagonal, chirp, and low ϵ_x waveform design methods. Measured P_{cc} versus waveform energy ϵ_x for AMD waveform and the high ϵ_x performance limit for the scenario where the two classes have identical covariance matrices and the clutter response is white Gaussian.

The figure also shows the measured performance of the AMD waveform. The measured performance is shown because the theoretical performance is not derived for the scenario where variance in target response is not negligible. The figure shows that the AMD does not perform well in comparison to other waveforms.

2) *Performance When the Covariance Matrices Are Identical (i.e., $\Sigma_{R_1} = \Sigma_{R_2}$) and Clutter Is Present ($\sigma_c^2 = 0.5$):* In this second scenario, we study the performance of the same four waveform design methods when the two classes also share the same covariance matrix but in the presence of clutter and noise. The clutter and noise vectors as mentioned before are both assumed white with a spectral variance of 0.5 and 1, respectively.

Fig. 2 shows the performance of the four waveforms in terms of P_{cc} versus ϵ_x . The figure shows no difference in the relative performances of the four waveforms between this scenario and the previous one. However, the overall performance of all waveforms has clearly degraded due to the presence of the clutter where, for example, the PS optimal and the diagonal waveform can only attain around $P_{cc} = 0.95$ at $\epsilon_x = 10$ dB, while it attained higher than that, at the same energy level, in the previous scenario where no clutter is present.

Fig. 3 shows the performance of the four waveforms in terms of P_{cc} versus ϵ_x where the covariance matrix of the clutter is not diagonal. The eigenvalue ratio of the clutter covariance matrix is not one and equals 0.54. Compared to the previous plot, the overall performance has slightly degraded.

Also, in this scenario, the low ϵ_x waveform achieves better performance than the previous scenario where the clutter is assumed white.

3) *Performance When the Covariance Matrices Are Different (i.e., $\Sigma_{R_1} \neq \Sigma_{R_2}$) and Clutter Is Negligible ($\sigma_c^2 = 0$):* Next, we study the more general scenario where the two classes have different mean vectors and covariance matrices. Also, the clutter is assumed negligible in this scenario.

Fig. 3. Theoretical P_{cc} versus waveform energy ϵ_x for PS optimal, diagonal, chirp, and low ϵ_x waveform design methods. Measured P_{cc} versus waveform energy ϵ_x for AMD waveform and the high ϵ_x performance limit for the scenario where the two classes have identical covariance matrices and the clutter response is colored Gaussian.

Fig. 4. Measured P_{cc} versus waveform energy ϵ_x for PS optimal, diagonal, chirp, and AMD waveform design method for the scenario where the two classes have different covariance matrices and no clutter is present.

Fig. 4 shows P_{cc} vs ϵ_x for the four main waveforms. The figure shows a noticeable degradation in the overall performance of all waveforms where the maximum expected P_{cc} at $\epsilon_x = 10$ dB is around $P_{cc} = 0.84$.

4) *Performance When the Covariance Matrices Are Different (i.e., $\Sigma_{R_1} \neq \Sigma_{R_2}$) and Clutter Is Present ($\sigma_c^2 = 0.5$):* The performance of the four waveforms is shown in Fig. 5. As expected, the performance of the waveforms looks like the previous scenario except that it is now degraded due to the presence of clutter. The maximum P_{cc} at $\epsilon_x = 10$ dB is now $P_{cc} = 0.82$ in comparison to around $P_{cc} = 0.84$ attained in the clutter-less scenario.

