Rate-Splitting Multiple Access for Intelligent Reflecting Surface aided Multi-User Communications

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Abstract—Intelligent reflecting surface (IRS) has recently emerged as a promising technology for 6G wireless systems, due to its capability to reconfigure the wireless propagation environment. In this paper, we investigate a Rate-Splitting Multiple Access (RSMA) for IRS-assisted downlink system, where the base station (BS) communicates with single-antenna users with the help of an IRS. RSMA relies on rate-splitting (RS) at the BS and successive interference cancellation (SIC) at the users and provides a generalized multiple access framework. We derive a new architecture called IRS-RS that leverages the interplay between RS and IRS. For performance analysis, we utilize an *on-off control technique* to control the passive beamforming vector of the IRS-RS and derive the closed-form expressions for outage probability of cell-edge users and near users. Moreover, we also analyze the outage behavior of cell-edge users for a sufficiently large number of reflecting elements. Additionally, we also analyze the outage performance of cooperative RS based decode-and-forward (DF)-assisted framework called DF-RS. Through simulation results, it is shown that the proposed framework outperforms the corresponding DF-RS, RS without IRS and IRS-assisted conventional non-orthogonal multiple access (NOMA) schemes. Furthermore, the impact of various system's parameters such as the number of IRS reflecting elements and the number of users on the system performance is revealed.

Index Terms—Intelligent reflecting surface (IRS), decode-and-forward (DF), rate-splitting (RS), downlink, multi-user.

I. INTRODUCTION

DUE to the advancement of radio frequency (RF) micro-electro-mechanical systems (MEMS) along with the plentiful applications of the reconfigurable and programmable metasurfaces [1], [2], intelligent reflecting surface (IRS) has recently attracted wide attention from both industry and academia. Multiple low-cost programmable reflecting elements are deployed at IRS, wherein each reflecting element has capability to adjust the phase of each passive antenna element. Consequently, the IRSs provide the reconfigurable reflections

of the impinging signals and thus, it gains huge advantages over conventional relaying protocols [3]–[5]. Moreover, [6] has pointed out IRS as one of the remedies for improving the spectrum efficiency and energy efficiency in 6G wireless communication network.

A. Related Work

Various schemes for IRS-assisted communications have been proposed to improve the energy and spectral efficiency [7]–[16]. In order to enhance the spectral efficiency and signal quality at the receiver, the authors in [7] have studied large intelligent surfaces (LIS)-space shift keying and LIS-spatial modulation schemes, while [8] has emphasized reconfigurable intelligent surfaces (RIS)-aided wireless networks and discussed the most important open research challenges in RIS-aided communications. An IRS-assisted non-orthogonal multiple access (NOMA) transmission scheme has been studied in [9] in order to serve more users. The authors in [10] have considered an IRS-assisted multi-user multiple-input single-output (MISO) system and proposed a robust beamforming design algorithm, whereas in [11] the achievable rate region has been identified for the coordinated IRSs-aided multi-user MISO interference channel. While [26] has focused on joint optimization of beamforming vector at the base station (BS) and the phase-shift matrix at the IRS to minimize an outage probability of IRS-assisted MISO network, resource allocation scheme has been investigated for IRS-aided MISO systems in [13]. Moreover, the authors in [14] have compared the analytical performance of IRS with conventional decode-and-forward (DF) relaying. More recently, the authors in [16] have studied the maximization of the average sum rate for two-user downlink scenario with NOMA and investigated two schemes for phase shift adjustments, while a weighted sum power minimization optimization problem has been formulated in [17] with the constraint of quality-of-service (QoS) for IRS-assisted MISO uplink network under the perfect and imperfect channel state information (CSI) knowledge. However, the works [7]–[16] have not analyzed the performance of rate-splitting multiple access (RSMA) based IRS-assisted communication network and thus, it motivates us to focus on this problem.

Recently, RSMA [18]–[25] and NOMA [9], [27] have emerged as very promising technologies which significantly improve the spectrum efficiency. Due to the flexibility of

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rate-splitting (RS) in controlling the interference in single-input single-output interference channels, the researchers have recently investigated the advantages of RS in multi-antenna MISO broadcast channels and have derived a novel multiple access framework, so-called RSMA, that encompasses Space Division Multiple Access (SDMA) and NOMA as special cases [18]–[21]. While the authors in [18] have studied the fundamental of RS and its application in massive MIMO and multi-cell scenario with respect to *weighted mean square error* (WMSE) and *weighted sum-rate* (WSR) performance metric, in [19], the *sum rate maximization* problem has been considered to design precoders with partial channel state information (CSI) and it has been illustrated that RS scheme boosts the achievable Degrees of Freedom (DoF). Moreover, the authors in [20] have adopted an unconventional RS transmission scheme to achieve max-min fairness in a multi-user MISO system and by utilizing the *Weighted Minimum Mean Square Error* (WMMSE) approach, they have highlighted that in the interference limited regimes, the RS scheme gives better performance than the conventional scheme without RS. The authors in [21] have demonstrated the advantages of RS over conventional NOMA (i.e., superposition coding with successive interference cancellation (SC-SIC)) scheme in multi-antenna settings in terms of *complexity and weighted sum rate performance*. By considering RS and common beamforming coordination, the joint optimization of beamforming and rate allocation to maximize both *energy efficiency and spectral efficiency* have been studied in [23]. In [24] RS has been adopted for a multigroup multicast downlink MISO systems and a generic *max-min fairness* optimization problem has been formulated and solved by developing a *modified WMMSE* approach together with an alternating optimization algorithm and significant performance enhancements have been reported over conventional precoding techniques. However, none of the RS works in [18]–[25] have derived the analytical outage behavior of the IRS-assisted downlink system.

RS can also be applied to scenarios with relaying. The cooperative RS (CRS) has been investigated in [28]–[31]. While the weighted sum rate maximization problem has been formulated in [28] to jointly design the precoder and the resource allocation for three-node CRS network, a successive convex approximation based scheme has been proposed in [30] for jointly optimizing the precoders, message split and time slot allocation in K -user CRS network. However, the authors in [28]–[31] have not explored the outage performance of the CRS network.

Though the above discussions highlighted the benefits of RS in multi-antenna settings (i.e., MIMO and MISO scenarios), RS also finds benefits in single-antenna settings (i.e., each user and BS have a single antenna). Indeed, although NOMA is capacity achieving in single-input single-output (SISO) broadcast channel (BC), it is very complex since the strong user has to decode the message of $K - 1$ users and therefore requires $K - 1$ layers of SIC. In contrast, [21] demonstrated that in SISO BC, RS can get similar performance to NOMA but with a single SIC, therefore significantly reducing the receiver complexity. This calls for strong benefits in marrying RS with IRS even in the presence of single-antenna nodes.

B. Motivation

The above works [7]–[17] have investigated about IRS-assisted communications in order to enhance the energy and spectral efficiency of MISO system, while the RS works have been explored in [18]–[25] to improve the spectrum efficiency of communication system. In addition, CRS system has been studied in [27]–[30]. Note that the works in [7]–[25], [27]–[30] have mainly focused on resource optimization in either IRS-assisted communication system or RS based system, however, none of the works [7]–[25], [27]–[30] have consider the joint benefit of integration of IRS and RSMA and analyzed the outage performance of RSMA based IRS-assisted communication network. Thus, it motivates us to focus on this problem. Though the works [7]–[25] have highlighted the benefits of RS in multi-antenna settings, RS also finds benefits in single-antenna settings. Indeed, although NOMA is capacity achieving in single-input single-output (SISO) broadcast channel (BC), it is very complex since the strong user has to decode the message of $K - 1$ users and therefore requires $K - 1$ layers of SIC. In contrast, [21] have demonstrated that in SISO BC, RS can get similar performance to NOMA but with a single SIC, therefore significantly reducing the receiver complexity. This calls for strong benefits in marrying RS with IRS even in the presence of single-antenna nodes. Motivated by the aforementioned discussions and considering the fact that IRS will play a pivotal role in the next-generation (i.e., Beyond 5G (B5G) and 6G) wireless networks as a cost, power and spectrum efficient technology, in this work, we consider a multi-user single-antenna downlink environment, where users are divided into a near users (users closer to the BS) and cell-edge users and IRSs are deployed near to cell-edge and only the cell-edge users are served by the IRSs equipped with N passive reflecting elements. In order to control these N passive reflecting elements, we propose an *on-off technique*. Moreover, we apply RS, particularly one-layer RS (1L-RS) [18], [19], [21] at the BS which splits the message of each of the K users into a common and a private part. The splitted common parts of all the K users are encoded into a single common stream (by using a codebook known to the users), which is decoded by all users. The splitted private part of a user is decoded only by the intended user. Accordingly, this work focuses on the outage performance analysis of the NUs and CEUs.

