Coarse-Fine Mixtures Subjected to Repetitive K_o Loading: Effects of Fines Fraction, Particle Shape, and Size Ratio

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Abstract:

This study explores the effects of fines fraction, particle shape, and size ratio on the long-term response of sand-silt mixtures with fines fraction $F_F = 0$ -to-100% during K_o -loading cycles $i = 10^4$. The void ratio for all specimens evolves toward the asymptotic terminal void ratio e_T that captures the transition from coarse-to-fine controlled behavior. Shear wave velocity versus intergranular e_C and equivalent intergranular $(e_C)_{eq}$ void ratios estimates the participation rate of fines-in-sand for the force chain. More fine particles contribute to the load-carrying skeleton as the particle shape becomes rounder and the size ratio decreases. In contrast, shear wave velocity against interfine e_F and equivalent interfine $(e_F)_{eq}$ void ratios reflects the reinforcing effect of coarse grains floating in the load-carrying fines matrix. The rounder sand in the fine-dominant matrix leads to more significant fabric changes. Clearly, there is a transition from coarse-to-fine controlled repetitive load-deformation response that promotes the reassessment of current soil classification systems

Keywords: Repetitive Mechanical Load, Fines Fraction, Particle Shape, Size Ratio.

1 INTRODUCTION

Soils often experience repetitive loading cycles. Repetitive loads originate from many different sources such as wind, waves, pore-water pressure changes, temperature cycles, and dry-wet cycles [1-5]. A large number of these loading cycles imposed on sediments alter their engineering properties and state parameters. Long-term repetitive load is likely to compact or loosen sediments depending on packing density, resulting in stiffness change [6] and the development of excess pore-water pressure that can weaken the strength of soils [7].

Previous studies have explored the long-term effect of repetitive mechanical loads on pure coarse or pure fine sediments [8-13]. However, soils are granular materials consisting of a broad range of grain sizes; therefore, the role of fines as a minor component in coarse-fine mixtures should be considered for the long-term performance of geosystems [14-17]. On the other hand, floating sands in fine-dominant mixtures reinforce force chains [18]. Furthermore, particle shape and size ratio determine the participation rate of fines-in-sand or sand-in-fines in the load-carrying skeleton, and so also contribute to changes in engineering properties. However, the effect of particle shape and size ratio on the formation of force chains remains unclear when coarse-fines mixtures are subjected to repetitive mechanical loads.

This study explores the effects of fines fraction, particle shape, and size ratio on the long-term response of sand-silt mixtures in terms of terminal void ratio and small strain stiffness. This study starts with a theoretical framework to describe conceptual void ratios that captures the role of fines-in-sand and sand-in-fines, followed by an experimental program. Experimental results and analyses highlight that there is a clear transitional zone from coarse-to-fine controlled mechanical response that depends on particle shape and size ratio.

2 THEORETICAL FRAMEWORK

The gravimetric-volumetric analysis of binary mixtures defines the threshold fines fraction and conceptual void ratios such as the intergranular, equivalent intergranular, interfine, and equivalent interfine void ratios. These are briefly summarized below.

2.1 Threshold Fines Fraction

Soils are granular materials consisting of a wide range of grain sizes; therefore, the engineering properties of soils depend on not only its major components, but also on its minor components [19-21]. Fines as the minor component in a binary mixture lead to the transition of void ratio at the threshold fines fraction F_{th} [14, 17-18]. The gravimetric-volumetric analysis of binary mixtures defines the threshold fines fraction as being when fine grains completely occupy pore spaces between coarse grains in terms of the void ratio of coarse grain e_C and fine grain e_F :

$$F_{th} = \frac{M_F}{M_T} = \frac{M_F}{M_C + M_F} = \frac{1}{1 + \frac{M_C}{M_F}} = \frac{1}{1 + \frac{V_{s,C} G_{s,C}}{V_{s,F} G_{s,F}}} \approx \frac{e_C}{1 + e_C + e_F} \quad F_C = 1 - F_F \quad (1)$$

