A reflection-based efficient wavefield inversion to retrieve a good initial background velocity

Chao Song and Tariq Alkhalifah, King Abdullah University of Science and Technology.

SUMMARY

Full-waveform inversion (FWI) requires either a good initial velocity model or low-frequency data to mitigate the cycle-skipping issue. Reflection-waveform inversion (RWI) uses a migration/demigration process to retrieve a background model that can be used as a good initial velocity in FWI. The drawback of the conventional RWI is that it requires the use of a least-squares migration, which is often computationally expensive. Efficient wavefield inversion (EWI) is a recently developed method from wavefield reconstruction inversion (WRI). EWI uses a modified source function to introduce the multiscattering components in the reconstructed wavefield. However, when the initial velocity is far from the true one, the wavefield can not be accurately reconstructed. We incorporate the RWI formulation into EWI by inverting for the Born scattered wavefield instead of the wavefield itself. In this case, we use the perturbation related to secondary sources as a modified source function. As the sources in the reflection-based EWI (REWI) are located in the subsurface, we are able to update the background model along the reflection wave path. We calculate the background model perturbation by a deconvolution process in each frequency like what is done in the original EWI. We demonstrate the validity of the proposed approach using synthetic data generated for the Marmousi model.

INTRODUCTION

Full-waveform inversion (FWI) has shown great potential in retrieving high-resolution velocity models of the Earth (Tarantola, 1984). However, FWI has specific requirements on the initial velocity and recorded data for convergence. It, specifically, suffers from the cycle-skipping problem if the data lacks low-frequency components or the available model for starting FWI is poor. Reflection-waveform inversion (RWI) has recently become a popular method for background model retrieval, as it is capable of updating the long wavelength components of the model along reflection wave paths (Xu et al., 2012; Zhou et al., 2012). However, it is often expensive mainly because we need to implement a least-squares reverse time migration (LSRTM) to obtain a true-amplitude image at each iteration. To improve the computational efficiency, a new objective function was proposed to separate the data into the scattering and background components (Alkhalifah and Wu, 2016b). A simultaneous inversion for the background model and the perturbation model using this new objective function was proposed by Wu and Alkhalifah (2015). This new objective function is easily adaptable to elastic media (Guo and Alkhalifah, 2017; Li et al., 2019).

Another way to mitigate the cycle-skipping issue is to formulate more robust objective functions for waveform inversion. Wavefield reconstruction inversion (WRI) relaxes the

wave equation accuracy by using it as a regularization term. By doing so, the data fitting is enhanced to avoid cycle skipping (Leeuwen and Herrmann, 2013). To reduce the computational cost, an efficient wavefield inversion (EWI) was proposed (Alkhalifah and Song, 2019). In EWI, they keep the modeling operator stationary in each frequency by using a modified source function as an independent parameter. The inner loop between the wavefield reconstruction and the modified source function update injects multiscattering components in the reconstructed wavefield. Using a deconvolution process, the velocity perturbation can be calculated directly in one step. This method proves to be effective in complex media, like acoustic VTI media and isotropic elastic media (Song et al., 2019a; Song and Alkhalifah, 2020a). WRI or EWI is able to provide both low and high wavenumber updates (Leeuwen and Herrmann, 2015; Song and Alkhalifah, 2020b). However, they are not totally immune from cycle skipping when the initial velocity is far away from the true one, regardless on how much we impose data fitting in the objective function (Alkhalifah and Song, 2019). Thus, we develop a new method to pursue a good background model to resolve this issue. We extend the original EWI by separating the reconstructed wavefield into a background wavefield and a scattered wavefield, utilizing the form of Alkhalifah and Wu (2016b) that naturally combines FWI and RWI. In this case, two regularization terms corresponding to the background wave equation and the Born scattered wave equation are used.

Thus, in this abstract, we develop a method to estimate the perturbation model and invert for the background model using a reflection-based efficient wavefield inversion (REWI). This method doesn't require an expensive LSRTM process within each iteration of updating the background model. The perturbation acts as secondary sources generating the scattered wavefield, and it can be easily estimated by reconstructing the scattered wavefield. Even though the estimated perturbation may deviate from its true depth in the early stages of the background model inversion, REWI is still able to match the reflection data by relaxing the requirement for the scattered wavefield to satisfy the Born scattered wave equation. Similar to the original EWI, we are able to calculate the background model perturbation directly. After obtaining a reasonable background model, a sequential EWI will inject detailed information to build a high-resolution model. We test the proposed approach on synthetic data that lacks frequencies below 5 Hz generated from the Marmousi model.

