Multiscale Phase Inversion of Anisotropic Data

Shihang Feng¹, Lei Fu², Zongcai Feng¹ and Gerard T. Schuster¹

¹King Abdullah University of Science and Technology (KAUST), Saudi Arabia

²Southern University of Science and Technology, China

SUMMARY

In this paper, we extend the multiscale phase inversion (MPI) methodology to anisotropic media. The MPI method is developed to avoid the cycle-skipping problem of full waveform inversion (FWI) by temporally integrating the traces several times. The input data are also filtered into different narrow frequency bands for the MPI implementation. The simultaneous inversion of anisotropic parameters v_{p0} and ε is performed while δ is set as 0. Numerical tests on synthetic data and field data demonstrate the feasibility of this method.

INTRODUCTION

Conventional full waveform inversion (FWI) inverts for a velocity model by using an objective function that minimizes the L2 norm of the residuals between the predicted and the observed traces (Tarantola, 2005; Virieux and Operto, 2009; Fu and Symes, 2017b). However, such a misfit function is highly non-linear and the iterations often get stuck in a local minimum.

In order to mitigate this problem, a skeletonized representation of the data such as first-arrival traveltimes (Luo and Schuster, 1991a,b; Zhou et al., 1995) can be inverted to obtain the low-to-intermediate wavenumber details of the background velocity model. The misfit function for skeletonized inversion is quasi-linear and enjoys better convergence properties than conventional FWI. The resulting tomograms provide a starting model for FWI and reduce the likelihood of cycle skipping. However, refraction arrivals only provide the low-intermediate wavenumber information of the shallow part of the model. In order to reach deeper depth, the inversion methods has been extended to reflection waves, such as reflection WT (Ma and Hale, 2013; Feng and Schuster, 2017, 2019) and migration velocity analysis (MVA) (Sava et al., 2005; Sava and Vlad, 2008). However, it is still challenging to invert the velocity distribution with the complex geology such as salt dome.

As an alternative, multiscale strategy can also mitigate the cycle skipping problem of FWI (Bunks et al., 1995; Fu and Symes, 2017a). Conventional multiscale FWI apply a low-pass filter on the data, so the objective function enjoy less local minimals. The drawback of this method is that sometimes low frequency components are weaker than the high frequency components.

To combine the multiscale strategy and skeletonized inversion, Sun and Schuster (1993) develop multiscale phase inversion method, where the recorded data are integrated several times in the time domain to boost the low-frequency components. This method had been successfully applied to crosswell data (Sun

and Schuster, 1993) and surface seismic data with isotropic media (Fu et al., 2017). In this paper, we extend the multiscale phase inversion methodology to anisotropic media.

After the introduction, the next section of this paper summarizes the theory of this method, and the workflow is in the third section. Numerical tests on synthetic data and field data are shown in the following sections and the conclusions are in the last section.

THEORY

Multiscale phase inversion can be extended to VTI medium by inverting for the anisotropy parameters v_{p0} , ε and δ using the MPI misfit function,

$$\begin{aligned}\zeta &= \frac{1}{2} \sum_{s,g} \int dt [I^n \bar{d}(\mathbf{g}, t|\mathbf{s}) - I^n \bar{d}(\mathbf{g}, t|\mathbf{s})^{obs}]^2, \\ &= \frac{1}{2} \sum_{s,g} \int dt [\Delta \bar{d}(\mathbf{g}, t|\mathbf{s})]^2,\end{aligned}\quad (1)$$

where $\Delta \bar{d}(\mathbf{g}, t|\mathbf{s}) = I^n \bar{d}(\mathbf{g}, t|\mathbf{s}) - I^n \bar{d}(\mathbf{g}, t|\mathbf{s})^{obs}$ is the data residual, I^n is an integration operator $I = \int dt$, and n indicates integration performed n times, and $\bar{d}(\mathbf{g}, \mathbf{s}, t)$ and $\bar{d}(\mathbf{g}, \mathbf{s}, t)^{obs}$ are predicted and observed modified traces for a point source at $\mathbf{s}$ and a geophone at $\mathbf{g}$.

The modified predicted $\bar{d}(\mathbf{g}, t|\mathbf{s})$ and observed $\bar{d}(\mathbf{g}, t|\mathbf{s})^{obs}$ traces are obtained by the following equation:

$$\begin{aligned}\bar{d}(\mathbf{g}, t|\mathbf{s}) &= \mathcal{F}^{-1}\{L(\omega)|D(\mathbf{g}, \mathbf{s})^{obs}|e^{i\phi(\mathbf{g}, \mathbf{s})}\}, \\ \bar{d}(\mathbf{g}, t|\mathbf{s})^{obs} &= \mathcal{F}^{-1}\{L(\omega)|D(\mathbf{g}, \mathbf{s})^{obs}|e^{i\phi(\mathbf{g}, \mathbf{s})^{obs}}\}.\end{aligned}\quad (2)$$