Then, the packing conditions of coarse and fine components lead to the two threshold fines fractions. We obtain the low threshold fines fraction $F_{th}|^L$ when coarse grains are densely packed, and fine grains are loosely packed [22-23]:

$$F_{th}|^L = \frac{e_C^{\min}}{1 + e_C^{\min} + e_F^{\max}} \quad (2)$$

On the other hand, we expect the high threshold fines fraction $F_{th}|^H$ when coarse grains are loosely packed, and fine grains are densely packed. However, extensive soil property data accumulated in the laboratory and the field require a correction factor $\lambda=1.3$ to compute the data-adjusted high threshold fines fraction [22]:

$$F_F|^{H*} = \frac{\lambda \cdot e_C^{\max}}{1 + \lambda \cdot e_C^{\max} + e_F^{\min}} \quad (3)$$

The low and data-adjusted high threshold fines fractions divide binary mixtures into three categories: coarse-dominant $F_F < F_{th}|^L$, transitional $F_{th}|^L < F_F < F_{th}|^{H*}$, and fines-dominant $F_F > F_{th}|^{H*}$ mixtures.

2.2 Intergranular and Equivalent Intergranular Void Ratios

The coarse-dominant mixtures at $F_F < F_{th}|^L$ allow us to consider the concepts of intergranular e_C and equivalent intergranular void ratio (e_C).

2.2.1 Intergranular void ratio e_C . Fines may not participate in the force chains of a load-bearing coarse skeleton and they do not disturb the load-bearing coarse skeleton when the fines fraction is smaller than the threshold value $F_F < F_{th}$ (i.e., ideal packing of binary mixtures - fines are very small and do not disturb the primary packing of the coarse grain and vice-versa, McGeary [24]; Marion et al. [25]; Koltermann and Gorelick [26]). Hence, we employ the concept of intergranular void ratio e_C (also known as the sand skeleton void ratio, Kuerbis et al. [27]) assuming the fines as void in the binary mixture (see detailed derivation in Choo and Burns [17]):

$$e_C = \frac{e + F_F}{1 - F_F} \quad \text{where } F_F < F_{th}|^L \quad (4)$$

where e is global void ratio ($e = V_V/V_S$). The intergranular void ratio e_C increases as the fines fraction F_F increases at $F_F < F_{th}|^L$. The concept of intergranular void ratio e_C built on the ideal packing condition involves several limitations:

- ideal packing is almost impossible in laboratory tests (Koltermann and Gorelick [26])

- the packing condition of mixtures becomes closer to the ideal packing when the relative size ratio R_d is greater than $R_d > 100$ ($R_d = D_{50}/d_{50}$ where D_{50} = median grain size of coarse grains and d_{50} = median grain size of fine grains, Marion et al. [25])
- the intergranular void ratio underestimates the active contact of fines in force chains or the disturbance effect of fines on the load-bearing coarse skeletons
- this void ratio does not reflect the fact that fines with a wide range of plasticity causes different packing conditions for coarse-fines mixtures

These limitations lead to the use of equivalent intergranular void ratio $(e_c)_{eq}$ addressed below.

2.2.2 Equivalent intergranular void ratio $(e_c)_{eq}$. Let us introduce the b -parameter in the intergranular void ratio e_c to reflect the limitations listed above, and then the equivalent intergranular void ratio $(e_c)_{eq}$ is (Thevanayagam [18]):

$$(e_c)_{eq} = \frac{e + (1-b)F_F}{1 - (1-b)F_F} \quad \text{where } F_F < F_{th}^L \quad (5)$$

The physical meaning of b -parameter follows:

- $b=0$: all fines fill completely pore spaces between coarse grains, or the relative size ratio R_d is very large, i.e. $R_d > 100$.
- $b=1$: all fines participate in force chains, or the relative size ratio is $R_d=1$.

In other words, the more fines participate actively in load-bearing skeletons with a higher b -parameter. The b -parameter ranges from 0-to-0.57 [14-15, 28-29].

2.3 Interfine and Equivalent Interfine Void Ratios

The fine-dominant mixtures at $F_F > F_{th}|^H$ lead to the concepts of interfine e_F and equivalent interfine void ratio $(e_F)_{eq}$.

2.3.1 Interfine void ratio e_F . Coarse grains in a binary mixture will disperse when the fines fraction is greater than the threshold value $F_F > F_{th}|^H$; thereafter, the effect of coarse grain on force chains is negligible. This conceptual packing defines the interfine void ratio e_F assuming the volume of coarse grains as zero $V_C = 0$ (see details in Thevanayagam [18]):

$$e_F = \frac{e}{F_F} \quad \text{where } F_F > F_{th}|^H \quad (6)$$

The interfine void ratio e_F indicates that the global void ratio e overestimates the intensity of active contacts. On the other hand, the interfine void ratio e_F underestimates the intensity of active contacts because the zero-volume assumption $V_C = 0$ cannot capture the reinforcing effect of dispersed coarse grains on the load carrying fines matrix.

