Slot Antenna Array Synthesis Using the Infinitesimal Dipole Model Technique: Theory and Experiment

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Abstract—The present paper proposes and validates a new general design methodology that can be used to automatically find proper positions and orientations of waveguide-based radiating slots capable of realizing any given radiation beam profile. The new technique combines basic radiation theory and waveguide propagation theory in a analytical model that permits the prediction of the radiation characteristics of generic slots without the need to perform full-wave numerical solution while completely avoiding using transmission-line models or the slot impedance method. The analytical model is then used to implement a low-cost objective function within a global optimization scheme and subsequently deployed to find optimum positions and orientations of clusters of radiating slots cut into the waveguide surface such that any desired beam pattern can be obtained, leading to very efficient and robust beamforming capability that can be applied to dynamic beamforming in real-time scenarios like MIMO systems. The proposed design method is verified using both full-wave numerical solution and experiment.

Index Terms—Slotted waveguide, Genetic Algorithm, Infinitesimal dipole model

I. INTRODUCTION

Slot arrays are among the earliest technologies originally developed for applications involving wireless communications [1]. They have been applied to domains as diverse as near-field focusing [2], beam shaping [3], and many others.¹ Many of their applications in radar and communication systems stem from this particular antenna's characteristics, such as low-profile design requirements, mechanical robustness, good efficiency, and the relative ease of realization compared with other conventional antenna arrays. More recently, it also found applications in the emerging field of mmWave antennas [4], where dynamic beamforming is expected to play a fundamental role in 5G wireless communication systems [5] and massive MIMO [6].

The present paper focus is on devising robust and highly efficient synthesis algorithms that can be used for beamforming applications in 5G and mmWave technology. In particular, we emphasize the need to develop slot antenna array technology capable of changing the shape of the main radiated beam in *real-time*. For the sake of definiteness, we concentrate in this paper on waveguide-based slot arrays, though the method proposed here is quite general and can be applied to any radiating slot configuration. The main idea

in waveguide-based slot array technology is to avoid direct feeding circuits at each array element by utilizing instead only one common 'substructure' supporting an illumination field impinging on the slots' open areas. According to the classic surface equivalence theorem, the impinging electric field will generate an equivalent magnetic current that can efficiently radiate into free space. Using classical array theory, beamforming can be attained by, for example, changing the amplitude and/or phases of each slot's excitation. In this case, no special matching circuit and electronic phase shifters are needed at each radiating slot antenna but the design challenge now shifts toward the problem of how to control the relative amplitude and phase excitation of each slot in order to achieve a desired radiation beam pattern.

In order to perform this change in the slot excitation in real-time, a mechanical means is usually required. In recent years, RF MEMS switches became available as a reliable method to electronically control the physical layout of microwave antennas and circuits [7], and applications of this promising technology to slot antenna arrays for beamforming have been already outlined [8], [9]. The present paper proposes and validate a general methodology that can help the antenna and communication engineer in finding the best locations and orientations of each slot in order to satisfy such general requirements imposed on the radiation pattern. Since waveguide-based slot antennas are energized by the propagating guided mode's field, the excitation information of each radiating slot can be ultimately reduced to knowledge of the position and orientation of each slot cut into the waveguide's broadside wall. However, while it is possible to change and control the slot excitation using RF MEMS switches, the real challenge remains in how to find the *proper values* of the required excitation in *real-time*. That is, for a given beam shape, it is required to construct an algorithm fast enough that can obtain optimum slot location and position while using the typically expensive full-wave analysis information of the radiation problem. We next review briefly some of the work done in this area before introducing our own contribution to the subject.

Early in the beginning of the slotted waveguide technology, methods have been proposed to determine the positions/orientations of the slots in order to achieve a given radiation pattern [10], [11]. More recently, the original approach by Elliot [12] was exploited to advance optimization-based algorithms for slot antenna arrays [13]–[15]. These

¹The literature on slot antennas is massive and only a selective list of references is given throughout this paper.

methods, however, could be too complicated and appear to be cumbersome to realize. The basic idea in Elliot [12] was to consider each half-wavelength waveguide section with one slot as a basic ‘slot module’ with variable load impedance attached to it. Afterwards, an equivalent circuit model for this ‘slot module’ is found, which includes mutual coupling with other slots and the slot self impedance. But in order to find its impedance, the slot is either replaced by the active impedance of an equivalent dipole strip (using Babinet principle) [1], or full-wave analysis is deployed in order to estimate each slot’s impedance, and subsequently tabulating the results for future use. The designer will then try to use a combination of proper load impedances and excitations at each ‘slot module’ in order to realize the desired excitation on the slot itself. Using conventional array theory, the radiation pattern of the slot array is then actualized through the array factor of classical array theory.

The present paper proposes a different approach to the problem of slot array design that is considerably easier to implement for this particular problem, but also much more general in its applicability. *The method is based on the empirical observation by the authors that by allowing the slot length to be slightly off the resonant length, the electric field inside each slot in an array of slots tends, to a very good approximation, to follow the exact theoretical field distribution corresponding to the mode excited in the waveguide.* Most importantly, it seems that mutual coupling between the slots does not alter the field distribution when several slots at various positions co-exist on the same waveguide’s wall. This implies that it is possible in principle to dispense with full-wave analysis or measurement in the design process itself if the field radiated by the slots can be evaluated in a simple manner. This we achieve by replacing the fields distribution in each slot by an equivalent magnetic dipole model with magnetic moment following the aforementioned theoretical field distribution. Such transition from the electric field in the slot to the equivalent magnetic dipole is effected via the classic surface equivalence theorem. In this way, there is no need to find the slot impedance or to consider the total impedance matrix of the entire slot array as has been done so far in the literature. Preliminary results about the synthesis of a slotted waveguide using this new technique have been reported in [16]. In the present paper, we first develop a general theoretical treatment of coupled radiating slots using the surface equivalence theorem and a new magnetic-type current Green’s function. After attaining a physical understanding of slot-slot interaction in the near field, the various stages of the design algorithm will be elaborated in details then carefully validated, each step individually, by extensive numerical simulations using the FDTD package Microwave Studio CST. The combined design algorithm will then be applied to a practical array synthesis problem and good results will be reported. Finally, the attained design is physically implemented in the lab and measurements collected, which confirms the theoretical and numerical results.

