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Snap-back instability of double cantilever beam with bridging

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ABSTRACT

Adhesive bonding community shows a continued interest in using bridging mechanisms to toughen the interface of secondary bonded joints, especially in the case of laminated composites. Due to snap-back instability that occurs during fracture, confusions may exist when identifying the toughening effect experimentally. The true toughening effect may be overestimated by lumping all energy contributions (kinetic energy included) in an overall effective toughness. Here, fundamentals for bridging to enhance fracture resistance are explored through the theoretical analysis of the delamination of a composite double cantilever beam (DCB) with bridging. Specifically, we establish a theoretical framework on the basis of Timoshenko beam theory and linear elastic fracture mechanics to solve the fracture response of DCB in the presence of discrete bridging phases. We elucidate the crack trapping and the snap-back instability in structural response during the crack propagation. We identify the contribution to the overall toughness observed numerically/experimentally of both the physical fracture energy and other types of dissipation. The associated toughening mechanisms are then unveiled. Furthermore, we study the effects of property of the bridging phases on the snap-back instability, based on which, we propose a dimensionless quantity that can be deployed as an indicator of the intensity of snap-back instability. Finally, we identify the role of geometrical properties, i.e. the substrate thickness and the arrangement spacing of the bridging phases, in the snap-back instability and the macroscopic fracture toughness of a DCB. This work provides, from a theoretical point of view, an essential insight into the physics related to the structural response of DCB with discrete toughening elements.

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1. Introduction

Toughening interfaces is of central importance, especially for laminates composites, because composite structures are vulnerable to interlaminar failure. To this end, many strategies have been proposed to address this vulnerability, such as seeding nanofibres (Lee et al., 2012; Daelemans et al., 2015), embedding interleaves (Nash et al., 2015) or interface heterogeneity (Sills and Thouless, 2015; Herráez et al., 2020), processing through-thickness reinforcement, e.g. z-pin (Cartié et al., 2006; Yasaee et al., 2014; Hoffmann and Scharr, 2018) or stitching (Jain and Mai, 1994; Yudhanto et al., 2015; Pappas et al., 2018), during manufacture, among others. All these strategies essentially provoke or introduce bridging, at various scales, to trap crack propagation and improve extrinsic dissipation. Recently, similar concepts have drawn attentions to enhance the fracture resistance of secondary bonded joints (Maloney and Fleck, 2019). Lubineau and coworkers designed a

series of architectural bondline by patterning interface heterogeneity with laser to trigger large-scale ligaments bridging (Tao et al., 2020a; Tao et al., 2020b) or by embedding 3D printed thermoplastic nets in bondline (Yudhanto et al., 2020), which showed remarkable potential to control and tailor the fracture resistance curve. Heide-Jørgensen et al. (Heide-Jørgensen et al., 2018) applied a two-phase composited bondline made of a polymer supporting carrier embedded in epoxy. An increased resistance to fracture was reported thanks to the bridging mechanisms promoted by the lattice carrier. The effectiveness of various bridging mechanisms on toughening laminates or joints has been evaluated, proving the potential of application. In the discipline of material design, inspired by biological structures, many efforts have also been made towards toughening general materials through promoting micro- or macro- scale bridging (Wegst et al., 2015). Noticeably, some of the developed strategies are introducing discretely distributed bridging phases, namely, the bridging events that happen discontinuously (Hoffmann and Scharr, 2018; Pappas et al., 2018; Tao et al., 2020b). In the presence of discrete large-scale bridging

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events, structural loading responses generally show oscillations manifesting a snap-back instability.

During crack propagation, a snap-back instability appears when the released energy due to the advancing crack exceeds the energy dissipated to form the new fracture surfaces (Bažant, 1987; Carpinteri, 1989; Carpinteri, 1989). In such scenarios, the applied load and displacement must decrease simultaneously to sustain equilibrium, leading to so called “snap-back” behavior. As a result, under deflection-control, the structure demonstrates a sudden drop in structural response (snap-through) and the emitted energy may give rise to a virtual catastrophic branch of structural response (Carpinteri and Accornero, 2018; Xu et al., 2019; Heide-Jørgensen and Budzik, 2017). The occurrence of snap-back is dependent on multiple factors, such as the geometric characteristics of structure and initial notch (Gajo, 2004; Salviato et al., 2019), material properties (brittle or ductile), boundary conditions (Carpinteri, 1989; Carpinteri and Accornero, 2018) and property heterogeneity (Xu et al., 2019; Heide-Jørgensen and Budzik, 2017; Xia et al., 2013; Xia et al., 2015; Morano et al., 2020). Implicit finite element codes are normally employed to numerically predict responses with non-linearity. However, once snap-back occurs, the convergence radius of the Newton–Raphson scheme corresponding to the snap-back point reduces to zero, which diverges the solution of the nonlinear equilibrium. To remove this numerical instability, the arc-length algorithm was developed (Crisfield, 1981). In general, the arc-length method adds a constraint to the equilibrium equations and thus maintains the numerical stability. However, implementing this technique is cumbersome, and sometimes the traced unstable equilibrium path does not necessarily lead to the physically correct solution when dealing with local snap-backs. More practical numerical schemes have been developed through controlling the loading process by a monotonically increasing index, typically the crack length. Examples includes the sequence linear analysis (SLA) (Invernizzi et al., 2011; Luo and Bažant, 2018) and the crack length control scheme (CLCS) (Carpinteri, 1989; Carpinteri and Accornero, 2018). For such techniques, the accuracy of predictions depends on the discretization of propagation.

