Wake Interactions Of A Tetrahedron Quadcopter

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Abstract—This paper studies the influence of the placement of a quadcopter’s rotors with a tetrahedron shape on its produced thrust. A tetrahedron quadcopter is a rotorcraft with four horizontal rotors, including an upper rotor and three lower rotors placed equidistant around the upper rotor on a lower plane. The goal of this aircraft design is to create an airframe that is structurally rigid in 3-dimensions while being as efficient as an aircraft that has its rotors on the same vertical plane. Due to the wake interaction between the top and bottom propellers, a reduction of thrust is expected compared to a placement of the rotors on the same plane when rotors are close enough. The results presented in this paper illustrate how the wake interactions impact the performance of several configurations of the tetrahedron quadcopter.

I. Introduction

The development of small Unmanned Air Vehicles (UAVs) has seen a considerable increase in the last decade thanks to the miniaturization and reduction in costs of electronics used for their fabrication. Most popular rotorcraft UAVs use four rotors to sustain flight, as it is the smallest number of identical rotor types that can induce controlled flight and hovering without the necessity of tilting the rotors. Different layouts exist for the placement of the four rotors and almost all of them position the rotors in the same horizontal plane. This is for obvious reasons of simplicity, symmetry, and wake interaction reduction. Multi-rotors with more than four rotors such as hexa-rotorcraft and octo-rotorcraft generally respect this principle as well. However, novel concepts of multi-rotor UAVs have used different spatial configurations of the rotors for different goals. For instance, Toratani presents an hexacopter whose rotors are placed on six of the faces of a double tetrahedron in order to provide translational control without rotation of the rotorcraft [14]. Brescianini and D’Andrea designed the Omnicopter, a hexa-rotorcraft with rotors of different inclinations that has six degrees of freedom [3]. In [7], Guerout et al. introduce a multirotor in the shape of a dodecahedron. Similar to what is presented in this paper, work by Otsuka and Nagatani studies the wake interaction of different octa-rotorcraft configurations for which the rotors are placed in two parallel planes [11]. In the authors’ recent work [6], a concept of a tetrahedron rotorcraft was introduced. The goal of the tetrahedron rotorcraft is to provide rigidity in all three dimensions while being as efficient as a traditional quadcopter. The

tetrahedron structure was chosen as the airframe of the vehicle because of its known structural strength and rigidity [4], allowing it to be used as the building block of a self-similar assembly inspired by fractals. Fig. 1 shows the prototype of the module, named Tetracopter, as built by the authors.

Fig. 1: The Tetracopter prototype.

The generative rule to construct a bigger assembly from four modules or from four sub-assemblies comes from the Sierpinski tetrahedron [5]. This generative rule is illustrated in Fig. 2.

The use of a tetrahedron airframe for the main module of an assembly can be dated back to 1904, when Bell designed several kites combining multiple smaller kites with a tetrahedron airframe [2]. In [2], Bell stated he was attracted to the tetrahedral airframe because “the tetrahedral cell possess rigidity or strength in three directions – that is to say, vertically, laterally, and longitudinally–

Fig. 2: Generation of an assembly from the Tetracopter module.

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so that internal bracing is entirely superfluous and is dispensed with”.