II. A THEORY OF MUTUALLY-COUPLED RADIATING SLOT ARRAYS

A. Theoretical Analysis of Radiating Waveguide Slots

It is well-known that a slot is equivalent to a magnetic dipole through the (surface equivalence theorem) relation [1]

$$\mathbf{M} = -\hat{n} \times \mathbf{E}, \quad (1)$$

where $\mathbf{M}$ is the magnetic current, $\mathbf{E}$ the electric current and $\hat{n}$ the normal vector (see Fig. 1). In order to facilitate the

Figure 1. Equivalent model of a slot.

various methodological and design aspects to be elaborated throughout, we introduce a general theoretical model describing the physics of excitation and radiation by slots, especially in light of the presence of strong mutual coupling between the slots themselves. The key ideas of this model are based on the antenna current Green’s function (ACGF) method [17]–[20], particularly the electromagnetic mutual coupling theory outlined in [18], [21], [22], [23] and further expanded in [24].

Assume a single slot is cut into a waveguide wall and let the surface (opening) of the slot be given by the region S_a . From (1), the radiating slot can be modeled by a magnetic current $\mathbf{M}$. Since such magnetic currents are related to the input excitation fields by linear operators [25], it is possible to use the methods introduced in [24] to prove that a *magnetic antenna current Green’s function* exists, which we here denote by $\bar{\mathbf{F}}_m(\mathbf{r}, \mathbf{r}')$. The key property enjoyed by such Green’s function is the following relation

$$\mathbf{M}(\mathbf{r}) = \int_{S_a} ds' \bar{\mathbf{F}}_m(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}_{\text{ex}}(\mathbf{r}'), \quad (2)$$

where $\mathbf{E}_{\text{ex}}(\mathbf{r}')$ is the tangential field excitation on the slot surface S_a . This relation can be interpreted as a statement of superposition of slots magnetic currents where the total value of the current at position $\mathbf{r}$ is seen as the sum of contributions emerging from the excitation fields at *all* other points $\mathbf{r}' \in S_a$ on the slot such that each contribution is mediated by a transfer function $\bar{\mathbf{F}}_m(\mathbf{r}, \mathbf{r}')$.

It is important to remember why the induced magnetic current $\mathbf{M}$ is *not in general* equal to $-\hat{n} \times \mathbf{E}_{\text{ex}}$, as would be inferred from an incautious application of (1). Indeed, since a waveguide with a slot is a new electromagnetic boundary value problem, the field inside waveguide + slot is not necessary the same as waveguide-only fields. This is because a slot large enough can introduce a disturbance or perturbation in the waveguide field [25]. Mathematically, we write the actual waveguide field $\mathbf{E}_a$ in terms of the excitation (ideal waveguide) field using the operator equation

$$\mathbf{E}_a = \mathcal{L}\mathbf{E}_{\text{ex}}, \quad (3)$$

where again $\mathbf{E}_{\text{ex}}$ denotes the unslotted waveguide field on S_a . The exact form of the operator $\mathcal{L}$ can be derived using the methods of [25]. Afterwards, the magnetic ACGF in (2) can be constructed using the techniques introduced in [17] and [24]. Because of the way it was constructed, the magnetic ACGF relation (1) is rigorous and exact. In what follows, we will not need to compute the ACGF itself, but work instead with magnetic currents measured directly using near field probes.

Next, the magnetic ACGF introduced above will now be used to describe how *electromagnetic mutual coupling* or *slot-slot interaction* can be described in order to develop the main topic of this paper, the slot-array reconfigurable beam synthesis algorithm. Indeed, as discussed in the Introduction, the main difficulty is to how to account for the actual magnetic current on the radiating slot since there is a mutual interaction between slots close enough to each other. Consider two slots, which we label as Slots 1 and 2, with surface regions S_{a_1} and S_{a_2} . The excitation fields $\mathbf{E}_{\text{ex}}$ will be referred to from now on as $\mathbf{E}_{\text{wg}}(\mathbf{r})$, $\mathbf{r} \in S_{a_1}, S_{a_2}$, i.e., the propagating field mode distribution actually excited in the waveguide system feeding the slots. The total magnetic current on the two slots can be written in terms of the magnetic ACGF (2) as

$$\mathbf{M}(\mathbf{r}) = \int_{S_{a_1} \cup S_{a_2}} ds' \bar{\mathbf{F}}_m(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}_{\text{wg}}(\mathbf{r}'). \quad (4)$$

If we define the individual currents on each slot by functional restriction operation

$$\begin{aligned} \mathbf{M}_1(\mathbf{r}) &= \mathbf{M}(\mathbf{r})|_{\mathbf{r} \in S_{a_1}}, \quad \mathbf{E}_{\text{wg}}^1(\mathbf{r}) = \mathbf{E}_{\text{wg}}(\mathbf{r}')|_{\mathbf{r} \in S_{a_1}}, \\ \mathbf{M}_2(\mathbf{r}) &= \mathbf{M}(\mathbf{r})|_{\mathbf{r} \in S_{a_2}}, \quad \mathbf{E}_{\text{wg}}^2(\mathbf{r}) = \mathbf{E}_{\text{wg}}(\mathbf{r}')|_{\mathbf{r} \in S_{a_2}}, \end{aligned} \quad (5)$$

it is possible to find using the general mutual coupling ACGF approach that [24]

$$\begin{aligned} \mathbf{M}_1(\mathbf{r}) &= \int_{S_{a_1}} ds' \bar{\mathbf{F}}_m^{11}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}_{\text{wg}}^1(\mathbf{r}') \\ &\quad + \int_{S_{a_2}} ds' \bar{\mathbf{F}}_m^{12}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}_{\text{wg}}^2(\mathbf{r}'), \end{aligned} \quad (6)$$