Crack propagation within two- or multi- phase materials, such as fiber reinforced polymers, is generally discontinuous, manifested as trapping and sudden advancing of crack tip (Bosco and Carpinteri, 1995; Xu et al., 1998). The hard phase featuring high strength, such as fibers, plays a role that traps the crack and promotes the accumulation of elastic energy in the system (Mower and Argon, 1995; Wang and Xia, 2017; Li et al., 2020; Wang et al., 2019). Snap-back instability is always concomitant with the discontinuous crack propagation resulting from crack-trapping, and this process has attracted significant attention in recent decades. In terms of theoretical analysis, Bosco and Carpinteri (Bosco and Carpinteri, 1995) developed a bridged-crack model with a perfectly plastic bridging response to describe the discontinuous constitutive flexural behavior of brittle fibrous composites. A dimensionless brittleness quantity was applied to characterize the extent of virtual catastrophe. They proved that such catastrophic failure tends to reproduce the classical Griffith instability for small fracture toughness and/or for large structure sizes. In addition to the models describing the discrete distributed bridging phases, analytical models that can be used to describe the continuous bridging effect were developed by the same authors (Carpinteri and Massabo, 1997) and other researchers (Wang and Xia, 2017; Cox and Marshall, 1991) through integral formulations of surface traction. Compared to numerical simulations, analytical methods provide snap-back solutions economically and fast, avoiding the cumbersomeness of numerical modeling.

As aforementioned, more recent works focus on toughening laminates joints/interface via engineering large-scale discrete

bridging mechanisms. The fracture propagation exhibiting snap-back instability may lead to brittle or even premature catastrophic failure, which poses a threat to the application of laminates and joints. Unfortunately, there is still a lack of universal design principles to suppress such instability. Moreover, conventional models/methods homogenize the fracture process zone, which may be invalid in analyzing the propagation test data with snap-back instability (Li et al., 2020). Therefore, the efficiency of bridging-enhanced toughness may be somewhat inadequately evaluated. An insight into the physics behind the snap-back instability contributing to the failure of laminates and joints with bridging can help to establish the design principle in the context of interface toughening via engineering bridging. By modeling the bridging fibers as elastic beams, Szekrényes and Uj (Szekrényes and Uj, 2005) developed a theoretical model to analyze the crack propagation in DCB (the most commonly utilized specimen for estimating the resistance to delamination) with fiber bridging, with which, they approximated the number of the bridging fibers and the bridging force. They also identified the inadequacy of the classical model in the evaluation of the propagation test data with bridging. Based on Euler beam theory, an iterative scheme was proposed (Ratcliffe and O'Brien, 2004) to find the analytical solution enabling the description of the fracture of DCB reinforced with discrete bridging actions. Typically, such structures present response curves of saw-tooth appearance which is, at macrostructural scale, the result of local unstable discontinuity in crack propagation. Though some efforts have been made to analyze the response of DCB with discrete bridging forces (Spearing and Evans, 1992; Robinson and Das, 2004; Allegrì et al., 2014), very limited research is dedicated to clarify the mechanisms of snap-back instability observed in such structures. With the aim of elucidating the unstable delamination and of exploring the fundamentals of snap-back instability, this study presents a theoretical model to accurately reproduce the complexity of delamination in a DCB with discretely distributed bridging actions. Applying the model, we investigate the underlying mechanisms behind bridging and the snap-back instability of the global response of DCB, and we identify the key properties, i.e. the bridging laws and the arrangement spacing of bridging actions, to minimize propensity toward snap-back instability. The results presented here will pave the way for engineers to pursue the transition from catastrophic (or brittle) failure to non-catastrophic (or ductile) failure, thereby facilitating the design of laminates or joints with bridging.

2. Theoretical framework

2.1. Problem statement

For different toughening strategies, the configuration of the triggered bridging elements varies, as in Yasaee et al. (2014) and Tao et al. (2020b). In most cases, these bridging elements take effect in mixed mode crack propagation. To avoid the complexity of analyzing mixed-mode fracture while clearly verifying the mechanisms of interest, we assume the bridging force/traction oriented perpendicular to the crack advance direction (influencing mode-I crack). The DCB with discretely arranged bridging actions is shown schematically in Fig. 1. The depth and width of the DCB are $2h$ and b , respectively. Referring to Bosco and Carpinteri (1995) and Ratcliffe and O'Brien (2004), the existence of bridging actions is idealized as discrete springs with initial length of zero (representative unit bridging cells). The bridging spring has an axial force F_i versus elongation (or extension) characteristic based on a specific uniaxial tensile response. Here a bilinear law is considered. The initial crack length is a_0 and the distance between the loading point and the i_{th} spring is denoted by a_i . The instant-

Fig. 1. The configuration and boundary conditions of the studied DCB with discretely distributed bridging actions.

neous crack length during crack propagation is denoted as a_c . Before the crack propagation bypasses the spring ($a_c < a_1$), the DCB behaves like a conventional DCB. Once the crack propagates to the bridging site, the spring starts to stretch and a contribution from the bridging is expected to be manifested by structural responses.

The bilinear behavior of the bridging springs can be described by the following expression:

$$F_i = T + T'w(a_i) \quad (1)$$

where $w(a_i)$ is the extension of the spring, T and T' are, respectively, the intercept and slope of the bridging curve controlling the mechanical behavior, expressed with:

$$\begin{cases} T = 0, T' = K, 0 < w(a_i) \leq w_0 \\ T = \frac{Kw_0}{w_f - w_0}, T' = -\frac{Kw_0}{w_f - w_0}, w_0 < w(a_i) < w_f \\ T = 0, T' = 0, w(a_i) \geq w_f \end{cases} \quad (2)$$

where K, w_0 and w_f are the initial stiffness of spring in linear regime, the elongation values corresponding to the failure onset and full failure of spring, respectively. Of course, we can also assume the evolution of bridging failure follows general paths of different shapes, such as an exponential path.

