Multi-stage elastic full-waveform inversion for tilted transverse isotropic media

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Abstract

Seismic anisotropy is an important physical phenomenon that significantly affects wave propagation in complex sedimentary basins. When geological structures exhibit steep dips or severe folding, the symmetry axis of the transversely isotropic (TI) representation of the region can be rotated, leading to tilted transversely isotropic (TTI) media. We seek to find the optimal full-waveform inversion (FWI) strategy to estimate both the seismic velocities and the anisotropic parameters, including the tilt angle, in the presence of elastic TTI media. We first formulate the forward and inverse problems for elastic TTI media and analyze the radiation patterns of the model parameters. Based on the analyses of the radiation patterns, we propose two similar multi-stage FWI strategies that add inversion parameters over three stages, beginning with the isotropic parameters (horizontal P- and vertical S-wave velocity) and moving to the anisotropic parameters; the tilt angle is directly inverted in the last stage. Since diving waves, which are useful for providing long-wavelength updates, are mainly controlled by horizontal motion in anisotropic media, it is reasonable to choose the horizontal P-wave velocity rather than the vertical P-wave velocity. Then, the anisotropic parameters are inverted mainly using the reflected waves based on the isotropic background model built in the first stage. The main difference between the two multi-stage FWI strategies is whether the anisotropic parameter η is inverted. Comparing the two multi-stage FWI strategies with the simultaneous inversion strategy for a downsized version of the synthetic BP TTI model, we confirm that the multi-stage FWI strategies yield better inversion results than the simultaneous inversion strategy. When we compare the two multi-stage FWI strategies with each other for surface seismic data, ignoring η during the FWI process (focused multi-stage FWI) yields better inversion results for the tilt angle than those obtained with the inversion of η because η has less influence on the FWI than the other parameters and is not recovered well, which plays a role

in degrading the tilt angle. Numerical examples support our conclusions that the focused multi-stage FWI strategy (neglecting η) is the optimal FWI strategy for TTI media and achieves computational efficiency for surface seismic data.

Keywords: Seismic anisotropy, Waveform inversion, Elasticity and anelasticity

Introduction

Full-waveform inversion (FWI) is a nonlinear optimization technique that estimates subsurface physical properties by fitting the modeled and field data for high-resolution seismic images (Tarantola, 1984; Pratt, 1999; Virieux and Operto, 2009). Due to the ill-posed nature of FWI and the large computational cost, most inversion applications have been conducted under the assumption that the subsurface media are isotropic. However, for an appropriate interpretation of seismic data acquired in a geologically complex region, more realistic assumptions considering anisotropic properties are needed.

Vertical transversely isotropic (VTI) media represent the most common occurrence of anisotropy in sedimentary basins. Since the Earth is dominated by horizontal layering, this type of anisotropy is reasonably representative of many regions. However, when sedimentary layers are deformed by regional tectonism (e.g., plate tectonics) or local forces (e.g., salt bodies), the corresponding rotation may tilt the TI (transversely isotropic) medium, forming a tilted transversely isotropic (TTI) medium.

To account for the rotation of anisotropic media, modeling and inversion algorithms for TTI media have been studied (Zhou et al. 2006; Operto et al., 2009; Hadden and Pratt, 2017; Rusmanogroho et al., 2017). Operto et al. (2009) developed an acoustic-approximated modeling algorithm using the frequency-domain finite-difference method to describe wave propagation in 2D TTI media and showed that this algorithm yields reliable simulation results in terms of kinematic and dynamic aspects compared with those obtained by elastic modeling algorithms, although S-wave artifacts are often present depending on the anisotropic parameters of the background media.

Previous FWI studies conducted on VTI media can be classified into two types, namely, those that employed either the 1) acoustic approximation (Plessix and Cao, 2011; Gholami et

al., 2013a, 2013b; Alkhalifah and Plessix, 2014; He and Plessix, 2017) or the 2) elastic approximation (Lee et al., 2010; Kim and Min, 2014; Oh and Min, 2014; Kamath and Tsvankin, 2016; He et al., 2018; He et al., 2019). Gholami et al. (2013a) analyzed certain parameterizations using the acoustic assumption for 2D VTI media depending on the source-receiver configuration. In this implementation, these researchers estimated two velocities and did not update δ because δ has a relatively insignificant effect on the partial reflections and diving wave (Gholami et al., 2013b).

In elastic VTI FWI, Lee et al. (2010) performed a sequential inversion using what they called a coupling method to avoid local minima by obtaining C_{11} from ε and C_{33} . Oh and Min (2014) introduced a new parameterization with Poisson's ratio by analyzing virtual sources to overcome the limited coverage of the conventional elastic VTI FWI parameterization, but their strategy increases the trade-offs between the model parameters. In 3D cases, we encounter both orthorhombic media, which, in addition to VTI, have horizontal transverse isotropy (HTI) due to vertical fractures, and triclinic media (Köhn et al., 2015). Orthorhombic anisotropy was also investigated under the assumptions of acoustic media (Masmoudi and Alkhalifah, 2016) and elastic media (Oh and Alkhalifah, 2016; Oh and Alkhalifah, 2018; Oh and Alkhalifah, 2019).

Multi-parameter FWI is prone to suffer from trade-offs between the model parameters. Finding the optimal parameterization can reduce the trade-off issue (Köhn et al., 2012). Alkhalifah and Plessix (2014) showed that using the horizontal P-wave velocity and ε benefits from inverting the horizontal P-wave velocity from diving waves and inverting ε from reflections with less trade-off. Alkhalifah (2016) analyzed the influences of anisotropic parameters on FWI in terms of the wavenumber component and showed that the parameter η (Alkhalifah, 2003) has less influence on FWI than the horizontal velocity and ε .

On the other hand, in TTI media, we need to consider the rotation of the symmetry axis as

well as the VTI parameters because all the model parameters can be affected by the rotation of the symmetry axis. Implementing FWI for TTI media is more complicated than that for VTI media because the number of parameters increases; moreover, the parameters are affected by each other. To properly recover TTI media, it is important to develop an appropriate FWI strategy by analyzing radiation patterns and trade-offs between parameters. Rusmanugroho et al. (2017) analyzed the radiation patterns of the Voigt parameters and a modified version of the elastic parameters (the Chen and Tromp parameters; Sieminski et al., 2007a) and tried to invert for the TTI properties by estimating the tilt angle indirectly from the inverted elastic constants; in their strategy, the success of estimating the tilt angle is governed by the inverted elastic constants in elastic TTI media.