2.3.2 Equivalent interfine void ratio $(e_F)_{eq}$. Equivalent interfine void ratio $(e_F)_{eq}$ overcomes the limitation of interfine void ratio e_F and takes into account the concept of equivalent number of fine grains N_{eq} that remains the same total number of contacts when fine grains replace one coarse grain (see detailed derivation in Thevanayagam [30]):

$$(e_F)_{eq} = \frac{e}{F_F + \frac{(1-F_F)N_{eq}}{R_d^3}} = \frac{e}{F_F + \frac{1-F_F}{R_d^m}} \quad \text{where } F_F > F_{th}|^H \quad (7)$$

where m is fitting parameter ($0 < m < 1$) and the R_d is relative size ratio. The equivalent interfine void ratio $(e_F)_{eq}$ indicates that the role of dispersed coarse grains on active force chains becomes

more significant with a smaller relative size ratio, i.e., $R_d \rightarrow 1$ and a lower fitting parameter m , i.e., $m \rightarrow 0$ that represents a higher equivalent number of fines grains N_{eq} .

3 EXPERIMENTAL STUDY

In this experimental study, we explore the evolution of void ratio and shear wave velocity in sand-silt mixtures for different particle shape and size ratio subjected to the repetitive K_o -loading.

3.1 Materials

We use the KAUST 1/2, KAUST 20/30, and Ottawa 20/30 sands, as well as silica flour, to prepare sand-silt mixtures (KAUST 1/2: diameter $D=1\sim 2\text{mm}$, roundness $R=0.53$, specific gravity $G_s=2.65$, maximum and minimum void ratios $e_{\max}=0.82$ and $e_{\min}=0.59$; KAUST 20/30: $D=0.60\sim 0.85\text{ mm}$, $R=0.60$, $G_s=2.65$, $e_{\max}=0.79$ and $e_{\min}=0.55$; Ottawa 20/30: $D=0.60\sim 0.85\text{ mm}$, $R=0.91$, $G_s=2.65$, $e_{\max}=0.74$ and $e_{\min}=0.50$; silica flour: $D < 0.075\text{mm}$, $G_s=2.65$, $e_{\max}=1.50$ and $e_{\min}=0.74$). Fig. 1 presents the macroscopic images for each material.

3.2 Specimen Preparation

We prepare sand-silt mixtures with six different fines fractions $F_F = 0\%, 10\%, 20\%, 30\%, 60\%$, and 100% by mass $F_F = M_F / (M_C + M_F)$ for three different sands. The maximum $e_{\max}$ and minimum $e_{\min}$ void ratios for the three different sands-silt mixtures help to attain the target initial relative densities $D_r = (e_{\max} - e) / (e_{\max} - e_{\min}) = 70\%$ (Fig. 2). The relative size ratio $R_d = D_{50}/d_{50}$ are $R_d = 20$ for KAUST 1/2 sand-silt mixtures and $R_d = 10$ for both KAUST 20/30 sand-silt and Ottawa 20/30 sand-silt mixtures.

3.3 Instrumented Oedometer Cell

This study uses a tall floating ring oedometer cell to minimize the friction effect on load-deformation response. (Fig. 3, floating ring: ID = 52.5 mm, wall thickness $t=3.9$ mm, and height $H=100$ mm; caps: OD = 50.8 mm and height $h = 50.8$ mm). Top and bottom caps involve parallel-type bender elements for both source and receiver to minimize crosstalk and attain better shielding [31]. The dimensions of the bender elements are $12.7 \times 8 \times 0.7$ (length $\times$ width $\times$ thickness in mm). A 5 mm cantilevered length of the bender elements ensures better contact in the soils. An LVDT (TransTek DC 0242) clamped to the top and bottom caps tracks changes in specimen height.