2.2. Deformation of the structure

The arms of the DCB are modeled as Timoshenko beams to incorporate the effects of shearing deformation. Considering the symmetry, half of the DCB is modeled as shown in Fig. 2, where the DCB is bridged by n discretely distributed springs. Based on Timoshenko's theory (Gere and Timoshenko, 1990), the governing equation of the system can be expressed as:

$$\begin{cases} PX = E_{xx}I \frac{d\varphi}{dx}, 0 \leq x \leq a_1 \\ PX - F_1(x - a_1) = E_{xx}I \frac{d\varphi}{dx}, a_1 < x \leq a_2 \\ \vdots \\ PX - \sum_{j=1}^i F_j(x - a_j) = E_{xx}I \frac{d\varphi}{dx}, a_i < x \leq a_{i+1} \\ \vdots \\ PX - \sum_{j=1}^n F_j(x - a_j) = E_{xx}I \frac{d\varphi}{dx}, a_n < x \leq a_c \end{cases} \quad (3)$$

and

Fig. 2. The schematic on the deformation of the half DCB corresponding to the studied case. a_c is the instantaneous crack length during crack propagation.

$$\begin{cases} -P = \kappa A G_{xy} \left(-\varphi + \frac{dw}{dx} \right), 0 \leq x \leq a_1 \\ -P + F_1 = \kappa A G_{xy} \left(-\varphi + \frac{dw}{dx} \right), a_1 < x \leq a_2 \\ \vdots \\ -P + \sum_{j=1}^i F_j = \kappa A G_{xy} \left(-\varphi + \frac{dw}{dx} \right), a_i < x \leq a_{i+1} \\ \vdots \\ -P + \sum_{j=1}^n F_j = \kappa A G_{xy} \left(-\varphi + \frac{dw}{dx} \right), a_n < x \leq a_c \end{cases} \quad (4)$$

where φ and w are the rotation and deflection of the beam arm, a_c is the length of the deformed beam, i.e., the instantaneous length of the propagated crack, P is prescribed load on the left end of the cantilever beam, $A = bh$ the cross section area of the beam, E_{xx} the longitudinal elastic modulus, G_{xy} the shear modulus, $I = bh^3/12$ the second moment of area, and κ is the Timoshenko shear coefficient to account for the shearing deformation of the beam arm.

In general, κ depends on the geometry of the beam section. Various shear coefficients have been proposed. Herein, we use the calculation proposed by Hutchinson (Hutchinson, 2001):

$$\kappa = - \frac{2(1+\nu)}{\left[\frac{144}{h^3 b} C + \nu \left(1 - \frac{b^2}{h^2} \right) \right]} \quad (5)$$

in which ν is the Poisson's ratio, and

$$C = \frac{bh^3}{180} \left(-3h^2 - \frac{15}{4} \nu h^2 + \frac{5}{4} \nu b^2 \right) + \sum_{n=1}^{\infty} \frac{\nu^2 b^5}{2} \left(\frac{n\pi h}{2} - \frac{b}{2} \right) \frac{\tanh \left(\frac{n\pi h}{b} \right)}{(n\pi)^5 (1+\nu)} \quad (6)$$

Solving the ordinary differential governing equation sets in Eq. (3) and (4), we obtain the deflection of the beam:

$$w(x) = \begin{cases} -\frac{Px}{\kappa A G_{xy}} + \frac{Px^3}{6E_{xx}I} + \frac{C_0 x}{2E_{xx}I} + D_0, x \leq a_1 \\ -\frac{P+F_1}{\kappa A G_{xy}} x + \frac{P-F_1}{6E_{xx}I} x^3 + \frac{a_1 F_1}{2E_{xx}I} x^2 + \frac{C_1}{2E_{xx}I} x + D_1, a_1 < x \leq a_2 \\ \vdots \\ -\frac{P+\sum_{j=1}^i F_j}{\kappa A G_{xy}} x + \frac{P-\sum_{j=1}^i F_j}{6E_{xx}I} x^3 + \frac{\sum_{j=1}^i a_j F_j}{2E_{xx}I} x^2 + \frac{C_i}{2E_{xx}I} x + D_i, a_i < x \leq a_{i+1} \\ \vdots \\ -\frac{P+\sum_{j=1}^n F_j}{\kappa A G_{xy}} x + \frac{P-\sum_{j=1}^n F_j}{6E_{xx}I} x^3 + \frac{\sum_{j=1}^n a_j F_j}{2E_{xx}I} x^2 + \frac{C_n}{2E_{xx}I} x + D_n, a_n < x \leq a_c \end{cases} \quad (7)$$

Imposing the boundary conditions and continuity conditions, the parameters to be determined in the solution can be calculated from:

$$\begin{cases} C_0 = \sum_{i=1}^n (a_c - a_i)^2 F_i - a_c^2 P \\ C_i = C_{i-1} - a_i^2 F_i, i = 1, \dots, n \end{cases} \quad (8)$$

and

$$\begin{cases} D_n = \frac{P-\sum_{i=1}^n F_i}{\kappa A G_{xy}} a_c - \frac{P-\sum_{i=1}^n F_i}{6E_{xx}I} a_c^3 - \frac{\sum_{i=1}^n a_i F_i}{2E_{xx}I} a_c^2 - \frac{C_n a_c}{2E_{xx}I} \\ D_i = D_{i+1} + \frac{F_{i+1} a_{i+1}}{\kappa A G_{xy}} + \frac{F_{i+1} a_{i+1}^3}{3E_{xx}I} - \frac{(C_i - C_{i+1}) a_{i+1}}{2E_{xx}I}, i = n-1, \dots, 0 \end{cases} \quad (9)$$

With the expressions of the beam deflection Eq. (7), we can obtain n equations with respect to the extensions of n bridging springs:

$$\begin{cases} w(a_n) = \frac{-P+\sum_{j=1}^n F_j}{\kappa A G_{xy}} (a_n - a_c) + \frac{P-\sum_{j=1}^n F_j}{6E_{xx}I} (a_n^3 - a_c^3) \\ \quad + \frac{\sum_{j=1}^n a_j F_j}{2E_{xx}I} (a_n^2 - a_c^2) + \frac{C_n}{2E_{xx}I} (a_n - a_c) \\ w(a_i) = w(a_{i+1}) + \left(\frac{\sum_{j=1}^i F_j - P}{\kappa A G_{xy}} + \frac{C_i}{2E_{xx}I} \right) (a_i - a_{i+1}) \\ \quad + \frac{P-\sum_{j=1}^i F_j}{6E_{xx}I} (a_i^3 - a_{i+1}^3) + \frac{\sum_{j=1}^i a_j F_j}{2E_{xx}I} (a_i^2 - a_{i+1}^2) \\ i = n-1, \dots, 1 \end{cases} \quad (10)$$

Note that the instantaneous load P needs to be determined to get the spring extensions $w(a_i)$.