In this paper, we first try to estimate the tilt angle of the symmetric axis of transverse isotropic media as a target parameter of FWI. To accomplish this, we seek to find the optimal FWI strategy for elastic TTI media that directly inverts the tilt angle of the symmetry axis along with the seismic velocities and the dimensionless anisotropic parameters. To achieve this goal, we first formulate the forward and inverse problems for elastic TTI media and analyze the scattering potential of each model parameter by plotting the radiation patterns of the partial-derivative wavefields. Based on a sensitivity analysis, we next propose two multi-stage FWI strategies (called ‘Strategy-II’ and ‘Strategy-III’) and then compare these strategies with the simultaneous inversion strategy (called ‘Strategy-I’) for the downsized synthetic BP TTI model. In both multi-stage strategies, the model parameters to be inverted are added over the stages. In the first stage, we invert for the horizontal P-wave velocity and the vertical S-wave velocity. Because diving waves are mainly controlled by horizontal motion in anisotropic media, we choose the horizontal P-wave velocity rather than the vertical P-wave velocity to obtain the benefits of long-wavelength updates from the diving wave. Then, the anisotropic parameters

are inverted mainly using the reflected waves based on the isotropic background model built in the first stage. The main difference between Strategies-II and III is that Strategy-III does not update the anisotropic parameter η , which has little influence on the inversion. Comparing the inversion results obtained by Strategies-II and III with each other, we demonstrate that Strategy-III is the optimal FWI strategy for TTI media.

Methodology

Forward problem for TTI media

The elastic wave equation for TTI media is expressed in terms of particle displacements (u_i), stresses (σ_{ij}), and strains (ε_{kl}) as follows (Tsvankin, 2001):

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sum_j \partial_j \sigma_{ij} + f_i, \quad (1)$$

and

$$\sigma_{ij} = \sum_{kl} \tilde{C}_{ijkl} \varepsilon_{kl}, \quad (2)$$

where ρ is the density and $\tilde{C}_{ijkl}$ represents the elastic constants for TTI media. In 2D elastic TTI media, the particle displacement, stress, and strain can be expressed by

$$\mathbf{u}^T = (u_x \quad u_z), \quad (3)$$

$$\boldsymbol{\sigma}^T = (\sigma_{xx} \quad \sigma_{zz} \quad \sigma_{xz}), \quad (4)$$

and

$$\boldsymbol{\varepsilon}^T = (\varepsilon_{xx} \quad \varepsilon_{zz} \quad \varepsilon_{xz}), \quad (5)$$

respectively. The stiffness matrix for 2D elastic TTI media can be derived from that for 2D elastic VTI media using the Bond transformation about the horizontal y -axis in a clockwise direction (Ivanov and Stovas, 2016):

$$\tilde{\mathbf{C}}^{TTI} = D_y(\theta_T) \mathbf{C}^{VTI} D_y^T(\theta_T), \quad (6)$$

where

$$\tilde{\mathbf{C}}^{TTI} = \begin{pmatrix} \tilde{C}_{11} & \tilde{C}_{13} & \tilde{C}_{15} \\ \tilde{C}_{13} & \tilde{C}_{33} & \tilde{C}_{35} \\ \tilde{C}_{15} & \tilde{C}_{35} & \tilde{C}_{55} \end{pmatrix}, \quad (7)$$

$$\mathbf{C}^{VTI} = \begin{pmatrix} C_{11} & C_{13} & 0 \\ C_{13} & C_{33} & 0 \\ 0 & 0 & C_{55} \end{pmatrix} \quad (8)$$

and

$$D_y(\theta_T) = \begin{pmatrix} \cos^2 \theta_T & \sin^2 \theta_T & 2 \sin \theta_T \cos \theta_T \\ \sin^2 \theta_T & \cos^2 \theta_T & -2 \sin \theta_T \cos \theta_T \\ -\sin \theta_T \cos \theta_T & \sin \theta_T \cos \theta_T & \cos^2 \theta_T - \sin^2 \theta_T \end{pmatrix}. \quad (9)$$

The parameter θ_T is the tilt angle of the symmetry axis measured from the vertical. As a result,

$\tilde{C}_{IJ}$ (Voigt notation) for 2D elastic TTI media can be explicitly defined (Macbeth, 2002) as

$$\begin{pmatrix} \tilde{C}_{11} \\ \tilde{C}_{33} \\ \tilde{C}_{13} \\ \tilde{C}_{55} \\ \tilde{C}_{15} \\ \tilde{C}_{35} \end{pmatrix} = \begin{pmatrix} C_{11} \cos^4 \theta_T + C_{33} \sin^4 \theta_T + (2C_{13} + 4C_{55}) \sin^2 \theta_T \cos^2 \theta_T \\ C_{11} \sin^4 \theta_T + C_{33} \cos^4 \theta_T + (2C_{13} + 4C_{55}) \sin^2 \theta_T \cos^2 \theta_T \\ (C_{11} + C_{33} - 4C_{55}) \sin^2 \theta_T \cos^2 \theta_T + C_{13} (\sin^4 \theta_T + \cos^4 \theta_T) \\ (C_{11} + C_{33} - 2C_{13}) \sin^2 \theta_T \cos^2 \theta_T + C_{55} (\cos^2 \theta_T - \sin^2 \theta_T)^2 \\ (C_{13} - C_{11} + 2C_{55}) \cos^3 \theta_T \sin \theta_T - (C_{13} - C_{33} + 2C_{55}) \cos \theta_T \sin^3 \theta_T \\ (C_{13} - C_{11} + 2C_{55}) \sin^3 \theta_T \cos \theta_T - (C_{13} - C_{33} + 2C_{55}) \sin \theta_T \cos^3 \theta_T \end{pmatrix}. \quad (10)$$

In this study, we solve the displacement-based elastic wave equations by substituting eq.