$$\begin{aligned} \mathbf{M}_2(\mathbf{r}) &= \int_{S_{a_1}} ds' \bar{\mathbf{F}}_m^{21}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}_{\text{wg}}^1(\mathbf{r}') \\ &\quad + \int_{S_{a_2}} ds' \bar{\mathbf{F}}_m^{22}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}_{\text{wg}}^2(\mathbf{r}'), \end{aligned} \quad (7)$$

where $\bar{\mathbf{F}}_m^{11}$ and $\bar{\mathbf{F}}_m^{22}$, respectively, play the role of Slots 1 and 2 self-interaction Green's (transfer function). On the other hand, the dyadic functions $\bar{\mathbf{F}}_m^{12}$ and $\bar{\mathbf{F}}_m^{21}$ represents the *mutual* or *cross* slot-slot interactions. Note that it was shown in [18], [22], [24] that even self-interactions ACGFs are affected by mutual coupling, but to simplify the analysis we focus on cross-interactions in what follows since eventually we will devise a method to eliminate mutual coupling altogether.

The relations (6) and (7) constitute the most general expression that can be written for the slot current excitation including mutual coupling. We quickly note the main advantages of this magnetic ACGF (m-ACGF) model:

- 1) The m-ACGF model is rigorous and exact because it is based on the ACGF, itself an exact concept [17].

- 2) The m-ACGF model is very general because it is applicable to *any* waveguide propagating mode field $\mathbf{E}_{\text{wg}}$. In particular, there is no need to find new m-ACGF every time we change the waveguide mode. The very idea of the ACGF is that the slot system electromagnetic system representation in terms of the m-ACGF is invariant to the input excitation field.
- 3) The m-ACGF model is exact for any slot position or shape since mutual coupling over the entire slot area is taken into account through the surface integrals in (6) and (7).

There are, however, serious difficulties encountering the use of the general model (6) and (7) above for the array slot synthesis algorithm proposed here. First, we note that measuring or estimating the m-ACGF is not currently available, though the relations (6) and (7) can be readily established using the general methods presented in [24]. Second, even if the m-ACGF is known, computing the slot magnetic currents by means of (6) and (7) is computationally expensive in real-time applications like reconfigurable slot array systems. For these two reasons, we leave the direct use of the full m-ACGF model to a future work. In what follows, we show that it is possible to completely eliminate mutual coupling in the slot array system using a simple trick.

B. Generalized Theory of Slot-Slot Mutual Coupling

It is clear from the surface equivalence theorem (1) that in the ideal situation when there is no interaction between the waveguide excitation field and the slot system, the magnetic current is nothing but the tangential component of this very excitation field itself. Therefore, with no mutual coupling with other slots or disturbance of the waveguide mode itself, the slot m-ACGF is nothing but the identity dyadic operator $\bar{\mathbf{I}}$, that is

$$\bar{\mathbf{F}}_m^0(\mathbf{r}, \mathbf{r}') \simeq \bar{\mathbf{I}}, \quad (8)$$

where in that case we have

$$\mathbf{M}(\mathbf{r}) \simeq \mathbf{E}_{\text{wg}}(\mathbf{r}'). \quad (9)$$

This provides a simple method to estimate the *mutual coupling magnetic ACGF* $\delta\bar{\mathbf{F}}_m(\mathbf{r}, \mathbf{r}')$, which has been introduced in [24] for electric-type ACGF, as the difference between the full m-ACGF (with mutual coupling) and the ideal no-interaction m-ACGF in (8). In other words, we may write

$$\delta\bar{\mathbf{F}}_m(\mathbf{r}, \mathbf{r}') := \bar{\mathbf{F}}_m(\mathbf{r}, \mathbf{r}') - \bar{\mathbf{F}}_m^0(\mathbf{r}, \mathbf{r}') = \bar{\mathbf{F}}_m(\mathbf{r}, \mathbf{r}') - \bar{\mathbf{I}}. \quad (10)$$

This general definition can be applied now to, say, Slots 1 and 2, with their individual m-ACGF given by

$$\begin{aligned} \bar{\mathbf{F}}_m^1(\mathbf{r}, \mathbf{r}') &= \bar{\mathbf{F}}_m^{12}(\mathbf{r}, \mathbf{r}') + \bar{\mathbf{F}}_m^{11}(\mathbf{r}, \mathbf{r}'), \\ \bar{\mathbf{F}}_m^2(\mathbf{r}, \mathbf{r}') &= \bar{\mathbf{F}}_m^{21}(\mathbf{r}, \mathbf{r}') + \bar{\mathbf{F}}_m^{22}(\mathbf{r}, \mathbf{r}'), \end{aligned} \quad (11)$$

which in light of (10) becomes

$$\begin{aligned} \delta\bar{\mathbf{F}}_m^1(\mathbf{r}, \mathbf{r}') &= \bar{\mathbf{F}}_m^{12}(\mathbf{r}, \mathbf{r}') + \bar{\mathbf{F}}_m^{11}(\mathbf{r}, \mathbf{r}') - \bar{\mathbf{I}}, \\ \delta\bar{\mathbf{F}}_m^2(\mathbf{r}, \mathbf{r}') &= \bar{\mathbf{F}}_m^{21}(\mathbf{r}, \mathbf{r}') + \bar{\mathbf{F}}_m^{22}(\mathbf{r}, \mathbf{r}') - \bar{\mathbf{I}}. \end{aligned} \quad (12)$$

For a two-slot array system, the mutual coupling m-ACGF (11) supplies a complete exact and rigorous description of

everything needed regarding how the slot system will respond to a generic waveguide excitation mode.