When the bridging actions distribute continuously, the bridging force can be achieved by integrating the traction t over the bridging zone. Then the governing equations of the system read:

$$\begin{cases} Px = E_{xx} I \frac{d\varphi}{dx}, 0 \leq x \leq a_b \\ Px - \int_{a_b}^x t(x) dx = E_{xx} I \frac{d\varphi}{dx}, a_b < x < a_c \end{cases} \quad (11)$$

and

$$\begin{cases} -P = \kappa A G_{xy} \left(-\varphi + \frac{dw}{dx} \right), 0 \leq x \leq a_b \\ -P + \int_{a_b}^x t(x) dx = \kappa A G_{xy} \left(-\varphi + \frac{dw}{dx} \right), a_b < x < a_c \end{cases} \quad (12)$$

where a_b is the distance between the loading point and the outermost within the bridging-affecting area from the crack tip. The bridging traction $t(x)$ is a function of coordinate x . The solution will be consistent with that from a beam based on a Winkler elastic foundation (Kanninen, 1973).

2.3. Crack propagation criterion

According to linear elastic fracture mechanics (LEFM), the energy release rate (ERR) G is defined as Irwin and Kies (1997):

$$G = - \frac{\partial \Pi}{\partial a_c} = \frac{1}{b} \left(\frac{dU_e}{da_c} - \frac{dU_s}{da_c} \right) \quad (13)$$

where Π is the total potential of the system, and U_e and U_s are the work done by external force and the stored strain energy, respectively. For the DCB configuration with bridging, the ERR during crack propagation can be decomposed into the ERR contributed from DCB to locally break the crack tip, denoted by G^{DCB} , and the ERR contributed from non-local events, i.e. the bridging actions, denoted by $G^{Bridging}$.

$$G = G^{DCB} + G^{Bridging} \quad (14)$$

Regarding G^{DCB} , Williams (1988) assumed a singular stress field at the crack tip according to LEFM and developed a general analytical solution based on the local characteristics of bending moments and loads shearing load (ignoring the axis load) at the crack tip, expressed as Hashemi et al. (1990):

$$G^{DCB} = G_M + G_Q \quad (15)$$

in which the moment contribution reads:

$$G_M = \frac{1}{16bE_{xx}I} \left[\frac{M_1^2}{\xi^3} + \frac{M_2^2}{(1-\xi)^3} - (M_1 + M_2)^2 \right] \quad (16)$$

and the shear force contribution reads:

$$G_Q = \frac{1}{4\kappa G_{xy}A} \left[\frac{Q_1^2}{\xi} + \frac{Q_2^2}{(1-\xi)} - (Q_1 + Q_2)^2 \right] \quad (17)$$

where M_i and Q_i are, respectively, the moment and shearing load applied on the crack tip; $i = 1, 2$ denotes the two beams arms of DCB and ξ is the thickness portion of the 1_{st} arm among the whole

DCB. For a standard DCB, we have $\xi = \frac{1}{2}$. Considering the system of DCB with multiple bridging actions subjected to the point load P on the left end of beam arms (see Fig. 1), the following equation can be achieved by applying the LEFM criterion ($G^{DCB} = G_{Ic}$):

$$\frac{1}{bE_{xx}I} \left(Pa_c - \sum_{j=1}^n F_j(a_c - a_j) \right)^2 + \frac{1}{\kappa G_{xy}A} \left(P - \sum_{j=1}^n F_j \right)^2 - G_{Ic} = 0 \quad (18)$$

where G_{Ic} is the intrinsic fracture energy of mode I fracture (critical ERR), which only depends on the materials/interfaces. Combining Eq. (10) and (18), we solve the simultaneous linear equations and obtain unknowns $w_1, \dots, w_n$ and P of the system. Then, the opening of the loading points of the DCB can be obtained as $\delta = 2w(0) = 2D_0$. The latter term in Eq. (14) is expressed as:

$$G^{Bridging} = \frac{2}{b} \sum_{j=1}^n \frac{1}{2} T \frac{dw(a_j)}{da_c} \quad (19)$$

Note that the proposed model is also applicable to cases that have more complicated loading conditions and bridging laws. Below, we apply the developed model to study the snap-back instability during the failure of the DCB toughened through bridging.

3. Results

In accordance with the solving strategy of ALS and CLCS, the crack length a_c is taken as the index driving fracture. With the theoretical model established in the last section, we can obtain, for a given crack length a_c , the response solutions, i.e., the applied load vs. the crack opening (P vs. δ) and the crack length vs. the crack opening (a_c vs. δ), of a DCB in the presence of bridging. Correspondingly, the R -curve behaviors can also be drawn according to the following calculation of ERR, which is a direct derivation from Eq. (13) (Irwin and Kies, 1997):

$$G = \frac{1}{2b} \left(P \frac{d\delta}{da_c} - \delta \frac{dP}{da_c} \right) \quad (20)$$

where $\frac{d\delta}{da_c}$ and $\frac{dP}{da_c}$ are obtained by a central-difference approximation. We can also use Eqs. (15) and (19) to derive the R -curve behavior of the system. The flow chart for the computing process is shown in Fig. 3. The distributed bridging actions have two states: 'awake' indicates the spring is functioning, either within its elastic or inelastic regime, and 'dead' indicates the spring is not yet surpassed by the crack tip or it has been damaged completely.

chart
Fig. 3. Flow chart of the computation process. The crack propagation is discretized, Δa is the crack increment, a_f is maximum crack length of interests.