3.4 Repetitive Loading System and Data Acquisition

Fig. 3 presents the stress-controlled loading system. It consists of a stiff reaction frame, the tall floating ring oedometer cell, a pneumatic cylinder, and peripheral electronics. The PID controller (Proportional-integral-derivative controller) activates the sinusoidal stress amplitudes for the repetitive vertical loadings (see details in Park and Santamarina [3]). The vertical deformation data are saved by the data logger (Keysight 34970A). The function generator sends step input signals ($V=10$ volt, $\text{freq}=20$ Hz, Keysight 33210A). The mechanically-transformed shear waves pass through the soils. The filter-amplifier establishes a 500 Hz high-pass and 200 kHz low-pass window (Krohn-Hite 3364). The computer saves the average signal from the oscilloscope after the stacking of 1024 signals (Keysight DSOX 2014A-stacking in Santamarina and Fratta [32]).

3.5 Test Procedures

The loading history involves four stages: (1) static step loading to $\sigma_o = 65\text{kPa}$, (2) repetitive K_o -loading with stress amplitude $\Delta\sigma = 100\text{kPa}$ by $N=10^4$ cycles, (3) static step loading to $\sigma_o = 465\text{kPa}$, and (4) unloading. Note that we measure the shear waves during the cyclic loading at the same level of vertical effective stress $\sigma_o + \Delta\sigma$ only at the number of cycles $i = 1, 10, 100, 300, 1000, 3000$, and 10000 . The period of the loading cycle is 12 seconds (i.e., frequency = 0.083 Hz).

4 EXPERIMENTAL RESULTS

This section presents the evolution of void ratio and shear wave velocity for the sand-silt mixtures during static and repetitive K_o -loadings. Detailed experimental results are below.

4.1 Vertical Effective Stress σ'_v vs. Void Ratio

Fig. 4 shows the change in void ratio for selected KAUST 1/2 sand-silt mixtures with fines fractions $F_F = 0\%, 10\%, 50\%$, and 100% during static-repetitive-static loading history followed by unloading. The repetitive loading cycles result in greater contraction with a higher fines fraction.

4.2 Void Ratio Evolution.

Fig. 5 presents the evolution of void ratio against the number of cycles for five KAUST 1/2 sand-silt mixtures. Most changes in the void ratio take place in the first one hundred cycles and the repetitive load-induced compaction becomes more significant as the fines fraction increases.

Fig 5 also plots a plastic strain accumulation model (dotted line) overlain on the experimental results [3]:

$$e_i = e_T + (e_1 - e_T) \left[1 + \left(\frac{i-1}{N^*} \right)^m \right]^{-1} \quad \text{for } i \geq 0 \text{ and } m > 0 \quad (8)$$

where e_1 is the void ratio measured after 1st cycle, and the terminal void ratio e_T corresponds to the asymptotic void ratio as the number of cycles reaches infinite $i \rightarrow \infty$. The model parameter N^* captures the number of cycles where the half of the asymptotic contraction $(e_0 - e_T)/2$ occurs and m is a fitting constant.

4.3 Shear Wave Signature

Fig. 6 presents the shear wave signals for FALST 1/2 sand-silt mixtures recorded during the “static-repetitive-static” loading followed by the unloading. The first arrival time decreases during the first static loading stage. Changes in the arrival time are not clear during the repetitive loading; therefore, we use Coda wave analyses to capture these minute changes in travel time [33].

4.4 Shear Wave Velocity

The shear wave velocity V_s is a power function of vertical effective stress σ'_v [34]:

$$V_s = \alpha \left(\frac{\sigma'_0}{1 \text{ kPa}} \right)^\beta \quad (9)$$

where the α -factor is the shear wave velocity at effective stress $\sigma'_v = 1 \text{ kPa}$ and the β -exponent captures the sensitivity of shear wave velocity to the effective stress. The shear wave velocity-stress relationship reflects both contact behavior and fabric changes.

Fig. 7(a) presents the evolution of shear wave velocity during the “static-repetitive-static” loading history. The α -factor and β -exponent estimated by fitting Eq. (9) before and after repetitive loading indicate that particle contacts increase, and the soil fabric becomes less sensitive to stress change after repetitive loading. Overall, the sand-silt mixture becomes stiffer.

Fig. 7(b) shows the evolution of shear wave velocity with the number of cycles. The shear wave velocity increases with number of cycles for all sand-silt mixtures. The linear trend in the log (i) reveals that the shear wave velocity at i^{th} cycles can be anticipated by power function of the number of cycles with shear wave velocity after first cycle (i.e., $V_{s|i} = V_{s|1} \cdot i^d$).