Next, in order to estimate how strong is electromagnetic coupling between the slots, we use a coupling estimation measure introduced in [23] and is based on the mutual coupling ACGF. The idea is that since the latter is a complete transfer function of the system's mutual coupling effects in space, the overall impact of electromagnetic interactions can be fully assessed by measuring the "operator strength" of the mutual coupling m-ACGF with respect to the total m-ACGF. Generalizing this idea from the electric- to magnetic- current type ACGF, we define

$$\gamma_l := \frac{\|\delta \bar{\mathbf{F}}_m^l(\mathbf{r}, \mathbf{r}')\|}{\|\bar{\mathbf{F}}_m^l(\mathbf{r}, \mathbf{r}')\|}, l = 1, 2, \dots, N, \quad (13)$$

where $\|\cdot\|$ denotes a suitable operator norm in functional analysis. In [22], this norm was approximated using a the matrix norm applied to a MoM-based matrix approximation of the exact electromagnetic operator. Note that for complete generality, we included in (13) multiple measures of mutual coupling strength evaluated separately for all slots indexed by $l = 1, 2, \dots, N$.

Moving now to the magnetic currents themselves, the above theory allows us to immediately write

$$\mathbf{M}_1(\mathbf{r}) = \mathbf{M}_1^0(\mathbf{r}) + \delta \mathbf{M}_1(\mathbf{r}), \quad (14)$$

where $\mathbf{M}_1^0$ is the no-mutual coupling slot current while $\delta \mathbf{M}_1$ is the mutual coupling perturbation.

Generalizing the m-ACGF formalism from two slots to N slots, we can now express this mutual coupling perturbation in terms of all coupled slots through

$$\begin{aligned} \delta \mathbf{M}_l(\mathbf{r}) = & \int_{S_{a_l}} ds' \delta \bar{\mathbf{F}}_m(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}_{wg}^l(\mathbf{r}') \\ & + \sum_{n=N, n \neq l}^N \int_{S_{a_n}} ds' \delta \bar{\mathbf{F}}_m(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}_{wg}^n(\mathbf{r}'), \end{aligned} \quad (15)$$

The relation (15) represents the most general form of coupled-slot array current. It shows that when mutual coupling is strong, it is expected that every waveguide field slot excitation will spill over into all other slots. When that happens, building efficient beamforming algorithms becomes very challenging because the computational cost of implementing (15) in the optimization procedure becomes prohibitively high. Finally, following the same idea in (13), we may quantify the degree of mutual coupling between slots using

$$\gamma_l^M := \frac{\|\delta \mathbf{M}_l(\mathbf{r})\|}{\|\mathbf{M}_l(\mathbf{r})\|}, \quad (16)$$

where $l = 1, 2, \dots, N$, characterize the degree of mutual coupling in every slot.

C. On Mutual Coupling in Waveguide-Based Slot Arrays

The authors have investigated mutual coupling between slots using full-wave analysis to measure the magnetic currents using near-field probes, then used the theory developed above in order to quantify mutual coupling. Using the observation first

predicted theoretically in [18], then proved numerically in [22], the slot-slot mutual coupling transfer function is decomposed into *resonant* and *nonresonant* parts as follows

$$\delta \bar{\mathbf{F}}_m^l(\mathbf{r}, \mathbf{r}') = \delta \bar{\mathbf{F}}_{m, \text{resonant}}^l(\mathbf{r}, \mathbf{r}') + \delta \bar{\mathbf{F}}_{m, \text{nonresonant}}^l(\mathbf{r}, \mathbf{r}'). \quad (17)$$

Here, the resonant interaction part corresponds to the propagating mode component of the interaction fields being fully observed by the slots during the process of mutual coupling. The nonresonant part, however, is dominated by evanescent modes that tend to localize in the space between slots [18], [22]. The authors noticed that in the slot array problem

$$\|\delta \bar{\mathbf{F}}_{m, \text{nonresonant}}^l(\mathbf{r}, \mathbf{r}')\| \simeq 0, \quad (18)$$

which implies that most interactions tend to be of resonant type. Therefore, one can write

$$\delta \bar{\mathbf{F}}_m^l(\mathbf{r}, \mathbf{r}') \simeq \delta \bar{\mathbf{F}}_{m, \text{resonant}}^l(\mathbf{r}, \mathbf{r}'), \quad (19)$$

Based on this mode, in order to destroy mutual coupling, one can operate the slot slightly off resonance. Therefore, we expect that

$$\gamma_l^M \rightarrow 0, \quad \text{when the } l\text{th slot length is slightly off-resonance.} \quad (20)$$

These theoretical considerations have been demonstrated computationally and experimentally as will be shown in the remaining parts of this paper.

III. THE SLOT ARRAY SYNTHESIS ALGORITHM

The proposed beamforming algorithm will utilize the conclusion arrived to above, namely that off-resonant slots don't couple in order to build an analytical model of the radiating slot using the Infinitesimal Dipole Method. To anticipate this analytical method, we need first to describe the non-coupled slot field first, which we describe in the next section.

A. A Model for the Waveguide Slot's Field: Implementation and Validation

We consider a standard rectangular waveguide with the internal field following that of the dominant TE₁₀ mode as in Fig. 2. In this case, the phase variation along the waveguide (in z -direction) (Fig. 3) is given by the exact formula [25]

$$\varphi_{wg} = \frac{2\pi f}{c} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} z, \quad (21)$$

where, f is the frequency (hereafter, the frequency $f = 10$ GHz is chosen and fixed throughout the rest of the present paper); f_c is the mode cutoff frequency. For a TE₁₀ mode, this is equal to $f_c = c/2a$, where a is the broadside dimension of the waveguide as in Fig. 2.

Transversal slots can be cut into the waveguide's walls to create a radiating antenna. While both sides of the waveguide may be utilized for this purpose, in this paper we consider slots on the broadside walls only. In this case, the slots will be inserted along the guided mode's distribution with special positions and orientations allowing a *potential differential* to build up between the two edges of the slot, which will result

Figure 2. Schematic view of a standard waveguide. At 10 GHz a standard waveguide operating at the fundamental TE_{10} mode has a broadside length $a = 23$ mm and a small sidelength $b = 10$ mm. The longitudinal length is $L = 169$ mm.

in inducing a significant electric field internal to the slot and hence lead to effective slot excitation. To obtain an ‘in-phase’ field, we have to alternate the slot position on both sides of the medial axis of its broadside as shown in Fig. 3. We should also mention that a slot inserted at the middle of the broad side will not radiate and the amplitude of the field inside the slot steadily increases as we move away from the medial axis of the waveguide.