C. Contributions

The main contributions are highlighted as follows:

- We explore for the first time the interplay between RSMA and IRS and show that RSMA is a promising multiple access strategy for IRS-assisted multi-user communications. Although RSMA is a general multiple access framework [21], we emphasize on analyzing the 1L-RS for considered IRS-assisted cellular network with single-antenna BS.
- Utilizing the *on-off control* at the IRS, we derive the closed-form expressions for outage probability of cell-edge users (CEUs) and near users (NUs) by considering both common as well as private data signal-to-

interference-plus-noise ratio (SINR). To get more insights, we study the impact of sufficiently large number of IRS reflecting elements on the outage behavior of a cell-edge user.

- In order to have a comparative analysis, we consider a cooperative RS based DF-assisted framework called DF-RS, wherein each CEU is assisted by a DF relay node. Under the DF-RS scenario, we derive the closed-form expression of outage probability for the cell-edge user with both the available and unavailable direct BS-CEU link.
- Moreover, we have compared the outage performance of the proposed IRS-RS framework with an IRS-assisted NOMA scheme [9], where a multiple-antenna BS applies NOMA in each orthogonal spatial dimension to serve a NU and a single IRS-assisted CEU. Through simulations, it is shown that the proposed IRS-RS transmission with single antenna BS provides significant performance gain as compared to the IRS-assisted NOMA framework of [9].
- Through extensive numerical results, it is shown that the proposed IRS-RS framework also outperforms the corresponding cooperative DF-RS framework and the conventional RS-based cellular network without IRS.

Structure: The rest parts of the paper is structured as follows. The system model is described in Section II. While Section III-A presents the proposed on-off scheme for controlling the passive beamforming vector at the IRS with practical phase shifts and outage analysis. The performance analysis of DF relay-assisted RS framework is illustrated in Section IV. In Section V, numerical results are presented. Finally, conclusions are drawn in Section VI.

Notations: The following notations are used throughout the paper. The lowercase and uppercase boldface letters (e.g., $\mathbf{a}$ and $\mathbf{A}$) are used to denote a vector and a matrix, respectively, while the operations of transpose, conjugate transpose, matrix inversion and element-wise conjugate are represented by $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$, and $(\cdot)^*$, respectively. The matrix $\mathbf{I}_M$ denotes an $M \times M$ identity matrix. Moreover, $\text{Diag}(\cdot)$ generates a diagonal matrix with the argument on the diagonal.

II. SYSTEM MODEL

Consider a downlink multi-user communication system as depicted in Fig. 1 consisting of a single-antenna based BS and K single-antenna users. Based on the average channel power gain, the users are divided into near users (NUs) and cell-edge users (CEUs) [9]. Furthermore, we assume that there is no direct link between the base station (BS) and the CEUs due to deep fading. In result, IRSs are deployed near to cell-edge in order to serve CEUs. Moreover, we assume that the BS and IRSs are fully coordinated with the help of a central control unit (CCU). The role of the control unit is to optimally control the reflection coefficients for obtaining optimum SINR. In order to achieve this, the control unit is assumed to be able to gather all the CSI [26]. Let K_1 and K_2 (such that $K_1 + K_2 = K$) denote the number of NUs having direct link with the BS and number of CEUs having no direct link with

Fig. 1: An IRS-assisted downlink communication system with K users and K_2 IRSs.

the BS, respectively. Moreover, the transmission of data from BS to each CEU takes place through an IRS equipped with N reflecting elements. The BS utilizes the so-called 1L-RS technique of [18], [19], [21] to serve all the users.

A. Transmission Protocol in 1L-RS

In the considered system model, RS is applied at the BS where each user message is split into a common and a private part. The common part of each user is encoded together into a common stream using a codebook shared by all users, while private messages are encoded into the private stream for each user. To illustrate this in details, let's consider a two-user example. There are two messages W_1 and W_2 intended for user-1 and user-2, respectively. The message of each user is split into two parts, $\{W_1^{12}, W_1^1\}$ for user-1 and $\{W_2^{12}, W_2^2\}$ for user-2. The messages W_1^{12}, W_2^{12} are encoded together into a common stream s_{12} using a codebook shared by both users. Hence, s_{12} is a common stream required to be decoded by both users. The messages W_1^1 and W_2^2 are encoded into the private stream s_1 for user-1 and s_2 for user-2, respectively. Thus, the BS splits the k -th user message (z_k) , $k = 1, 2, \dots, K$ into two sub-messages known as private $(z_{k,p})$ and common $(z_{k,c})$ messages. The common messages of all the users are then jointly encoded into a common data symbol x_c . The private message of each user is separately encoded into a data symbol x_k , $k = 1, 2, \dots, K$. The combined data vector at BS will be $\mathbf{x} = [x_c, x_1, \dots, x_K]^T$. It is assumed that $\text{Tr}[\mathbf{x}\mathbf{x}^H] = 1$, where $\text{Tr}[\cdot]$ is the trace of a matrix. It should be noted that x_c will be decoded by all the users however, x_k will only be decoded by the k -th user.

Let $\mathcal{P}_B$ is total transmit power available at the BS and α_c, α_k denote the fraction of total power used for common and k -th private data, respectively, then the data symbol broadcasted by the BS will be $s = \sqrt{\alpha_c \mathcal{P}_B} x_c + \sum_{k=1}^K \sqrt{\alpha_k \mathcal{P}_B} x_k$. The coefficients α_c, α_k are chosen such that $0 \leq \alpha_c, \alpha_k \leq 1$ and

$\alpha_c + \sum_{k=1}^K \alpha_k = 1$. Utilizing the RS at the BS, the received signal at NU_{k_1} , $k_1 = 1, 2, \dots, K_1$ can be written as

$$y_{k_1} = h_{k_1} s + e_{k_1}, \quad (1)$$

where $h_{k_1} \sim \mathcal{CN}(0, \Omega)$ with Ω representing the average channel power; and $e_{k_1} \sim \mathcal{CN}(0, \sigma^2)$ denotes the noise at NU_{k_1} . The use of IRSs enables BS to serve a larger number of CEUs. Similar to [9], we assume that each IRS is placed near to cell-boundary in such a way that only desired CEU can hear from it. Although, the reflected signal from a particular IRS can be treated as *interference* to the users associated with other IRSs, however, the interference power will be very low due to long distance and hence, we do not consider this interference at other users. Moreover, we only consider first signal reflection at IRS and completely ignores the other reflections of the same signal as they will have very low power content as compared to the first reflected signal [7]-[16]. Thus, the received signal at CEU_{k_2} , $k_2 = K_1 + 1, K_1 + 2, \dots, K$ can be written as

$$y_{k_2} = \mathbf{g}_{k_2}^H \mathbf{\Phi}_{k_2} \mathbf{h}_{k_2} s + e_{k_2}, \quad (2)$$

where $\mathbf{h}_{k_2}$ represents the $N \times 1$ channel vector between the BS and IRS_{k_2} , $\mathbf{g}_{k_2}$ denotes the channel vector between IRS_{k_2} and CEU_{k_2} . Matrix $\mathbf{\Phi}_{k_2} = \text{Diag}[\beta_1 e^{j\phi_1}, \beta_2 e^{j\phi_2}, \dots, \beta_N e^{j\phi_N}]$ is an $N \times N$ diagonal matrix containing the reflection coefficients at IRS_{k_2} , where β_n and ϕ_n , for $n = 1, 2, \dots, N$ represents the controllable reflection amplitude and phase shift of n -th reflecting element. All the channel coefficients are assumed to be independent and identically distributed (i.i.d.) as $\mathcal{CN}(0, 1)$; $e_{k_2} \sim \mathcal{CN}(0, \sigma^2)$ denotes the noise at CEU_{k_2} . The transmit signal-to-noise ratio (SNR) can therefore be defined as $\rho_B = \frac{P_B}{\sigma^2}$. Without loss of generality, we assume $\Omega > 1$ as a BS-NU link will always have more average power as compared to the BS-CEU link.

It should be noted that while the RS transmit signal model resembles a broadcasting system with unicast (private) streams and a multicast stream, the role of the common message is fundamentally different. The common message in a unicast-multicast system carries public information intended as a whole to all users in the system, while the super common message in RS encapsulates parts of private messages, and is not entirely required as by all users, although decoded by them all for interference mitigation purposes. The scenario when users don't need a common message, we simply allocate no power to common message x_c and treat multi-user interference as noise $s = \sum_{k=1}^K \sqrt{\alpha_k P_B} x_k$. In this case, the rate-splitting multiple access (RSMA) strategy resembles as a super-positioned signal transmission (SST), while it works as a Space Division Multiple Access (SDMA) when the base station has multiple antennas [18]-[21].

B. SINR Characterization

At first, all the users decode the common message x_c by treating the interference from all private messages as noise.