where $\mathcal{F}^{-1}$ is the inverse Fourier transform and $L(\omega)$ is a low-pass filter. $|D|$ is the magnitude spectrum and ϕ is the phase spectrum that are obtained by the Fourier transform of the predicted data $d(\mathbf{g}, t|\mathbf{s})$ and observed data $d(\mathbf{g}, t|\mathbf{s})^{obs}$ as following

$$\begin{aligned}\mathcal{F}[d(\mathbf{g}, t|\mathbf{s})] &= |D(\mathbf{g}, \mathbf{s})|e^{i\phi(\mathbf{g}, \mathbf{s})}, \\ \mathcal{F}[d(\mathbf{g}, t|\mathbf{s})^{obs}] &= |D(\mathbf{g}, \mathbf{s})^{obs}|e^{i\phi(\mathbf{g}, \mathbf{s})^{obs}}.\end{aligned}\quad (3)$$

The decoupled P-wave equation for vertical transverse isotropic (VTI) media in the time and wavenumber domain is given by (Zhan et al., 2012)

$$\frac{1}{v_{p0}^2} \frac{\partial^2 P}{\partial t^2} = - \left[(1 + 2\varepsilon)k_x^2 + k_z^2 - \frac{2(\varepsilon - \delta)k_x^2 k_z^2}{k_x^2 + k_z^2} \right] P, \quad (4)$$

where k_x and k_z are the spatial wavenumber corresponding to point $\mathbf{x}$. The anisotropic parameters $m(\mathbf{x}) \sim (v_{p0}, \varepsilon, \delta)$ can be estimated using any gradient-based method. To update the

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anisotropic parameters, the steepest descent method gives

$$m(\mathbf{x})^{(iter+1)} = m(\mathbf{x})^{(iter)} - \alpha \sum_{s,g} \Delta \bar{d}(\mathbf{g}, t | \mathbf{s}) \frac{\partial I^n \bar{d}(\mathbf{g}, t | \mathbf{s})}{\partial m(\mathbf{x})}, \quad (5)$$

where α is the step length and $iter$ is the iteration index.

The phase inversion gradient is calculated by the crosscorrelation of the forward-propagated $\bar{p}(\mathbf{x}, t | \mathbf{s}, 0)$ and the backward-propagated wavefield $\bar{g}(\mathbf{x}, t | \mathbf{g}, 0)$ of the residual $\Delta \bar{d}(\mathbf{g}, t | \mathbf{s})$. The gradients of the misfit function with respect to the three anisotropy parameters v_{p0} , ε and δ .

$$\begin{aligned} \gamma_{v_{p0}}^{mpi}(\mathbf{x}) &= \sum_{s,g} \int_0^T dt \left[\frac{2}{v_{p0}^3(\mathbf{x})} I^n \bar{g}(\mathbf{x}, t | \mathbf{g}, 0) \star I^n \bar{p}(\mathbf{x}, t | \mathbf{s}, 0) \right] \\ \gamma_{\varepsilon}^{mpi}(\mathbf{x}) &= \sum_{s,g} \int_0^T dt [I^n \bar{g}(\mathbf{x}, t | \mathbf{g}, 0) \star I^n \bar{U}(\mathbf{x}, t | \mathbf{s}, 0)] \\ \gamma_{\delta}^{mpi}(\mathbf{x}) &= \sum_{s,g} \int_0^T dt [I^n \bar{g}(\mathbf{x}, t | \mathbf{g}, 0) \star I^n \bar{V}(\mathbf{x}, t | \mathbf{s}, 0)] \end{aligned} \quad (6)$$

where $\star$ denotes the temporal convolution, $\bar{U}$ and $\bar{V}$ are given in following.

$$\begin{aligned} \bar{U}(\mathbf{x}, t | \mathbf{s}, 0) &= -2\mathcal{F}^{-1} \left(\frac{k_x^4}{k_x^2 + k_z^2} \right) \star \bar{p}(\mathbf{x}, t | \mathbf{s}, 0), \\ \bar{V}(\mathbf{x}, t | \mathbf{s}, 0) &= -2\mathcal{F}^{-1} \left(\frac{k_x^2 k_z^2}{k_x^2 + k_z^2} \right) \star \bar{p}(\mathbf{x}, t | \mathbf{s}, 0), \end{aligned} \quad (7)$$

where $\star$ denotes the spatial convolution.

Workflow

The workflow of the multiscale phase inversion is shown in Figure 1. The predicted and recorded data are filtered into different frequency bands by applying different bandpass filters. We start with the data with lower frequency contents. The data are integrated N times in the time domain to boost the low-frequency components. The low-boost data are inverted till the misfit change is less than a misfit threshold (ζ_1). Then the integration times decrease by 1. When the integration time is less than 0, the data will be appended with higher frequency content and the MPI procedure is repeated.

SYNTHETIC TESTS

The observed and predicted data are simulated by the rapid expansion method (REM) (Pestana et al., 2011) based on the pseudo-acoustic P-wave VTI wave-equation in equation 4. δ cannot be accurately recovered from such data (Cheng et al., 2014), because surface seismic data are weakly sensitive to variations in the anisotropic parameter δ . Thus δ is set to 0 in the test.