3.1. The toughening mechanisms of bridging and the snap-back instability

3.1.1. Crack-trapping and snap-back instability

Fig. 4 demonstrates the typical responses of a DCB, where a single bridging action is arranged head of the crack tip. The configuration characteristics and input properties of the model are listed in Tables 1 and 2. Fig. 4a plots the P vs. δ and a_c vs. δ curves of the studied DCB, along with the response of the pristine DCB as a reference case for comparison. Fig. 4b plots the R -curve with regard to the corresponding crack procedure. The crack lengths corresponding to points O, A and B of the loading branch are denoted by a_1 , a_1^0 and a_1^u , respectively. As indicated by the curves, the crack propagation can be subdivided into four stages. Stage I, for $a_0 < a_c < a_1$, concerns the crack propagating procedure in the plain interface without the presence of bridging, which has a stable global response dictated by the intrinsic toughness only. Stage II, for $a_1 < a_c < a_1^0$, concerns the process after the crack-tip passes by the spring site. The triggered bridging mechanism starts to take effects in the wake of crack in terms of a linearly elastic traction

Table 1

The properties and geometrical parameters of the beams of DCB.

E_{xx} (Mpa)	G_{xy} (Mpa)	b (mm)	h (mm)	a_0 (mm)	G_{IC} (N/mm)
125000	5100	1	2	50	0.4

Table 2

The properties of the bridging action and the distance between the loading point and the bridging action.

K (N/mm)	w_0 (mm)	w_f (mm)	a_1 (mm)
15	0.2	0.25	70

while essentially the bridging action remains intact. Note from the a_c vs. δ curve, this estimated case experiences the essential crack-trapping phenomena where the crack propagation obviously slows down as the crack opening increases, as compared to the case of pristine DCB. The crack is trapped by the bridging force up to point A, at which the peak load of the bridging regime is

Fig. 4. Fracture responses of the DCB with single bridging action. (a) The P vs. δ curve (the left coordinate axis) and the a_c vs. δ curve (the right coordinate axis) showing how the bridging actions taking effect on the structural response of the DCB during crack propagation and the crack trapping mechanisms of bridging and the post peak crack propagation process. (b) R -curves of the studied DCB in the cases of snap-back and snap-down ($70 \text{ mm} < a_c < 92 \text{ mm}$), showing how the transition from pure intrinsic toughness to a combination of extrinsic and intrinsic toughness. (c) The closeup of P vs. δ curve ranging from $\delta = 6.5 \text{ mm}$ to $\delta = 12.5 \text{ mm}$, together with the snap-down P vs. δ response when the DCB is subjected to deflection controlled loads. The bridging energy and emitting energy are highlighted. (d) The closeup of a_c vs. δ curve ranging from $\delta = 6.5 \text{ mm}$ to $\delta = 12.5 \text{ mm}$, together with the snap-up a_c vs. δ response when the DCB is subjected to deflection controlled loads. The crack propagation shows a discontinuous acceleration in the latter case.

reached. The slopes of P vs. δ demonstrates a significant increase, from negative to positive with a promoting slope, which represents an enhancement of structural stiffness and a significant accumulation of elastic energy. In contrast, the R -curve still matches with the input interface toughness (intrinsic fracture energy), indicating that the spring remains intact during Stage II and the toughness is only contributed to by breaking the plain interface. The cuspidal point A of P vs. δ curve corresponds to the moment when the extension of the bridging spring reaches its critical value, w_0 , and the softening onsets. The propagation enters into Stage III for $a_0^I < a_c < a_1^I$, during which, the spring undergoes a softening mechanism. A sharp snap-back instability occurs and a discontinuity is identified in the a_c vs. δ response. Both P and δ are decreasing to sustain the equilibrium state during the softening of the bridging action. Meanwhile, we observe a steep increasing in the R -curve behavior. Thus the toughness of the DCB is enhanced as a consequence of the participation of G^{bridging} , which is substantially higher than that of the pristine counterpart. Once the spring is fully damaged (corresponding to Point B), the bridging action has no effect thereafter. As the system enters into Stage IV ($a_c > a_1^I$), P drops onto the lower branch with negative slope (baseline response). The R -curve behaves the same. The above evaluated case clarifies the trapping mechanisms of bridging and demonstrates that the developed model is capable of producing the snap-back unstable response of a DCB in the presence of bridging.

In experiments where the loading is controlled by deflection, rather than undergoing snap-back, the loading capacity will show a discontinuity with a negative jump and the representative response point will drop onto the fourth branch of the P vs. δ curve, exhibiting a snap-down response, as illustrated by the vertical path (AB') accompanying the snap-back branch in Fig. 4c. Accordingly, an instantaneous acceleration of crack is observed after the occurrence of the crack-trapping event until the crack-tip reaches the propagation path of baseline; see the vertical rise (snap-up) of a_c vs. δ curve in Fig. 4d. Note that the above expected response is based on the premise that the surplus energy (the energy gap highlighted with the gray shadow and denoted as U_{emitting}), emitted in other forms, e.g. acoustic emission, kinetics, thermal and etc., is not recoverable. Under this circumstance, the snap-back instability transfers to a snap-down behavior and the extrinsic dissipation is thus the superposition of U_{bridging} (the consumed energy by the failure of bridging actions) and U_{emitting} . The corresponding R -curve behavior is shown in Fig. 4b. The length of toughening segment is obviously higher than that of the snap-back process, although the instantaneous ERR is higher for the snap-back process. However, in practice, the emitted energy (the kinetic energy at least) is essentially recoverable or at least partially recoverable, and the associated dynamic effect may accelerate a fast formation of more fracture surfaces (Kanninen, 1974), which features the structure of such behaviors with a premature catastrophic failure, providing U_{emitting} is sufficiently large. In conventional implicit numerical schemes, artificial viscosity is introduced to compensate the energy gap to sustain a stable propagation (Gao and Bower, 2004). In so doing, an identical response with the dash line can be achieved, which overestimates a structure's crack resistance. Therefore, the numerically observed toughening effect is, to some extent, an artifact due to the involvement of artificial compensation of dissipation.