Therefore, the SINR at NU_{k_1} and CEU_{k_2} for decoding the common data symbol x_c will be given, respectively, by

$$\gamma_{k_1,c} = \frac{\alpha_c |h_{k_1}|^2}{\sum_{k=1}^K \alpha_k |h_{k_1}|^2 + \frac{1}{\rho_B}}, \quad (3)$$

$$\gamma_{k_2,c} = \frac{\alpha_c |\mathbf{g}_{k_2}^H \mathbf{\Phi}_{k_2} \mathbf{h}_{k_2}|^2}{\sum_{k=1}^K \alpha_k |\mathbf{g}_{k_2}^H \mathbf{\Phi}_{k_2} \mathbf{h}_{k_2}|^2 + \frac{1}{\rho_B}}. \quad (4)$$

After successful decoding of x_c , the common message is removed from the received signals and the private message is then decoded by assuming all other private symbols as noise. Thus, the SINR for decoding the private messages of NU_{k_1} and CEU_{k_2} can be respectively written as

$$\gamma_{k_1,p} = \frac{\alpha_{k_1} |h_{k_1}|^2}{\sum_{k=1, k \neq k_1}^K \alpha_k |h_{k_1}|^2 + \frac{1}{\rho_B}}, \quad (5)$$

$$\begin{aligned} \gamma_{k_2,p} &= \frac{\alpha_{k_2} |\mathbf{g}_{k_2}^H \mathbf{\Phi}_{k_2} \mathbf{h}_{k_2}|^2}{\sum_{k=1, k \neq k_2}^K \alpha_k |\mathbf{g}_{k_2}^H \mathbf{\Phi}_{k_2} \mathbf{h}_{k_2}|^2 + \frac{1}{\rho_B}} \\ &= \frac{\alpha_{k_2} |\phi_{k_2}^H \mathbf{G}_{k_2} \mathbf{h}_{k_2}|^2}{\sum_{k=1, k \neq k_2}^K \alpha_k |\phi_{k_2}^H \mathbf{G}_{k_2} \mathbf{h}_{k_2}|^2 + \frac{1}{\rho_B}}, \end{aligned} \quad (6)$$

where vector $\phi_{k_2}^H$ is a row vector of size $1 \times N$ containing main diagonal elements of matrix $\mathbf{\Phi}_{k_2}$. The matrix $\mathbf{G}_{k_2}$ is an $N \times N$ diagonal matrix with diagonal elements obtained from $\mathbf{g}_{k_2}^H$.

III. PERFORMANCE ANALYSIS OF IRS-ASSISTED RS

In order to analyze the performance of IRS-assisted RS framework for multi-user downlink communication system, it is important to choose the IRS beamforming matrix $\mathbf{\Phi}_{k_2}$ or equivalently, vector $\phi_{k_2}^H$. There are many design solutions such as Zero Forcing (ZF) beamforming (an ideal solution that require infinite resolution), discrete fourier transform (DFT) based beamforming (a finite resolution based design), and an on-off control etc. However, due to low-cost implementation for controlling the IRS-RS, we consider *on-off control* method [9].

A. On-Off Technique to Control Passive Beamforming of IRS

In on-off controlling, it is assumed that the elements of beamforming vector $\phi_{k_2}^H$ are either 1 (on) or 0 (off). Let us define a matrix $\mathbf{F} = \mathbf{I}_L \otimes \mathbf{v}$, $L \in \mathbb{I}^+$, where $\mathbb{I}^+$ represents the set of positive integers and $\mathbf{v} = \{v_d\}_{d=1}^D$ is a unit norm row vector of length D , $D \in \mathbb{I}^+$ such that $N = LD$ and $v_d = \frac{1}{\sqrt{D}}$; and $\otimes$ denotes the Kronecker product. It can be noticed that all the row vectors in $\mathbf{F}$ are orthonormal vectors. Vector $\phi_{k_2}^H$ is selected as ℓ -th row vector $\mathbf{f}_\ell$, $\ell = 1, 2, \dots, L$ of $\mathbf{F}$ such

that $\mathbf{f}_\ell$ maximizes the SINRs at CEU given in (4) and (6). Therefore, the SINRs at CEU are evaluated as

$$\begin{aligned}\tilde{\gamma}_{k_2,c} &\triangleq \max \left(\{ \gamma_{k_2,c}(\mathbf{f}_\ell) \}_{\ell=1}^L \right), \\ \tilde{\gamma}_{k_2,p} &\triangleq \max \left(\{ \gamma_{k_2,p}(\mathbf{f}_\ell) \}_{\ell=1}^L \right),\end{aligned}\quad (7)$$

where

$$\begin{aligned}\gamma_{k_2,c}(\mathbf{f}_\ell) &= \frac{\alpha_c |\mathbf{f}_\ell \mathbf{G}_{k_2} \mathbf{h}_{k_2}|^2}{\sum_{k=1}^K \alpha_k |\mathbf{f}_\ell \mathbf{G}_{k_2} \mathbf{h}_{k_2}|^2 + \frac{1}{\rho_B}}, \\ \gamma_{k_2,p}(\mathbf{f}_\ell) &= \frac{\alpha_{k_2} |\mathbf{f}_\ell \mathbf{G}_{k_2} \mathbf{h}_{k_2}|^2}{\sum_{k=1, k \neq k_2}^K \alpha_k |\mathbf{f}_\ell \mathbf{G}_{k_2} \mathbf{h}_{k_2}|^2 + \frac{1}{\rho_B}}.\end{aligned}\quad (8)$$

Observation 1: It should be noted that random variables (RVs) $\gamma_{k_2,c}(\mathbf{f}_1), \gamma_{k_2,c}(\mathbf{f}_2), \dots, \gamma_{k_2,c}(\mathbf{f}_L)$ are independent to each other. Similarly, the instantaneous SINRs $\gamma_{k_2,p}(\mathbf{f}_1), \gamma_{k_2,p}(\mathbf{f}_2), \dots, \gamma_{k_2,p}(\mathbf{f}_L)$ are also independent to each other. Furthermore, the RV $\gamma_{k_2,c}(\mathbf{f}_\ell)$ is dependent on $\gamma_{k_2,p}(\mathbf{f}_\ell)$, for all ℓ . However, the pair $\{\gamma_{k_2,c}(\mathbf{f}_i), \gamma_{k_2,p}(\mathbf{f}_i)\}$ remains independent of the pair $\{\gamma_{k_2,c}(\mathbf{f}_j), \gamma_{k_2,p}(\mathbf{f}_j)\}$, for $i \neq j, i, j = 1, 2, \dots, L$.

We have numerically and analytically verified that a vector $\mathbf{f}_\ell$ that maximizes the common SINR at CEU also maximizes the private SINR of that CEU. The analytical proof for the same is given in Appendix A.

B. Outage Analysis for CEUs

A CEU will not suffer outage if and only if the SINR values for common and private symbols are simultaneously more than the corresponding threshold SINRs (equivalently, the targeted quality-of-service (QoS)). Hence, the outage probability at CEU _{k_2} is defined as,

$$\mathbb{P}_{k_2}^{\text{out}} = 1 - \Pr \{ \tilde{\gamma}_{k_2,c} > \tau_c, \tilde{\gamma}_{k_2,p} > \tau_p \}, \quad (9)$$

where $\tau_c = 2^{\bar{R}_c} - 1$ and $\tau_p = 2^{\bar{R}_p} - 1$ are threshold SINRs for common and private messages, respectively, with $\bar{R}_c$ and $\bar{R}_p$ being their targeted data rates, respectively. $\Pr\{\cdot, \cdot\}$ represents the joint probability. Using the concepts of joint probabilities [32], the outage probability in (9) can be further simplified as,

$$\mathbb{P}_{k_2}^{\text{out}} = \mathbb{F}_{\tilde{\gamma}_{k_2,c}}(\tau_c) + \mathbb{F}_{\tilde{\gamma}_{k_2,p}}(\tau_p) - \mathbb{F}_{\tilde{\gamma}_{k_2,c}, \tilde{\gamma}_{k_2,p}}(\tau_c, \tau_p), \quad (10)$$

where $\mathbb{F}_X(\cdot)$ is the marginal cumulative distribution function (CDF) of RV X and $\mathbb{F}_{X,Y}(\cdot, \cdot)$ is the joint CDF of X and Y . Following the discussion in Observation 1, we can rewrite the outage probability in (10) as

$$\begin{aligned}\mathbb{P}_{k_2}^{\text{out}} &= \prod_{\ell=1}^L \mathbb{F}_{\gamma_{k_2,c}(\mathbf{f}_\ell)}(\tau_c) + \prod_{\ell=1}^L \mathbb{F}_{\gamma_{k_2,p}(\mathbf{f}_\ell)}(\tau_p) \\ &\quad - \prod_{\ell=1}^L \mathbb{F}_{\gamma_{k_2,c}(\mathbf{f}_\ell), \gamma_{k_2,p}(\mathbf{f}_\ell)}(\tau_c, \tau_p).\end{aligned}\quad (11)$$