THEORY

Efficient wavefield inversion

We simulate wave propagation using a frequency domain acoustic wave equation with a constant density, given by:

$$\frac{\omega^2}{v^2}u(\mathbf{x}, \omega) + \nabla^2 u(\mathbf{x}, \omega) = f(\mathbf{x}, \omega), \quad (1)$$

Reflection-based efficient wavefield inversion

where ω represents the angular frequency, and $u(\mathbf{x}, \omega)$ denotes the pressure wavefield. The source function $f(\mathbf{x}, \omega)$ is given in the frequency domain, and v denotes the velocity. Equation 1 can be expressed in compact form as: $L(m)u(\mathbf{x}, \omega) = f(\mathbf{x}_s, \omega)$. $L(m) = \omega^2 m + \nabla^2$ is the impedance matrix, which is also referred to as the modeling operator. Here, we use m to represent the squared slowness. Conventional FWI is a highly non-linear optimization problem, as the wavefield u depends nonlinearly on the model parameters. EWI was proposed to solve the problems of conventional FWI, and its objective function is stated as:

$$E(u_i, f_{ei}) = \min \frac{1}{2} \sum_i \|d_i - Cu_i\|_2^2 + \frac{\alpha^2}{2} \|L_0 u_i - f_{ei}\|_2^2, \quad (2)$$

where i is the source index, and d_i denotes the recorded data. C is a mapping operator extracting the wavefield at the sensor locations. α^2 is a weighting factor used to balance the data fitting with the need to satisfy the background wave equation. At each frequency, EWI uses a background modeling operator $L_0(m_0) = \omega^2 m_0 + \nabla^2$ corresponding to the background squared slowness: m_0 . The modified source function f_{ei} starts with the true source function f_i . We solve a linear equation for the wavefield, u , to simultaneously satisfy the data fitting objective and the wave equation for the wavefield, u , as follows:

$$\begin{pmatrix} \alpha L_0 \\ C \end{pmatrix} u_i = \begin{pmatrix} \alpha f_{ei} \\ d_i \end{pmatrix}. \quad (3)$$

The modified source function can be evaluated using:

$$f_{ei} = L_0 u_i \quad (4)$$

With the help of perturbation theory, the squared slowness m can be split into a background part m_0 and a perturbation part dm , so the wave equation is given by:

$$Lu_i = f_i \rightarrow (L_0 + \omega^2 dm)u_i = f_i \rightarrow L_0 u_i = f_{ei} \rightarrow f_{ei} = f_i - \omega^2 dm u_i. \quad (5)$$

This equation shows that the modified source function f_{ei} is made up of the true source function f_i and the perturbation component. As a result, inner iterations between equations 3 and 4 will add multiscattering components to the reconstructed wavefield. After obtaining a well reconstructed wavefield u_i and a modified source function f_{ei} , the squared slowness perturbation can be calculated using the following division:

$$dm = \sum_i \frac{(f_i - f_{ei})u_i^*}{\omega^2 u_i u_i^* + \lambda}, \quad (6)$$

where λ is a small positive real number to avoid dividing over zero. As the squared slowness includes multiscattering components, the convergence rate in inverting each parameter accelerates (Alkhalifah and Wu, 2016a; Song et al., 2019b). However, when the perturbation is very large and the low frequency components in the data are missing, even EWI is not powerful enough to completely overcome the cycle-skipping issue. That is because when the initial velocity is far from the true velocity, the wavefield can not be accurately reconstructed. In this case, the derivation in equation 5 breaks down.