Fig. 5 compares the R -curves obtained by Eq. (20) and those from the simple beam theory (SBT), which is a conventional data reduction method applicable for standard DCB tests (Škec et al., 2019). During $a_1 \leq a_c \leq a_0^I$, a gradual increase in ERR is described by SBT, which overestimates the instantaneous toughness, whereas after the initiation of bridging failure, an underestimation is observed. This error is due to the fact that, in the presence of

bridging, the loading on beam arm differs from that of the standard DCB subjected to point load. Note that, overall, the predicted extra dissipation volume shows fewer differences. The reason for this remarkable discrepancy of R -curve behaviors is that the applied non-linear bridging force on the beam arm due to the presence of bridging action jeopardizes the validity of the classical methods, such as SBT, corrected beam theory and compliance calibration (they all produce similar results in the case of DCB without bridging) that are derived for a DCB prescribed with a single point load at the edge. This suggests that the conventional data reduction methods are inapplicable in cases when the crack is propagating in the presence of large-scale, discrete bridging events. Therefore, caution must be exercised when selecting data reduction methods to derive R -curve behaviors of DCB.

3.1.2. Bridging actions in series and parallel

The concept of implementing bridging actions in parallel or series to enhance the toughening effect is straightforward, e.g., by thickening the bridging actions or installing a cascade of multiple bridging actions. Fig. 6 is a schematic to demonstrate the basic concept of the two strategies. The combination of bridging's physical attributes leads to an equivalent bridging action with upgraded properties. In the following, we deploy a dimensionless parameter, ζ , to quantify the intensity of systems' snap-back instability:

$$\zeta = \frac{U_{\text{emitting}}}{U_{\text{bridging}} + U_{\text{emitting}}} \quad (21)$$

U_{emitting} is calculated by performing numerical integration after we complete the computation process in Fig. 3.

Fig. 7 plots the obtained responses when implementing the bridging actions in parallel and in series. U_{emitting} and ζ of the two studied scenarios are listed in Table 3. All the response diagrams in Fig. 7 converge towards the same asymptotic tail (baseline). In the case of parallel bridging, with more paralleled actions/thicker bridging action at the bridging site, both U_{bridging} and U_{emitting} increase. The converging point is independent of the parallel number because w_f keeps constant, whereas the cuspidal points of snap-back, at which P locally reaches peak, are dependent on the parallel number. Note that the snap-back index, ζ , in the case of parallel bridging actions, exhibits an increasing trend with increasing number of parallel bridging actions. This reveals that, despite the increasing extrinsic dissipation resulted from more failure work from bridging actions, parallel bridging is not an efficient strategy to enhance the toughening effect due to the deterioration of the fracture stability. A cascade of bridging can increase the critical extending values corresponding to both the onset and ultimate of bridging's failure. From an energetic point of view, the extrinsic dissipation contributed to by bridging shows a significant increase, while U_{emitting} experiences a moderate increase only, as listed in Table 3. As a consequence, the snap-back index exhibits an obvious decreasing trend, which indicates that implementing series connecting bridging actions is a suitable strategy to enhance the toughening effect compared to the parallel counterpart strategy.

3.1.3. The effect of bridging action's property

Engineering bridging actions, i.e., optimizing the medium of bridging, is another potential route to maximize the efficiency of bridging toughening. Thus, here we focus on the effects of bridging action property, including the constitutive responses and the implementing site of bridging determined by the parameter a_1 . Keeping K and w_0 constant, we firstly increase w_f , which essentially improves the ductility of bridging actions. A dimensionless ratio can be used to quantify the difference between the studied scenarios:

Fig. 5. The comparison of the R -curve behaviors drawn by using Eq. (20) and SBT (a conventional approach). The R -curves for both loading branches $O-A-B$ and $O-A-B'$ are presented.

Fig. 6. The schematic demonstration of bridging actions in parallel and series.

$$\lambda_1 = \frac{w_f}{w_0} \quad (22)$$

Fig. 8 plots the solved structural responses for the evaluated cases. It can be seen that P at the cuspidal point during the bridging event is not changed attributed to the same critical traction for all the bridging actions in the four solved cases. Whereas the instant at which the bridging reaches complete failure is delayed with the increase of w_f . Table 4 lists the computed $U_{emitting}$ and ζ for different λ_1 . As indicated by the solutions, the emitted energy monotonously decreases with increasing λ_1 , such that the snap-back index decreases accordingly. The improving ductility of bridging action gradually alters the structural response from brittle fracture

with snap-back instability to a relatively stable softening behavior. Starting with the studied case of $\lambda_1 = 2.5$, an almost stable softening behavior is achieved (a branch of structural response presents a negative softening slope). It suggests that, a substantial improvement of bridging actions' ductility may produce more energy dissipation as well as stabilize the crack propagation. Therefore, engineers should design bridging actions with more ductile responses to eliminate the snap-back instability and achieve a robust design.

We alter the bridging law by varying w_0 and w_f simultaneously while keeping K and λ_1 constant. Fig. 9a plots the solved structural responses for the evaluated cases. Table 5a lists the computed

(a) Bridging actions in parallel

(b) Bridging actions in series

Fig. 7. The structural responses (P vs. δ) of the DCB (a) with bridging actions in parallel and (b) with bridging actions in series. Insets: the constitutive behaviors of the equivalent bridging actions.

$U_{emitting}$ and ζ . Here we deploy a dimensionless factor λ_2 to distinguish the calculations, defined by:

$$\lambda_2 = \frac{t_0}{P_0} \quad (23)$$

in which $t_0 = Kw_0$ is the critical traction of spring and P_0 is the prescribed load on DCB at the instant of crack tip surpassing the bridging site, expressed by the classical DCB solution (Gere and Timoshenko, 1990):

Table 3

The performance of the DCB with bridging actions in parallel and series.

