Robust Wave-equation Surface-wave Skeletonized Inversion for Near-surface Environments

Jing Li^{1,2}, Zhaofa Zeng¹, and Gerard Schuster²

¹ College of geo-exploration Sci. &Tech, Jilin University, Changchun, Jilin 130026, China

² King Abdullah University of Science and Technology (KAUST)

Summary

A robust imaging technology is reviewed that provide subsurface information in near-surface environments: wave-equation skeletonized inversion of surface waves for the S-velocity model and quality factor (Qs) structure. We demonstrate the benefits and liabilities of the method with synthetic seismograms and field data. The benefit of skeletonized inversion method is that it has higher resolution than the conventional 1D method and mostly avoids getting stuck in local minima. The synthetic and field data examples demonstrate that it is a robust method so that the complex near-surface structure can be reliably obtained with seismic surveys over near-surface environment. It is easily extended to anisotropic media or rugged topography seismic survey. The liability is that it is almost as expensive as full-waveform inversion (FWI) and the resolution almost lower than FWI method.

Introduction

In near-surface seismic survey, how to obtain accuracy velocity model and quality factor (Q) are the challenge works in seismic data process. The conventional method falls into two categories: 1) the classical method of 1D dispersion curves (Evison et al., 1959; Park et al., 1998) for inverting S-velocity, the velocity analysis to estimate Qs structure; and 2) full waveform inversion (FWI) (Groos et al., 2014; Solano et al., 2014; Yuan et al., 2015) for complex 2D or 3D media. The classical 1D dispersion-curve method accurately inverts for a 1D Vs model but becomes less accurate with increasing lateral heterogeneity in the subsurface. The 1D assumption is not satisfied for some practical applications, so partial remedies are spatial interpolation of 1D velocity models (Xia et al., 1999) and laterally constrained inversion (Socco et al., 2010; Bergamo et al., 2012). In comparison, full waveform inversion (FWI) can theoretically account for any lateral heterogeneity, but it is computationally expensive and can easily get stuck in local minima in land seismic data (Tarantola, 1984). For Q factor estimation, after determining S-velocities by inverting Rayleigh-wave phase velocities (Xia et al., 1999), the dissipation factors (γ) and (η) can be inverted directly by the relationship between Rayleigh-wave attenuation coefficients and the quality factors (Q) for P and S waves of a layered model were given by Anderson et al. (1965).

The wave-equation skeletonized inversion is a robust inversion strategy for seismic near-surface survey. What is the meaning of skeletonized inversion? It only remains the key information of seismic data to invert the parameter structure. An example is shown in Figure 1, in FWI

iteration, the gradient method stops at the local minimum nearest to the starting model (the rightmost red circle in Figure 1b) and cannot proceed further unless special adjustments are made. One way to do this is to find the velocity model that minimizes the sum of the squared traveltimes as depicted in Figure 1c. Here, there is only one simple traveltime for each trace, rather than thousands of amplitudes that need to be explained by the velocity model. In this case, the velocity model will be much simpler than the actual velocity model because it only has to explain traveltimes. Therefore, the complicated waveform misfit function is replaced by the skeletonized data misfit function. Another simplification, i.e., skeletonization, of the traces, is the dispersion curves of surface wave as shown in Figure 1e. In this case, the waveform misfit function in Figure 1f is much simpler than that for the original data in Figure 1b. This strategy of simplifying the data (traveltime and/or dispersion curves or frequency shift) is defined as skeletonization or multi-scaling of the data (Luo and Schuster, 1991; Li et al, 2017a, 2017b).

Figure 1: Panels (a, c, and e) are synthetic data, traveltime data, and dispersion-curve data, respectively, and (b, d, and f) are the associated misfit functions plotted against the hypothetical velocity value of the first layer in the two-layer model. Notice the fewer local minima (red circles) (Illustration from Schuster, 2017).

In this abstract, we present the wave-equation skeletonized inversion of surface waves to obtain robust inversion result for S-velocity and Qs structure (Li et al., 2016, Li, et al.,

Surface-wave Skeletonized Inversion

2017). Both the synthetic and field data tests demonstrate that it is a robust way to obtain high-resolution tomogram in near-surface environment.

Wave-equation dispersion Vs inversion (WD)

We will introduce the theories for the wave-equation dispersion inversion (WD). The mathematical details for WD theory are presented in Li et al. (2017).