If marginal and joint statistics of $\gamma_{k_2,c}(\mathbf{f}_\ell)$ and $\gamma_{k_2,p}(\mathbf{f}_\ell)$ are same for all ℓ , the outage probability in (11) can be given as

$$\begin{aligned}\mathbb{P}_{k_2}^{\text{out}} &= \left[\mathbb{F}_{\gamma_{k_2,c}(\mathbf{f}_\ell)}(\tau_c) \right]^L + \left[\mathbb{F}_{\gamma_{k_2,p}(\mathbf{f}_\ell)}(\tau_p) \right]^L \\ &\quad - \left[\mathbb{F}_{\gamma_{k_2,c}(\mathbf{f}_\ell), \gamma_{k_2,p}(\mathbf{f}_\ell)}(\tau_c, \tau_p) \right]^L.\end{aligned}\quad (12)$$

Lemma 1: For an arbitrary CEU in multi-user IRS-assisted RS technique, the joint CDF of common and private SINRs for a fixed passive beamforming vector $\mathbf{f}_\ell$, is given as

$$\mathbb{F}_{\gamma_{k_2,c}(\mathbf{f}_\ell), \gamma_{k_2,p}(\mathbf{f}_\ell)}(\tau_c, \tau_p) = 1 - \frac{2}{\Gamma(D)} \left(\frac{D\tau}{\rho_B} \right)^{\frac{D}{2}} K_D \left(2\sqrt{\frac{D\tau}{\rho_B}} \right), \quad (13)$$

where $K_v(\cdot)$ is the v -th order modified Bessel's function of second kind, $\Gamma(\cdot)$ is the Gamma function, and $\tau > 0$ is given as $\tau = \min(\delta_1(\tau_c), \delta_2(\tau_p))$ with

$$\delta_1(\tau_c) = \frac{\tau_c}{\alpha_c - \tau_c \sum_{k=1}^K \alpha_k}, \quad \delta_2(\tau_p) = \frac{\tau_p}{\alpha_{k_2} - \tau_p \sum_{k=1, k \neq k_2}^K \alpha_k}. \quad (14)$$

Proof: See Appendix B for the proof. ■

Corollary 1: The marginal CDFs $\mathbb{F}_{\gamma_{k_2,c}(\mathbf{f}_\ell)}(\tau_c)$ and $\mathbb{F}_{\gamma_{k_2,p}(\mathbf{f}_\ell)}(\tau_p)$ can be obtained by using $\tau_c \rightarrow \infty$ and $\tau_p \rightarrow \infty$ in the joint CDF expression given in (13), as

$$\begin{aligned}\mathbb{F}_{\gamma_{k_2,c}(\mathbf{f}_\ell)}(\tau_c) &= 1 - \frac{2}{\Gamma(D)} \left(\frac{D\delta_1(\tau_c)}{\rho_B} \right)^{\frac{D}{2}} K_D \left(2\sqrt{\frac{D\delta_1(\tau_c)}{\rho_B}} \right), \\ \mathbb{F}_{\gamma_{k_2,p}(\mathbf{f}_\ell)}(\tau_p) &= 1 - \frac{2}{\Gamma(D)} \left(\frac{D\delta_2(\tau_p)}{\rho_B} \right)^{\frac{D}{2}} K_D \left(2\sqrt{\frac{D\delta_2(\tau_p)}{\rho_B}} \right).\end{aligned}\quad (15)$$

The overall outage probability of CEU _{k_2} can thus be evaluated by using (13)-(16) in (12). It is shown through numerical results that the choice of $L = N$ (or $D = 1$) gives the best outage performance for all the values of BS transmit power. The intuitive reason for the same and the information regarding which IRS elements are turned on are given in the following remarks.

Remark 1: It should be noted that the on-off control at IRS does not considers all possible combinations of selecting D elements out of N (i.e., $\binom{N}{D}$), however, it only focuses on $\frac{N}{D} = L$ disjoint subsets of reflecting elements, each with cardinality D . Thus, the random variable Z_ℓ , $\ell = 1, 2, \dots, L$ (defined before (49)) is a sum of D i.i.d. random variables (refer (42)). Intuitively, the sum of D i.i.d. random variables results in similar statistical advantages as achieved for single random variable (i.e., $D = 1$). Therefore, we do not get any diversity benefits for $D > 1$ as the SNR is proportional to $|Z_\ell|^2$. Moreover, obtaining the maximum of L independent SNRs (as defined in (7)) always provides the best results for the largest possible L (i.e., $L = N$). Therefore, the on-off control at IRS for RS based BS gives the best results for $L = N$ (or $D = 1$).

Remark 2: To identify which D elements should be turned on in general on-off scenario, an index should be assigned to all the reflecting elements and L non-overlapping sets of D reflecting elements should be formed. A set having the

maximum sum of its elements will give the information about active reflecting elements, i.e., all those reflecting elements, whose index are present in the set, will be turned on provided that the sum of composite BS-IRS-CEU channel gains for those elements is maximum among all possible disjoint sets.

Observation 2: It can be observed from (14) that for given τ_c and τ_p (equivalently, $\bar{R}_c$ and $\bar{R}_p$), there is always a constraint for α_c and α_{k_2} as

$$\alpha_c > \frac{\tau_c}{1 + \tau_c}, \quad \text{and} \quad \alpha_{k_2} > \frac{\tau_p(1 - \alpha_c)}{1 + \tau_p}, \quad (17)$$

such that $\delta_1(\tau_c) > 0$ and $\delta_2(\tau_p) > 0$. From the valid range of α_c and α_{k_2} , we can choose the optimum values of these power allocation coefficients for which the outage probability of a CEU is minimized. Moreover, the condition for α_{k_1} will be same as that for α_{k_2} provided τ_p is same for both NU and CEU.

In Figs. 2 (a) and (b), we have considered 2-user scenario with single NU and CEU and have shown the outage probability variations of the CEU for varying α_c and α_2 . Initially, we set the target data rate for common and private messages as $\bar{R}_c = \bar{R}_p = 0.25$ bits per channel use (BPCU). We also assume that out of the total available power for private messages of all the users, 60% power is allocated to the CEU, i.e., $\alpha_2 = 0.6(1 - \alpha_c)$. Under these settings, it can be observed from Fig. 2(a) that (i) the valid range for α_c is $0.17 < \alpha_c < 1$, and (ii) the outage probability is minimized at $\alpha_c \approx 0.4$. However, if we increase the target data rate $\bar{R}_c$ to 0.5 BPCU (by keeping $\bar{R}_p$ fixed), we observe an increase in the optimum value of α_c which minimizes the outage probability of the CEU. In Fig. 2(b), we have used $\alpha_c = 0.4$ and varied α_2 from 0 to $(1 - \alpha_c)$. For $\bar{R}_c = \bar{R}_p = 0.25$ BPCU, it can be noticed from Fig. 2(b) that (i) $0.1 < \alpha_2 < 0.6$, and (ii) the outage probability of CEU is minimum at $\alpha_2 \approx 0.36$, which is 60% of $(1 - \alpha_c)$. Further, on raising the target data rate $\bar{R}_p$ for the CEU, the optimum power allocation coefficient for the CEU increases accordingly.

In the numerical results section, the optimum values of power allocation coefficients (for given target data rates of common and private data) are chosen in similar fashion as described in Observation 2.

C. Asymptotic Analysis for large N

To obtain further insights, we investigate the outage behavior of IRS-assisted CEU for sufficiently high reflecting elements, i.e., $N \rightarrow \infty$.

Lemma 2: For a multi-user downlink network employing 1L-RS scheme, the joint CDF of optimum common and private SINRs at an arbitrary IRS-assisted CEU can be given for large number of reflecting elements as,

$$\begin{aligned} \mathcal{F}_{\tilde{\gamma}_{k_2,c}, \tilde{\gamma}_{k_2,p}}(\tau_c, \tau_p) &= \lim_{N \rightarrow \infty} \mathbb{F}_{\tilde{\gamma}_{k_2,c}, \tilde{\gamma}_{k_2,p}}(\tau_c, \tau_p) \\ &= \exp \left\{ -\frac{2N}{\Gamma(D+1)} \left(\frac{D\tau}{\rho_B} \right)^{\frac{D}{2}} K_D \left(2\sqrt{\frac{D\tau}{\rho_B}} \right) \right\}, \end{aligned} \quad (18)$$

where $\tau = \min(\delta_1(\tau_c), \delta_2(\tau_p))$ with $\delta_1(\tau_c)$ and $\delta_2(\tau_p)$ are given in (14).

Fig. 2: Outage performance of CEU with varying α_c and α_2 for $K = 2$ and $N = 4$.

