Regularized elastic passive equivalent source inversion with
full waveform inversion: application to a field monitoring
microseismic dataset

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Running head: **Elastic passive source FWI**

ABSTRACT

One of the key goals of microseismic processing is the accurate estimation of the source location. Utilizing full waveform information in passive-source datasets can potentially delineate microseismic sources. The accuracy of both P- and S-wave velocities has a strong influence on the estimation of source locations and hence the reliability of the fracture detection. We propose a methodology for passive source and velocity inversion, in which the conventional source term of the elastic wave equation is represented by an equivalent source. The equivalent source term is composed of source images and source functions, as it is inspired by elastic reflection waveform inversion. Thus, we update the source locations, source functions and velocities simultaneously by using a waveform inversion scheme. In the 2D isotropic case, the source terms are defined by two source image components and three source function components. They provide an alternative representation of the source mechanism, usually defined by the moment tensor. Waveform inversion of passive events has severe nonlinearity due to the unknown source locations in space and their functions in time. We, thus, use a source-independent objective function, based on convolving reference traces with both modeled and observed data, to avoid cycle skipping caused by the unknown sources. We first synthetically test our method on a modified Marmousi model. Then, by applying a nested inversion for these variables, the proposed method also produces good estimation of the source and background velocity for real microseismic monitoring data. We use a ball-drop event to test the accuracy as the inverted source location should match the ball-seat location. For the uncontrolled events, the estimated source distribution using waveform inversion agrees with the local stress potential information. Though the proposed method has higher computational cost than traveltimes or migration based methods, the estimated event locations have significantly improved accuracy.

INTRODUCTION

There has been an explosive growth of research in the field of passive seismicity, because hydraulic fracturing has been a common technique in unconventional gas and oil fields, especially prevalent in reservoirs with dense rocks such as shale. The microseismic events are generated naturally or manually by hydraulic stimulation (Duncan, 2005). High pressure conditions caused by hydraulic injections crack the dense rock and generate pathways for oil and gas to flow. Such procedures can induce small-magnitude earthquakes, that are known as microseismic events. The location of microseismic events is typically interpreted as the fracture region. The fracture image is typically shown as a cluster of dots that represent source locations of those microseismic events. The accuracy of the fracture image highly depends on the quality of the estimated passive source locations. Therefore, accurate methods to locate these events are critical for guiding the hydraulic process (Warpinski et al., 2009).

Previous studies have shown that many factors can influence the location estimation (Eisner et al., 2009; Maxwell, 2009; Mueller, 2013). The velocity accuracy is one of the important factors. Due to the trade-off between the source location and the velocity model, velocity errors could mislead the source location estimation.

A widely used technique to build the velocity model is to interpret a $1D$ velocity model through the well log information and then calibrate the velocity model by perforation shots (Warpinski et al., 2005; Pei et al., 2008, 2009). Such techniques simply use the traveltimes information to build a $1D$ approximation of the true velocity model. Manual picking of arrivals, however, can hardly be done accurately because the data usually has low signal-to-noise (S/N) ratio (Bose et al., 2009; Kummerow, 2010; Song et al., 2010).

An alternative option is to use a time-reversed imaging (TRI) technique to focus the

recorded microseismic signal energy to the source location (Larmat et al., 2006, 2010; Lu et al., 2008; Steiner et al., 2008). TRI requires less human interaction and it allows for higher accuracy by utilizing the waveform information. We can back-propagate recorded seismograms and use a proper imaging condition, such as the maximum energy imaging condition (Artman et al., 2010; Yang and Zhu, 2019), the geometric mean imaging condition (Nakata and Beroza, 2016) and maximum variance imaging condition (Wang and Alkhalifah, 2017), to extract the source location. One advantage of TRI is that it does not require manual picking, especially when dealing with poor S/N data. If a dense and uniform acquisition system is available, the wavefield will focus at the correct source locations. If, however, only limited traces are available, TRI is not a robust method to accurately locate the sources. Many advanced strategies are proposed to mitigate the issue of irregular and sparse receivers (Fink et al., 2000; Tanter et al., 2000, 2001; Aubry et al., 2001; Jonsson et al., 2004; Larmat et al., 2010; Sava, 2011; Gallot et al., 2012).

