A Novel Carriers Selection Scheme for OFDM with Index Modulation

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Abstract—In this paper, we investigate the subcarriers combination selection for OFDM-IM system, which fully utilize transfer rate and diversity gain characteristics of OFDM-IM system based on the lexicographic ordering principle. Aiming at the two optimization problems, we give the corresponding algorithms. Finally, we verify the low complexity and high efficiency of the two algorithms.

Keywords—OFDM-IM; selection; optimization; algorithm

I. INTRODUCTION

Communication systems with ultra-low latency, faster speeds, greater throughput, and higher spectral efficiency have become challenging targets for researchers. Index modulation (IM) is a derivative form of spatial modulation (SM) in MIMO systems [1-2], which has great attraction in spectrum efficiency and energy efficiency. Literatures [3-4] proposed the combination of IM and OFDM transmission technology, called OFDM-IM. And the system will achieve higher reliability or transmission rate than the traditional OFDM system under certain conditions [5-6]. As a kind of modulation scheme, how to encode the input bits into the modulation symbol and the index of subcarriers becomes the key problem to improve the superiority of the system. M. Wen proposed that the system could generate a certain coding gain by using an equiprobable subcarrier activation (ESA) scheme [7], but without any diversity gain. S. Dang proposed two mapping selection schemes based on on-off keying (OOK) to provide more flexible mapping relations and frequency diversity gain [8-9]. However, the varying

number of activated subcarriers increases the difficulty of receiver detection. In [10], another diversity scheme based on source coding and redundancy is presented, but it leads to spectral efficiency loss. In [11], a novel codebook design scheme for OFDM-IM is proposed, which is proved to be simpler without increasing the block error rate (BLER). However, the scheme lacks a complete subcarriers combination selection algorithm and ignores the power allocation problem. In order to solve the above problems, we propose a robust algorithm and power allocation modification to further improve the system performance.

II. SYSTEM MODEL

The most significant difference between OFDM-IM and classical OFDM is that the OFDM-IM system modulates constellation and subcarriers simultaneously. It is an effective way to improve the coding gain and expand the modulation resource of the space domain. The system first sorts the subcarriers based on lexicographic order of performance, and then selects the subcarriers participating in constellation modulation according to index bits '0'(silent) - '1'(active).

Figure 1. OFDM-IM system's transmitter design.

A. Signal Transmission Model

As shown in Figure 1, the transmitter contains N subcarriers, corresponding to N points IFFT. Assuming that m bits are input for each frame, including P_i index bits and P_m constellation modulation bits. The first P_i bits are input as the activation vector to determine the number and combination of activated subcarriers based on lexicographic order. And then the remaining P_m bits are modulated into constellation symbols by the activated subcarriers and input to the next level. Without loss of generality, we represent the input of the system as $\mathbf{x}(\mathbf{c}, \mathbf{b})$.

$\mathbf{x}(\mathbf{c}, \mathbf{b}) = [x(m_1, 1), x(m_2, 2), \dots, x(m_N, N)]^T$, (1) where $(\cdot)^T$ denotes the matrix transpose operation, and the element of the block $\mathbf{x}(\mathbf{c}, \mathbf{b})$ is given by

$$x(m_{n_c}, n_c) = \begin{cases} \chi_{m_{n_c}}, & v(n_c) = 1 \\ 0, & v(n_c) = 0 \end{cases} \quad (2)$$

where $v(n_c)$ is either 0 or 1 representing whether the state of the n_c -th ordered subcarrier is inactive or active, respectively, and $\chi_{m_{n_c}}$ is the normalized M -ary constellation symbol modulated according to the input bits when M -ary phase shift keying (M -PSK) is used. For the M -order constellation modulation, the transmission rate B can be expressed as $B = \lfloor \log_2 \binom{N}{K} \rfloor + K \log_2 M$, where $(\cdot)$ is the binomial coefficient and K represents the number of activated subcarriers. It can prove that B is a non monotonic function. After the index mapping and constellation mapping, the OFDM-IM system converts the data into parallel series, transforms the frequency-domain data into time-domain data through N -points IFFT, and then adds CP to complete the baseband data processing. After sampling, discarding CP and performing N -point FFT, we can express the received OFDM block $\mathbf{y}(\mathbf{c}, \mathbf{b})$ as follows

$$\mathbf{y}(\mathbf{c}, \mathbf{b}) = \sqrt{\frac{P_t}{K}} \mathbf{H} \mathbf{x}(\mathbf{c}, \mathbf{b}) + \mathbf{w} \quad (3)$$

where P_t is the total transmission power; $\mathbf{w}$ denotes N independently combined additive white Gaussian noise (AWGN) with N_0 noise power density sampled vector on each subcarrier; $\mathbf{H}$ is an $N \times N$ diagonal channel state matrix characterizing the channel quality.