Bridging actions in parallel				
Spring number	1	2	3	4
$U_{emitting}$ (N · mm)	0.398	1.761	3.941	6.824
ζ	0.347	0.540	0.637	0.695
Bridging actions in series				
Spring number	1	2	3	4
$U_{emitting}$ (N · mm)	0.398	0.695	0.946	1.168
ζ	0.347	0.317	0.296	0.280

$$P_0 = \sqrt{\frac{G_{Ic}}{\frac{a_1^2}{bE_{ext}} + \frac{1}{\kappa bG_{xy}A}}} \quad (24)$$

Obviously, this dimensionless quantity accounts for the effects of both t_0 and a_1 , for a specific DCB. It can be seen, from Fig. 9a, increasing t_0 , leading to an increase of λ_2 , can significantly improve the extrinsic dissipation. However, at the onset of spring failure, the stored elastic energy in the system can be considerable, as well as the emitted energy resulting from the snap-back behavior. Consequently, the kinetics energy transferred from the emitted elastic energy may promote the failure process, giving rise to more brittleness in the fracture and possibly proceeding to premature catastrophic failure. This is exemplified by the DCB with the highest λ_2 among the studied cases, as illustrated by the response of the case with $\lambda_2 = 2.30$ in Fig. 9a. To further check the effect of λ_2 , we calculate the responses when we implement the bridging action at different sites, that is, we vary λ_2 by changing P_0 while keeping t_0 constant according to Eqs. (23) and (24). The obtained P vs. δ curves are shown in Fig. 9b and the corresponding $U_{emitting}$ and ζ are listed in Table 5b. It can be seen from Table 5b that the

Table 4

The performance of the DCB with bridging of different λ_1 .

$a_1 = 15$ N/mm, $w_0 = 0.2$ mm, $\lambda_1 = \frac{w_f}{w_0}$				
λ_1	1.25	1.56	1.88	2.5
$U_{emitting}$ (N · mm)	0.398	0.211	0.080	0.018
ζ (N · mm)	0.347	0.184	0.066	0.012

variation trend of both $U_{emitting}$ and ζ with λ_2 agree with the observations when we change the t_0 . With the increasing a_1 , the bridging behaviors are similar for all cases, and $U_{bridging}$ is identical, yet the snap-back behavior is getting deteriorated, as presented in Fig. 9b.

Based on the above findings, we propose a dimensionless factor λ to indicate the effect of bridging property on the intensity of snap-back instability, expressed as:

$$\lambda = \frac{\lambda_1^2}{\lambda_2} = \frac{P_0}{t_0} \left(\frac{w_f}{w_0} \right)^2 \quad (25)$$

The variation of ζ with the sampled λ is presented in Fig. 10. It can be observed that, a nonlinear function that decreases monotonically can be fitted, proving the validity of the proposed dimensionless quantity. The properties of bridging, i.e. t_0 , w_f , w_0 and a_1 , affect the snap-back instability in a coupled fashion. Generally, the intensity of snap-back instability is weakening with the decrease of λ_1 and increase of λ_2 . It should be noted that this dimensionless factor is applicable in the case that the initial stiffness of the spring is a constant because, obviously, the function is not single-valued in the case that the bridging is parallelly implemented as presented in Table 3. In practice, the dimensionless quantity λ can be used as an index for predicting the intensity of snap-back instability in a fast manner.

Fig. 8. The structural responses (P vs. δ) of the DCB with bridging actions of changing w_f (w_0 and a_1 are kept) to evaluate the effect of λ_1 . All the calculated cases are with the same initial stiffness of bridging. Insets: the constitutive behaviors of the bridging actions.

Fig. 9. The structural responses (P vs. δ) of the DCB (a) with bridging actions of changing t_0 (λ_1 is kept) and (b) with bridging actions of different a_1 to evaluate the effect of λ_2 . All the calculated cases are with the same initial stiffness of bridging. Insets: the constitutive behaviors of the bridging actions.

3.2. The snap-back instability of DCB in the presence of discrete bridging actions

Generally, toughening strategies are implemented through periodical arrangement of bridging actions that produces a self-similar fracture process. The bridging events are expected to be

triggered successively during crack propagation, as with the trapping mechanisms. Thus, the spacing of bridging actions, quantified by the distance between the neighboring bridging actions, η , is of key importance in the design of such structures. Herein, we predict the structural response of DCB with discretely distributed bridging actions, where η is varied to identify the effect. We set $a_1 = 50$ mm

Table 5
The performance of the DCB with bridging of different λ_2 .

(a) $\lambda_1=1.25$, $a_1=70$ mm, λ_2 varied by changing t_0				
λ_2	1.15	1.44	1.73	2.30
$U_{emitting}$ (N · mm)	0.398	0.787	1.349	3.070
ζ	0.347	0.402	0.444	0.506
(b) $\lambda_1=1.25$, $t_0=3$ N, λ_2 varied by changing a_1				
λ_2	0.89	0.99	1.15	1.28
$U_{emitting}$ (N · mm)	0.257	0.310	0.398	0.469
ζ	0.255	0.292	0.347	0.385

and $w_f = 0.3125$ mm, all the other parameters are in accordance with those listed in Tables 1 and 2.

Fig. 11a shows the P vs. δ responses of DCB with different bridging spacing η . To obtain more general insights, we deploy $\lambda_3 = \frac{\eta}{h}$ to describe the spacing characteristic. As long as the characteristic spacing parameter λ_3 is smaller than the critical value that corresponds to the crack length required to fully damage a single bridging action, there can be multiple bridging simultaneously participating in the crack trapping events, as seen in the successive occurrence of spring failure. In the case of $\lambda_3 = 1.57$ ($\eta = 3.14$ mm for the current DCB), at most, six springs are awaking simultaneously. The P vs. δ solutions, with saw-toothed profiles, suggest that, with decreasing λ_3 /increasing bridging density, the crack propagation tends to be more stable because, obviously, the emitted energy decreases for the local upheavals subjected to similar DCB opening. The snap-back behaviors are diminished with more dense bridging actions. The snap-back instability may be fully eliminated provided the bridging actions are arranged densely enough, and unremitting trapping effects can be produced during crack propagation; see the case of $\lambda_3 = 1.57$. The cusps of the 'saw teeth' in P vs. δ represent the failure initiation of the temporal outermost spring, which establishes the upper and lower bounds of the envelope of the saw-toothed responses. With decreasing λ_3 , the envelope narrows and the load P exhibits an overall increasing trend, thus indicating the improving crack resistance of the system. These saw-toothed