The positions of the slot along the x -axis will determine the amplitudes of the fields inside the slot. To estimate analytically this amplitude as a function of the slot position, we rely on the fundamental condition (20) (no mutual coupling between slots), after which it follows that the slot field is equated with the guided mode field given by [25]

$$E_y = C \sin\left(\frac{\pi x}{a}\right) e^{j(\omega t - \beta_g z)}, \quad (22)$$

$$H_x = -C \sin\left(\frac{\pi x}{a}\right) e^{j(\omega t - \beta_g z)}, \quad (23)$$

$$H_z = \frac{j\pi Y_{TE}}{\beta_g a} \cos\left(\frac{\pi x}{a}\right) e^{j(\omega t - \beta_g z)}, \quad (24)$$

where $\beta_g = 2\pi/\lambda_g$ and Y_{TE} are the propagation constant and the wave admittance of the guided mode TE_{10} . As can be inferred from Fig. 3, the amplitude variation of the field inside the slot will depend on the amplitude of the H_z field. Consequently, this variation can be predicted using (22)-(24).

Figure 3. Field distribution on a slotted waveguide and the proper alternating locations for the slot in order to produce broadside radiating antenna system.

To verify our fundamental assumption regarding the slot field predicted by (20), we use full-wave FDTD electromagnetic solver CST. A near-field probe will be deployed to measure the field’s intensity and phase variations everywhere inside the slot. In Fig. 4, the theoretical amplitude variation predicted based on use of (22)-(24) is compared with the actual slot’s amplitude variation calculated by CST as a function of

the distance from the medial axis of the broadside when the condition (20) is applied. Excellent agreement is observed for any location of the slot along the waveguide wall verifies the overall theoretical scheme of Sec. II-C. Therefore, with the help of (21), (22)-(24), it is possible now to predict *analytically* the amplitude and the phase inside the slot as function of the transversal (x, z) position (of the slot’s center) within the waveguide’s wall.

Figure 4. Variation of the maximum field amplitude inside the slot as function of the distance from the medial axis of the broad side

B. The Magnetic Infinitesimal Dipole (MID) Model Method

Once the slot has been modeled by a magnetic current, the current itself can be discretized by replacing it by an array of *magnetic infinitesimal dipole* (MIDs) as will be explained next. The general method of the infinitesimal dipole model (IDM) has been used successfully in many applications, ranging from fundamental antenna modeling [26], mutual coupling and antenna modeling [27], near-field focusing [28], and MIMO antenna design [29]. In the present paper we propose applying the IDM method to develop a slot array design algorithm.

It can be shown that an array of N magnetic infinitesimal dipoles produces the following field expression [27]

$$\mathbf{H}(\mathbf{r}) = \sum_{n=1}^N \frac{1}{4\pi\epsilon_0} (k_0^2 (\mathbf{n}_n \times \mathbf{p}_n) \times \mathbf{n}_n) \frac{e^{-jk_0 r_n}}{r_n}, \quad (25)$$

$$\mathbf{E}(\mathbf{r}) = \sum_{n=1}^N \frac{c_0 k_0^2}{4\pi} (\mathbf{n}_n \times \mathbf{p}_n) \frac{e^{-jk_0 r_n}}{r_n}, \quad (26)$$

where

$$\mathbf{n}_n = \left(\frac{x - x_d^n}{r_n}, \frac{y - y_d^n}{r_n}, \frac{z - z_d^n}{r_n} \right)^T \quad (27)$$

is the unit vector from the location of the n th infinitesimal dipole to the observation point O , and

$$\mathbf{p}_n = A_d^n \begin{pmatrix} \cos(\phi_d^n) \sin(\theta_d^n) \\ \sin(\theta_d^n) \sin(\phi_d^n) \\ \cos(\theta_d^n) \end{pmatrix} \quad (28)$$

is the moment of the n th dipole. Here, A_d^n is the n th dipole (complex) excitation and

$$r_n = \sqrt{(x - x_d^n)^2 + (y - y_d^n)^2 + (z - z_d^n)^2} \quad (29)$$

is the distance from the center of the magnetic infinitesimal dipole $\mathbf{r}_d^n$ to the observation point O , k_0 is the wave-number in free space ($k_0 = 2\pi/\lambda$), ε_0 is the vacuum electric permittivity, c_0 is the velocity of the light, and N is the number of dipoles in the discrete approximation of the continuous current. The angles ϕ_d and θ_d prescribe the orientation of the infinitesimal dipole; x_d, y_d, z_d their positions; and A_d the dipole's complex current. Based on the analytical expressions (25)-(29), the fields radiated by the slot array can be obtained directly and without numerical computation if the orientations, amplitudes and phases of the individual MIDs $\mathbf{p}_n$ are known. Since, as was shown in Sec. III-A, the field inside the slot is well approximated with excellent accuracy by the waveguide mode corresponding to the un-slotted or normal waveguide boundary condition, the dipole model amplitude and phase excitation are directly determined by (22)-(24). Based on numerous simulations, the authors concluded that it is adequate to replace a typical slot (half-wavelength) by a proper array of 15 MIDs. The amplitude and phase excitation of each infinitesimal dipole will manually be adjusted to match the theoretical amplitude and phase distribution of the field's mode inside a slot. In conformity with the theory of Sec. II, We have considered off-resonant slots with each slot having a length slightly above or below $\lambda/2$.