All imaging methods using TRI to locate microseismic sources suffer from a misfit between the true velocity model and the one used for the back-propagation, which may lead to misfocused spatial images. Some techniques, such as migration velocity analysis (MVA) (Witten and Shragge, 2015, 2017; Song et al., 2019a), can be introduced to improve the TRI robustness.

Some other approaches, like full waveform inversion (FWI) based methods (Kaderli et al., 2015; Behura, 2015; Sun et al., 2016; Kamei and Lumley, 2017; Song and Alkhalifah, 2019), make it possible to update the microseismic source locations and velocity models by minimizing the differences between the observed and modeled data. The resulting source location is what we refer to as the inverted source location. However, these existing FWI methods focus on the acoustic case and require a good initial velocity model to mitigate the

cycle skipping problem caused by the unknown source onsets, which may be a limitation in practice. Wang and Alkhalifah (2018) used a source independent objective function in microseismic waveform inversion to mitigate the cycle skipping problem caused by unknown onsets. However, the method ignores the elastic effects in real world, which may cause artifacts in source images.

We extend the work proposed by Wang and Alkhalifah (2018) to the elastic medium by utilizing an elastic FWI based method that separately updates the source term and the velocity model with the help of a nested approach. By minimizing the L2-norm data misfit, the gradients of each parameter are derived from the adjoint-state method (Plessix, 2006) under a 2D case assumption. We use a new equivalent source term incorporated into the conventional elastic wave equation. It is inspired by the concept of equivalent sources of elastic reflection waveform inversion (Guo and Alkhalifah, 2017; Li et al., 2019). The new source expression contains separate components of the source spatial and temporal information, named as the source images ($\delta\alpha$ and $\delta\beta$) and source functions ($[w_{ij}]$), respectively. The proposed modeling can be considered as a post-stack exploding reflector modeling extended to elastic media (Loewenthal et al., 1976; Claerbout, 1985). A source independent FWI approach is used to update the velocity models in order to mitigate the cycle skipping problem of the unknown source origin time. By inverting for all these unknowns, our method is able to provide accurate source estimation with initial velocity models, though the initial velocities are very different from the true ones. Testing on a modified Marmousi model shows the effectiveness and accuracy of the proposed method. Then, applying our method on a field monitoring dataset highlights the features of inverting for source images, source functions and velocity models, iteratively, as opposed to the conventional moment tensor inversion, which requires to first locate the source correctly. We first illustrate the

performance of the introduced method by using the ball-drop events recorded during the hydraulic fracture treatment, since these events are generated at the ball-seats whose locations are known. Then we apply the method to all detected microseismic events in the field monitoring data.

THE EQUIVALENT SOURCE TERM IN ELASTIC PASSIVE SOURCE CASE

The passive equivalent source

Full waveform inversion is highly nonlinear, especially under the elastic assumption, due to the complexity of wave propagation in the subsurface medium. Reflection full waveform inversion (RFWI) is one way to reduce such nonlinearity, by splitting the model estimation into low-wavenumber components, which control the kinematics, and high-wavenumber scatters (Hicks and Pratt, 2001; Wang et al., 2013; Alkhalifah and Wu, 2016).

In the isotropic, constant density case, elastic wave propagation satisfies the following wave equation:

$$\rho \ddot{\mathbf{u}} = \mathbf{f} + (\lambda + 2\mu) \nabla (\nabla \cdot \mathbf{u}) - \mu \nabla \times (\nabla \times \mathbf{u}), \quad (1)$$

where $\mathbf{u}$ is the elastic displacement wavefield, λ and μ are Lamé constants; the density, ρ , is considered constant here and f_i is the source term.