In the case of rotorcraft with a tetrahedron frame, the question of the placement of the rotors comes to mind as one wants to optimize certain criteria such as the thrust-to-weight ratio or power consumption at hovering. Typically, a more compact airframe is desirable and for that reason an equilateral triangle base with a placement of the rotors that respects the symmetries of the triangle base seems natural. The only way to achieve such symmetry with four rotors in a same plane is to place three of them at the vertices of an equilateral triangle and one at the center of it. The distance between the center rotor and the three side ones must then be superior to the diameter of the rotors in order to separate them. It is also possible to place the center rotor in a different plane parallel to the other rotors, allowing them to overlap for an even more compact airframe. The vehicle would then experience a loss of efficiency incurred by the wake interaction between the top and bottom rotors, increasing with the area of overlap. The distance between the top and bottom plane also has the potential of impacting this efficiency loss because of the changing velocity and cross section area of the airflow exiting the top rotor. Let us notice that for practical reason, even with non overlapping rotors, it could be hard to place all four of them in the same plane. Space on the frame must indeed be reserved for other necessary components, and their placement can have a significant impact on the dynamics of the rotorcraft, especially when they have a significant weight of volume like batteries. To balance the rotorcraft, ensure an equally shared thrust between the rotors at hover, and minimize the effort required for pitching and rolling the aircraft, these components must be placed between the three side rotors, imposing that the center rotor be placed in a different plane. In order to find the optimal balance between the different criteria to optimize in the design of a tetrahedron rotorcraft, it is necessary to characterize the impact of interactions between overlapping rotors on the thrust output and power consumption of the tetrahedron rotorcraft. This work focuses on the analysis of the efficiency of several rotor configurations inspired by the Tetracopter.

For each configuration, by measuring the total thrust produced by the rotors and their power consumption, we can study their efficiency at hovering, defined as the ratio of thrust to power consumption. The studied configurations are based on a placement of the rotors at the vertices of a triangular pyramid, a special type of tetrahedron. A triangular pyramid has an equilateral triangle base and three isosceles triangle sides. Fig. 3 shows a representation of a triangular pyramid. A regular tetrahedron is a special case of a triangular pyramid. In all the configurations considered, the rotors have an identical horizontal inclination.

The purpose of studying different configurations is to integrate the study’s findings to the design process

Fig. 3: Four views of the same triangular pyramid.

of a more efficient Tetracopter module. These findings will be considered along other technical requirements and parameters to optimize, such as the weight of the resulting module and its ability to form an assembly with good structural rigidity.

As mentioned in [11], the number of studies on the rotor wake interaction for small UAVs is limited in comparison with the number of studies completed for full-size rotorcraft. Most of the few existing works focus either on coaxial rotors or flat quad-rotorcraft. For instance, Lakshminarayan and Baeder investigate the aerodynamics of small coaxial rotorcraft in simulations by changing the distance between rotors and computing the output thrust of each rotor [9]. In [1], Aleksandrov and Penkov study the impact of inter-rotor distance on the lift generated by a quadrotor. The optimal distance they find is a trade-off between limiting wake interactions by increasing the gap between rotors and reducing weight by decreasing this gap. Hwang, Jung, and Kwon study the aerodynamic interactions of the rotors of a quadcopter during hovering and forward flight via numerical methods [8]. They find that there is an overall small reduction in thrust in all configurations and that downwash and upwash in forward flight create unequal thrusts and moments between rotors. Zhou et al. perform an experimental investigation of the aerodynamic interaction of two rotors placed side-by-side as the distance between them is changed [15]. The result of their experiment show that as this distance is reduced, a very small decrease in thrust is observed with, more importantly, a significant increase in the fluctuation of the forces produced by the rotors. Flow visualizations are performed experimentally by Shukla et al. in [13] on coaxial and quadrotor configurations. These visualizations allow the studies of vortices and interactions between rotors for the mentioned configurations. Further work by Shukla and Komerath studies the performance of side-by-side rotors for different separation distances and Reynolds numbers. Their experiment show a decrease in performance when rotors are very close to each other at low Reynolds numbers. In [11], Otsuka and Nagatani introduce a novel octo-rotorcraft configuration with two layers of overlapping rotors. The top rotors

are not directly above the bottom rotors as in coaxial configurations, such that the overlap is limited to provide a trade-off between reduced performance from wake interaction and compactness. The work in [11] relates to ours in the sense that it focuses on assessing the performance of a configuration of a rotorcraft for which the rotors lie in two different parallel planes. However, we focus on the efficiency of the studied configuration by looking at the relation between thrust and power, while work by [11] focuses on the thrust output at different angular velocities of the rotors in the presence of overlap.