(2) into eq. (1) as follows:

$$\begin{aligned} \rho \frac{\partial^2 u_x}{\partial t^2} = & \frac{\partial}{\partial x} \left[\tilde{C}_{11} \frac{\partial u_x}{\partial x} + \tilde{C}_{13} \frac{\partial u_z}{\partial z} + \tilde{C}_{15} \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) \right] \\ & + \frac{\partial}{\partial z} \left[\tilde{C}_{15} \frac{\partial u_x}{\partial x} + \tilde{C}_{35} \frac{\partial u_z}{\partial z} + \tilde{C}_{55} \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) \right] + f_x, \end{aligned} \quad (11)$$

and

$$\begin{aligned} \rho \frac{\partial^2 u_z}{\partial t^2} = & \frac{\partial}{\partial x} \left[\tilde{C}_{15} \frac{\partial u_x}{\partial x} + \tilde{C}_{35} \frac{\partial u_z}{\partial z} + \tilde{C}_{55} \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) \right] \\ & + \frac{\partial}{\partial z} \left[\tilde{C}_{13} \frac{\partial u_x}{\partial x} + \tilde{C}_{33} \frac{\partial u_z}{\partial z} + \tilde{C}_{35} \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) \right] + f_z. \end{aligned} \quad (12)$$

We solve these elastic wave equations using a finite-element method (FEM) in the frequency

domain with the CPML (convolutional perfectly matched layer; Rodenm and Gedney, 2000) to remove artificial reflections from the model boundaries. The sparse matrix solver UMFPACK (Davis, 2004) is applied to solve the forward problem with the MPI (Message Passing Interface).

Inverse problem for TTI media

The objective function based on the l_2 norm (Tarantola, 1986), which measures the misfit between the observed data ($\mathbf{d}$) and modeled data ($\mathbf{u}$), can be expressed in the frequency domain as follows:

$$E(\mathbf{p}) = \frac{1}{2} \sum_{\omega} \sum_s \left[(\mathbf{u}_s(\omega, \mathbf{p}) - \mathbf{d}_s(\omega))^T (\mathbf{u}_s(\omega, \mathbf{p}) - \mathbf{d}_s(\omega))^* \right], \quad (13)$$

where ω and s are the angular frequency and source index, respectively. The model parameter vector, $\mathbf{p}$, consists of seven parameters: the six elastic constants and the density. The modeled data, $\mathbf{u}$, can be represented in a matrix form in the frequency domain as follows (Pratt et al., 1998):

$$\mathbf{u}_s(\omega, \mathbf{p}) = \mathbf{S}^{-1}(\omega, \mathbf{p}) \mathbf{f}_s(\omega). \quad (14)$$

The gradient direction for the model parameter class (m) at the n^{th} nodal point that minimizes the objective function in eq. (13) can be obtained by

$$\frac{\partial E}{\partial p_{m,n}} = \sum_{\omega} \sum_s \text{Re} \left[\mathbf{J}_{m,n}^T [\mathbf{u}_s(\omega, \mathbf{p}) - \mathbf{d}_s(\omega)]^* \right], \quad (15)$$

where $\mathbf{J}_{m,n}$ corresponds to the partial-derivative wavefields (the Jacobian or sensitivity matrix) for each model parameter perturbation. Each column of the Jacobian matrix can be obtained by

$$\frac{\partial \mathbf{u}_s}{\partial p_{m,n}} = \mathbf{S}^{-1} \left(-\frac{\partial \mathbf{S}}{\partial p_{m,n}} \mathbf{u}_s \right) = \mathbf{S}^{-1} \mathbf{f}_{m,n}^v, \quad (16)$$

where $\mathbf{f}_{m,n}^v$ is the virtual source generated by perturbing the model parameter m at the n^{th} nodal point. Using the adjoint-state method (Pratt et al., 1998; Plessix, 2006), the gradient direction can be calculated by multiplying the virtual source and backpropagated wavefields as follows:

$$\frac{\partial E}{\partial p_{m,n}} = \sum_{\omega} \sum_s \text{Re} \left[\left[\mathbf{f}_{m,n}^v(\omega, \mathbf{p}) \right]^T \left[\mathbf{S}^{-1}(\omega, \mathbf{p}) \right]^T \left[\mathbf{u}_s(\omega, \mathbf{p}) - \mathbf{d}_s(\omega) \right]^* \right]. \quad (17)$$

Note that the backpropagated wavefield (represented by the product of the last two brackets in eq. (17)) is independent of the perturbation in the model parameter m , which means that the sensitivities of different model parameter classes are determined only by the virtual source (Oh and Alkhalifah, 2016). For this reason, to succeed in multi-parameter FWI for elastic TTI media, we need to analyze the radiation patterns of the partial-derivative wavefields due to the virtual source, from which we can understand the sensitivity of each parameter and design the optimal parameterization (Operto et al., 2013) and FWI strategy (Oh and Alkhalifah, 2018). For optimization, we precondition the gradient in eq. (17) with the diagonal of the pseudo-Hessian matrix (Shin et al., 2001) and the conjugate gradient method (Fletcher and Reeves, 1964).

Model parameterization

Understanding the trade-off between the parameters and the FWI sensitivity coverage of each parameter is the key to determining the optimal parameterization (Operto et al., 2013; Gholami et al., 2013a). For elastic TTI media, Rusmanugroho et al. (2017) chose the Voigt parameters and Chen and Tromp parameters (Sieminski et al., 2007a, Sieminski et al., 2007b), which include six independent parameters (excluding the density). In this parameterization, the

tilt angle is indirectly estimated from the other inverted parameters.

In this study, we directly invert the tilt angle by calculating the gradient direction of the tilt angle as a least-squares solution. To accomplish this calculation, we first choose the hierarchical VTI parameterization, which is the VTI version of the optimal orthorhombic parameterization suggested by Oh and Alkhalifah (2016), as follows:

$$(v_h \ v_s \ \varepsilon \ \eta \ \rho \ \theta_T), \quad (18)$$

where v_h and v_s represent the horizontal P-wave and the vertical S-wave velocity in VTI media, respectively, and ε and η are the anisotropic parameters (Thomsen, 1986; Alkhalifah and Tsvankin, 1995). The anisotropic parameter η is a parameter to correct the NMO velocity in VTI media ($v_h = v_{NMO} \sqrt{1 + 2\eta}$ and $\eta = (\varepsilon - \delta) / (1 + 2\varepsilon)$; Alkhalifah and Tsvankin, 1995). The choice of the horizontal P-wave velocity (v_h) allows the inversion of the horizontal P-wave velocity from diving waves, which are required to build smooth background models and to invert ε from small-angle reflections featuring fewer trade-offs with the P-wave velocity (Alkhalifah and Plessix, 2014; He and Plessix, 2017). The S-wave velocity as an FWI target is mainly sensitive to intermediate-angle reflections, as we will discuss later in the radiation pattern analysis (Tarantola, 1986).