At the receiver, due to the slow fading channel model adopted in this paper, it is assumed that the optimized codebook and CSI, namely $\mathbf{S}(\mathbf{c})$ and $\mathbf{H}$, are known. Due to this prior information, the maximum-likelihood (ML) detection method can be used to perform block detection by the following criteria:

$$\hat{\mathbf{x}}(\mathbf{c}, \hat{\mathbf{b}}) = \arg \min_{\mathbf{x}(\mathbf{c}, \mathbf{b}) \in \mathcal{X}(\mathbf{c})} \left\| \hat{\mathbf{y}}(\mathbf{c}, \hat{\mathbf{b}}) - \sqrt{\frac{P_t}{K}} \mathbf{H} \mathbf{x}(\mathbf{c}, \mathbf{b}) \right\|_F. \quad (4)$$

where $\hat{\mathbf{x}}(\mathbf{c}, \hat{\mathbf{b}})$ represents the estimated input signal after ML detection, and $\hat{\mathbf{y}}(\mathbf{c}, \hat{\mathbf{b}})$ is one of all possible output signals.

Obviously, the subcarriers combination number S must be the power of 2, that is $S = 2^{\lfloor \log_2 \binom{N}{K} \rfloor}$. However, mathematically there are $L = \binom{N}{K}$ combinations of subcarriers in total. In other words, $\Delta(N, K)$ combinations need to be filtered, where $\Delta(N, K) = L - S$. It leads to the

problem of how to select subcarriers from so many possibilities. In order to quantify the quality of the selected scheme, we introduce the concept of the diversity gain d_0 to OFDM-IM system:

$$d_o = \begin{cases} 1, & \Delta(N, K) < \Omega(N, K, 1) \\ \nu, & \Omega(N, K, \nu - 1) \leq \Delta(N, K) < \Omega(N, K, \nu), \end{cases} \quad (5)$$

where $\Omega(N, K, \nu) = \sum_{\xi=1}^{\nu} \binom{N-K}{\xi}$ represents the sum of the number of combinations out of L with ones in the ν most significant bits.

For the perspective of the system model, the diversity gain represents the system's ability to screen high-quality subcarriers, but the relationship between the diversity gain and the total number of subcarriers and the number of active subcarriers are non-monotonic and highly complex. And the transmission rate of the OFDM-IM no longer simply varies with N monotonically. So we put forward two optimization problems, Maximize Diversity Gain, Maximize Transmission Rate.

B. Maximize Diversity Gain (MaxD)

In particular, for scenarios that only need to meet the minimum transmission rate but have higher requirements on the quality of the subcarriers, our goal is to maximize the diversity gain. We can express this optimization problem mathematically as follows:

$$[N, K] = \arg \max \{d_o\}, \quad (6)$$

$$s.t. \quad 2 \leq N \leq N_{max}, 1 \leq K \leq N - 1,$$

where N_{max} is the maximum subcarriers number. The problem must meet the requirements of $B > B_{min}$, where B_{min} respectively represent the basic transmission rate requirements.

C. Maximize Transmission Rate (MaxB)

Similarly, for scenarios that only need to meet the minimum diversity gain but have higher requirements on the transmission efficiency, our goal is to the transmission rate of OFDM-IM system. We can express this optimization problem mathematically as follows:

$$[N, K] = \arg \max \{U = d_o^w B^{1-w}\}, \quad (7)$$

$$s.t. \quad 2 \leq N \leq N_{max}, 1 \leq K \leq N - 1,$$

which must meet the requirements of $d_0 > d_{min}$, where d_{min} represent the essential diversity gain requirements.

III. ALGORITHM FOR OPTIMIZATION

Obviously, the exhaustive search algorithm can help to solve an NP hard problem, but the computational complexity is extremely high so as to make it not suited for practical applications. Therefore, Shuping Dang's team introduced genetic algorithm (GA) to effectively reduce the search space [25], which had important instructive. However, parameters determination is a challenging problem, which can easily lead to premature convergence or even no convergence even if the cost is huge. We will explore and propose two specific algorithms, reducing complexity and ensuring the robustness.