Reflection-waveform inversion (RWI) provides smooth updates of the background velocity which requires utilizing the reflectivity (Yao and Wu, 2017). Alkhalifah and Wu (2016b) use an

objective function to invert for the perturbation and the background model. This new objective function separates the total wavefield u into the background wavefield u_0 and the scattered wavefield δu , given by:

$$E(\delta m_0, \delta m)_{RWI} = \min \frac{1}{2} \sum_i \|Cu_{0i} + C\delta u_i - d_i\|_2^2 \quad (7)$$

such that : $L_0 u_{0i} = f_i, L_0 \delta u_i = s_i$,

where $s_i = -\omega^2 \delta m u_0$ represents secondary sources in the subsurface courtesy of the Born approximation, with δm representing the reflectivity-like perturbations, and δm_0 is the background model perturbations between the desired good background model and the available poor initial models. With the help of this new objective function, we effectively match the diving waves generated from the true sources and reflections generated from the perturbations, simultaneously. To incorporate this idea in the EWI framework, we also extend the reconstructed wavefield into two parts: the background wavefield u_0 and the scattered wavefield δu . In this case, we include two regularization terms corresponding to u_0 and δu satisfying the wave equation. In this case, the extended EWI objective function can be stated as:

$$E(u_{0i}, \delta u_i, s_{ei}) = \min \frac{1}{2} \sum_i \|Cu_{0i} + C\delta u_i - d_i\|_2^2 + \frac{\alpha^2}{2} \|L_0 u_{0i} - f_i\|_2^2 + \frac{\varepsilon^2}{2} \|L_0 \delta u_i - s_{ei}\|_2^2, \quad (8)$$

where s_{ei} is the modified secondary source function, and it is given by s_i as we initiate the problem. As we will see later, this source function will provide smooth background velocity updates along reflection wave paths, similar to RWI. The weighting factor ε^2 controls how much the scattered wavefield is required to satisfy the Born wave equation. If we focus on minimizing the reflection data misfit like RWI, the reflection-based EWI (REWI) objective function can be stated as:

$$E(\delta u_i, s_{ei}) = \min \frac{1}{2} \sum_i \|Cu_{0i} + C\delta u_i - d_i\|_2^2 + \frac{\varepsilon^2}{2} \|L_0 \delta u_i - s_{ei}\|_2^2, \quad \text{such that : } L_0 u_{0i} = f_i. \quad (9)$$

Analogy to the idea of the original EWI, this new objective function aims to minimize the misfit in the scattered wavefield, which partially satisfies the wave equation. The scattered wavefield δu can be solved using a modified augmented wave equation, given by:

$$\begin{pmatrix} \varepsilon L_0 \\ C \end{pmatrix} \delta u_i = \begin{pmatrix} \varepsilon s_{ei} \\ d_i - Cu_{0i} \end{pmatrix}. \quad (10)$$

As s_{ei} starts with s_i at each frequency, we will need to calculate the perturbation δm prior to updating the background velocity. Next, we will introduce how we calculate δm .

Perturbation model calculation

To have a pure reconstruction of the scattered wavefield δu using equation 10, we need to subtract the data corresponding to

Reflection-based efficient wavefield inversion

the background wavefield u_0 from the recorded data. Theoretically, this background wavefield should correspond to a good background velocity model, so we cannot calculate it directly using the background wave equation, which may correspond to a poor initial velocity. Luckily, we can still use equation 3 to reconstruct the background wavefield, even with a poor initial velocity model. After we get the background wavefield u_0 at each frequency, we set $\delta m = 0$ which means $s = 0$ in the beginning of δm calculation. Even under this condition, we can still use equation 10 to initially reconstruct the scattered wavefield δu . This is equivalent to the microseismic source estimation, because s_i here acts as secondary sources in the subsurface (Song and Alkhalifah, 2019). Following the Born approximation (Zhang and Duan, 2012):

$$L_0 \delta u = -\omega^2 \delta m u_0, \quad (11)$$

the squared slowness perturbation δm can be evaluated using:

$$\delta m = -\Re \sum_i \left(\frac{L_0 \delta u_i}{\omega^2 u_{0i}} \right) \approx -\Re \sum_i \left[\frac{(L_0 \delta u_i) u_{0i}^*}{\omega^2 u_{0i} u_{0i}^* + \lambda} \right], \quad (12)$$

To further improve the accuracy of the scattered wavefield δu , we plug the calculated δm back into equation 10, and repeat equations 11 and 12. In the perturbation calculation, we don't update the modified source function s_{ei} , and it is equivalent to s_i . As we implement our method in the frequency domain, we calculate the perturbation δm_{ifre} at each frequency, and stack them over the full frequency band to get the final perturbation inversion result δm , which in the case be generally artifact free. During the process of the δm calculation, we don't update the background velocity model. The δm calculation is an essential step in each outer iteration of inverting for the background model, as the background model matures, δm quality improves as well. We share the algorithm for inverting for the background model in the next section.