B. Results Generated Using Real Dataset

In this section, the synthetic target data are replaced with real data from the Moving and Stationary Target Acquisition and Recognition (MSTAR) dataset [38]. The dataset is made

Fig. 5. Measured P_{cc} versus waveform energy ϵ_x for PS optimal, diagonal, chirp, and AMD waveform design method for the scenario where the two classes have different covariance matrices and the clutter is white Gaussian.

up of extended complex TFRs of ten civilian vehicles. The TFRs are captured at four different elevation angles θ_{el} and 5760 azimuth angles covering the 360° of the target. The statistical properties of the classes of targets are estimated from the data and the rest of the data are used in results generation. Target mean vectors are estimated by averaging classes' TFRs from the training data, while covariance matrices are estimated using a sample covariance matrix estimator. Clutter and noise vectors are generated synthetically from white complex Gaussian random fields. We assume that the set of TFRs in a sector from the 360° of the target consists of realizations from the same given distribution that can be estimated from the responses in the sector. We use ρ_θ to refer to the sector width in degrees.

The three main waveforms to be studied in this scenario are as follows.

- 1) *Diagonal Waveform:* The waveform generated from computing the optimal waveform using the diagonal solution proposed in this article.
- 2) *Chirp Waveform:* A wideband flat spectrum linearly frequency-modulated waveform.
- 3) *PS Optimal Waveform:* Obtained using MATLAB optimization toolbox to solve (12) where the algorithm “particle swarm” is used.

1) *Performance of Classifying the Target Into “Toyota Tacoma” or “Toyota Avalon” With $\theta_{el} = 60^\circ$, $\rho_\theta = 4^\circ$, and no Clutter Is Present $\sigma_c^2 = 0$:* In this scenario, we study the classification performance of the diagonal solution against the PS optimal waveform and the chirp waveform. The radar is to classify an already detected target into either a “Toyota Tacoma” or “Toyota Avalon” using its extended TFR [38]. The radar-target orientation is assumed unknown, but it is given that it is within the first four degrees of the target (in the civilian vehicle (CV) dataset, $\theta_{az} = 0^\circ$ is at the front of the vehicle) in azimuth ($\rho_\theta = 4^\circ$) at $\theta_{el} = 60^\circ$. The only type of interference present, in this scenario, is noise, which is generated synthetically to be white with unity spectral variance σ_n^2 .

Fig. 6. Probability of correct classification P_{cc} versus waveform energy ϵ_x for different waveform design methods for the scenario where the radar is to classify the target into “Toyota Tacoma” or “Toyota Avalon” where $\theta_{el} = 60^\circ$, $\rho_\theta = 4^\circ$, and no clutter is present $\sigma_c^2 = 0$ where the optimal waveform is designed using particle swarm.

Fig. 7. Probability of correct classification P_{cc} versus waveform energy ϵ_x for different waveform design methods for the scenario where the radar is to classify the target into “Toyota Tacoma” or “Toyota Avalon” where $\theta_{el} = 60^\circ$, $\rho_\theta = 4^\circ$, and clutter is present $\sigma_c^2 = 0.5$ where the optimal waveform is designed using particle swarm.

The classification performance in terms of P_{cc} versus ϵ_x for the three waveforms considered here is shown in Fig. 6. The figure shows that on average, the PS optimal and the diagonal waveform perform better than the chirp waveform. It also shows how the diagonal waveform attains a performance close to the PS optimal waveform.

2) *Performance of Classifying the Target Into “Toyota Tacoma” or “Toyota Avalon” With $\theta_{el} = 60^\circ$, $\rho_\theta = 4^\circ$, and Clutter Is Present $\sigma_c^2 = 0.5$:* When the same scenario as Section IV-B1 is considered but clutter is present, the resulting classification performance is shown in Fig. 7. In general, we see an expected degradation due to the presence of clutter, but the relative difference between the waveforms is almost the same as the previous scenario of Section IV-B1.