Radon and FFT transforms applied to CSGs:

The fundamental-mode Rayleigh waves are windowed in a common shot gather (CSG), and an FFT and Radon transform are applied to get the spectrum $D(c, \omega)$, where c is the phase velocity and ω is frequency. From this spectra the fundamental dispersion curve $C(\omega)$ is picked to get $\kappa(\omega) = \omega / C(\omega)$. We use the superscript *obs* if the data are recorded, otherwise it is predicted data computed by a finite-difference solution to the 2D elastic wave equation.

The misfit function is computed to get

$$\varepsilon = \frac{1}{2} \sum_{\omega} (\kappa(\omega) - \kappa(\omega)^{obs})^2, \quad (1)$$

where an extra summation is over shot indexes if there is more than one shot gather.

The gradient with respect to the shear-slowness model $s(x)$ is computed to get

$$\gamma(x) = \frac{\partial \varepsilon}{\partial s(x)} = \sum_{\omega} \Delta \kappa(\omega) \frac{\partial \kappa}{\partial s(x)}, \quad (2)$$

and the steepest descent formula is

$$s(x)^{k+1} = s(x)^k - \alpha \sum_{\omega} \Delta \kappa(\omega) \frac{\partial \kappa}{\partial s(x)}, \quad (3)$$

where the Fre'chet derivative is computed using solutions to the 2D elastic wave equation. If there is more than one shot gather then there is an extra summation over different indices in equation 3. Equation 3 is used to get the shear-velocity slowness model $s(x)^k$ until the data residual falls below some accepted criterion. This methodology is valid for both 2D and 3D velocity models and eliminates the layered medium assumption in traditional dispersion inversion methods.

WD Model Test

The WD method is tested on the S-velocity model shown in Figure 2a, where the starting velocity model for WD is depicted in Figure 2b. The input data were 100 shot gathers (2 m shot intervals) with 100 traces (2 m spacing) per shot gather, and each shot gather was transformed into the k - ω domain in order to pick the fundamental dispersion curve $\kappa(\omega)^{obs}$. Finite-difference solutions to the 2D elastic wave equation are computed to get the predicted dispersion

values $\kappa(\omega)$. Inverting all of these dispersion curves simultaneously (without assuming a 1D model) gives the 2D S-velocity tomogram in Figure 2c. It is obvious that this tomogram is an accurate rendering of a smoothed version of the actual model in Figure 2a.

Figure 2. a) Actual and b) starting S-velocity models, c) 2D WD tomogram, and d) 1D WD tomogram that assumes a 1D layered model beneath each shot point (Figures from Li et al. (2017)).

The WD method is also applied to near-surface field data recorded over the Qademah fault, which is a possible fault because it is characterized by a low-velocity zone that linearly trends for many kilometres. A COG is shown in Figure 3a with the source-receiver offset of 50 m. The dashed lines in Figure 3a indicate the location of the Qademah fault, which is consistent with the lateral velocity decrease in the P-wave velocity tomogram of Figure 3b. The P-wave velocity tomogram is computed by inverting the P-wave first-arrival traveltimes with a ray-based tomography method. Then, all shot gathers are transformed into the κ - ω domain by the application of the Fourier and

Surface-wave Skeletonized Inversion

linear Radon transforms, and the maximum energy values of the dispersion curves are picked. The 2D WD method does not assume a horizontal layered medium, and it is used to invert the picked dispersion curves to give the 2D S-wave velocity tomogram shown in Figure 3c.

Figure 3. Qademah COG and associated tomograms: (a) The COG (offset = 50 m), (b) P-wave velocity tomogram inverted from the first-arrival traveltimes, (c) S-wave velocity tomogram inverted from the 2D WD inversion of the dispersion-curves data (Li and Schuster, 2016).

Wave-equation frequency shift Q inversion (WQs)

Here, $Q(x)$ is the Rayleigh-wave attenuation parameter, and Δf is the residual between the predicted and observed frequencies of the spectral peaks (Dutta and Schuster, 2016a; Dutta, 2016b) of a fundamental Rayleigh wave (see Figure 4c). The spectral peaks are computed by windowing the computed (or observed) surface waves in the shot gather (see Figure 4a), applying a Fourier transform, and picking the peak frequencies of the spectra. Now the complicated surface-wave arrivals are skeletonized as simpler data, namely, the amplitude spectrum of the windowed fundamental-mode Rayleigh-wave arrivals. The centroid or peak frequency shifts between the observed and the predicted Rayleigh-wave arrivals are the skeletal residuals, and the attenuation model is sought that minimizes the sum of the squared frequency residuals

$$\varepsilon = \frac{1}{2} \sum_{s,g} (\Delta f(s, g))^2, \quad (4)$$

The optimal attenuation model is found by the steepest descent method:

$$\eta(x)^{k+1} = \eta(x)^k - \alpha \sum_{s,g} (\Delta f(s, g))^k \frac{\partial \Delta f(s, g)^k}{\partial \Delta \eta(x)}, \quad (5)$$

where $\eta(x)$ is the relaxation rate, and approximately

equals $1/Q(x)$ for realistic geologic models (Li et al., 2017b). The connective function is defined as the derivative of the cross-correlation between the predicted and the observed spectral peak frequencies of the fundamental Rayleigh-wave arrivals

$$\Phi(Q(x), \gamma_{\Delta f}) = \int df D(s, g, f) \mathcal{B}(s, g, f + \nabla f)_{obs}^* = 0, \quad (6)$$

where $\gamma_{\Delta f}$ represents the frequency lag in the correlation,

$D(s, g, f)$ and $D(s, g, f + \Delta f)_{obs}$ are the predicted and the observed spectral peak frequencies of the fundamental Rayleigh wave at the specified receiver g and source s locations, and

$$\mathcal{B}(s, g, f + \Delta f)_{obs}^* = \partial D(s, g, f + \Delta f)_{obs}^* / \partial f. \quad (7)$$

If $\gamma_{\Delta f} = \Delta f$, which means the frequency shift Δf aligns the predicted and observed spectra with one another, the cross-correlation achieves the maximum value, and its frequency derivative becomes zero. Solutions to the viscoelastic wave equation are used to compute the predicted Rayleigh-wave arrivals and the gradient at every iteration (Li et al., 2017b).

Figure 4. (a) A common-shot gather comparing the fundamental-mode Rayleigh-wave arrivals with and without Q. The blue dashed line indicates the window used to extract these arrivals for WQ, (b) comparison between a predicted and an observed surface-wave arrival, and (c) comparison between their amplitude spectra.

A numerical example with a complex near-surface model is illustrated in Figure 5. The true Vs and Q models are shown in Figure 5a and 5b, respectively. The smoothed true S-wave velocity model shown in Figure 5c is used as the background velocity. The observed data are generated by 40 evenly distributed shots and 70 receivers at intervals of 2 m on the surface. The Q tomogram after 21 iterations is shown in Figure 5d, which roughly agrees with the true Q model in Figure 5b.

The proposed method is now tested on the same field data used in the WD section. The starting Q model is

Surface-wave Skeletonized Inversion

homogeneous with $Q = 1000$, and the inverted Q tomogram is shown in Figure 6a. A geologic relation between the S-wave velocity and the Q tomograms says that the high attenuation regions in the Q tomogram (low Q values) correspond to the low S-wave velocity regions (De Meersman, 2013; Zhu and Harris, 2015). Previous work by Zhang et al. (2015) demonstrates that areas with high V_p/V_s ratios tend to have low Q values (or high attenuation), which mostly agrees with what can be observed in Figure 6a. To check the feasibility of the inverted Q tomogram, it is compared with a COG profile (Hanafy et al., 2015). Figure 6c shows a COG profile using the processed data for an offset of 50 m. The black dashed line in this figure shows the location of the fault, which is between 250 and 300 m. The locations of the Q anomalies in the Q tomogram and the low-velocity area in the S-wave tomogram are consistent with the location of the fault, as delineated in the COG profile.

Figure 5. (a) True velocity, (b) Q models used for generating the observed data, (c) velocity model used for WQ, and (d) inverted Q tomogram from WQ (Figures from Li et al. (2017b)).

Figure 6. (a) WQ Q_s tomogram, (b) the ratio (V_p/V_s) of the seismic field data (Figures from Li et al. (2017b)).

Conclusions

We presented an overview of wave-equation surface wave skeletonized inversion for V_s and Q_s . The skeletal form a simplified objective function that largely is devoid of local

minima and complexity. This results in an iterative gradient optimization method that can converge to the proximity of the global minimum. The benefit is that there is robust convergence to the vicinity of the actual model without, typically, getting stuck in a local minima. The liability is that the resulting tomogram is only of moderate because of the reduced complexity of the skeletal data. This model with moderate resolution can be used as a good starting model for full-wave inversion to give more detailed estimates of the model. The S-velocity and Q_s tomogram can be used to estimate velocity statics for multicomponent data and assess locations of near-surface faults for drilling hazard assessment.

Acknowledgements

We thank the financial support from the sponsors of the Consortium of Subsurface Imaging and Fluid Modeling (CSIM). We thank KAUST for funding this research. This work is supported by the Natural Science Foundation of China (41874134) and the China Postdoctoral Science Foundation 2106T902503 and 2015M571366.