Proof: Following (10)-(13), we can write the joint CDF of $\tilde{\gamma}_{k_2,c}$ and $\tilde{\gamma}_{k_2,p}$ as

$$\mathbb{F}_{\tilde{\gamma}_{k_2,c}, \tilde{\gamma}_{k_2,p}}(\tau_c, \tau_p) = \left[1 - \frac{2}{\Gamma(D)} \left(\frac{D\tau}{\rho_B} \right)^{\frac{D}{2}} K_D \left(2\sqrt{\frac{D\tau}{\rho_B}} \right) \right]^L. \quad (19)$$

For sufficiently large number of reflecting elements (i.e., $N \rightarrow \infty$), we can have $L \rightarrow \infty$ provided D is fixed. Using the standard approximation $\lim_{n \rightarrow \infty} (1-x)^n \approx e^{-nx}$ with some algebra, we obtain (18). ■

Corollary 2: For asymptotically high N , the marginal CDFs $\mathbb{F}_{\tilde{\gamma}_{k_2,c}}(\tau_c)$ and $\mathbb{F}_{\tilde{\gamma}_{k_2,p}}(\tau_p)$ can be obtained by using $\tau_c \rightarrow \infty$ and $\tau_p \rightarrow \infty$ in the asymptotic joint CDF expression given in (18), as

$$\begin{aligned} \mathcal{F}_{\tilde{\gamma}_{k_2,a}}(\tau_a) &= \lim_{N \rightarrow \infty} \mathbb{F}_{\tilde{\gamma}_{k_2,a}}(\tau_a) \\ &= \exp \left\{ -\frac{2N}{\Gamma(D+1)} \left(\frac{D\delta_i(\tau_a)}{\rho_B} \right)^{\frac{D}{2}} K_D \left(2\sqrt{\frac{D\delta_i(\tau_a)}{\rho_B}} \right) \right\}, \end{aligned} \quad (20)$$

where $(i, a) \in \{(1, c), (2, p)\}$.

Utilizing (18) and (20) in (10), we can evaluate the asymptotic outage probability of CEU $_{k_2}$ for $N \rightarrow \infty$ as

$$\begin{aligned} \mathcal{P}_{k_2}^{\text{out}} &= \lim_{N \rightarrow \infty} \mathbb{P}_{k_2}^{\text{out}} \\ &= \mathcal{F}_{\tilde{\gamma}_{k_2,c}}(\tau_c) + \mathcal{F}_{\tilde{\gamma}_{k_2,p}}(\tau_p) - \mathcal{F}_{\tilde{\gamma}_{k_2,c}, \tilde{\gamma}_{k_2,p}}(\tau_c, \tau_p), \end{aligned} \quad (21)$$

We introduce a performance metric known as *Log-Asymptotic Outage Order* for an arbitrary CEU $_{k_2}$, denoted by $\mathcal{O}_{k_2,N}$. It is defined as the negative ratio of natural logarithm of asymptotic outage probability (for large N) of CEU $_{k_2}$ and number of reflecting elements N , i.e.,

$$\mathcal{O}_{k_2,N} = -\frac{1}{N} \ln \mathcal{P}_{k_2}^{\text{out}}, \quad (22)$$

Fig. 3: A DF-RS downlink communication system showing a single CEU assisted with single DF relay.

where $\mathcal{P}_{k_2}^{\text{out}}$ is given in (21). The insights obtained from the Log-Asymptotic Outage Order are discussed in the numerical results section.

D. Outage Analysis for NUs

For NUs, the outage probability can be defined as

$$\begin{aligned} \mathbb{P}_{k_1}^{\text{out}} &= 1 - \Pr\{\gamma_{k_1,c} > \tau_c, \gamma_{k_1,p} > \tau_p\}, \\ &= \mathbb{F}_{\gamma_{k_1,c}}(\tau_c) + \mathbb{F}_{\gamma_{k_1,p}}(\tau_p) - \mathbb{F}_{\gamma_{k_1,c}, \gamma_{k_1,p}}(\tau_c, \tau_p). \end{aligned} \quad (23)$$

Since $h_{k_1} \sim \mathcal{CN}(0, \Omega)$, we can write that $|h_{k_1}|^2 \sim \text{Exp}(\Omega)$. After some simple mathematical manipulations, we get

$$\mathbb{F}_{\gamma_{k_1,c}}(\tau_c) = 1 - \exp\left(-\frac{\delta_1(\tau_c)}{\Omega \rho_B}\right), \quad (24)$$

$$\mathbb{F}_{\gamma_{k_1,p}}(\tau_p) = 1 - \exp\left(-\frac{\delta_2(\tau_p)}{\Omega \rho_B}\right), \quad (25)$$

$$\mathbb{F}_{\gamma_{k_1,c}, \gamma_{k_1,p}}(\tau_c, \tau_p) = 1 - \exp\left(-\frac{\min(\delta_1(\tau_c), \delta_2(\tau_p))}{\Omega \rho_B}\right). \quad (26)$$

Using (24)–(26) in (23), we get the outage probability of a NU.

IV. PERFORMANCE ANALYSIS OF DF-RS FRAMEWORK

In order to compare the performance of IRS-RS framework with a DF-RS network, we analyze the outage performance of the considered multi-user downlink communication system by replacing the IRS_{k₂} with a single-antenna based DF relay R_{k₂} as shown in Fig. 3. Contrary to the cooperative RS models considered in [28] and [30], we assume that the relay R_{k₂} does not receive its own data from BS. It completely decodes the common message followed by the decoding of private message of CEU_{k₂}, received from BS and forwards them to CEU_{k₂} only by utilizing RS. In the following subsections, we consider two different scenarios based on the availability of direct BS-CEU link and obtain the outage probability under both scenarios.

A. With Direct BS-CEU Link

We assume that each CEU is served directly by the BS as well as through a DF-based cooperative link as shown in Fig. 3. Let $h_{0,k_2} \sim \mathcal{CN}(0, \Omega')$ and $h_{1,k_2} \sim \mathcal{CN}(0, 1)$ represent the channel coefficients of direct BS-CEU_{k₂} link (with average power Ω') and the BS-R_{k₂} link, respectively. **Since the BS has a larger separation from the CEU as compared to the NU,**

the average channel power of BS-CEU link is assumed to be smaller than that of the BS-NU link, (i.e., $\Omega' < \Omega$). Thus, the common and private message SINRs at CEU_{k₂} and R_{k₂} will be given as

$$\begin{aligned} \gamma_{i,k_2,c} &= \frac{\alpha_c |h_{i,k_2}|^2}{\sum_{k=1}^K \alpha_k |h_{i,k_2}|^2 + \frac{1}{\rho_B}}, \\ \gamma_{i,k_2,p} &= \frac{\alpha_{k_2} |h_{i,k_2}|^2}{\sum_{k=1, k \neq k_2}^K \alpha_k |h_{i,k_2}|^2 + \frac{1}{\rho_B}}, \end{aligned} \quad (27)$$

where $i = 0, 1$ corresponds to BS-CEU_{k₂} and BS-R_{k₂} links, respectively. The k_2 -th relay node employs RS with the decoded common data stream and one private data streams (for CEU_{k₂}) and forwards the combined data stream to CEU_{k₂} after re-encoding. If the channel coefficient of the k_2 -th R-CEU link is denoted by $h_{2,k_2} \sim \mathcal{CN}(0, 1)$, then we have

$$\begin{aligned} \gamma_{2,k_2,c} &= \frac{\hat{\alpha}_c |h_{2,k_2}|^2}{\hat{\alpha}_{k_2} |h_{2,k_2}|^2 + \frac{1}{\rho_R}}, \\ \gamma_{2,k_2,p} &= \hat{\alpha}_{k_2} |h_{2,k_2}|^2 \rho_R, \end{aligned} \quad (28)$$

where $\hat{\alpha}_c$ and $\hat{\alpha}_{k_2}$ are the fraction of the relay transmit power ($\mathcal{P}_R$) associated with common and k_2 -th private data streams, respectively, such that $\hat{\alpha}_c + \hat{\alpha}_{k_2} = 1$, and $\rho_R = \frac{\mathcal{P}_R}{\sigma^2}$ is the transmit SNR by the relay. Utilizing the concepts of a DF-based cooperative relay network with direct link, the end-to-end SINRs of common and private data streams at CEU_{k₂} can be written as

$$\begin{aligned} \tilde{\gamma}_{k_2,c} &= \gamma_{0,k_2,c} + \min(\gamma_{1,k_2,c}, \gamma_{2,k_2,c}), \\ \tilde{\gamma}_{k_2,p} &= \gamma_{0,k_2,p} + \min(\gamma_{1,k_2,p}, \gamma_{2,k_2,p}), \end{aligned} \quad (29)$$

and the outage probability of CEU_{k₂} under the considered DF-RS cooperative scenario can be obtained as

$$\begin{aligned} \mathbb{P}_{k_2}^{\text{out,DF}} &= 1 - \Pr\{\tilde{\gamma}_{k_2,c} > \tau_c, \tilde{\gamma}_{k_2,p} > \tau_p\}, \\ &\triangleq 1 - \mathcal{I}. \end{aligned} \quad (30)$$