V. CONCLUSION

In this article, we introduced signal-dependent interference to the frequency-based radar waveform design for optimal

binary target identification. The two main scenarios considered here are when the TFRs are complex, random, and normally distributed with unequal mean vectors, but their covariance matrices are either identical or unequal. The objective function for the first scenario was derived to minimize the probability of misclassification and derived for the second scenario to maximize Fisher’s separability function. Based on the new signal model and objective functions, we updated the optimization problems and the waveform and receiver design methods for the two scenarios. The optimal waveforms can be then designed using general optimization solvers. This article also studied special situations at extreme (high or low) waveform energy levels where a solution for the optimal waveforms can be obtained. Also, a closed-form expression was derived for an approximate solution of the optimization problems for both scenarios at any waveform energy level. The closed-form solution was tested against the chirp waveform and the waveforms were obtained by solving the optimization problems using two global optimization algorithms. The closed-form solution showed an overall better performance than the chirp waveform in all scenarios while performing close if not the same as the waveforms designed using global optimization algorithms. Synthetic and real data were used to obtain simulation results.

REFERENCES

- [1] M. R. Bell, “Information theory and radar waveform design,” *IEEE Trans. Inf. Theory*, vol. 39, no. 5, pp. 1578–1597, Sep. 1993.
- [2] S. Kay, “Optimal signal design for detection of Gaussian point targets in stationary Gaussian clutter/reverberation,” *IEEE J. Sel. Topics Signal Process.*, vol. 1, no. 1, pp. 31–41, Jun. 2007.
- [3] S. Z. Alshirah and B. Mulgrew, “Improved 2-class target classification performance using radar waveform design,” in *Proc. IEEE Radar Conf. (RadarConf)*, Apr. 2018, pp. 0458–0461.
- [4] S. D. Blunt and E. L. Mokole, “Overview of radar waveform diversity,” *IEEE Aerosp. Electron. Syst. Mag.*, vol. 31, no. 11, pp. 2–42, Nov. 2016.
- [5] L. K. Patton and B. D. Rigling, “Modulus constraints in adaptive radar waveform design,” in *Proc. IEEE Radar Conf.*, May 2008, pp. 1–6.
- [6] D. A. Garren, A. C. Odom, M. K. Osborn, J. S. Goldstein, S. U. Pillai, and J. R. Guerci, “Full-polarization matched-illumination for target detection and identification,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 38, no. 3, pp. 824–837, Jul. 2002.
- [7] D. A. Garren, M. K. Osborn, A. C. Odom, J. S. Goldstein, S. U. Pillai, and J. R. Guerci, “Optimal transmission pulse shape for detection and identification with uncertain target aspect,” in *Proc. IEEE Radar Conf.*, May 2001, pp. 123–128.
- [8] D. A. Garren, M. K. Osborn, A. C. Odom, J. S. Goldstein, S. U. Pillai, and J. R. Guerci, “Enhanced target detection and identification via optimised radar transmission pulse shape,” *IEE Proc. Radar, Sonar Navigat.*, vol. 148, no. 3, pp. 130–138, Jun. 2001.
- [9] A. Nehorai, F. Gini, M. S. Greco, A. P. Suppappola, and M. Rangaswamy, “Introduction to the issue on adaptive waveform design for agile sensing and communication,” *IEEE J. Sel. Topics Signal Process.*, vol. 1, no. 1, pp. 2–5, Jun. 2007.
- [10] J. Bae and N. A. Goodman, “Target recognition with high-fidelity target signatures and adaptive waveforms in MIMO radar,” in *Proc. IEEE 6th Int. Workshop Comput. Adv. Multi-Sensor Adapt. Process. (CAMSAP)*, Dec. 2015, pp. 285–288.
- [11] J. Bae and N. A. Goodman, “Widely separated MIMO radar with adaptive waveform for target classification,” in *Proc. 4th IEEE Int. Workshop Comput. Adv. Multi-Sensor Adapt. Process. (CAMSAP)*, Dec. 2011, pp. 21–24.
- [12] J. Bae and N. A. Goodman, “Automatic target recognition with unknown orientation and adaptive waveforms,” in *Proc. IEEE RadarConf (RADAR)*, May 2011, pp. 1000–1005.
- [13] R. Romero and N. A. Goodman, “Improved waveform design for target recognition with multiple transmissions,” in *Proc. Int. Waveform Diversity Design Conf.*, Feb. 2009, pp. 26–30.