C. IDM Radiation Field Expression

The radiation pattern produced by the slot array can be written as a single function $\mathbf{E}^{\text{rad}} = F(\mathbf{x}_s, \mathbf{o}_s)$ depending only on the slot's positions $\mathbf{x}_s$ and orientations $\mathbf{o}_s$ (each vector $\mathbf{x}_s$ and $\mathbf{o}_s$ is of length M , where M is the number of slots.) More specifically, using the formula (26) for the MD model of one slot, the electric field radiated by an array of M slots, where the m th slot is modeled by N_m MDs, is given by

$$\mathbf{E}^{\text{rad}}(\theta, \phi; \mathbf{x}_s, \mathbf{o}_s) = \sum_{m=1}^M \sum_{n=1}^{N_m} \frac{c_0 k_0^2}{4\pi} (\mathbf{n}_n^m \times \mathbf{p}_n^m) \frac{e^{-jk_0 r_n^m}}{r_n^m}. \quad (30)$$

The data $\mathbf{x}_s(n, m)$ and $\mathbf{o}_s(n, m)$ will determine the location and orientation, respectively, of the n th dipole belonging to the MD's model of the m th slot. Explicitly, one has the following functions

$$\begin{aligned} r_n^m &= r_n^m(\theta, \phi; \mathbf{x}_s, \mathbf{o}_s), \\ \mathbf{n}_n^m &= \mathbf{n}_n^m(\theta, \phi; \mathbf{x}_s, \mathbf{o}_s), \mathbf{p}_n^m = \mathbf{p}_n^m(\mathbf{x}_s, \mathbf{o}_s), \end{aligned} \quad (31)$$

where the explicit functional forms of r_n^m and $\mathbf{n}_n^m$ can be found obtained with the help of (27)-(29). For the complex n th dipole moment belonging to the MD's model of the m th slot, i.e., $\mathbf{p}_n^m$, the direction is given by the data $\mathbf{o}_s(n, m)$ while the amplitude and phase follows from the theoretical model of Sec. II. Therefore, the formula (30) represents a completely analytical expression for the far field radiated by the slot array as accurate as full wave solution. This field-theoretic-based magnetic dipole model will be next combined with a global optimization algorithm in order to provide an automatic method for determining proper positions for each slot in order to reach any target radiation pattern.

D. Design Cost Function Evaluation

Using the theoretical model of the radiating slots described above, we now proceed to the construction of the cost function pertinent to the design problem under consideration, i.e, far-field beam shaping. Because of its extremely low complexity, the fully-analytical model culminating in the radiation formula (30) can be used profitably in an optimization algorithm to design a radiating slot array with a radiation pattern falling below a desired target or radiation mask.

We postulate that a well-designed slot array can be obtained by minimizing an error function $e(\mathbf{x}_s, \mathbf{o}_s)$ with the following form

$$e(\mathbf{x}_s, \mathbf{o}_s; \mathbf{E}^{\text{target}}) = \int_{\Omega_0} d\Omega |\mathbf{E}^{\text{rad}}(\theta, \phi; \mathbf{x}_s, \mathbf{o}_s) - \mathbf{E}^{\text{target}}(\theta, \phi)|, \quad (32)$$

where $\mathbf{E}^{\text{rad}}(\theta, \phi; \mathbf{x}_s, \mathbf{o}_s)$ is the trial far field produced by the array with slot's positions $\mathbf{x}_s$ and orientations $\mathbf{o}_s$. On the other hand, $\mathbf{E}^{\text{target}}$ is the target (desired) radiation pattern. The angular integration is performed over the following azimuthal/elevation region

$$\Omega_0 := \{\Omega : \mathbf{E}^{\text{rad}}(\theta, \phi) > |\mathbf{E}^{\text{target}}(\theta, \phi)|\}. \quad (33)$$

That is, the integral of (34) is computed only over those special angles where the obtained trial field *overshoots* or *fails* to be bounded by the (target) desired field. In other words, the error is set to zero when $|\mathbf{E}^{\text{rad}}(\theta, \phi)| < |\mathbf{E}^{\text{target}}(\theta, \phi)|$, i.e., when the obtained beam is already below the target radiation mask. Note that since here we focus on optimization of radiation pattern or directivity, the mask is normalized since only the shape is relevant. In order to facilitate the construction of the error function to be defined shortly through comparisons between the mask and the attained radiation fields, all field quantities are normalized.

The objective of the slot array synthesis algorithm is to find the optima $\mathbf{x}_s^*$ and $\mathbf{o}_s^*$ such that

$$e^*|_{\mathbf{x}_s, \mathbf{o}_s = \mathbf{x}_s^*, \mathbf{o}_s^*} = \min_{\mathbf{x}_s, \mathbf{o}_s} e(\mathbf{x}_s, \mathbf{o}_s; \mathbf{E}^{\text{target}}). \quad (34)$$

More precisely, the synthesis algorithm can be summarized compactly as the solution to the following general optimization problem

$$(\mathbf{x}_s^*, \mathbf{o}_s^*) = \arg \min_{\mathbf{x}_s, \mathbf{o}_s} e(\mathbf{x}_s, \mathbf{o}_s; \mathbf{E}^{\text{target}}). \quad (35)$$

The waveguide slot antenna array design problem is now reduced to solving the optimization problem (35), which is in general a highly nonlinear with respect to the desired optimization parameters (slot positions and orientations), making the search for best values a formidable problem. Cost functions as those defined by (34) tend to have multiple local minima and maxima, and probably more than one global minimum. Therefore, a powerful global search strategy is needed.

Although the fields radiated by a generic array have been successfully put into the simple and efficient analytical form (30) using the idea of equivalent magnetic dipole model, the resulting error function (35) is impossible to solve using only analytical methods. In particular, local search methods, e.g. gradient-based techniques, though very efficient, tend to get

Table I
MAIN CHARACTERISTICS OF THE GENETIC ALGORITHM

Characteristics	Value
Number of infinitesimal dipoles	15 per slot (10 slots on the waveguide)
Number of variables	2 (x and z positions of the slots)
Number of bits for each variable	9
Number of chromosomes	20
Mutation rate	0.5%
Goal	Minimize the cost function with a maximum of 100 iterations

trapped very quickly in local minima, leading to long search times or premature convergence. On the other hand, in generic antenna design problems, especially beam shaping problems where the requirements of the desired radiation mask may vary considerably from one application to another (often in real-time), it is not possible to afford such long wait time or poor convergence results. To cope with challenging problem, we utilize in this paper a global search method instead of local techniques. We choose here the Genetic Algorithm (GA), which is well known to be stable, robust, and less immune to the pre-tuning of the relevant routine parameters. In this paper, our goal is to obtain optimum slot arrangements which give the best cost function based on a required radiation pattern. The physically relevant optimization parameters are reported in Table I, where each slot is modeled by an array of 15 infinitesimal dipoles. We will consider 10 slots on the waveguide, leading to a total of 150 dipoles. As mentioned in the previous section only the (x, z) position of the slot will be considered. To avoid too lengthy optimization time, we will consider a maximum of 100 iterations.