Utilizing (29) in (30), we can write

$$\begin{aligned} \mathcal{I} &= \Pr\{\gamma_{0,k_2,c} + \min(\gamma_{1,k_2,c}, \gamma_{2,k_2,c}) > \tau_c, \\ &\quad \gamma_{0,k_2,p} + \min(\gamma_{1,k_2,p}, \gamma_{2,k_2,p}) > \tau_p\}, \\ &= \Pr\{\gamma_{0,k_2,c} + \gamma_{1,k_2,c} > \tau_c, \gamma_{0,k_2,c} + \gamma_{2,k_2,c} > \tau_c, \\ &\quad \gamma_{0,k_2,p} + \gamma_{1,k_2,p} > \tau_p, \gamma_{0,k_2,p} + \gamma_{2,k_2,p} > \tau_p\}. \end{aligned} \quad (31)$$

If we assume that $\gamma_{0,k_2,c}$ and $\gamma_{0,k_2,p}$ are known, then the probability term in (31) can be conditionally written as

$$\begin{aligned} \mathcal{I}(\gamma_{0,k_2,c} = \mu, \gamma_{0,k_2,p} = \nu) &= \Pr\{\gamma_{1,k_2,c} > \tau_c - \mu, \gamma_{1,k_2,p} > \tau_p - \nu\}, \\ &\times \Pr\{\gamma_{2,k_2,c} > \tau_c - \mu, \gamma_{2,k_2,p} > \tau_p - \nu\}. \end{aligned} \quad (32)$$

Utilizing (27) in (32) along with the use of $|h_{1,k_2}|^2 \sim \text{Exp}(1)$, we get

$$\begin{aligned} &\Pr\{\gamma_{1,k_2,c} > \tau_c - \mu, \gamma_{1,k_2,p} > \tau_p - \nu\}, \\ &= \exp\left[-\frac{\max(\delta_1(\tau_c - \mu), \delta_2(\tau_p - \nu))}{\rho_B}\right], \end{aligned} \quad (33)$$

where $\delta_1(\cdot)$ and $\delta_2(\cdot)$ are defined in (14). Similarly, we can write

$$\Pr\{\gamma_{2,k_2,c} > \tau_c - \mu, \gamma_{2,k_2,p} > \tau_p - \nu\} = \exp\left[-\frac{\max(\delta_3(\tau_c - \mu), \delta_4(\tau_p - \nu))}{\rho_R}\right], \quad (34)$$

where

$$\delta_3(\tau_c - \mu) = \frac{\tau_c - \mu}{\hat{\alpha}_c - (\tau_c - \mu)\hat{\alpha}_{k_2}}, \quad \delta_4(\tau_p - \nu) = \frac{\tau_p - \nu}{\hat{\alpha}_{k_2}}. \quad (35)$$

Substituting (33) and (34) in (32) and jointly averaging over $\gamma_{0,k_2,c}$ and $\gamma_{0,k_2,p}$, we can evaluate (31) as

$$\mathcal{I} = \int_0^\infty \int_0^\infty \mathcal{I}(\gamma_{0,k_2,c} = \mu, \gamma_{0,k_2,p} = \nu) \times f_{\gamma_{0,k_2,c}, \gamma_{0,k_2,p}}(\mu, \nu) d\mu d\nu, \quad (36)$$

where $f_{\gamma_{0,k_2,c}, \gamma_{0,k_2,p}}(\mu, \nu)$ is the joint PDF of common and private SINRs at CEU_{*k*₂} received through the direct link from BS, and can be obtained as

$$f_{\gamma_{0,k_2,c}, \gamma_{0,k_2,p}}(\mu, \nu) = \frac{\partial^2}{\partial \mu \partial \nu} \mathbb{F}_{\gamma_{0,k_2,c}, \gamma_{0,k_2,p}}(\mu, \nu). \quad (37)$$

The joint CDF $\mathbb{F}_{\gamma_{0,k_2,c}, \gamma_{0,k_2,p}}(\mu, \nu)$ can be evaluated from (26) by replacing Ω with Ω' and (τ_c, τ_p) with (μ, ν) . Hence, the joint PDF in (37) will be

$$f_{\gamma_{0,k_2,c}, \gamma_{0,k_2,p}}(\mu, \nu) = \frac{1}{\Omega' \rho_B} \exp\left[-\frac{\min(\delta_1(\mu), \delta_2(\nu))}{\Omega' \rho_B}\right] \times \frac{\partial^2}{\partial \mu \partial \nu} [\min(\delta_1(\mu), \delta_2(\nu))]. \quad (38)$$

Using (36)-(38), we numerically obtain the outage probability values in (30).

B. Without Direct BS-CEU Link

If we assume that the direct path between the BS and the CEU is not available due to high rise buildings and other obstacles, then the end-to-end SINRs of common and private data streams at CEU_{*k*₂} (given in (29)) can be re-written as

$$\begin{aligned} \tilde{\gamma}_{k_2,c} &= \min(\gamma_{1,k_2,c}, \gamma_{2,k_2,c}), \\ \tilde{\gamma}_{k_2,p} &= \min(\gamma_{1,k_2,p}, \gamma_{2,k_2,p}), \end{aligned} \quad (39)$$

where $\gamma_{1,k_2,a}$ and $\gamma_{2,k_2,a}$, $a \in \{c, p\}$ are defined in (27) and (28), respectively. Substituting (39) in (30) followed by steps performed in (31)-(32), we can obtain the closed-form expression of end-to-end outage probability of *k*₂-th CEU (assisted by a DF-RS relay only) as

$$\mathbb{P}_{k_2}^{\text{out,DF}} \triangleq 1 - \exp\left[-\frac{\max(\delta_1(\tau_c), \delta_2(\tau_p))}{\rho_B} - \frac{\max(\delta_3(\tau_c), \delta_4(\tau_p))}{\rho_R}\right], \quad (40)$$

where $\delta_i(\cdot)$, $i = 1, 2, 3, 4$, is same as given in (14) and (35).

Fig. 4: Outage performance of IRS-assisted cell-edge user with RS based downlink environment having $K = 2$ users.

V. NUMERICAL RESULTS

The simulation and analytical results for the outage behavior of IRS-assisted multi-user downlink communication system with RS are presented in this section. We adopt the IRS placement policy given in [9]. Accordingly, we deploy the IRSs near to cell-edge boundary such that only one CEU can hear from its associated IRS. The average channel powers of BS-IRS and IRS-CEU links are assumed to be unity. Further, we have assumed the ideal reflection scenario at each IRS, i.e., the reflection amplitude of all the active reflecting elements is unity. Additionally, it is assumed that 40% of the total transmit power ($\mathcal{P}_B$) is assigned to common data, i.e., $\alpha_c = 0.4$. The remaining power is divided among all private messages such that $\sum_{k=1}^K \alpha_k = 1 - \alpha_c$. Throughout the numerical results, it is considered that $\bar{R}_c = \bar{R}_p = 0.25$ BPCU. Without loss of generality, we consider $K_1 = K_2 = 0.5K$, i.e., there are equal number of NUs and CEUs. However, our results are applicable to any arbitrary distribution of K_1 and K_2 .

Fig. 4 shows the outage performance of the NU and the CEU in the IRS-assisted downlink two-user system utilizing 1L-RS at the BS. For the outage performance of NU, we assume $\Omega = 10, 15$ dB. It can be seen in Fig. 4 that the derived analytical results coincide with the simulated outage probability values, which confirms the accuracy of our analysis. It can be observed from Fig. 4 that for two-user scenario, the IRS-assisted CEU outperforms the NU beyond certain ρ_B . However, the SNR value beyond which the CEU outperforms the NU, depends on different operational conditions such as number of reflecting elements at IRS, number of NUs and CEUs in the system, average channel powers of BS-NU and BS-IRS-CEU links, target data rate, and the power allocation coefficients of the common and private messages, etc.

Example 1: Under single NU and single CEU case considered in Fig. 4 with BS-NU average channel power of 10 dB, the SNR value after which the CEU outperforms the NU, is $\rho_B \approx 20$ dB for $N = 2$, whereas for $N = 4$, this value reduces to $\rho_B \approx 2$ dB. However, if we have $\Omega = 15$ dB, the

Fig. 5: Outage performance comparison of the proposed IRS-RS downlink communication system and IRS-NOMA framework [9] for $K = 4, 6$ users and $N = 8$.

CEU outperforms the NU after $\rho_B \approx 6$ dB and $\rho_B \approx 27$ dB for $N = 2$ and $N = 4$, respectively.