Background model inversion

After obtaining a reasonable estimation of the perturbation δm , we begin our background model inversion. This new formulation of REWI has two important modifications to the original EWI. The first one is that we are fitting the scattered components of the wavefield to the data instead of the whole wavefield. The second is that we use a modified secondary source s_{ei} instead of f_{ei} . In order to accurately reconstruct the scattered wavefield, it is important to mitigate the influence of the background wavefield u_0 on the process. So we formulate a sub-problem to solve u_0 according to equation 9, given by:

$$E(u_{0i}) = \frac{1}{2} \sum_i \|C u_{0i} + C \delta u_i - d_i\|_2^2 + \frac{\alpha^2}{2} \|L_0 u_{0i} - f_i\|_2^2, \quad (13)$$

such that : $L_0 \delta u_i = s_i$.

As a result, the background wavefield u_0 satisfies the following linear equation:

$$\begin{pmatrix} \alpha L_0 \\ C \end{pmatrix} u_{0i} = \begin{pmatrix} \alpha f_i \\ d_i - C \delta u_i \end{pmatrix}. \quad (14)$$

Initially, we set $\delta u_i = 0$. After we obtain u_0 from equation 14, the secondary source can be updated using $s_i = -\omega^2 \delta m u_{0i}$.

We solve linear equation 10 to get the scattered wavefield δu_i . Then the calculated δu_i is used in equation 14 again to remove the scattered components in the reconstructed background wavefield u_0 , and the secondary source s_i will also be updated. Now, all the ingredients for REWI given by equation 10 are described. Specifically, we first solve equation 10 with $s_{ei} = s_i$, then we update the modified secondary source function using: $s_{ei} = L_0 \delta u_i$. We use cheap inner iterations to calculate δu_i and s_{ei} to include multi-scattering components in the reconstructed scattered wavefield (Alkhalifah and Song, 2019). We use two inner iterations in all the examples shown in this abstract, as multi-scattering energy higher than the second order is usually weak. Analogy to how we calculate the δm in equation 5, the background model perturbation is evaluated as follows:

$$\delta m_0 = \sum_i \frac{(s_i - s_{ei}) \delta u_i^*}{\omega^2 \delta u_i \delta u_i^* + \lambda}. \quad (15)$$

Finally, the background model can be updated by:

$$m_0 = m_0 + \delta m_0. \quad (16)$$

At each frequency, we update the background model once, and we use outer iterations sweeping the full frequency band to refine the background model (Song and Alkhalifah, 2020b).

NUMERICAL EXAMPLES

We test the proposed method on the Marmousi model, and we show the true and initial velocity models in Figures 1a and 1b, respectively. The initial velocity is linearly increasing with depth. The size of the model is 370×121 with a grid interval of 25 m in both vertical and horizontal directions. We use a 8 Hz Ricker wavelet to generate the synthetic data.

Figure 1: (a) The true the Marmousi velocity model, and (b) the initial model.

Figure 2: The inverted perturbation corresponding to the initial velocity at the first outer iteration.

Reflection-based efficient wavefield inversion

Figure 3: (a) The inverted background model after 10 outer iterations using extended EWI, and (b) the corresponding inverted perturbation.

We ignite 45 sources on the surface with a horizontal interval of 0.02 km, and all the grid points on the surface act as receivers. The selected frequency band used in this example is from 5 Hz to 10 Hz with the sampling interval of 0.5 Hz. We still use 10 outer iterations to implement our proposed method. Before the background model inversion, the inverted perturbation corresponding to the initial model at the first outer iteration is shown in Figure 2. We can see that the general structure is recovered, but the events are shifted from their true depth. After 10 outer iterations of REWI, the inverted background model is shown in Figure 3(a). We see that the background model is reasonably inverted, especially the shallow part. The perturbation model is enhanced, as shown in Figure 3(b). Using the well recovered background velocity model as the initial velocity to perform a sequential EWI, the final inverted velocity model after 30 outer iterations is shown in Figure 4. As a result, the details in the true Marmousi model are generally well reconstructed. For comparison, we use the linearly increasing with depth model in Figure 1b as the initial model to perform conventional FWI and the original EWI. After 30 outer iterations sweeping from 5 Hz to 10 Hz, the final inverted velocity models are shown in Figures 5a and 5b, respectively. FWI severely suffers from the cycle-skipping as expected. Though EWI improves the inversion result, especially in the shallow part, it doesn't work well in recovering the deep part. The vertical velocity profile at location 4.625 km clearly demonstrates that the sequential EWI is able to generate a better inversion result than using EWI directly, as shown in Figure 6.