5 ANALYSES AND DISCUSSION

In this section, we reproduce the results to analyze the effects of fines fraction, particle shape, and size ratio on the transition from sand-controlled to fines-controlled load-deformation behavior in terms of terminal void ratio e_T followed by variation of shear wave velocity.

5.1 Terminal Void Ratio e_T

Fig. 8 describes the terminal void ratio e_T for three sands-fines mixtures versus initial void ratio e_o [Eq. (8)]. The terminal void ratio for the fines fraction $F_F=0\text{-to-}20\%$ decreases as the initial void ratio decreases while the terminal void ratio for the fines fraction $F_F=60\text{-to-}100\%$ increases with an increase in the initial void ratio. Clearly, the fines fraction $F_F \sim 30\%$ captures the transition from decreasing terminal void ratio in sand-dominant mixtures ($F_F=0\text{-to-}20\%$) to increasing terminal void ratio in silt-dominant mixtures ($F_F=60\text{-to-}100\%$) as a function of initial void ratio e_o .

5.2 Shear Wave Velocity: Fines Fraction, Particle Shape, and Size Ratio

Fig. 9 plots the shear wave velocity V_s for the KAUST 1/2, KAUST 20/30 and Ottawa 20/30 sands-silt mixtures measured after the first loading cycles versus global e , intergranular e_C , interfine e_F , equivalent intergranular $(e_C)_{eq}$, and equivalent interfine $(e_F)_{eq}$ void ratios.

- global void ratio e - Fig. 9(a) presents the shear wave velocity versus global void ratio e . The shear wave velocity for all sands-silt mixtures with $F_F < 30\%$ decreases as the global void ratio decreases while the shear wave velocity for the mixtures with $F_F > 30\%$ decreases with higher global void ratio. In particular, this binary trend captures the sand-to-fines controlled shear stiffness at the fines fraction $F_F \sim 30\%$.
- intergranular e_C and interfine e_F void ratios - Fig. 9(b), shows the shear wave velocity plotted in terms of intergranular ($F_F \leq 30\%$) or interfine ($F_F > 30\%$) void ratios [Eqs. (4) and (6)]. The intergranular and interfine void ratios lead to the linear trend of shear wave velocity being limited only for the fines fractions $F_F = 0\%$, 10% , and 100% . Note that the shear wave velocity for the fines fraction $F_F = 10\%$ better fits the linear line with more angular particle shape and with higher relative size ratio (i.e., $R_d = 20$). The shear wave velocity for the fines fractions 20% , 30% , and 60% shows the limitations of concepts of intergranular void ratio e_C and the interfine void ratio e_F . The silt-in-sand for $F_F = 20\%$ and 30% actively participate in force chains and the volume of silt should not be considered as void. On the other hand, the sand-in-silt for $F_F = 60\%$ reinforces the load-bearing fine matrix and contributes to the small strain stiffness; therefore, they cannot be assumed as zero-volume material.
- equivalent intergranular $(e_C)_{eq}$ and equivalent interfine $(e_F)_{eq}$ void ratios - Fig. 9(c) illustrates the shear wave velocity versus equivalent intergranular $(e_C)_{eq}$ ($F_F \leq 30\%$) and equivalent interfine $(e_F)_{eq}$ ($F_F > 30\%$) void ratios by introducing the b -parameter and m -value [Eqs. (5)

and (7)]. Once again, the coarse-fine mixture attains a higher b -parameter as more fines participate in force chains. The b -parameter for the mixtures with $F_F = 10\%$, 20% , and 30% increases as the fines fraction increases, the roundness of sand increases, and the relative size ratio decreases. The rounder Ottawa 20/30 sand tends to attain dense-packing configuration in comparison to angular sands [35]. Furthermore, the rounder Ottawa sand is prone to slip at particle contacts. Thus, fines between the rounder sand particles readily participate in force chain and it results in the higher b -parameter. In contrast, the m -value applied to the mixtures with $F_F = 60\%$ indicates that floating sand-in-silt still contributes to the fine-dominant force chain. The m -value decreases as the sand becomes rounder. This observation indicates that the rounder Ottawa 20/30 sand leads to a higher equivalent number of fine grains N_{eq} and has a greater reinforcing effect in the fine-dominant matrix than angular KAUST 20/30 sand.