An anisotropic marmousi model is shown in Figure 2, where the model size is 4.6 km in the X direction and 2.84 km in

Figure 1: The workflow of multiscale phase inversion. Images adapted from (Fu et al., 2017)

the Z direction with a grid spacing of 10 m. One hundred and fifteen sources are located on the surface with a spacing of 40 m, and the traces are recorded by 460 receivers spaced at an interval of 10 m on the free surface. The source wavelet is a Ricker wavelet with a peak frequency of 15 Hz.

Monoparameter case

We invert for v_{p0} only with the true ε model. The initial model is shown in Figure 3. The maximal integration time is equal to 2. Five different bandpass filters that the center frequency ranging from 2.5 to 22.5 Hz are applied to the data. Anisotropic and isotropic MPI are applied and the inversion results after 50 iterations are shown in Figure 4. The anisotropic MPI tomograms is consistent with the true model. The quality of isotropic MPI tomogram is poor since the kinematics of the waves are affected by anisotropic effect.

Multiparameter case

Then v_{p0} and ε are inverted simultaneously. The initial models are shown in Figure 5a and 5b. The maximal integration time is 2 and same bandpass filters are used as in the monoparameter case. The MPI tomograms after 65 iterations are shown in Figure 5c and 5d, the v_{p0} tomogram is consistent with the true v_{p0} model. For ε tomogram, only shallow portion are inverted correctly due to its low sensitivity in the deep area. Conventional multiscale FWI gets stuck in a local minimum as shown in Figure 5e and 5f. The reflectors in the box A are mispositioned and the value of ε is incorrect in the box B.

GOM DATA TESTS

The proposed method is also tested on a 2D GOM data set. The input data consist of 100 shot gathers with a shot interval of 37.5 m, and each shot is recorded by a 6 km long cable with

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480 receivers having a 12.5 m receiver interval. The shortest offset is 200 m and the data are processed by a 25-Hz Wiener filter (Boonyasiriwat et al., 2009). Here the source wavelet is extracted from the raw data by stacking the time-shifted reflection events together from 200 to 250 m offset in the shot gather. The reflection traveltimes are then used to time-shift the traces so the reflection events are flattened. These flattened reflection events are stacked together to get an estimate of the source wavelet.

Traveltime tomography is used to provide an initial v_{p0} model, and the ϵ initial model is set as a homogeneous model with $\epsilon = 0.02$ below the sea bottom as shown in Figures 6a and 6b. The maximal integration time is equal to 2. The input data are filtered into five different narrow frequency bands that center frequency ranges from 2.5 to 31.5 Hz for the MPI implementation. The MPI tomograms after 60 iterations are shown in Figures 7a and 7b. The resolution of the MPI tomograms is much higher than the initial v_{p0} and ϵ model.

Figure 8 shows the RTM images constructed from the initial model and MPI tomograms. The zoom views of these images are compared in Figures 9, in which the red arrows point to the areas with noticeable improvements in focusing and continuity. To validate the effectiveness of MPI, conventional multiscale FWI is also tested on this data. However, conventional multiscale FWI falls in local minimum in the first few iterations.

CONCLUSION

An anisotropic MPI method is presented that inverts for the anisotropic parameters v_{p0} and ϵ in a VTI medium. This method integrates the traces several times to boost the low-frequency component of the seismograms. Numerical tests on the synthetic and field data validate that the anisotropic MPI method is more robust than the conventional FWI method. Synthetic examples show that when the initial anisotropic models are far from the true model, the anisotropic MPI method provides more accurate tomograms than FWI. In the GOM case, the MPI method successfully invert the data to obtain tomograms that are more accurate than the initial model while FWI fails to obtain the tomogram. The limitation of this procedure is that the computational cost of anisotropic MPI is about twice that of anisotropic FWI because of the extra iterations associated with the integration operations.

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Figure 2: The true a) v_{p0} and b) ϵ Marmousi models.

Figure 3: The initial v_{p0} model for the monoparameter MPI.

Figure 4: The monoparameter MPI results: the a) anisotropic MPI v_{p0} tomogram after 75 iterations; b) the isotropic MPI v_{p0} tomogram after 40 iterations.

Figure 5: The initial a) v_{p0} and b) ϵ model. The multiparameter MPI results: the anisotropic MPI a) v_{p0} and b) ϵ tomogram after 65 iterations; the anisotropic FWI c) v_{p0} and d) ϵ tomogram after 35 iterations.

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Figure 6: a) The initial a) v_{p0} and b) ϵ models for the GOM data

Figure 8: The RTM image with the a) initial model and b) the MPI tomogram using the GOM data as the input traces.

Figure 7: The anisotropic MPI a) v_{p0} and b) ϵ tomograms after 60 iterations.

Figure 9: The zoom views of the red boxes in the RTM image with the a) initial models and b) the MPI tomograms; the zoom views of the red boxes in the RTM image with the c) initial models and d) the MPI tomograms using the GOM data as the input traces.

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