profiles appear more regular and tend to become smooth as the bridging actions becoming continuously distributed, and the global response gradually shifts from brittle (as indicated by the case $\lambda_3 = 5$) to stable (for $\lambda_3 = 1.57$), as shown in Fig. 11a. The response approaches that of the cohesive-crack model of the Dugdale type homogeneous interface when the density of the bridging elements tends to infinity. The characteristic saw-toothed profile is in consistency with the experimental measurements from DCB reinforced by thickness z-pins (Cartie, 2000) and adhesive ligaments induced by laser patterning (Tao et al., 2020b). Based on the above analysis, in the context of designing tougher joint by including bridging features, a dense arrangement may provide significantly more crack resistance enhancement with less snap-back instability. Fig. 11b plots the R -curves corresponding to the computed P vs. δ solutions shown in Fig. 11a. Clearly, the representative piecewise characteristics of the R -curve behavior indicate that the fracture process experiences periodical bridging mechanisms. The contribution to the total toughness from spring failure increases with decreasing λ_3 . The higher segments of R -curve corresponds to the crack propagation with multiple events of bridging actions' failure.

The responses, shown in Fig. 12, are computed for the DCB with different substrates thickness h . The arrangement of bridging actions is coarse enough ($\eta = 20$ mm), to avoid the scenario of two or multiple bridging events awaking simultaneously. Although the bridging laws with regard to all the three local upheavals of P vs. δ are identical, entailing an equal volume of bridging work $U_{bridging}$, aggravating snap-back instability is presented in the structural response due to the increasing $U_{emitting}$ with increasing distance from the bridging site to the loading point. In particular, the outermost bridging even show near-zero snap-back instability. The crack propagation in the presence of bridging is thus not strictly a self similar process. This agrees with the observations when we change the implementing site of single bridging in the previous section. The reason for this observation is that more beam volumes are available to store strain energy, thereby transferring to $U_{emitting}$ as the crack length increases. Comparing the structural

Fig. 10. Variation of ζ with the proposed dimensionless quantity λ .

Fig. 11. The structural responses of the DCB in the presence of discretely distributed bridging actions with different characteristic spacing parameter λ_3 of bridging. $h = 2 \text{ mm}$ for the present DCB.

responses from DCB with $h = 2 \text{ mm}$ and those from DCB with $h = 3 \text{ mm}$, we see that the increase in the substrate thickness stabilizes the fracture. Therefore, when designing laminates or joints with relatively thick substrates or sub-laminates, designers may extend the design margin and attempt to utilize bridging actions with relatively high risks of snap-back instability, such as bridging actions

with high strength, to maximize the toughening effects while avoiding the occurrence of premature catastrophe. Referring to Eqs. (23) and (24), the beam thickness h affects the structural response, e.g. the snap-back behavior, by changing the bending stiffness of and shearing stiffness of the beam. Noticeably, the dimensionless quantity λ proposed in the last section is still appli-

(a) Substrate thickness $h = 2$ mm ($\lambda_3 = 10$)

(b) Substrate thickness $h = 3$ mm ($\lambda_3 = 6.67$)

Fig. 12. The structural responses (P vs. δ) of the DCB with different beam thickness in the presence of discretely distributed bridging phases. The insets show the crack length vs. DCB opening (a_c vs. δ) corresponding to the processes presented in the main curves. The responses with snap-down are obtained from the cases that DCB is subjected to deflection controlled load.

cable when we are considering the effect of h on the intensity of snap-back instability.

The above analysis indicates that the snap-back instability is not only an attribute depending on the bridging actions' response, but is also relating to geometrical characteristics, such as the distance between the bridging site and the loading points and the thickness of substrates. It is essentially a material-structure prop-

erty, an interesting feature of bridging mechanisms adding to existing knowledge.

4. Conclusion

Based on Timoshenko beam theory and LEFM criterion, we established a theoretical framework to derive the delamination

of DCB in the presence of discretely distributed bridging actions, with which the response characteristics in such structures, e.g. crack-trapping events and snap-back instability, are reproduced. Detailed derivation is presented, along with an extensive discussion on the crack trapping mechanisms and the discontinuous crack propagation due to bridging. Our results suggest that the fracture resistance of interface can be toughened through the crack trapping mechanisms due to the involvement of bridging forces. The associated snap back instability, occurring with crack extension during the softening of bridging, makes the structures prone to premature catastrophe.

The property of bridging actions affects the snap-back instability. Notably, increasing the ductility of the bridging actions leads to less instability and a more stable softening structural response. A dimensionless quantity formulated by the bridging action properties is proposed that can be deployed as an indicator for the snap-back intensity of DCB with bridging. The macro-scale responses of DCB linked to successive occurrence of bridging events emphasize the importance of the bridging spacing. A dense arrangement can efficiently stabilize the crack propagation and meanwhile produce more extrinsic toughness. In particular, the snap-back instability can be completely addressed for the arrangement spacing smaller than a definite value. The geometrical characteristics of the DCB with bridging are also determining factors of snap-back behaviors, indicating the structure-material attribute of snap-back instability.

The numerically predicted toughening mechanism in the presence of snap-back instability, by the implicit solving scheme, is an artifact due to the involvement of artificial viscosity. In practice, the kinetic energy transferred from the emitted energy is recoverable. This part of energy may make a significant contribution to forming new fracture, which makes the structure vulnerable to premature catastrophic failure and thus casts doubts upon the effectiveness of the crack arrest features.

This study provides fundamental insights into the bridging mechanisms and the associated physics behind snap-back instability in laminates or joints that are instrumental for the development of rational concepts for toughening interfaces by triggering bridging. The established theoretical framework can be used in the evaluation and design of laminated composites or joints with bridging in a reliable, more accurate and fast manner. The developed model uses springs with bi-linear responses to mimic the characteristic bridging behaviors when the bridging events occur discretely along the interface. More sophisticated responses of bridging actions will be considered in future studies.

Additional information – Competing interests

The authors declare no financial or non-financial competing interests.

Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Author contribution

G.L, X.L.: Conceptualization, Methodology, Supervision X.L. and S.L.: investigation, Data Curation All authors: Writing - Original Draft, Writing - Review and Editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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