The remainder of this paper is structured as follows: In Section II, the spatial configuration considered in this work is explained in more details, and an intuition about the expected results is provided. Section III describes the experiments performed and their results. Section IV provides a brief discussion about the results. Finally, Section V concludes this work by recapitulating the results obtained the thrust tests introduced in this paper.

II. The triangle pyramid configuration

A. Spatial arrangement of the rotors

As explained in the introduction, a triangle pyramid is a special case of tetrahedron with a triangle base and three isosceles triangle sides. A triangle pyramid can be entirely described by two parameters: the length of its base side a and its height h . These two parameters are shown in Fig. 4. For the special case of the regular tetrahedron, $h = \sqrt{6}a/3$. Let x be defined as the 2-dimensional distance, viewed from the top of the vehicle, from the center of the upper rotors to the center of one of the outer rotors, thus $x = \sqrt{3}a/3$.

In this work, for a given triangle pyramid, four rotors are placed horizontally at each of its vertices in order to analyse the thrust of the resulting spatial configuration. The rotors are grouped by pairs of clockwise rotating rotors and counterclockwise rotating rotors.

Fig. 4: Representation of the parameters a , h , and $x = \sqrt{3}a/3$ describing a triangle pyramid.

The minimum practical value for a is equal to $2r$, where r is the radius of a rotor disk. Such a configuration is shown in Fig. 5a. This is to ensure that the bottom rotors do not touch each others. On the other side, a can be arbitrary big. A configuration with a big enough is expected to produce the maximum possible thrust, as wake interactions become null. h can be reduced to a value of 0, which corresponds to a flat layout as shown

in Fig. 5c. In this case the minimum value for a has to be increased to $2\sqrt{3}r$ to ensure separation between the "top" rotor now in the middle with the three remaining ones. h can also be arbitrary big, implying null interactions between the top and bottom rotors. It would be possible to consider negative values of h , but this is not in the scope of this paper. Positive values of h enable the resulting quad-rotorcraft to be supported when on the ground, facilitating landing.

Fig. 5: Different possible configurations of the rotors.

B. Wake interaction theory

It is common knowledge that the induced power of overlapping rotors is found to be higher than that of isolated rotors [10]. Any rotor operating in the slipstream of another will result in higher induced power to produce the same thrust of a rotor operating in undisturbed air. This will clearly result in a degradation in efficiency of any Tetracopter configuration where the radius of the bottom three rotors is less than half of the blade's radius. Calculating the induced power of the three rotors operating in the slipstream and comparing it to that of an isolated rotor can show how the efficiency of the lower rotors decrease. A simple analysis of the induced power of the upper and lower rotors can be completed using the Rankine-Froude momentum theory [10].

1) Single rotor analysis: In the case of a single rotor of area A producing a thrust T at hover, we consider a circular cross section of the rotor's wake at a distance z from the rotor. Let $r(z)$ be its radius and $v(z)$ the velocity of the air going through that cross section, assumed uniform. The conservation of momentum implies that the quantity $\pi r(z)^2 v(z)$ is constant. By defining v_i as the incident velocity of the air flow through the rotor, v_∞ the final velocity of the air flow, and A_∞ the final area of the wake, we have

$$Av_i = A_\infty v_\infty. \quad (1)$$

The produced thrust is also equal to the mass flow in the rotor multiplied by its final velocity, that is

$$T = \rho A v_i v_\infty, \quad (2)$$

where ρ is the air density assumed constant. Since the induced power $P = T v_i$ of the rotor must equate the

rate of created kinetic energy,

$$P = \rho A v_i v_\infty v_i = \frac{1}{2} \rho A v_i v_\infty^2 \quad (3)$$

and

$$\begin{aligned} v_\infty &= 2v_i, \\ A_\infty &= A/2, \\ v_i &= \sqrt{\frac{T}{2\rho A}}, \\ P &= 2\rho A v_i^3 = \frac{T^{3/2}}{\sqrt{2\rho A}}. \end{aligned}$$

A contraction of the wake, vena contracta, is therefore observed as the airflow accelerates under the rotor and is shown in Fig. 6.