A. Algorithm Solving Problem MaxD

On the assumption that $d_0 = v$, the condition given by definition (5) can be equivalent to an expression as follows:

$$\binom{N-v}{K} < 2^{\lfloor \log_2 \left(\frac{N}{K} \right) \rfloor} \leq \binom{N-v+1}{K}, \quad (8)$$

where it inspires us that the solution to the problem lies in finding the largest positive integer satisfying the left half. According to the property of combinatorial number, for the OFDM-IM system, we have:

$$\lfloor \log_2 \left(\frac{T_r}{K_r} \right) \rfloor > \log_2 \left(\frac{T_r}{K_r} \right) - 1, \quad (9)$$

$$\lfloor \log_2 \left(\frac{T_r}{K_r} \right) \rfloor \leq \log_2 \left(\frac{T_r}{K_r} \right). \quad (10)$$

The above equation facilitates us to obtain further the upper bound of the diversity gain of the OFDM-IM system, as follows:

$$\text{Sup}\{d_o\} = \arg \max_{[N,K]} \left\{ \frac{N-K+1}{N-d_o+1} \geq 2 \right\}. \quad (11)$$

So far, through the calculation and analysis in this section, we can get that the optimal combination of subblocks $[N,K]$. Now, the upper bound of the diversity gain $\text{Sup}\{d_0\}$ can be finally transformed into

$$\text{Sup}\{d_o\} = \lceil \frac{N}{K+1} \rceil. \quad (12)$$

Besides, we need to add the conclusion that according to the physical meaning of the diversity gain, when K is fixed, within a certain range, the diversity gain can be obtained quickly, such as

$$d_o(N, K) = d_o(N_o, K) - (N_o - N), \quad \forall N \in [N_o - d_o(N_o, K) + 1, N_o]. \quad (13)$$

Combined with the analysis of this section, we design the optimization process in **Algorithm 1**. As can be seen from Algorithm 1, no matter the size of the search space, the computational complexity can be continuously reduced through a small number of iterations and the dynamic restricted bounds.

B. Algorithm Solving Problem MaxB

The transmission rate is related to N and K , but is not positively related. For a fixed N , B increases monotonously under the premise of K that is not larger than $\lfloor \frac{N-1}{2} \rfloor$. In consideration of the constraint d_{min} , we can obtain the range of K as follows:

$$\text{Sup}\{K\} = \lfloor N_{max}/(1 + d_{min}) \rfloor - 1. \quad (14)$$

In other words, we just need to find the first combination $[N,K]$ that satisfies the constraint d_{min} condition. We design the process of the optimization MaxB in **Algorithm 2**. So far,

we have finally completed the algorithm design for two kinds of optimization problems.

IV. PERFORMANCE ANALYSIS

We focus on analyzing the performance of this two algorithms.

A. Computational Complexity

For **Algorithm 1**, according to the conclusion of the previous chapter, we take the constant complexity, $O(1)$, to get the K first satisfying basic requirement of the transmission rate, and then get the best combination of candidates through a finite number of recurrences and a little multiplication. Obviously, the algorithm 1 has the advantages of very little search space and significantly fewer multiplications and comparisons. In other words, the total computational complexity of the algorithm 1 is constant, $O(1)$, independent of parameters, including the total number of subcarriers N_{max} , modulation order M , the constraints B_{min} , and the preset maximum diversity gain d_{max} .

The Algorithm 2 adopts the backward searching. Based on relevant corollary, Algorithm 2 gives the upper bound of subcarrier number K that may activate $\text{Sup}\{K\}$, and then compute the lower bound of subblock subcarriers number for corresponding K and then quickly provides the maximum diversity gain corresponding to each K by non-ergodic optimization and constant time. Therefore, the time complexity of Algorithm 2 is also constant.

Algorithm 1 Proposed Algorithm for solving optimization MaxD for OFDM-IM systems applying the lexicographic codebook design scheme.

Input: The total number of subcarriers N_{max} , modulation order M , the constraints B_{min} , and the preset maximum diversity gain v_{max} .