5.3 Repetitive Loading: b -parameter and m -value

Fig. 10(a) presents the changes in b -parameter Δb during the repetitive loading cycles $i = 1 \rightarrow 10,000$ for the all three sand-silt mixtures in terms of fines fraction F_F . The b -parameter for all mixtures with $F_F = 10\%$, 20% and 30% increases as the loading cycle increases ($i = 1 \rightarrow 10,000$). This observation captures that repetitive mechanical loads yield the additional volume contraction of entire sand-fines mixtures; thereafter, more silt-in-sand participates actively in force chain and contributes to wave propagation in a small strain level. Furthermore, the change in b -parameter Δb reflects the initial packing condition of a coarse-fines mixture subjected to repetitive mechanical loads. The change in b -parameter Δb tends to be more significant as the coarse-fines mixture becomes closer to the ideal packing conditions (i.e., a higher relative size

ratio R_d , a smaller fine fraction, and more angular shape). Thus, the change in b -parameter Δb appear to be a good indicator to represent the initial fabric of the coarse-fines mixtures.

Fig. 10(b) shows that the fitting parameter m for all three sands-silt mixtures with $F_F = 60\%$ decreases during the loading cycles $i = 1 \rightarrow 10,000$. The reduction in fitting parameter m reflects that repetitive mechanical loads lead to the increase in contact number between sand and silt. In particular, the rounder Ottawa 20/30 sand is subjected to more significant fabric changes associated with particle rolling, sliding, and rotation, and attains the greatest contact number. Note that angular sand hinders particle mobility that promotes dense-packing configuration, and it frustrates particle rearrangement during repetitive loading [36]. Clearly, the particle shape of floating sands plays a critical role in fine-dominant fabric stability rather than relative size ratio.

5.4 Toward the Revised Soil Classification System RSCS.

We reemploy the variation of the maximum and minimum void ratios for three sands-fines mixtures with different fines fractions by mass as shown in Fig. 2. The estimations of the low F_F^L and data-adjusted high threshold F_F^{H*} fines fraction are superimposed on Fig. 2 [Eqs. (2) and (3)] and properly capture the transitions from coarse-to-fine controlled behavior. Either coarse, fines, or both fractions can control the mechanical response of transitional mixture at $F_F^L < F_F < F_F^{H*}$. Clearly, the particle shape of sands and the relative size ratio affect the two threshold fines fractions.

Fig. 11 presents the shear wave velocity versus the fines fraction for Ottawa 20/30 sand-silt mixtures. The RSCS boundaries superimposed on Fig. 11 also anticipate the soil fraction(s) that controls the small strain stiffness of sand-silt mixtures, i.e. either sand S , sand with fines SF , or fines F (See details in Park and Santamarina [22]; Park et al. [23]).

Experimental results and data analyses reveal that the silt-in-sand has a marked effect on repetitive loading-induced asymptotic contraction and small strain sediment stiffness even when the fines fraction F_F is significantly lower than the $F_F = 50\%$ used for the first split of coarse-grained or fine-grained soil in the Unified Soil Classification System USCS. Moreover, the sands in the $F_F = 60\%$ mixtures still influence load-deformation responses even though the sand-in-silt float in a fine-dominant matrix. These observations encourage us to use the physics-inspired and data-driven soil classification boundaries in the Revised Soil Classification System RSCS.

6 CONCLUSIONS

This study explores the effects of fines fraction, particle shape, and size ratio on the load-deformation response of sand-silt mixtures subjected to repetitive K_o -loading. Experimental study monitors the evolution of void ratio and small strain stiffness of three sands-silt mixtures with fines fraction $F_F = 0$ -to-100% for different particle shapes and relative size ratios. All specimens are subjected to repetitive loading cycles $i = 10,000$. Notable conclusions made from this study follow:

- A gravimetric-volumetric analysis defines low $F_{th}|^L$ and data-corrected high $F_{th}|^{H*}$ threshold fines fractions. Ideal packing conditions lead to the definition of intergranular void ratio e_C by assuming the fines as void when the fines fraction is lower than the low threshold fines fractions $F_{th}|^L$. In contrast, a zero-volume assumption of coarse grains results in the interfine void ratio e_F when the fines fraction is larger than data-corrected high threshold fines fractions $F_{th}|^{H*}$.
- The void ratio for all specimens tends to evolve toward an asymptotic void ratio. The strain accumulation model fitted to the void ratio data anticipates the terminal void ratio when the

number of loading cycles $i \rightarrow \infty$. Changes in the terminal void ratio for all sand-silt mixtures with fines fraction $F_F = 0$ -to-100% exhibit the transition from coarse-to-fine controlled behavior at $F_F \sim 30\%$.