Fig. 6: Illustration of the wake contraction under a single rotor.

2) Overlapping rotors analysis: In the case of two overlapping rotors, several assumptions must be made to perform a similar analysis based on momentum theory. First, the total flow is partitioned into three regions. The first region consists in the flow that travels through the upper rotor only, the second is the flow traveling through both rotors, and the third the flow traveling through the lower rotor only. We assume that each of these regions has a uniform flow along any horizontal cross section. Then, we assume that the flow entering the upper rotor is undisturbed by the lower rotor and remains uniform, with an incident velocity v^u . Each of two distinct regions of the flow crossing the lower rotor have respective induced velocities v_1^l and v_2^l . Fig. 7 represents these velocities and the areas to which they apply. We also assume that the total flow is uniform at

Fig. 7: Illustration of the wake interaction between two overlapping rotors.

infinity and reaches a final velocity w . We introduce m^u and m^l , such that $m^u A$ and $m^l A$ represent the cross sectional areas of the flow going through both rotors, respectively at the upper and lower one.

For a given total thrust T produced for hovering, we are interested in finding the power consumption P of the overlapping rotors. Using only the conservation of momentum and energy is not enough to solve for all the variables just introduced. We then also assume that the total power is shared equally between the two rotors and uniformly along their surface. Therefore,

$$\frac{1}{2} P = T^u v^u \quad (4)$$

$$\frac{m^l}{2} P = T_1^l v_1^l \quad (5)$$

$$\frac{(1 - m^l)}{2} P = T_2^l v_2^l. \quad (6)$$

The total thrust can be written

$$T = T^u + T_1^l + T_2^l \quad (7)$$

and by considering separately the flow that goes only through the lower rotor and the rest of the flow, we obtain

$$T_2^l = (1 - m^l) \rho A v_2^l w \quad (8)$$

$$T^u + T_1^l = \rho A v^u w. \quad (9)$$

We also know that the conservation of momentum gives

$$v^u m^u = v_1^l m^l \quad (10)$$

and the total power can be written

$$P = \frac{1}{2} \rho A (v^u + (1 - m^l) v_2^l) w^2. \quad (11)$$

Equations (4) to (11) can be used to solve P and give

$$P = \frac{T^{3/2}}{C\sqrt{\rho A}} \quad (12)$$

with

$$C = 1 - m^l + \sqrt{1 + \frac{m^{l^2}}{m_u}}.$$

In the case with no vertical separation between both rotors, $m^u = m^l = m$ and

$$C = 1 - m + \sqrt{1 + m}.$$

m can also be determined geometrically given the distance between the rotors axes x and the rotors radius r :

$$m = \frac{2}{\pi} \left[\arccos\left(\frac{x}{r}\right) - \left(\frac{x}{r}\right) \sin\left(\arccos\left(\frac{x}{r}\right)\right) \right],$$

Finally, if $m = 0$, $C = 2$ and

$$P = \frac{T^{3/2}}{2\sqrt{\rho A}}$$

which corresponds to what we would find by considering two independent rotors generating each a thrust $T/2$. An estimation of the efficiency of the overlapping configuration is therefore given by the induced power factor $\kappa_{ov} = 2/C$. In general, other assumptions would be required to compute the ratios m^u and m^l .