Output: $i_{opt} = [N, K]$

```

1:  $\text{Sup}\{K\} \leftarrow \lfloor N_{max}/(1 + v_{max}) \rfloor - 1$ 
2: if  $B(N, K, M) < B_{min}$  and  $K \leq \text{Sup}\{K\}$  then
3:    $K \leftarrow K + 1$ 
4: end if
5:  $K_{min} \leftarrow K$ 
6: if  $K \leq \text{Sup}\{K\}$  then
7:   if  $N \geq (K + 1)(v_{max} + 1)$  and  $B \geq B_{min}$  then
8:      $v \leftarrow d_o(N, K)$ 
9:     update  $v_{max}$ 
10:    update  $i_{opt}$ 
11:     $N \leftarrow N - v$ 
12:   end if
13: else
14:    $K \leftarrow K + 1$ 
15:    $N \leftarrow N_{max}$ 
16:    $\text{Sup}\{K\} \leftarrow K_{min} + 1$ 
17: end if
18: return  $i_{opt} = [N, K]$ 

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Algorithm 2 Proposed Algorithm for solving optimization MaxB for OFDM-IM systems applying the lexicographic codebook design scheme.

Input: The total number of subcarriers N_{max} , modulation order M , constraint d_{min} , and the preset maximum diversity gain ν_{max} .

Output: $i_{opt} = [N, K]$

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1:  $Sup\{K\} \leftarrow \lfloor N/(1 + \nu_{max}) \rfloor - 1$ 
2:  $K \leftarrow Sup\{K\}$ 
3: for  $K \leq Sup\{K\}$  do
4:    $N \leftarrow N_{max}$ 
5:    $\nu_{max} \leftarrow constraints\ d_{min}$ 
6:   while  $N \geq (K + 1)(\nu_{max} + 1)$  do
7:      $\nu \leftarrow d_o(N, K)$ 
8:     if  $\nu > constraints\ d_{min}$  then
9:        $update\ i_{opt}$ 
10:       $update\ B_{max}$ 
11:       $K \leftarrow Sup\{K\} + 1$ 
12:       $\nu_{max} \leftarrow +\infty$ 
13:    else
14:       $N \leftarrow N - \nu$ 
15:       $B \leftarrow B - 1$ 
16:       $update\ \nu_{max}$ 
17:    end if
18:  end while
19: end for
20: return  $i_{opt} = [N, K]$ 

```

For the GA-based algorithm introduced in [8], the computational complexity of is $O(G_e V_p R_o)$, where is G_e the termination evolution algebra corresponding to the genetic algorithm; V_p is the population size; and R_o is the number of cycles to ensure robustness. However, V_p is related to the size of N and G_e is comparable to N in fact. The most critical problem is that the current GA-based algorithm cannot guarantee the ideal global optimization rate. In contrast, the algorithms we provide are more efficient in computation.

B. BLER Performance

The proposed algorithms give the optimal convergence results but do not change the system architecture based on the lexicographic codebook, so the block error rate (BLER) can be no less than other schemes [8-10]. The analytical formulae shows in (15),

$$P_e(\hat{\mathbf{x}}(c, \hat{\mathbf{b}}) \rightarrow \hat{\mathbf{x}}(c, \hat{\mathbf{b}})) \leq \sum_{i=1}^2 \sum_{\hat{\mathbf{x}}(c, \hat{\mathbf{b}}) \neq \hat{\mathbf{x}}(c, \hat{\mathbf{b}})} \prod_{m=1}^N \epsilon \frac{N!}{(N-m)!} \frac{\alpha_{i,m}^{\alpha_{i,m}-\frac{1}{2}}}{\beta_{i,m}^{\beta_{i,m}-\frac{1}{2}}} \exp(\beta_{i,m} - \alpha_{i,m}). \quad (15)$$

where $\epsilon = 1.02$, $\lambda = \{\frac{1}{12}, \frac{1}{4}\}$, $\eta = \{\frac{1}{2}, \frac{2}{3}\}$, $\delta(m)$ is equal to $|\hat{x}(\hat{m}) - \hat{x}(\hat{m})|^2$ and $\beta_{i,m}$ is determined by following equation:

$$\beta_{i,m} = N + 1 + \eta_i \frac{\mu P_t}{NK} \delta(m). \quad (16)$$

And $\alpha_{i,m}$ is different from equation(27), but it satisfies this equation: $\alpha_{i,m} = \beta_{i,m} - m$.

V. NUMERICAL RESULTS

Considering the global optimization rate and complexity, we improved the genetic algorithms given in literature to accelerate convergence and reduce the probability of falling into local optimization. The GA algorithm introduces simulation binary to execute chromosome coding and crossing, and adopts the optimal saving strategy. We set the limited numbers of subcarriers $N_{max} \in [16, 240]$, the order $M \in [2, 4]$, $P_c = 0.90$, $P_m = 0.01$, $G_e = 100$, $V_p = 10$, $R_o = 1$ and $N_s = 10^4$. And the results are shown in Table I and Table II.

In the tables, NCGA is the average number of calculations by GA, and GORGA represents global optimization rate by GA, NCA1 is the number of computations by the algorithm 1, and NCA2 is the number of computations by the algorithm 2.