E. The proposed Array Synthesis Algorithm

Based on the detailed description provided in the previous section, we are in a position now to assemble the various steps of the proposed beamforming algorithm as follows:

- 1) Based on the surface equivalence theorem, the electric field in the slot will be converted into a radiating *magnetic current*.
- 2) Using the analytical fields' expressions of the *magnetic infinitesimal dipole* (MID), we approximate the equivalent radiating slot magnetic current by a small number of MIDs placed in the position and the orientation of the actual slot.
- 3) Using waveguide theory, the mode field is used directly to provide information on how the excitations of the various MIDs in the model of each slot will be chosen. *This provides a complete analytical model of the entire waveguide-slot array that is as accurate as fullwave solutions, making its subsequent use in extensive optimization algorithms possible.*
- 4) The far field produced by the slot array is computed *analytically* from the formula of the MID and a square error function is formed to compare with a desired radiation pattern.

- 5) Finally, through a global optimization algorithm (GA in this case), we search for the best location and orientation of the slots resulting in the best fit of the radiation pattern.

IV. NUMERICAL AND EXPERIMENTAL VALIDATION

A. ID modeling and Simulation

As an example, consider the beamforming goals summarized in Table II. To attain this, we chose slots with off-resonant lengths around 0.49λ (14.7 mm at 10 GHz). The GA was deployed to optimize the positions of the slots modeled by an array of infinitesimal dipoles described above. Table III shows the attained optimized position of the slots on the waveguide. We also provide the radiation pattern results with the axis positions generated using CST Microwave Studio.

Table II
MAIN CHARACTERISTICS OF THE EXAMPLE

Characteristics	Example
HPBW (degrees)	10
Side lobe level (dB)	-10
Beam direction (degrees)	15

In Fig. 5, we illustrate the radiation pattern along θ for $\varphi = 90$ after optimization. We notice a very good agreement between the actual antenna pattern (obtained using FDTD) and the analytical prediction made by the magnetic IDM. The side lobe specification are reached with a maximum side lobe level equal to -11 dB. The slight differences between the IDM and the actual antenna are due to the edge radiation in the simulation, which is not considered in the IDM. If we compare the directivity in both cases, we obtain for the analytical model a directivity of 12.9 dBi, while the FDTD simulated waveguide gives a directivity of 14 dBi.

Table III
OPTIMIZED POSITIONS OF THE SLOTS

Slot	x (mm)	z (mm)
1	2.96	0
2	-1.48	19.9
3	3.71	30.58
4	-2.41	40
5	3.15	60
6	-1.85	75.3
7	2.96	91.7
8	-1.67	100
9	2.59	120
10	-3.15	130.64

B. Experiment and Measurement

Based on the optimized design obtained after the optimization process, the slotted waveguide has been realized in the lab. The system was printed by Object260 Connex1 3D printer by Stratasys. LPKF Pro-conduct silver paste was manually screen-printed to metalize the inner surface of waveguide. The 3D model of the slotted waveguide antenna was made into 2 pieces, as shown in Fig. 6(a), which were joined together after being metalized through their inner sides as in Fig. 6(b). The

Figure 5. Radiation pattern of the optimized array of infinitesimal dipole (solid line) and of the slotted waveguide simulated on CST (dashed line)

critical dimensions of the waveguide antenna i.e. a and b , the positions and locations of the slots, were extracted from CST model and the flanges were added on the input and output ports of the slotted waveguide in order to excite the waveguide and absorb the non-radiated RF signal respectively. Final prototype (in the form of one piece) is shown in Fig. 7(a).

Figure 6. Two pieces of 3D printed antenna (a) - Internal metalized waveguide antenna (b)

The slotted waveguide antenna was excited with the help of an SMA-to-waveguide adapter, while the other end of the antenna was terminated by a matched 50Ω load so that a non-radiated RF signal will be absorbed by that port. In this way, the matched load acts like a perfect absorber, which is needed in order to avoid reflections from port 2. The measure-

ment setup is shown in 7(b). The slotted waveguide antenna performance was measured in anechoic chamber within the frequency range 8 GHz-10 GHz. The 3D radiation pattern has been measured and compared with the simulated one. Fig. 8 illustrates the radiation pattern predictions along θ for $\varphi = 90^\circ$ for the cases of the analytical IDM, the FDTD simulated model, and the measured waveguide. Good agreement between simulation and measurement can be observed. Table IV summarizes the key features of this comparison. Differences in the radiation patterns can be explained by the presence of high $\epsilon_r = 2.8$ of the 3D printed material of 1.5 mm thickness outside the metallic coating of slotted waveguide antenna (which in the case of simulations is the air medium with $\epsilon_r = 1$).