Instead of δ , the parameter η is chosen to reduce the number of parameters that significantly influence the FWI (Alkhalifah, 2016). Using the model parameters listed in eq. (18), C_M along the symmetry axis ($\theta_T = 0$) in eq. (10) can be expressed as

$$\begin{pmatrix} C_{11} \\ C_{33} \\ C_{13} \\ C_{55} \end{pmatrix} = \begin{pmatrix} \rho v_h^2 \\ \frac{\rho v_h^2}{(1+2\varepsilon)} \\ \rho \left[\sqrt{\left(\frac{v_h^2}{(1+2\varepsilon)} - v_s^2 \right) \left(\frac{v_h^2}{(1+2\eta)} - v_s^2 \right)} - v_s^2 \right] \\ \rho v_s^2 \end{pmatrix}. \quad (19)$$

Note that C_{11} and C_{55} include only velocities, but C_{33} and C_{13} additionally include the anisotropic parameters one by one. As Oh and Alkhalifah (2018) and Masmoudi and Alkhalifah (2018) showed, this hierarchical parameterization enables the sequential inversion of the subsurface model parameters starting from the isotropic assumption and moving to the complex anisotropic assumption by adding the anisotropic parameters. In addition, for a surface seismic survey, we can reduce the number of influential parameters from seven to four in the elastic TTI case. A detailed explanation of this hierarchical parameterization in TTI media is presented in the following section. Ignoring the density, the virtual sources of an arbitrary parameter, m , can be obtained from the virtual sources of $\tilde{C}_{IJ}$ in eq. (16) using the chain rule as follows:

$$\mathbf{f}_{m,n}^v(\omega, \mathbf{p}) = \sum_{IJ} \frac{\partial \tilde{C}_{IJ}}{\partial m} \mathbf{f}_{\tilde{C}_{IJ},n}^v(\omega, \mathbf{p}). \quad (20)$$

Sensitivity analysis

Radiation pattern analysis

The radiation pattern of a partial-derivative wavefield provides intuitive information for the FWI sensitivity coverage and trade-offs between the model parameters depending on the parameterization (Operto et al., 2013). We derive the radiation patterns (amplitude versus angle) for the partial-derivative wavefields due to P-wave incidence. To avoid the complexities arising from anisotropic propagation effects on both incident (from the seismic source) and scattered (from the model parameter perturbation) ray paths, which depend on the given case-specific background anisotropic model, we assume an isotropic background model. For this reason, our analysis is constrained to the early stage of FWI when we still have isotropic or weak anisotropic velocity models. However, anisotropic effects on the propagation of partial-derivative wavefields are common for all model parameter perturbations, while a change in the source mechanism is the main factor for identifying radiation pattern changes depending on the model parameter. Therefore, this analysis provides some insights into the radiation pattern changes due to the different source mechanisms of model parameter perturbations. The anisotropic propagation effects on the radiation patterns from a point source depending on different background anisotropic parameters are listed in Gajewski (1993).

The particle displacements of P- and SV-waves from a point source can be described as (referring to eq. (4.97) in Aki and Richards, 1980)

$$\mathbf{u}^P(\theta_S) = (\mathbf{e}_P^T \mathbf{M} \mathbf{e}_P) \mathbf{e}_P = R^P(\theta_S) \mathbf{e}_P \quad (21)$$

and

$$\mathbf{u}^{SV}(\theta_S) = (\mathbf{e}_{SV}^T \mathbf{M} \mathbf{e}_P) \mathbf{e}_{SV} = R^{SV}(\theta_S) \mathbf{e}_{SV}, \quad (22)$$

respectively. The angle θ_S is the scattering angle from the source. The matrix $\mathbf{M}$ is the moment tensor source. The two vectors $\mathbf{e}_P$ and $\mathbf{e}_{SV}$ are the polarization vectors for the P-wave and SV-wave in the isotropic background media, respectively, and are expressed by

$$\mathbf{e}_P^T = (\sin \theta_S \quad \cos \theta_S) \quad (23)$$

and

$$\mathbf{e}_{SV}^T = (\cos \theta_S \quad -\sin \theta_S), \quad (24)$$

respectively. The radiation patterns of P- and SV-waves ($R^P(\theta_S)$ and $R^{SV}(\theta_S)$, respectively) are the vector norm of particle displacement.

The partial-derivative wavefield is the secondary wavefield from a model parameter perturbation in response to the incident wavefield. The radiation pattern analysis of the partial-derivative wavefields in multi-parameter FWI is important because each model parameter in anisotropic elastic media acts like a different moment tensor source (a so-called virtual source). In the vertical section that includes a reflection ray path from a horizontal reflector, the P-P and P-SV radiation patterns of the partial-derivative wavefields due to the incident P-wave can be described by

$$R_m^{P-P}(\theta_I, \theta_S) = \mathbf{e}_P^T \mathbf{M}_m^P \mathbf{e}_P \quad (25)$$

and

$$R_m^{P-SV}(\theta_I, \theta_S) = \mathbf{e}_{SV}^T \mathbf{M}_m^P \mathbf{e}_P, \quad (26)$$

respectively. The two angles θ_I and θ_S are the incidence angle from the seismic source and the scattering angle from the model parameter perturbation, respectively (Figure 1). The matrix $\mathbf{M}^P$ is the virtual source (Oh and Alkhalifah, 2016) for the incident P-wave in moment-tensor

form and is expressed with $\tilde{C}_{IJ}$ parameters as follows:

$$\mathbf{M}_{\tilde{C}_{IJ}}^P = \begin{pmatrix} \frac{\partial \tilde{C}_{11}}{\partial \tilde{C}_{IJ}} \hat{\varepsilon}_{xx}^P + \frac{\partial \tilde{C}_{13}}{\partial \tilde{C}_{IJ}} \hat{\varepsilon}_{zz}^P + \frac{\partial \tilde{C}_{15}}{\partial \tilde{C}_{IJ}} \hat{\varepsilon}_{xz}^P & \frac{\partial \tilde{C}_{15}}{\partial \tilde{C}_{IJ}} \hat{\varepsilon}_{xx}^P + \frac{\partial \tilde{C}_{35}}{\partial \tilde{C}_{IJ}} \hat{\varepsilon}_{zz}^P + \frac{\partial \tilde{C}_{55}}{\partial \tilde{C}_{IJ}} \hat{\varepsilon}_{xz}^P \\ \frac{\partial \tilde{C}_{15}}{\partial \tilde{C}_{IJ}} \hat{\varepsilon}_{xx}^P + \frac{\partial \tilde{C}_{35}}{\partial \tilde{C}_{IJ}} \hat{\varepsilon}_{zz}^P + \frac{\partial \tilde{C}_{55}}{\partial \tilde{C}_{IJ}} \hat{\varepsilon}_{xz}^P & \frac{\partial \tilde{C}_{13}}{\partial \tilde{C}_{IJ}} \hat{\varepsilon}_{xx}^P + \frac{\partial \tilde{C}_{33}}{\partial \tilde{C}_{IJ}} \hat{\varepsilon}_{zz}^P + \frac{\partial \tilde{C}_{35}}{\partial \tilde{C}_{IJ}} \hat{\varepsilon}_{xz}^P \end{pmatrix}. \quad (27)$$

The parameters $\hat{\varepsilon}_{kl}$ are the strains in spherical coordinates and are expressed by

$$\begin{pmatrix} \hat{\varepsilon}_{xx}^P \\ \hat{\varepsilon}_{zz}^P \\ \hat{\varepsilon}_{xz}^P \end{pmatrix} = \begin{pmatrix} \sin^2 \theta_I \\ \cos^2 \theta_I \\ \sin \theta_I \cos \theta_I \end{pmatrix}. \quad (28)$$

The moment-tensor source for an arbitrary parameter, m , can also be obtained from the moment-tensor sources of $\tilde{C}_{IJ}$ using the chain rule as follows:

$$\mathbf{M}_m^P = \sum_{IJ} \frac{\partial \tilde{C}_{IJ}}{\partial m} \mathbf{M}_{\tilde{C}_{IJ}}^P. \quad (29)$$

Finally, the P-P and P-SV radiation patterns of the hierarchical parameterization can be obtained using eqs. (25) and (26), respectively, with appropriate linear combinations of the moment-tensor sources for the $\tilde{C}_{IJ}$ parameters in eq. (27).

Figures 2 and 3 show the theoretical radiation patterns obtained from eqs. (25) and (26) and the numerical radiation patterns of the partial-derivative wavefields using the virtual source in eq. (20) for an isotropic background. The numerical radiation patterns of the partial-derivative wavefields are obtained by solving the first-order elastic wave equations in eqs. (1) and (2) with the virtual sources in eq. (29) using the time-domain staggered-grid finite-difference method (Graves, 1996). Although we perform frequency-domain FWI throughout this paper, we choose a time-domain modeling engine for the radiation pattern analysis to

clearly show the patterns in snapshots. To calculate the numerical radiation patterns, we plot a norm of the displacement vectors ($\sqrt{u_x^2 + u_z^2}$) at each grid point. We assume seven different incidence angles to examine the variations according to the P-wave incidence angle. For comparison, the density is included in this example.

In isotropic media, the numerical radiation patterns (snapshots at the center) of the perturbations in the P-wave velocity, the S-wave velocity, and the density agree well with the theoretical radiation patterns depending on the incidence angles. The radiation patterns for the perturbation in the P-wave velocity are isotropic, which means that we can invert the P-wave velocity at practically all available data (considering the frequency band and the recording coverage). However, the isotropic radiation pattern can cause trade-offs between the P-wave velocity and the other parameters (Oh and Alkhalifah, 2018). On the other hand, the P-P and P-SV radiation patterns due to the perturbation in the S-wave velocity behave as $\sin^2 \theta_s$ and $\sin \theta_s \cos \theta_s$ at an incidence angle of zero, respectively. The scattering patterns are rotated around the source so that the influence of the S-wave velocity is the strongest at 45° with respect to the incidence angle of the horizontal reflector. The density perturbation has a backscattering pattern directed toward the seismic source, and this pattern dominates at small offsets. Figure 2 agrees well with previous work (Tarantola, 1986) with respect to the isotropic parameters. Figure 3a shows that the P-P scattering patterns of the anisotropic parameter ε behave as a cosine function and that the P-P scattering radiates the majority of the energy along the vertical direction. In Figure 3b, the P-P scattering patterns of the anisotropic parameter η behave as a cosine function for large incidence angles but as a sine function for small incidence angles, which indicates that we cannot use diving waves and small-angle partial reflections to invert for η when the background model is isotropic or weakly VTI. The P-SV radiation

patterns of the anisotropic parameters are similar to each other (i.e., $\sin \theta_s \cos \theta_s$).

Influence of tilt angle on the FWI procedure

We further investigate the influences of the tilt angle on the source mechanisms of VTI parameters and the behavior of the tilt angle as an inversion target to be updated. To ignore the complex propagation effects in the anisotropic background model, we assume a local TTI media where the background model is still isotropic (same as Figures 2 and 3), but only seven perturbed points have tilted transverse isotropic properties with ε , η , and θ_T of 0.2, 0.1, and -30° , respectively. This unusual assumption is required to focus on the changes in the source mechanisms of the VTI parameters depending on elastic TTI properties. Although the radiation pattern of each parameter in this example may be far from the real radiation patterns in elastic TTI media that we encounter in TTI FWI (due to anisotropic propagation effects), this analysis provides us with insights into the variations in the source mechanism of the virtual source at a perturbed point.

Figure 4 shows that the radiation patterns of the seismic velocities and the density still maintain the patterns for the isotropic background (Figure 2), which means that the source mechanisms for perturbations in the seismic velocities and the density are less sensitive to the variation in the tilt angle. This result also implies that the seismic velocities and the density can be recovered under the assumption of isotropic media even though the true Earth may have elastic TTI properties. In contrast, Figure 5 shows that the virtual sources of the anisotropic parameters as moment-tensor sources are rotated in terms of both the scattering and the incidence angles due to the TTI properties at the perturbed points; hence, these two anisotropic parameters are strongly affected by the rotation of the symmetry axis in TTI media.

These observations support the idea that the hierarchical parameterization, which first

inverts the velocities and the density under the isotropic assumption and then inverts the anisotropic parameters, can be a reasonable choice because poorly estimated anisotropic parameters with an incorrect tilt angle may adversely affect the inversion of the isotropic parameters due to cross-talk. For this reason, estimating the isotropic parameters, which are less affected by the tilt angle, prior to inverting the anisotropic parameters is a safe way to reduce the risk in complex elastic TTI inversion, although the inversion of the isotropic parameters in this stage may suffer from the trade-off artifacts caused by ignoring the anisotropic parameters.