Compared with the listed data, it can be seen that the complexity of the algorithm 1 and the algorithm 1 is far lower than the complexity of the improved GA-based algorithm. And the computational complexity of algorithms we proposed is almost independent of factors of the total number of subcarriers N_{max} , modulation order M , the constraints B_{min} , and the preset maximum diversity gain d_{max} . At the same time, we can ensure the optimal results. For the original algorithm, the complexity is independent of N_{max} , but the results are disadvantageous, and the search space is enlarged even when N_{max} is small. It can also be seen from the comparison that the global optimization rate of GA-based algorithm is unstable, and when ω is small, the global optimization rate is significantly reduced. According to the statistical idea, in order to improve the global optimization rate, the number of original algorithm R_o can be set. If the non-global optimization rate is reduced to 10^{-4} , the number of cycles R_o needs at least 41 times, which means that the calculation cost rises sharply.

TABLE 1: Average solution performance vs. 2 algorithms with different N_{max} , M and B_{min}

	$M=2, B_{min}=5$			$M=2, B_{min}=10$			$M=4, B_{min}=5$			$M=4, B_{min}=10$		
N	NCGA	GORGA	NCA1	NCGA	GORGA	NCA1	NCGA	GORGA	NCA1	NCGA	GORGA	NCA1
16	6030	0.99	2	6032	0.99	2	6036	0.99	2	6031	0.99	2
32	6322	0.90	4	6261	0.53	5	6165	0.69	4	6232	0.60	5
48	6401	0.59	5	6318	0.78	6	6340	0.72	5	6393	0.62	6
96	6810	0.63	7	6508	0.43	5	6573	0.53	6	7021	0.53	7
112	7095	0.52	10	6603	0.60	10	6567	0.60	9	7162	0.61	10
128	7210	0.46	11	6577	0.66	9	6772	0.56	11	6977	0.52	10
144	7071	0.34	10	6682	0.64	11	6689	0.60	10	6883	0.62	9
160	7215	0.27	12	6723	0.30	12	6721	0.42	12	6926	0.42	11
176	7151	0.37	11	6701	0.36	13	6980	0.33	11	6928	0.40	10
192	7041	0.22	12	6743	0.31	12	7078	0.30	12	7077	0.31	13
208	7483	0.25	13	6872	0.27	15	6946	0.37	12	6954	0.37	12
224	7445	0.24	15	6949	0.47	15	6742	0.45	15	6864	0.41	15
240	7309	0.13	14	6902	0.51	16	6746	0.47	14	7204	0.43	13

TABLE 2: Average solution performance vs. 2 algorithms with different N_{max} , M and d_{min}

$\bar{N}$	$M=2, d_{min}=1$			$M=2, d_{min}=2$			$M=4, d_{min}=1$			$M=4, d_{min}=2$		
	NCGA	GORG	NCA2	NCGA	GORG	NCA2	NCGA	GORG	NCA2	NCGA	GORG	NCA2
16	6030	0.99	2	6023	0.99	2	6803	0.99	2	6003	0.99	2
32	6182	0.9	4	6146	0.83	5	6916	0.86	4	6126	0.86	5
48	6304	0.81	5	6535	0.81	5	6392	0.71	5	6354	0.82	5
96	6547	0.73	7	6495	0.63	5	6469	0.53	7	6557	0.73	5
112	6536	0.72	7	6759	0.66	7	6575	0.6	7	6586	0.76	7
128	6727	0.76	8	6447	0.76	9	6786	0.58	8	6797	0.72	9
144	6698	0.74	7	6690	0.64	10	6754	0.62	7	6768	0.66	10
160	6702	0.67	10	6987	0.53	11	6791	0.41	10	6872	0.62	11
176	6928	0.57	11	6987	0.56	11	6998	0.33	11	6998	0.53	11
192	7007	0.42	10	6977	0.61	12	6901	0.32	10	7037	0.51	12
208	6994	0.52	9	6895	0.57	13	6885	0.4	9	6984	0.47	13
224	7014	0.46	11	6940	0.47	13	6849	0.45	11	6994	0.45	13
240	6984	0.56	13	6798	0.41	15	6924	0.46	13	6894	0.47	15

VI. CONCLUSION

In combination with the transmission rate and diversity gain characteristics of the system OFDM-IM based on lexicographic codebook, two corresponding optimization algorithms are proposed, which ensure the global optimization rate and the satisfactory computational complexity, and solve the problems such as the difficulty in parameter setting, high complexity and unsatisfactory results based on genetic algorithm. Finally, the reliability and efficiency of the algorithm are verified by numerical simulation. In the future, adaptive segmentation algorithm, energy allocation algorithm and low complexity detection algorithm are very promising.

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