- Shear wave velocity measured during the repetitive loading cycles $i = 10^4$ is plotted in terms of equivalent intergranular $(e_c)_{eq}$ void ratios and anticipates the participation rate of fines-in-sand for the force chain. Overall, repetitive loading cycles enhance the role of fines on the load-carrying skeleton: more fine particles contribute to the force chain as the particle shape becomes rounder and the size ratio decreases.
- On the other hand, repetitive loading cycles improve the reinforcing effect of floating coarse grains on a load-carrying fine matrix. The shear wave velocity expressed by equivalent interfine void ratio $(e_F)_{eq}$ indicates that the effect of floating sand-in-fines on small strain stiffness is primarily controlled by particle shape rather than size ratio. Relevant phenomena include that the rounder sand in a fine-dominant matrix leads to more significant fabric change associated with particle rolling, sliding, and rotation; thereafter, the enhanced particle level shear stiffness between the sand and silt facilitates the wave propagation in a small strain level.
- In particular, silt-in-sand or sand-in-fines cannot be negligible and plays a crucial role in the asymptotic terminal void ratio and small strain stiffness; then, there is the clear transitional fines fraction range that is much lower than 50% and varies with soil index properties. This observation promotes the reassessment of current soil classification systems for engineering purposes.

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NOTATION

b	parameter of fines participation
D	grain diameter
D_r	Relative density
D_{50}	median grain size of coarse grain
d_{50}	median grain size of fine grain
e	global void ratio
e_C	intergranular void ratio
e_C^{max}	maximum void ratio of coarse grain
e_C^{min}	minimum void ratio of coarse grain
e_F	interfine void ratio
e_F^{max}	maximum void ratio of fine grain
e_F^{min}	minimum void ratio of fine grain
e_i	void ratio measured after i^{st} cycle
e_{max}	maximum void ratio
e_{min}	minimum void ratio
e_T	terminal void ratio
e_1	void ratio measured after 1 st cycle
$(e_C)_{eq}$	equivalent intergranular void ratio
$(e_F)_{eq}$	equivalent interfine void ratio
F_C	coarse grain fraction
F_F	fines fraction
$F_{th }^H$	high threshold fines fraction
$F_{th }^{H*}$	data-adjusted high threshold fines fraction
$F_{th }^L$	low threshold fines fraction
G_s	specific gravity
$G_{s,C}$	specific gravity of coarse grain
$G_{s,F}$	specific gravity of fine grain
M_C	mass of coarse grain
M_F	mass of fine grain
M_T	mass of total grain
m	fitting parameter
N_{eq}	equivalent number of fine grain
N^*	model parameter
R	grain roundness
R_d	relative size ratio
V_S	volume of soil
$V_{S,C}$	volume of coarse grain
$V_{S,F}$	volume of fine grain
V_V	volume of void
V_s	shear wave velocity
λ	correction factor
η	fitting constant