We can update the tilt angle by computing the gradient direction of the tilt angle. Figure 6 shows the radiation patterns of the partial-derivative wavefields for the tilt angle depending on the VTI parameters at each perturbed point. We assume three different cases of VTI parameters for each perturbed point, whose anisotropic parameters (ϵ, η) are (0.2, 0), (0, 0.2) and (0.2, 0.2). The first model corresponds to elliptical anisotropy. In Figures 6a and 6b, we observe that the background ϵ and η provide different radiation patterns for the partial-derivative wavefields due to the perturbation in the tilt angle, which means that ϵ and η change the source mechanism of the tilt angle as virtual sources. From these observations, we can obtain certain insights into the optimal parameter-update strategy in elastic TTI media, which is discussed in the following subsection.

TTI FWI strategies

Considering the radiation patterns of the partial-derivative wavefields for each parameter, we propose two hierarchical parameterizations (Strategy-II and Strategy-III) and then compare these strategies with the simultaneous inversion technique (Strategy-I). Table 1 shows the three TTI FWI strategies. For all the strategies, we ignore the density because the density is a

dynamic parameter. That is, the density affects only the amplitudes of wavefields. This characteristic makes it difficult to estimate the density in the FWI of real field data.

In Strategy-I, we invert all the model parameters simultaneously. We expect strong trade-offs between the P-wave velocity and the other parameters due to the wide coverage of the partial-derivative wavefields for the perturbation in the P-wave velocity (see Figure 2). For this reason, the dimensionless anisotropic parameters might suffer from trade-off artifacts when the initial P-wave velocity is not sufficiently accurate. To reduce trade-off artifacts, in Strategy-II, we first invert the P- and S-wave velocities, and then, we add the anisotropic parameters to the FWI procedure when the inverted P-wave velocity is close enough to the true P-wave velocity. When we invert the isotropic P- and S-wave velocities from the anisotropic data in the first stage, we might have an issue in that fitting the diving waves mainly estimates the horizontal P-wave velocity, while fitting the reflections mainly estimates the vertical P-wave velocity (He and Plessix, 2017). For this reason, when the diving waves are used mainly in the first stage, deeper subsurface layers can be shifted with the incorrectly estimated P-wave velocity. To avoid this limitation, we require either low-frequency data or a good initial estimate for the vertical P-wave velocity.

According to the radiation patterns for the anisotropic parameters and the tilt angle shown in Figure 5, there are also certain trade-offs between the anisotropic parameters and the tilt angle. In addition, the tilt angle affects the wave propagation when the model has strong VTI. Finally, we add the tilt angle to the FWI procedure after we establish the VTI background model. In the last stage, we invert all the parameters together using the velocities and the anisotropic parameters recovered in the second stage as initial estimates. We expect that the tilt angle can be estimated from the inverted VTI model and that the inverted VTI model can also be further improved as the tilt angle is updated.

Strategy-III is a cost-effective version of Strategy-II. The main difference between Strategy-II and Strategy-III is that we ignore the anisotropic parameter η during the FWI in Strategy-III. Physically, the inverted model of Strategy-III should have elliptical anisotropy, in which η equals 0 ($\varepsilon = \delta$); otherwise, we rely on a predetermined η from, for example, tomography that is not updated in the FWI. For this reason, Strategy-III can be called the elliptical multi-stage FWI strategy if we keep η equal to 0 during the FWI or, more generally, a focused multi-stage FWI strategy, as we focus the inversion on the parameters to which the FWI is sensitive considering conventional acquisition. We predict that this focused multi-stage FWI strategy can provide results similar to those of Strategy-II because the dynamic influence of η on surface seismic data is weak. However, the background η is important if we have the tomographic means to extract this parameter because η is an efficient parameter for characterizing nonhyperbolic moveout (Alkhalifah and Tsvankin, 1995). For this reason, if we have a good background η model, Strategy-II can be a better choice than Strategy-III.

Considering another aspect of the tilt angle inversion, the influence of η on the radiation pattern of the tilt angle is also generally weak (Figure 6). Therefore, we can invert the tilt angle without the need for η in the background media. In addition, inaccurate estimates of η from Strategy-II may degrade the FWI result of the tilt angle because η is a parameter that can change the source mechanism of the tilt angle. For this reason, Strategy-III can be a practical TTI FWI strategy, particularly for real data.

Numerical examples: the downsized BP TTI model

We investigate the feasibility of the three aforementioned TTI FWI strategies for a downsized version of the BP TTI model (Figure 7). To avoid the difficulty caused by the fact that large salt bodies in the original model do not invert well without low-frequency components, we downscale the original model. The dimensions of the downsized model are $12.6 \text{ km} \times 1.8 \text{ km}$ with a grid spacing of 20 m. The S-wave velocity model is built by applying two different scaling factors to the original P-wave velocity model; scaling factors of 0.54 and 0.67 are applied to the salt bodies and the background media, respectively. From the tilt angle shown in Figure 7e, we see that only four parts (expressed by Areas 1, 2, 3, and 4) have TTI properties: Area 1 corresponds to inclined shale layers pushed upward by the salt body; Area 2 is related to an anticline at a distance of 8 km; Area 3 is associated with certain dipping layers; and Area 4 is not related to the inclination of the geological layers in the velocity model. As a source-time function, we use the first derivative of a Gaussian function with peak and maximum frequencies of 2.5 Hz and 10 Hz, respectively. UMFPACK (Davis, 2004) is used to solve the forward problem in the frequency domain. Each single-frequency gradient direction is obtained by the cluster based on the system Cray XC40 (named ‘Shaheen II’) to perform parallel computing over the frequencies. A total of 315 vertical-component sources with a spacing of 40 m are applied, and a total of 630 2-component geophones with a spacing of 20 m are placed on the surface. As an initial model, we use isotropic P- and S-wave velocity models obtained by smoothing the true models (Figure 8). To avoid instability on the CPML boundaries of the anisotropic media (Bécache et al., 2003), we apply lower and upper constraints to the model parameters during the FWI, such as (1.3, 5.2), (0.7, 3.0), (0, 0.2), (0, 0.2), and $(-45^\circ, 45^\circ)$ for ν_h , ν_s , ε , η and θ_T , respectively. In anisotropic elastic FWI, it is difficult to estimate the proper step-length for each model parameter because of the different physical