The results for the proposed IRS-assisted 1L-RS¹ scheme are compared with IRS-assisted NOMA, where the BS utilizes power domain NOMA by combining all the user messages using superposition coding (SC). The communication between the BS and the CEU takes place through an IRS and the CEU performs SICs before decoding its own data. Fig. 4 demonstrates the superiority of the proposed IRS-assisted 1L-RS system over corresponding IRS-assisted NOMA framework² with fixed receiver complexity. The RS scheme achieves multiplexing gain advantage as compared to the NOMA scheme, however, the rate of decay of outage probability curves (i.e., the diversity order) remain same for the two schemes. The performance gain of 1L-RS is consistent with different values of N . Furthermore, the outage performance of the considered IRS-assisted RS framework is compared with the conventional RS framework without IRS, where the BS directly communicates with the CEUs using 1L-RS. Fig. 4 depicts that the use of IRS near to the CEU enhances the performance significantly and the performance gain increases drastically with the increase in N .

Fig. 5 shows the outage performance of IRS-assisted 1L-RS scheme for four-user and six-user downlink scenarios, where number of cell-edge users are 2 and 3, respectively. The simulation results exactly matches with outage values obtained through derived analytical expressions for both the scenarios. Further, the outage behavior of the proposed IRS-RS framework is compared with an IRS-assisted NOMA

¹In RS, a fraction of the interference is decoded and the remaining fraction is treated as noise. Thus, RS serves as a soft bridge between SDMA (fully treats interference as noise) and NOMA (fully decodes interference) to achieve required QoS with low complexity designs.

²If we force one user to completely decode the other user message in a two-user downlink scenario, the RS reduces to NOMA. This can be achieved by encoding the second user message into common message and allocating zero power to the corresponding private message [34].

Fig. 6: Impact of L on the outage performance of the proposed IRS-assisted RS framework for $K = 2$ and $N = 10$.

framework considered in [9], where the multiple antenna BS utilizes NOMA in each orthogonal spatial dimension to serve one NU and one IRS-assisted CEU. Both the considered IRS-RS and IRS-NOMA frameworks employ on-off controlling scheme at the IRS.

It is clear from Fig. 5 that the proposed IRS-RS technique with single antenna BS outperforms the IRS-NOMA framework of [9] having multiple antenna BS (with $M = 4$) by a huge margin. It can be noticed from Fig. 5 that the performance of IRS-NOMA framework with orthogonal user groups saturates at high transmit SNR for all the system parameters considered in the figure. However, the outage probability of the proposed IRS-RS system decays very rapidly under both four-user and six-user scenarios.

Example 2: It can be noted from Fig. 5 that under 6-user scenario, the outage probability values of the proposed IRS-RS framework at 20 dB and 22 dB are 9.236×10^{-7} and 7.33×10^{-8} , respectively. Using the relation $d = -\lim_{\rho_B \rightarrow \infty} \frac{\log_{10} \mathbb{P}_{k_2}^{\text{out}}}{\log_{10} \rho_B}$, we get $d \approx 5.5$. Thus, it can be deduced that the proposed framework achieves a diversity order of approximately 5.5.

Fig. 6 illustrates the impact of L on the outage performance of IRS-assisted 1L-RS framework with $K = 2$ and $N = 10$. It can be observed from Fig. 6 that for all the transmit SNR values, the outage performance improves with increasing L and when $L = N$ we get the best outage performance. This is intuitive from (12) as each term in outage probability expression decreases with increasing value of exponent $L (\geq 1)$. It can also be deduced from the discussion given in Remark 1 that the best outage performance is achieved with maximum possible value of L , (i.e., $L = N$). In addition, it can be seen in Fig. 6 that for $L = 5$, the outage probability reduces from 3.1×10^{-7} to 3.58×10^{-8} for ρ_B increasing from 14 dB to 16 dB only. Whereas, for a transmit SNR increase of 2 dB (from 26 dB to 28 dB) for $L = 2$, the outage probability decays from 4.68×10^{-6} to 1.87×10^{-6} only. This clearly shows that the diversity gain improves significantly as L increases.

Fig. 7: Outage probability versus N with $K = 2, 4$ and $\rho_B = 5, 10$ dB.

Fig. 8: Comparison of outage probability for a CEU under the proposed IRS-RS and cooperative DF-RS frameworks with $\rho_R = \rho_B$.

In Fig. 7, we show the outage performance versus the number of IRS reflecting elements N for two-user ($K = 2$) and four-user ($K = 4$) downlink scenarios with $\rho_B = 5$ dB and 10 dB. It can be observed in Fig. 7 that we achieve the better performance gain with increasing N . The main reason behind this is that increasing the number of reflectors at the IRS provides more degrees of freedom (DoFs) for the signal transmitted by the BS. Furthermore, when we increase the number of users from $K = 2$ to $K = 4$, the performance deteriorates due to increase in interference power.

Fig. 8 demonstrates the comparison of CEU outage performance under the IRS-RS downlink network and the cooperative DF-RS communication network for $K = 2$. We set $\alpha_c = 0.4$, $\alpha_2 = 0.36$, and $\rho_B = \rho_R$. It can be observed from Fig. 8 that performance of an IRS-assisted CEU is superior as compared to the performance of a dual-hop DF relay-assisted CEU (without direct link between the BS and the CEU) for all

Fig. 9: Log Asymptotic Outage Order for a CEU under sufficiently large number of reflecting elements at IRS with 8 user scenario, i.e., 4 NUs and 4 CEUs.

values of ρ_B and N considered in the figure. If a direct BS-CEU link is available (with average channel gain $\Omega' = -6$ dB) along with DF relay based cooperative link, the outage performance of CEU significantly improves and outperforms the IRS-assisted scenario for $N = 2$. However, for $N > 2$, the IRS-assisted CEU experiences substantial performance gain in comparison to DF relay based CEU with direct link. It can also be observed from Fig. 8 that the outage performance of CEU is worst in the case when the CEU is only receiving the signal through direct BS-CEU link. Simulation results for outage probability of DF relay-assisted framework corroborates the analytical results derived in Section IV.

Fig. 9 shows the variations of Log-Asymptotic Outage Order for large values of N with respect to transmit SNR at the BS. For this figure, we consider a cellular network of 8 users with equal NUs and CEUs. We assume that power allocated to the private data stream of each CEU is same. We consider two different values of threshold SINRs as $\tau_c = \tau_p = 0.1$ and $\tau_c = \tau_p = 0.2$ along with $D = 1$. In Fig. 9, the Log-Asymptotic Outage Order of k_2 -th CEU, $k_2 = K_1 + 1, K_1 + 2, \dots, K$ is examined for different settings of common and private power allocation coefficients. It can be noticed from Fig. 9 that under considered parameter settings, $\mathcal{O}_{k_2, N}$ saturates at 0.5 for high transmit SNR. Furthermore, it can be observed from Fig. 9 that more power to CEU's private data stream provides better $\mathcal{O}_{k_2, N}$ performance for a fixed common data stream power under both the threshold conditions. However, if we reduce the common power by 10% of the total transmit power (i.e., $0.1\mathcal{P}_B$) by maintaining a fixed ratio between α_{k_1} and α_{k_2} , the Log-Asymptotic Outage Order increases. Thus, it can be deduced from Fig. 9 that the private data stream power allocation coefficient significantly controls the overall performance of the CEU, specially under the conditions of large number of reflecting elements.

Fig. 10 shows the impact of estimation errors of BS-IRS channel on the outage performance of the considered network

Fig. 10: Impact of channel estimation error of BS-IRS link on the outage performance of IRS-assisted CEU under 4 user scenario (2 NUs and 2 CEUs) and $N = 4, 8$.

under 4 user scenario ($K_1 = K_2 = 2$) for different values of N . We have assumed equal power allocation coefficient for common message and CEU private messages as $\alpha_c = \alpha_{k_2} = 0.25$. The channel estimation error (CEE) for BS-IRS $_{k_2}$ channel is defined as $\text{CEE} = \mathbb{E}[||\mathbf{h}_{k_2} - \hat{\mathbf{h}}_{k_2}||^2]$, where $\hat{\mathbf{h}}_{k_2}$ denotes the estimated channel vector corresponding to the exact channel vector $\mathbf{h}_{k_2}$. We have considered different values for percentage CEE of BS-IRS $_{k_2}$ channel (10%, 20%, and 30%) and have compared the outage performance under imperfect channel state information (CSI) scenario with the perfect CSI case (i.e., 0% CEE). Although it is intuitive that the system performance will degrade under imperfect CSI scenario, however, it can be observed from Fig. 10 that the performance degradation for $N = 8$ is small as compared to that for $N = 4$. For example, to attain an outage probability of 0.0001 with 30% CEE, an additional transmit SNR (with respect to 0% CEE) of approximately 3 dB and 6 dB is required for $N = 8$ and $N = 4$, respectively. Moreover, it should be noted that the transmit SNR requirement further increases as we lower the allowable limit for outage probability.