Figure 4: The sequential EWI inverted model from the inverted background model.

Figure 5: (a) The inverted FWI and (b) EWI velocity models using the initial velocity model shown in Figure 1(b).

Figure 6: The velocity vertical profile at location 4.625 km.

CONCLUSIONS

We use a reflection-based efficient wavefield inversion (REWI) to perform a background velocity inversion using data free of low frequencies. We calculate the perturbation in an efficient way before we start the background inversion. The reflectors can be located at the wrong depth, which results in reflection data mismatch. In REWI, we use the wave equation for the Born scattered wavefield as a regularization term to enhance the reflection data match. Inner iterations between the scattered wavefield reconstruction and the update of the modified secondary source allow the multiscattering components to contribute to the background model update. We update the background once per frequency, and use the outer iterations to refine the background model. Finally, a sequential EWI is used to introduce the detailed structures to obtain the final inverted model. Applications on synthetic data generated from the Marmousi model demonstrate that REWI can recover reasonably good background models. We will also show the application of the proposed method on the real data at the conference.

ACKNOWLEDGMENTS

We thank KAUST for sponsoring this research. We thank our SWAG colleagues for their helpful suggestions.

REFERENCES

- Alkhalifah, T., and C. Song, 2019, An efficient wavefield inversion: Using a modified source function in the wave equation: *Geophysics*, **84**, R909–R922.
- Alkhalifah, T., and Z. Wu, 2016a, Multiscattering inversion for low-model wavenumbers: *Geophysics*, **81**, R417–R428.
- , 2016b, The natural combination of full and image-based waveform inversion: *Geophysical Prospecting*, **64**, 19–30.
- Guo, Q., and T. Alkhalifah, 2017, Elastic reflection-based waveform inversion with a nonlinear approach: *Geophysics*, **82**, R309–R321.
- Leeuwen, T. V., and F. Herrmann, 2013, Mitigating local minima in full-waveform inversion by expanding the search space: *Geophysical Journal International*, **195**, 661–667.
- , 2015, A penalty method for pde-constrained optimization in inverse problems: *Inverse Problems*, **32**, 015007.
- Li, Y., Q. Guo, Z. Li, and T. Alkhalifah, 2019, Elastic reflection waveform inversion with variable density: *Geophysics*, **84**, R553–R567.
- Song, C., and T. Alkhalifah, 2019, Microseismic event estimation based on an efficient wavefield inversion: *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, **12**, 4664–4671.
- , 2020a, An efficient wavefield inversion for transversely isotropic media with a vertical axis of symmetry: *Geophysics*, **85**, 1–51.
- , 2020b, Efficient wavefield inversion with outer iterations and total variation constraint: *Transactions on Geoscience and Remote Sensing*, 1–11.
- Song, C., T. Alkhalifah, G. Wang, and Q. Yang, 2019a, An efficient wavefield inversion for isotropic elastic media, *in* SEG Technical Program Expanded Abstracts 2019: Society of Exploration Geophysicists, 1575–1579.
- Song, C., Z. Wu, and T. Alkhalifah, 2019b, Passive seismic event estimation using multiscattering waveform inversion: *Geophysics*, **84**, KS59–KS69.
- Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation: *Geophysics*, **49**, 1259–1266.
- Wu, Z., and T. Alkhalifah, 2015, Simultaneous inversion of the background velocity and the perturbation in full-waveform inversion: *Geophysics*, **80**, R317–R329.
- Xu, S., D. Wang, F. Chen, and Y. Zhang, 2012, Full waveform inversion for reflected seismic data: 74th Annual International Conference and Exhibition, EAGE, Extended Abstracts, W024.
- Yao, G., and D. Wu, 2017, Reflection full waveform inversion: *Science China Earth Sciences*, **60**, 1783–1794.
- Zhang, Y., and L. Duan, 2012, Predicting multiples using a reverse time demigration, *in* SEG Technical Program Expanded Abstracts 2012: Society of Exploration Geophysicists, 1–5.
- Zhou, H., L. Amundsen, and G. Zhang, 2012, Fundamental issues in full waveform inversion: 82nd Annual International Meeting, SEG, Expanded Abstracts, 1–5.