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FIGURE CAPTION LIST

- Fig.1.** Microscopic images for tested materials. (a) KAUST 1/2 sand ($D=1\sim 2\text{mm}$), (b) KAUST 20/30 sand ($D=0.60\sim 0.85\text{mm}$), (c) Ottawa 20/30 sand ($D=0.60\sim 0.85\text{mm}$), and (d) Silica flour ($D<0.075\text{mm}$).
- Fig.2.** Maximum and minimum void ratios for sand-silt mixture versus fines fraction F_F by mass. (a) KAUST 1/2 sand-silt mixture, (b) KAUST 20/30 sand-silt mixture, and (c) Ottawa 20/30 sand-silt mixture. Low threshold fines fraction $F_F^L = e_C^{\min}/(1+e_C^{\min}+e_F^{\max})$ and data adjusted high threshold fines fraction $F_F^{H*} = \lambda \cdot e_C^{\max}/(1+\lambda \cdot e_C^{\max}+e_F^{\min})$ are superimposed on void ratio data [Input for (a): $e_C^{\max}=0.82$, $e_C^{\min}=0.59$, $e_F^{\max}=1.50$, $e_F^{\min}=0.74$, $\lambda=1.3$, $F_F^L=19.2\%$, $F_F^{H*}=38.1\%$, input for (b): $e_C^{\max}=0.79$, $e_C^{\min}=0.55$, $e_F^{\max}=1.50$, $e_F^{\min}=0.74$, $\lambda=1.3$, $F_F^L=18.0\%$, $F_F^{H*}=37.1\%$, and input for (c): $e_C^{\max}=0.74$, $e_C^{\min}=0.50$, $e_F^{\max}=1.50$, $e_F^{\min}=0.74$, $\lambda=1.3$, $F_F^L=16.8\%$, $F_F^{H*}=35.7\%$].
- Fig.3.** Repetitive K_o loading system. The peripheral devices measure deformation and shear waves. Notation: BE= Bender elements, LVDT= Linear variable displacement transducer
- Fig.4.** Change in void ratio of KAUST 1/2 sand-silt mixture during the static-repetitive-static loading history followed by unloading. (a) Fines fraction $F_F=0\%$, (b) Fines fraction $F_F=10\%$, (c) Fines fraction $F_F=60\%$, and (d) Fines fraction $F_F=100\%$.
- Fig.5.** Evolution of void ratio for KAUST 1/2 sand-silt mixture with the number of cycles. Dotted lines indicate the strain accumulation model.
- Fig.6.** Shear wave signals of KAUST 1/2 sand-silt mixture during the static-repetitive-static loading followed by unloading.
- Fig.7.** Changes in shear wave velocity during the loading history. (a) Shear wave velocity versus vertical effective stress during the static-repetitive-static loading, (b) Shear wave velocity versus the number of loading cycles.
- Fig.8.** Terminal void ratio e_T against initial void ratio e_o . (a) KAUST 1/2 sand-silt mixture, (b) KAUST 20/30 sand-silt mixture, and (c) Ottawa 20/30 sand-silt mixture.
- Fig.9.** Shear wave velocity measured after first loading cycle for the KAUST 1/2 sand, KAUST 20/30 sand, and Ottawa 20/30 sand-silt mixtures versus void ratios. (a) Global void ratio, (b) Intergranular e_C and interfine e_F void ratios, and (c) Equivalent intergranular $(e_C)_{eq}$ and equivalent interfine $(e_F)_{eq}$ void ratios.
- Fig.10.** Variation of soil parameters as the number of cycle $i=1\rightarrow 10,000$. (a) b -parameter versus fines fraction F_F , (b) m -value versus fines fraction F_F . Note: open circle indicates the parameter corresponding to the $i=1$ and closed circle indicates the parameter after $i=10,000$.
- Fig.11.** Shear wave velocity for Ottawa 20/30 sand-silt mixture versus fines fraction F_F . Note: open circle indicates the parameter corresponding to the $i=1$ and closed circle indicates the parameter after $i=10,000$. Transition boundaries $F_F^L = e_C^{\min}/(1+e_C^{\min}+e_F^{\max})$ and $F_F^{H*} = \lambda \cdot e_C^{\max}/(1+\lambda \cdot e_C^{\max}+e_F^{\min})$ are superimposed on data (input: $e_C^{\max}=0.74$, $e_C^{\min}=0.50$, $e_F^{\max}=1.50$, $e_F^{\min}=0.74$, $\lambda=1.3$, $F_F^L=16.8\%$, $F_F^{H*}=35.7\%$).

Credit Author Statement

Sang Yeob Kim: Methodology, Data curation, Writing- Original draft.

Junghee Park: Conceptualization, Investigation, Writing- Review & Editing.

Jong-Sub Lee: Validation, Supervision, Funding acquisition.

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Highlights

- Fines fraction, particle shape, size ratio affect soil response under repetitive load
- Experimental study tracks void ratio and shear wave velocity of binary mixtures
- Results show clear transition from coarse-to-fine controlled behavior
- Equivalent intergranular void ratio captures fines participation rate in force chain
- Equivalent interfine void ratio reflects reinforcing effect of floating coarse grains

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

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Figure 8

Figure 9

Figure 10

Figure 11