units of the model parameters and various wave modes, including surface waves with strong amplitudes. Estimating the step-length in an elastic inversion for land data does not guarantee an accurate step-length for each model parameter due to relatively strong Rayleigh waves (He et al., 2019). This is because each model parameter uses a different part of the data domain, and each wave mode is located in a different part. For example, v_h can be separated from Rayleigh waves if we keep only large scattering angles (Alkhalifah, 2015; Ramos-Martinez et al., 2016), while v_s easily suffers from contamination by Rayleigh waves. The parameter ε can be extracted if we keep only small angle reflections when the dispersion of Rayleigh waves is not severe, while η also easily overlaps with Rayleigh waves in the data domain because it diffracts energy mainly at intermediate diffraction angles (Figure 3).

For this reason, to mitigate the influences of strong Rayleigh waves, we multiply the preconditioned gradient direction by a depth-dependent scaling factor ($\beta(z) = z^2$; a modified version of Wang and Rao, 2009) to enhance the model updates at larger depths. To determine the step-length of each parameter for elastic TTI FWI in a safe way, we first normalize the preconditioned gradient direction of each parameter with the maximum gradient value of each parameter so that every gradient direction has values between -1 and 1. Then, we fix the step-length for each parameter throughout the FWI considering their natural range in the Earth, such as 0.05 km/s, 0.015 km/s, 0.005, 0.005 and 0.1 radians for v_h , v_s , ε , η and θ_T , respectively. For anisotropic dimensionless parameters (ε and η), we use a sufficiently small step-length to reduce trade-off artifacts in the early stage of FWI. For Strategy-I, the maximum number of iterations is 100, while for Strategy-II and Strategy-III, the maximum number of iterations is 50 at each stage. We empirically determine this number with the above fixed step-lengths. The frequency band used in the FWI is from 2 Hz to 8 Hz with an interval of 0.2 Hz for all strategies.

Figure 9 shows the models inverted using Strategy-I, where all the model parameters are

simultaneously inverted. We observe that the anisotropic parameters are not properly estimated due to strong trade-offs with the P-wave velocity. Because of the poorly estimated VTI background model, the tilt angle is not properly recovered (see Figure 9e).

Figures 10 and 11 show the models inverted at stage 1 and stage 3, respectively, of Strategy-II, which is the multi-stage FWI strategy that inverts the isotropic parameters first and then inverts the complex anisotropic parameters. Owing to the sufficiently good estimation of the isotropic parameters in stage 1 (Figure 10), the anisotropic parameter ε is recovered well in stage 2. However, because η is not sensitive to diving waves and small-offset reflections, this parameter is not properly recovered in most areas (refer to the parts represented by the arrows in Figure 11d). In Figure 11e, we see that the tilt angle can be inverted to a certain extent, although unwanted artifacts are also observed (see the parts indicated by the red arrows in Figure 11e). The inaccurate estimation of the tilt angles is closely associated with inaccurate updates of η , which means that failing to estimate η can degrade the quality of the inverted tilt angles.

Figure 12 shows the models inverted using the focused multi-stage FWI strategy (Strategy-III), in which we maintain the multi-stage strategy of Strategy-II but ignore η during the FWI. Without estimating η , we obtain a good background elliptical VTI model, and the tilt angle is estimated properly. The inverted VTI models from Strategy-III have slightly poorer features than those from Strategy-II. This is because the inverted η model from Strategy-II shows good features in many areas (pink arrows in Figure 11d) and provides positive influences on the FWI for those regions.

Comparing the tilt angle inverted by Strategy-III (Figure 12d) with that obtained using Strategy-II (Figure 11e), we observe that the artifacts (white arrows in Figure 11e) from the inaccurate estimation of η do not appear in Figure 12d. From these results, we can say that the

success of recovering the tilt angle depends on the quality of the inverted elliptical VTI model. However, in Figure 12d, we observe that the left part of Area 1 is not recovered well compared with the other areas. This result is attributed to the weak anisotropy of that area (Figure 7c), which implies that the tilt angle can be properly estimated only in cases of strong anisotropy. Otherwise, the influence of the tilt angle on the data is so weak that the TTI FWI is not needed. Figure 13 shows shot gather comparisons between the observed data and modeled data using the initial and final inverted models following Strategy-I, stage 1 of Strategy-II, stage 3 of Strategy-II and Strategy-III. The shot gather from Strategy-I shows the poorest match with the true data and is quite similar to that from the isotropic inversion (stage 1 of Strategy-II and Strategy-III). It seems that Strategy-I and the isotropic inversion converge on similar levels of the objective function. We hypothesize that this is because the anisotropic parameter inversion fails in Strategy-I due to trade-offs with the seismic velocities. Compared with Strategy-I, the cycle skipping in diving waves and some reflections are significantly reduced by Strategy-II owing to the multi-stage approach. We also observe that Strategy-III, in which we ignore η during the FWI, successfully provides modeled data quite similar to those from Strategy-II.

Conclusions

As the number of FWI parameters increases, the uncertainties and the trade-offs between parameters become large, and it is difficult to obtain good inversion results. Compared with VTI media, TTI media require an additional parameter, i.e., the tilt angle, to be correctly described. As a result, FWI for TTI media is more difficult than that for VTI media. For the successful FWI for TTI media, both the optimal parameterization and the inversion strategy are essential. To mitigate the trade-off between parameters, which is the major problem in multi-parameter FWI, we need to analyze the sensitivity of each model parameter. A sensitivity analysis was generally performed by investigating the radiation patterns of the partial-derivative wavefields due to the perturbation in each model parameter. The radiation pattern analysis in this paper was performed based on isotropic and local TTI background media, where only perturbation points have TTI properties, to focus on the source mechanism changes and to ignore the effects of complex anisotropic propagation.

The observations from the analyses of the radiation patterns of the model parameters in TTI media are summarized as follows: 1) the virtual sources of the seismic velocities and the density are not strongly affected by tilted anisotropy; 2) the radiation patterns from the virtual sources of dimensionless anisotropic parameters tend to be rotated by the background tilt angle; 3) the shape of the virtual source for the tilt angle is mainly affected by the background ε and η ; and 4) the influence of η , which mainly affects data at intermediate incidence angles, is relatively weaker than that of ε in FWI.