- [14] N. A. Goodman, "Closed-loop radar with adaptively matched waveforms," in *Proc. Int. Conf. Electromagn. Adv. Appl.*, Sep. 2007, pp. 468–471.
- [15] R. A. Romero, J. Bae, and N. A. Goodman, "Theory and application of SNR and mutual information matched illumination waveforms," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 47, no. 2, pp. 912–927, Apr. 2011.
- [16] J. Bae and N. A. Goodman, "Evaluation of modulus-constrained matched illumination waveforms for target identification," in *Proc. IEEE Radar Conf.*, May 2010, pp. 871–876.
- [17] R. A. Romero and N. A. Goodman, "Waveform design in signal-dependent interference and application to target recognition with multiple transmissions," *IET Radar, Sonar Navigat.*, vol. 3, no. 4, pp. 328–340, Aug. 2009.
- [18] R. Romero and N. A. Goodman, "Information-theoretic matched waveform in signal dependent interference," in *Proc. IEEE Radar Conf.*, May 2008, pp. 1–6.
- [19] T. B. Butler and N. A. Goodman, "Multistatic target classification with adaptive waveforms," in *Proc. IEEE Radar Conf.*, May 2008, pp. 1–6.
- [20] J. H. Bae and N. A. Goodman, "Adaptive waveforms for target class discrimination," in *Proc. Int. Waveform Diversity Design Conf.*, Jun. 2007, pp. 395–399.
- [21] N. A. Goodman, P. R. Venkata, and M. A. Neifeld, "Adaptive waveform design and sequential hypothesis testing for target recognition with active sensors," *IEEE J. Sel. Topics Signal Process.*, vol. 1, no. 1, pp. 105–113, Jun. 2007.
- [22] P. R. Venkata and N. A. Goodman, "Novel iterative techniques for radar target discrimination," in *Proc. Int. Waveform Diversity Design Conf.*, Jan. 2006, pp. 1–7.
- [23] Q. J. O. Tan and R. A. Romero, "Jammer nulling adaptive waveforms with cognitive radar for aircraft RCS recognition in presence of frequency sweep and base jammers," in *Proc. 52nd Asilomar Conf. Signals, Syst., Comput.*, Oct. 2018, pp. 1339–1343.
- [24] Q. J. O. Tan and R. A. Romero, "Ground vehicle target signature identification with cognitive automotive radar using 24–25 and 76–77 GHz bands," *IET Radar, Sonar Navigat.*, vol. 12, no. 12, pp. 1448–1465, Dec. 2018.
- [25] S. Z. Alshirah, S. Gishkori, and B. Mulgrew, "Frequency-based optimal radar waveform design for classification performance maximization using multiclass Fisher analysis," *IEEE Trans. Geosci. Remote Sens.*, vol. 59, no. 4, pp. 3010–3021, Apr. 2021.
- [26] A. Aubry, A. DeMaio, A. Farina, and M. Wicks, "Knowledge-aided (potentially cognitive) transmit signal and receive filter design in signal-dependent clutter," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 49, no. 1, pp. 93–117, Jan. 2013.
- [27] A. Aubry, A. De Maio, and M. M. Naghsh, "Optimizing radar waveform and Doppler filter bank via generalized fractional programming," *IEEE J. Sel. Topics Signal Process.*, vol. 9, no. 8, pp. 1387–1399, Dec. 2015.
- [28] A. Aubry, V. Carotenuto, A. De Maio, and L. Pallotta, "High range resolution profile estimation via a cognitive stepped frequency technique," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 55, no. 1, pp. 444–458, Feb. 2019.
- [29] A. Aubry, A. De Maio, M. A. Govoni, and L. Martino, "On the design of multi-spectrally constrained constant modulus radar signals," *IEEE Trans. Signal Process.*, vol. 68, pp. 2231–2243, 2020.
- [30] P. Addabbo, A. Aubry, A. De Maio, L. Pallotta, and S. L. Ullo, "HRR profile estimation using SLIM," *IET Radar, Sonar Navigat.*, vol. 13, no. 4, pp. 512–521, Apr. 2019.
- [31] A. Aubry, G. Cui, A. De Maio, and X. Du, "Hidden convexity in robust waveform and receive filter bank optimization for range unambiguous clutter," in *Proc. IEEE Int. Radar Conf. (RADAR)*, Apr. 2020, pp. 885–889.
- [32] A. Farina, S. Haykin, and A. De Maio, *The Impact of Cognition on Radar Technology*. Kolkata, India: Institution of Engineering & Technology, 2017.
- [33] A. Maio, A. Farina, and J. Li, *Radar Waveform Design Based on Optimization Theory*. Anaheim, CA, USA: SciTech Publishing, 2020.
- [34] S. Z. Alshirah, S. Gishkori, and B. Mulgrew, "Frequency-domain based waveform design for binary extended-target classification," in *Proc. IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP)*, May 2019, pp. 4450–4453.
- [35] R. O. Duda, *Pattern Classification*, 2nd ed. New York, NY, USA: Wiley, 2000.
- [36] C. F. Van Loan and G. H. Golub, *Matrix Computations*. Baltimore, MD, USA: Johns Hopkins Univ. Press, 1983.
- [37] L. Ingber, "Adaptive simulated annealing (ASA): Lessons learned," 2000, *arXiv:cs/0001018*. [Online]. Available: <https://arxiv.org/abs/cs/0001018>
- [38] *Civilian Vehicle Data Dome Overview*. Accessed: Sep. 3, 2019. [Online]. Available: <https://www.sdms.af.mil/index.php>