3) Triangle pyramid analysis: The transition from the two overlapping rotors to the four rotors in a triangle pyramid configuration is done easily by changing eqs. (4) to (6) to

$$\frac{1}{4}P = T^u v^u \quad (13)$$

$$\frac{3m^l}{4}P = 3T_1^l v_1^l \quad (14)$$

$$\frac{3(1 - m^l)}{4}P = 3T_2^l v_2^l, \quad (15)$$

(7) to

$$T = T^u + 3(T_1^l + T_2^l), \quad (16)$$

(9) to

$$T^u + 3T_1^l = \rho A v^u w, \quad (17)$$

and (11) to

$$P = \frac{1}{2} \rho A (v^u + 3(1 - m^l)v_2^l) w^2. \quad (18)$$

In this case we find

$$P = \frac{T^{3/2}}{C\sqrt{\rho A}} \quad (19)$$

with

$$C = 3(1 - m^l) + \sqrt{1 + \frac{3m^{l^2}}{m_u}}.$$

and the induced power factor is

$$\kappa_{ov} = 4/C.$$

Fig. 8: Plot of the induced power factor (κ_{ov}) versus the normalized distance between rotors ($\frac{x}{d}$) = ($\frac{x}{2r}$) as estimated from momentum theory.

In the case with no vertical separation, κ_{ov} is plotted in Fig. 8.

It should be noted that the Rankine-Froude momentum theory analysis is a first level analysis of rotor performance. There are several assumptions made during this analysis such as, the rotor is assumed to be an infinitesimally thin actuator disk, the flow through the rotors are one-dimensional, quasi-steady, inviscid and incompressible.

III. Experimental results

A. Experimental setup

1) Thrust stand: In the performed experiments, a custom thrust stand is used to perform thrust measurements of different triangle pyramid configurations. The thrust stand consists in a adjustable frame holding the four rotor together in a rigid structure. The frame allows each of bottom propeller mounts to slide along a horizontal rail in order to choose their distance from the center. When the desired distance is obtained, the sliding mounts are locked in place by tightening the screws that maintain them of the rails, as seen in Fig. 9. The top rotor is fixed on a mount centered on the frame. The distance between the lower rotors and upper rotor can be adjusted by loosening two collars around the center motor's mount. The whole frame is shown on Fig. 9. It is made of T-slot extruded aluminum, 3D printed parts and an aluminum rod.

The frame is held by a horizontal piece of T-slot extruded aluminum that is attached to a second piece of of T-slot extruded aluminum via two load cells whose purpose is to measure the torque about the z -axis of the system. Lastly, the second piece of T-slot extruded aluminum is used to attached to the thrust stand to a base via two more load cells whose purpose is to measure the thrust of the system. The load cells used to measure the torque and thrust generated by the system have a capacity of 1 kg and 20 kg, respectively.

2) Propeller assembly: For the propellers, we choose the HQProp Ethix S5, just as we did for the Tetracopter prototype. They are powered by Flywoo NIN 2207

Fig. 9: The custom thrust stand.

2450KV motors and controlled by a via EMAX Formula Series 45A electronic speed controllers (ESCs).

B. Experiment description

1) Propeller arrangements: Thrust tests are performed for a set of propeller configurations with different spatial arrangements. These arrangements, at the number of 64, correspond to combinations of x and h with x ranging between 115 mm and 190 mm and h ranging between 15 mm and 295 mm.

2) Experimental procedure: For each configuration of the rotors a series of thrust, torque, current, voltage and angular velocity measurements are recorded at 11 different throttle inputs. The throttle is measured by the duty cycle of the pulse-wide modulation (PWM) 50 Hz signal sent to the ESCs, ranging from 0 % to 100 %. The nominal throttle values that are considered ranged from 10 % to 60 % in increments of 5 %. For a given nominal throttle value, all motors are initially set to that value. The downwash of the top propeller creates a unbalance in the torques produced by the top and bottom propellers, resulting in a small but non-negligible yawing moment.

C. Experimental Results

A sample of the results illustrating the significance of the experiments seen in Fig.10 to Fig.13. The full set of results can be found at <https://b.gatech.edu/3e9TMba>. In Fig.10 and Fig.11 show the relationship between thrust production and power consumption of the configuration at the maximum h and x value respectively. The data set labeled Isolated Rotors in Fig. 10 and Fig. 11 are the addition of each of the motors power consumption found by individually testing each rotor on the thrust stand.