VI. CONCLUSIONS

This paper has proposed a novel IRS-RS framework to enhance the performance of multi-user downlink communication system by employing an IRS near to cell-edge users as well as 1L-RS at the BS. To analyze the system performance, we have derived the closed-form expressions of outage probability for the users. In addition, we also studied the performance of DF-RS framework. Next, through extensive simulation results, we have shown that with 1L-RS IRS-assisted downlink multi-user communication system gives better performance than that of the IRS-NOMA. Moreover, the performance of IRS-RS system has been also compared with the corresponding DF-RS framework. Furthermore, we have demonstrated the impact of BS-IRS channel estimation errors, employing higher number

of IRS reflecting elements N and the number of users K on the system performance in terms of outage probability.

APPENDIX A

Since, the ℓ -th row vector of $\mathbf{F}$ will have the following form

$$\mathbf{f}_\ell = D^{-\frac{1}{2}} [\underbrace{0, 0, \dots, 0}_{(\ell-1)D \text{ Terms}}, \underbrace{1, 1, \dots, 1}_D, \underbrace{0, 0, \dots, 0}_{(L-\ell)D \text{ Terms}}], \quad (41)$$

the term $\mathbf{f}_\ell \mathbf{G}_{k_2} \mathbf{h}_{k_2}$ can be expressed as

$$\begin{aligned} \mathbf{f}_\ell \mathbf{G}_{k_2} \mathbf{h}_{k_2} &= D^{-\frac{1}{2}} \sum_{n=(\ell-1)D+1}^{\ell D} g_{k_2,n} h_{k_2,n}, \\ &\triangleq D^{-\frac{1}{2}} \sum_{d=1}^D g_{k_2,d} h_{k_2,d}, \end{aligned} \quad (42)$$

where the second line utilizes $d = n - (\ell-1)D$. Considering a case with $L = 2$ and $D = 1$, let us find an $\mathbf{f}_\ell$, $\ell = 1, 2$ that maximizes the common SINR $\gamma_{k_2,c}(\mathbf{f}_\ell)$ as

$$\gamma_{k_2,c}(\mathbf{f}_1) \underset{\mathbf{f}_2}{\overset{\mathbf{f}_1}{\gtrless}} \gamma_{k_2,c}(\mathbf{f}_2), \quad (43)$$

Using (42) with $D = 1$, we may write $\mathbf{f}_1 \mathbf{G}_{k_2} \mathbf{h}_{k_2} = g_{k_2,1} h_{k_2,1}$ and $\mathbf{f}_2 \mathbf{G}_{k_2} \mathbf{h}_{k_2} = g_{k_2,2} h_{k_2,2}$. Substituting the same in the expression of $\gamma_{k_2,c}(\mathbf{f}_\ell)$ (given in (8)), we can rewrite (43) as

$$|g_{k_2,1} h_{k_2,1}| \underset{\mathbf{f}_2}{\overset{\mathbf{f}_1}{\gtrless}} |g_{k_2,2} h_{k_2,2}|. \quad (44)$$

Similarly, on obtaining a condition for the maximization of private SINR $\gamma_{k_2,p}(\mathbf{f}_\ell)$, we again (44). This implies that both common and private SINRs are maximized by same $\mathbf{f}_\ell$. Following similar steps, we generalize the condition in (44) for arbitrary L and $D = 1$ as

$$\ell^* = \arg \max_{\ell=1,2,\dots,L} |g_{k_2,\ell} h_{k_2,\ell}|, \quad (45)$$

such that resulting $\mathbf{f}_{\ell^*}$ maximizes both common as well as private SINRs. Now, consider another case of $L = 2$ and $D = 2$. Using (42), we may write

$$\begin{aligned} \mathbf{f}_1 \mathbf{G}_{k_2} \mathbf{h}_{k_2} &= \frac{1}{\sqrt{2}} (g_{k_2,1} h_{k_2,1} + g_{k_2,2} h_{k_2,2}), \\ \mathbf{f}_2 \mathbf{G}_{k_2} \mathbf{h}_{k_2} &= \frac{1}{\sqrt{2}} (g_{k_2,3} h_{k_2,3} + g_{k_2,4} h_{k_2,4}). \end{aligned} \quad (46)$$

Using (46) in (8) for both common and private SINRs, we again get the same result as

$$|g_{k_2,1} h_{k_2,1} + g_{k_2,2} h_{k_2,2}| \underset{\mathbf{f}_2}{\overset{\mathbf{f}_1}{\gtrless}} |g_{k_2,3} h_{k_2,3} + g_{k_2,4} h_{k_2,4}|, \quad (47)$$

which again verifies that same row vector $\mathbf{f}_\ell$ maximizes both the SINRs given in (8). Based on these observations, we finally generalize the condition for arbitrary L and D as

$$\ell^* = \arg \max_{\ell=1,2,\dots,L} \left| \sum_{n=(\ell-1)D+1}^{\ell D} g_{k_2,n} h_{k_2,n} \right|. \quad (48)$$

Thus, it is clear from the above discussion that a row vector with index ℓ^* will always maximize both common and private SINRs of a CEU, where ℓ^* is obtained by following (48).

APPENDIX B

PROOF OF LEMMA 1

If $\mathbf{f}_\ell \mathbf{G}_{k_2} \mathbf{h}_{k_2} \triangleq Z_\ell$ and $\{h_{k_2,d}\}_{d=1}^D$ are known, it immediately follows that

$$Z_\ell | \{h_{k_2,d}\}_{d=1}^D \sim \mathcal{CN}\left(0, D^{-1} \sum_{d=1}^D |h_{k_2,d}|^2\right). \quad (49)$$

and

$$|Z_\ell|^2 | \{h_{k_2,d}\}_{d=1}^D \sim \text{Exp}\left(D^{-1} \sum_{d=1}^D |h_{k_2,d}|^2\right). \quad (50)$$

Since $h_{k_2,d} \sim \mathcal{CN}(0, 1)$, the RV $W_\ell \triangleq D^{-1} \sum_{d=1}^D |h_{k_2,d}|^2$ follows Chi-square distribution with $2D$ DoFs. The probability density function (PDF) of W_ℓ can be given as

$$f_{W_\ell}(w) = \frac{D^D}{\Gamma(D)} e^{-Dw} w^{D-1} \quad w \geq 0. \quad (51)$$

Thus, the PDF of $|Z_\ell|^2$ can be evaluated using (51) in $f_{|Z_\ell|^2}(z) = \int_0^\infty \frac{1}{w} e^{-\frac{z}{w}} f_{W_\ell}(w) dw$ followed by the use of [33, Eq. 3.471.9] as

$$f_{|Z_\ell|^2}(z) = \frac{2D^{\frac{D+1}{2}}}{\Gamma(D)} z^{\frac{D-1}{2}} K_{D-1}\left(2\sqrt{Dz}\right). \quad (52)$$

The CDF $\mathbb{F}_{|Z_\ell|^2}(z) = \int_0^z f_{|Z_\ell|^2}(v) dv$, can be obtained by using (52) and [33, Eq. 6.561.8] as

$$\mathbb{F}_{|Z_\ell|^2}(z) = 1 - \frac{2}{\Gamma(D)} (Dz)^{\frac{D}{2}} K_D\left(2\sqrt{Dz}\right). \quad (53)$$

Substituting $\mathbf{f}_\ell \mathbf{G}_{k_2} \mathbf{h}_{k_2} \triangleq Z_\ell$ in (8), we may now write the joint CDF of $\gamma_{k_2,c}(\mathbf{f}_\ell)$ and $\gamma_{k_2,p}(\mathbf{f}_\ell)$ as

$$\begin{aligned} & \mathbb{F}_{\gamma_{k_2,c}(\mathbf{f}_\ell), \gamma_{k_2,p}(\mathbf{f}_\ell)}(\mu, \nu) \\ &= \Pr \left\{ \frac{\alpha_c |Z_\ell|^2}{\sum_{k=1}^K \alpha_k |Z_\ell|^2 + \frac{1}{\rho_B}} \leq \mu, \frac{\alpha_{k_2} |Z_\ell|^2}{\sum_{k=1, k \neq k_2}^K \alpha_k |Z_\ell|^2 + \frac{1}{\rho_B}} \leq \nu \right\}, \\ &= \Pr \left\{ |Z_\ell|^2 \leq \min \left(\frac{\delta_1(\mu)}{\rho_B}, \frac{\delta_2(\nu)}{\rho_B} \right) \right\}, \\ &= \mathbb{F}_{|Z_\ell|^2} \left(\min \left(\frac{\delta_1(\mu)}{\rho_B}, \frac{\delta_2(\nu)}{\rho_B} \right) \right). \end{aligned} \quad (54)$$

where $\delta_1(\mu)$ and $\delta_2(\nu)$ are defined in (14). Using (53) in (54) and substituting $\mu = \tau_c$ and $\nu = \tau_p$, we get (13).

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