and target classification.

The University of Edinburgh, Edinburgh, U.K. Since March 2020, he has been working as a Research Fellow at King Abdullah University of Science and Technology, Thuwal, Saudi Arabia. His research interests include signal and image processing and machine learning.

lectureship in 1986, promoted to Senior Lecturer in 1994, and became a Reader in 1996. At The University of Edinburgh, he was appointed a Personal Chair in October 1999 (Professor of signals and systems). He is a coauthor of three books on signal processing. His research interests are in adaptive signal processing and estimation theory and their application to radar and sensor systems.

Sultan Z. Alshirah received the B.Sc. degree (Hons.) in electrical engineering from King Saud University, Riyadh, Saudi Arabia, in 2011, and the M.Sc. degree (Hons.) in communication and signal processing and the Ph.D. degree in digital communications from The University of Edinburgh, Edinburgh, U.K., in 2016 and 2020, respectively.

He is currently a Research Assistant Professor at King Abdulaziz City for Science and Technology (KACST), Riyadh. His research interests include adaptive signal processing, radar waveform design,

Shahzad Gishkori (Senior Member, IEEE) received the B.Sc. degree in electrical engineering from the University of Engineering and Technology Lahore, Lahore, Pakistan, in 2002, and the M.Sc. (*cum laude*) and Ph.D. degrees in electrical engineering from Delft University of Technology, Delft, The Netherlands, in 2009 and 2014, respectively.

From 2014 to 2015, he was a Research Associate with Imperial College London, London, U.K. From 2016 to 2020, he was a Research Associate with

Bernard (Bernie) Mulgrew (Fellow, IEEE) received the B.Sc. degree from Queen's University Belfast, Belfast, U.K., in 1979, and the Ph.D. degree from The University of Edinburgh, Edinburgh, U.K., in 1987.

After graduation, he worked for four years as a Development Engineer at the Radar Systems Department, Ferranti, Edinburgh. From 1983 to 1986, he was a Research Associate with the Department of Electrical Engineering, The University of Edinburgh. He was appointed to