To compare the experimental results to the momentum theory analysis of overlapping propellers Fig. 12 was developed by creating a polynomial fit of each configuration's 11 data points and evaluating the polynomial using an arbitrary thrust value of 1.4 kg. The summation

Fig. 10: Thrust vs. power at the maximum allowable h

Fig. 11: Thrust vs. power at the maximum allowable x

of each isolated rotors power consumption at a collective thrust value of 1.4 kg was found using the same method. To find the optimal configuration using the data devel-

Fig. 12: $\frac{x}{d}$ vs. power at a thrust value of 1.4 kg

oped from these experiments a fifth degree polynomial model was fit to the data as seen in Fig. 13. Using a nonlinear programming solver the optimal configuration was found to be $h : 295$ mm, $x : 190$ mm.

IV. Discussion

Although the optimal configuration suggested by the surface plot in Fig. 13 is trivial, the results shown in Fig. 12 suggest that power consumption increases as $\frac{x}{d}$ decreases, which agrees with the momentum theory analysis. However, the power consumption does not plateau as smoothly as momentum theory suggests. It can be seen in Fig 12 that the power required does not match that of the isolated rotors until $\frac{x}{d} \approx 1.2$. The

Fig. 13: Surface fit of h and x vs. power at 1.4 kg of thrust

power consumption as the position of the rotors move past the overlap limit is not as sensitive to a change in $\frac{x}{d}$, but the increase in power consumption is significant enough to impact the design of the Tetracopter due to its purpose of being an element of a larger self-similar assembly. Fig. 13 also shows that the power consumption of closely spaced non-overlapping rotor configurations, $1.00 < \frac{x}{d} < 1.2$, with $h > 135$ mm is higher than the closely spaced rotor configurations with smaller distances between the upper and lower rotors. This may be due to the expansion of the rotor's wake further down the rotor's slipstream, which could be impacting the inflow of the lower rotors when h is large. When h is small, the lower rotors operate in a plane where the upper rotor's wake is contracted, therefore not significantly impacting the inflow of the lower rotors. Fig. 10, Fig. 11 and Fig. 13 suggest that the change in x impacts the efficiency of the system more than a change in h . These results also suggest that some configurations perform better than four isolated rotors. The isolated rotor experiments were conducted in the configuration seen in Fig. 9, which may have lead to interference between the upper rotor and the thrust stand frame causing the results seen in Fig. 10 and Fig. 11. The experiments' results lead to the question of how the degradation of thrust propagates as the number of Tetracothers in an assembly grows. Another question left unanswered is the impact of the change in mass of the frame required to accommodate the configurations. If taken into account, the change in mass would make configurations with larger rotor separations unfavorable. These question are left to be answered by future research.

V. Conclusion

The novel Tetracopter configuration can be created out of several possible rotor configuration, some of which cause the lower three propellers to operate in the contracting wake of the upper rotor. As shown through the use of the Rankine-Froude momentum theory, the propellers operating in the wake of the upper rotor require more power to produce the same thrust as a rotor

operating in undisturbed air. Although the results of the experiments support the momentum theory analysis for configurations where the rotors are overlapped, these results also show that closely spaced non-overlapping rotors experience a degradation in thrust, which leads to an increase in power consumption. The results show that the most efficient Tetracopter rotor configuration does not have the lower rotors operating in the contracting wake of the upper rotor and should be sufficiently spaced to maintain efficiency. The next step of the optimizing of the Tetracopter is to show how the weight of the aircraft increases as the height(h) and the distance(x) between the upper and lower rotors changes. This will allow for a conservative thrust to weight ratio comparison of each Tetracopter configuration. The discussed research illustrates that a Tetracopter aircraft can have the benefit of rigidity and strength of a 3-dimensional tetrahedron shape while maintaining the efficiency of a rotorcraft with a co-planar rotor configuration.

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