Figure 7. Fabricated waveguide antenna (a) and Measurement setup (b)

Figure 8. Radiation pattern of the optimized array of infinitesimal dipole (solid line) and of the slotted waveguide simulated on CST (dashed line)

Concerning the gain, the FDTD prediction of the gain at 10 GHz is equal to 10 dB while the measured gain is equal to 4 dB (Fig. 9). This difference between the simulated and measured gain can be due to two factors:

- Conductor losses, coming from the surface roughness and the smaller conductivity of the metallic coating used in fabrication as compared to that in the FDTD model.
- Presence of high $\epsilon_r = 2.8$ for the 3D printed material of 1.5 mm thickness outside the metallic coating of slotted waveguide antenna. Indeed this material induces a shift in the operational frequency with a maximum gain around

Table IV
COMPARISONS OF THE OPTIMIZED, THE SIMULATED AND THE MEASURED
RESPONSE OF THE SLOTTED WAVEGUIDE

Property	Desired pattern	Analytic model	Simulation	Measurement
Beam direction (degrees)	15°	15°	16°	18°
HPBW (degrees)	10°	10.5°	10°	10.5°
Side lobe level (dB)	-10 dB	-11 dB	-10.5 dB	-7.5 dB
Side Lobe location	Good agreement			

8.8 GHz. A retro-simulation on CST has shown that if we consider this material, the gain in simulation is equal to 4.6 dB which is closer to the measured one.

Figure 9. Measured Gain of the slotted waveguide

In order to calculate the radiation efficiency based on a 0.49λ slot length, we present in Fig. 10 the simulated and measured S-parameters of the slotted waveguide. As can be seen there, and due to the absorbing material at the output of the waveguide, the antenna is well matched with a S_{11} parameter equal to -17 dB at 10 GHz. However as a result of the slot length modification, the transmission coefficient (i.e. the energy absorbed by the output port) is equal to -2 dB at 10 GHz, which corresponds to a radiation efficiency of about 37%. But by considering the frequency shift due to the waveguide material, the efficiency is around 68% at 8.8 GHz. Moreover the S_{21} coefficient at 10 GHz is lower in simulation which may explain the higher radiation efficiency, hence the higher gain, obtained on CST.

V. TRADEOFF BETWEEN COMPUTATIONAL AND RADIATION EFFICIENCY

We note that the entire mathematical formulation of the design process given above has focused exclusively on controlling the radiation pattern by using an optimization mask function (target radiation pattern). Therefore, we have neither optimized over losses nor even included that in the basic cost function (34). This is quite natural given that as we know from

Figure 10. Measured S-parameters of the slotted waveguide

experience, directivity and losses (which jointly determine the gain) are often in conflict. Therefore, to optimize the gain, two things must be done: (i) introduce an analytical model to accurately estimate losses similar in ease of computation to the novel analytical model of directivity introduced in this paper using the IDM technique; (ii) use multi-objective global optimization algorithm to solve the problem of minimizing the directivity cost (34) and losses simultaneously. Each one of these two steps will take us significantly outside the scope of this paper. Regarding (i), no accurate analytical expression of losses is known to exist in full-wave electromagnetic scenarios. Regarding (ii), multi-objective codes exist but are substantially more complex to use and hence will require separate treatment in other publications. For these reasons, our stated goal from the beginning was to control the radiation pattern (directivity), not gain (which requires optimizing over losses.)

Moreover, and as mentioned previously, the slot length is a fundamental parameter in the analytical dipole model of slot radiation. However, there is a need to strike a compromise between the accuracy of the model and the efficiency of the antenna. In the present paper, for purposes of simplifying fabrication and measurement process, we selected an off-resonant slot length of 0.49λ and demonstrated that choice leads to a rather good model accuracy comparable with the FDTD simulated and the measured antenna, where the desired directivity pattern has been fully achieved while the attained is around 37% (up to 68% by considering the frequency shift). To highlight the effect of the slot length on these two parameters, we perform a parametric study, where the slot length was varied and the corresponding error between theoretical infinitesimal dipole model and the simulated antenna was reported, together with the radiation efficiency. The error is estimated using Mean Square Error (MSE) e calculation as follows

$$e = \sqrt{\frac{\sum_{\Omega} |F_{\text{fdd}} - F_{\text{th}}|^2}{\sum_{\Omega} |F_{\text{fdd}}|^2}}, \quad (36)$$

where F_{fdd} the simulated radiation pattern and F_{th} the theoretical radiation pattern obtained with the infinitesimal dipole model. The error defined above averages over all angles

$\Omega := \theta, \phi$. The results of this parametric study are shown in Fig. 11. We can clearly see that for perfect resonant slots (around 0.5λ), the efficiency is maximal but the model accuracy is very poor (because of mutual coupling), with an error around 14%. On the other hand, at nonresonant frequency (length), the efficiency may drop to as low as 5% while the model accuracy is very good. This parametric study shows that a compromise has to be established regarding the slot length. Computational and radiation efficiencies may be traded off for

Figure 11. Efficiency and accuracy of the model as function of the slot length (in units of wavelength).

each other in our method. The main goal of this paper has been established then, which is to provide the working engineering with a method to improve beamforming algorithms speed by sacrificing some radiation power whenever this is profitable for the overall wireless communication system. However, we note that more precise fabrication tools can build slots closer to the resonant length, leading to both good model and radiation efficiency. The example given in this paper was mainly a proof of concept but the fundamental design methodology is quite general and can also be used to investigate similar functions with other antenna types.

VI. CONCLUSION

A novel method to efficiently design waveguide-based slot arrays was proposed. Through a careful theoretical analysis of slot-slot coupling using a magnetic-type current Green's function, we predicted that mutual coupling between slots can be eliminated by operating the system at off-resonance length due to the smallness of the nonresonant interactions mediated by evanescent near-field modes. This prediction was verified by demonstrating that such slots can be analytically modeled by a proper array of infinitesimal dipole model (IDM), implemented here as a magnetic-type IDM, with accuracy approaching full-wave FDTD simulations. The infinitesimal dipole model can be used to very efficiently optimize a slotted waveguide in order to achieve any desired radiation pattern (in near-field or in far-field). An experimental proof of concept was provided for 0.49λ slots. It was found that a tradeoff exists between radiation efficiency and computational efficiency. However, if very precise manufacturing process is used to fabricate slighter deviations in slot length from resonance, it is expected that both radiation and computational efficiency can be improved.

In many applications where communication is done in short range, or when higher power is available for short time intervals, e.g., ultra-dense networks and mmWave networks, lower radiation efficiency can be traded-off with improved beamforming algorithm speed. Therefore, we expect that the proposed method will play a role in the forthcoming 5G and mmWave technology.

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