From these observations, we designed two TTI FWI strategies (Strategy-II and Strategy-III) that first invert the isotropic parameters (i.e., the seismic velocities), then recover the anisotropic parameters (i.e., ε and η), and finally reconstruct the tilt angle. Strategy-III (focused multi-stage FWI) is different from Strategy-II in that this strategy does not consider η to be an

inversion parameter because η does not influence the FWI of surface seismic data significantly and is not recovered well with the $(v_h \ v_s \ \varepsilon \ \eta \ \rho \ \theta_T)$ parameterization.

Numerical examples showed that the good quality of the inverted ε is an important requirement for a successful TTI inversion. In the parameterization we used, the virtual source of ε diffracts most energy vertically; thus, we lose low wavenumber updates for ε at large scattering angles (Oh and Alkhalifah, 2016). For this reason, we had to rely on a relatively low-frequency band (peak frequency of 2.5 Hz) to guarantee a sufficiently good elliptical VTI model. As the FWI results using Strategy-I show, the quality of the inverted ε is also affected by the P-wave velocity model, which means that we require a sufficiently good P-wave velocity model; otherwise, the ε gradients will suffer from trade-off imprints from v_h differences. For this reason, multi-stage approaches (Strategy-II and Strategy-III) from isotropic parameters to anisotropic parameters can be better solutions than a single-stage approach (Strategy-I).

Numerical examples also showed that the poor estimation of η degrades the recovery of the tilt angle. We compared the performance of the two multi-stage FWI strategies with that of the simultaneous FWI strategy (Strategy-I) for the downsized BP synthetic TTI model. The numerical examples showed that the two multi-stage FWI strategies mitigate the trade-off artifacts from the P-wave velocity in the inverted anisotropic parameters. In addition, while the poor estimation of η in Strategy-II degraded the recovery of the tilt angle, assuming $\eta=0$ (although this assumption is incorrect) in Strategy-III yielded better results for the tilt angle.

From another perspective, because the number of parameters is smaller in the focused multi-stage FWI strategy than in the original strategy, we can also achieve better computational efficiency using Strategy-III. It is well known that pseudo-acoustic modeling techniques for anisotropic media generate shear-wave artifacts unless the media are elliptic. We expect that the focused multi-stage FWI strategy can be applied to pseudo-acoustic TTI FWI not suffering

from shear-wave artifacts.

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Table 1 Three elastic TTI FWI strategies

	Stage 1	Stage 2	Stage 3
Strategy I	Simultaneously, update $(v_h, v_s, \varepsilon, \eta, \theta_T)$		
Strategy II	v_h, v_s	$v_h, v_s, \varepsilon, \eta$	$v_h, v_s, \varepsilon, \eta, \theta_T$
Strategy III	v_h, v_s	v_h, v_s, ε	$v_h, v_s, \varepsilon, \theta_T$

Figure 1 Schematic diagram illustrating the generation of partial-derivative wavefields due to model parameter perturbations (virtual sources) in isotropic background media. The red star indicates the seismic source, and the seven blue dots denote the virtual sources, which are the same distance from the seismic source. The red dashed lines show the fast and slow axes of the VTI anisotropy at the perturbed point. This information is required to interpret the radiation patterns in Figures 2 to 6.

Figure 2 Radiation patterns of the partial-derivative wavefields due to perturbations in (a) v_{ph} , (b) v_s , and (c) the density. The central figures show the scattering patterns obtained from numerical modeling using virtual sources. The left and right figures show the PP (upper) and P-SV (bottom) scattering patterns when the incidence angles are positive and negative, respectively. The yellow dashed lines show the fast and slow axes of the VTI.

Figure 2 (continued)

Figure 3 Same as Figure 2 but for (a) ϵ and (b) η .

Figure 4 Radiation patterns of the partial-derivative wavefields due to perturbations in (a) v_{ph} , (b) v_s , and (c) the density. Compared with Figure 2, each perturbed point is locally TTI, where ε , η , and θ_T are 0.2, 0.1, and -30° , respectively.

Figure 4 (Continued)

Figure 5 Same as Figure 4 but for (a) ϵ , (b) η , and (c) θ_r .

Figure 5 (Continued)

Figure 6 Radiation patterns of the partial-derivative wavefields due to the perturbation in the tilt angle. Compared with Figures 2 and 3, the background medium is isotropic, and each perturbed point is locally VTI, where ε and η are (a) (0.2, 0), (b) (0, 0.2), and (c) (0.2, 0.2).

Figure 6 (Continued)

Figure 7 True model parameters for the downsized synthetic BP model: (a) horizontal P-wave velocity, (b) S-wave velocity, (c) ϵ , (d) η , and (e) tilt angle.

Figure 7 (Continued)

Figure 8 Initial (a) horizontal P-wave and (b) S-wave velocity models. Note that we used isotropic initial models.

Figure 9 Model parameters inverted using Strategy-I, in which we update all 5 model parameters simultaneously.

Figure 9 (Continued)

Figure 10 Inverted seismic velocities at stage 1 of Strategy-II and Strategy-III, in which we update only the P- and S-wave velocities simultaneously.

Figure 11 Inverted model parameters at stage 3 of Strategy-II. Note that we first update the isotropic P- and S-wave velocities and then estimate the anisotropic parameters (ϵ and η) together. Finally, the tilt angle is estimated based on the VTI model inverted in the former stage. The pink arrows in (d) indicate the area where η is well estimated. The white arrows in (e) indicate artifacts in the tilt angle caused by the poor estimation of η indicated by white arrows in (d).

Figure 11 (Continued)

Figure 12 Inverted model parameters at stage 3 of Strategy-III. Note that we first update the isotropic P- and S-wave velocities and then estimate the anisotropic parameter (only ϵ). Finally, the tilt angle is estimated based on the inverted elliptical VTI model (η is 0, which means $\epsilon = \delta$).

Figure 12 (Continued)

Figure 13 Shot gather displaying interleaved observed and modeled data using the initial and final inverted models following Strategy-I, stage 1 of Strategy-II, stage 3 of Strategy-II, and Strategy-III: (a) horizontal and (b) vertical displacement components. For each shot gather, we intersperse 40 traces starting from the modeled data followed by observed data (Kalita and Alkhalifah, 2019).