In isotropic cases, P- and S-wave velocities (denoted by V_p and V_s , respectively) can be defined as

$$V_p^2 = \alpha = \frac{\lambda + 2\mu}{\rho} \text{ and } V_s^2 = \beta = \frac{\mu}{\rho}. \quad (2)$$

It is assumed that the model parameters can be separated into two parts,

$$\alpha = \alpha_0 + \Delta\alpha \text{ and } \beta = \beta_0 + \Delta\beta, \quad (3)$$

where α_0 and β_0 are the low-wavenumber (background) parts; $\Delta\alpha$ and $\Delta\beta$ are the perturbations of the V_p and V_s velocities, respectively. The total wavefield $\mathbf{u}$ can then be split into two parts correspondingly,

$$\mathbf{u} = \mathbf{u}_0 + \delta\mathbf{u}, \quad (4)$$

where $\mathbf{u}_0$ is the wavefield propagating in the background medium, and $\delta\mathbf{u}$ is the scattered wavefield controlled by the model perturbations. In our 2D implementation, we denote the horizontal direction as the X axis and the vertical direction as the Z axis. Then the perturbations in stress tensor ($\delta\tau$) can be derived, following Hooke's law, as

$$\begin{aligned} \delta\tau_{xx} &= (\alpha_0 + \Delta\alpha) \frac{\partial\delta u_x}{\partial x} + (\alpha_0 + \Delta\alpha - 2\beta_0 - 2\Delta\beta) \frac{\partial\delta u_z}{\partial z} + \tilde{\tau}_{xx}; \\ \delta\tau_{zz} &= (\alpha_0 + \Delta\alpha - 2\beta_0 - 2\Delta\beta) \frac{\partial\delta u_x}{\partial x} + (\alpha_0 + \Delta\alpha) \frac{\partial\delta u_z}{\partial z} + \tilde{\tau}_{zz}; \\ \delta\tau_{xz} &= (\beta_0 + \Delta\beta) \left(\frac{\partial\delta u_x}{\partial z} + \frac{\partial\delta u_z}{\partial x} \right) + \tilde{\tau}_{xz}, \end{aligned} \quad (5)$$

where $\tilde{\tau}_{ij}$ is described as

$$\begin{aligned} \tilde{\tau}_{xx} &= \Delta\alpha \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) - 2\Delta\beta \frac{\partial u_z}{\partial z}; \\ \tilde{\tau}_{zz} &= \Delta\alpha \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) - 2\Delta\beta \frac{\partial u_x}{\partial x}; \\ \tilde{\tau}_{xz} &= \Delta\beta \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right). \end{aligned} \quad (6)$$

These stress components are injected into the system of equation 5, as equivalent sources, to produce the reflected waves at the model perturbations, which is a de-migration proce-

ture. The de-migration requires estimation of the the scatter image (perturbation) and the incident wavefield at the scatter points as source wavelets (Wu and Zheng, 2014; Guo and Alkhalifah, 2017).

We borrow these concepts from the derivation of elastic reflection waveform inversion (ERWI). Passive sources are naturally buried deeply in the subsurface, which can be considered as a single scatter image point in the de-migration procedure. We now consider the passive source mechanism in a form of the equivalent source, which produces the passive seismic events.

The equivalent source in the passive seismic case is now a combination of source images ($\delta\alpha$ and $\delta\beta$), which correspond to a single scatter for de-migration in ERWI, and source functions ($[w_{ij}]$), which is equal to the wavefield at the scatter position in ERWI. It is an alternative representation of the source mechanism to the classic seismic moment tensor.

Now, we rewrite the source term on the RHS of equation 1 by the passive equivalent source term:

$$f_i = \partial_j \tau_{ij}^0, \quad (7)$$

where τ_{ij}^0 is the equivalent source term determined by the source spatial and temporal components. In the $2D$ case, each component of tensor τ_{ij}^0 can be written as:

$$\begin{aligned} \tau_{11}^0 &= \delta\alpha (w_{11} + w_{22}) - 2\delta\beta w_{22}; \\ \tau_{22}^0 &= \delta\alpha (w_{11} + w_{22}) - 2\delta\beta w_{11}; \\ \tau_{12}^0 &= \tau_{21}^0 = \delta\beta (w_{12} + w_{21}). \end{aligned} \quad (8)$$

$\delta\alpha$ and $\delta\beta$ are denoted as source images and $[w_{ij}]$ is the multi-component source time function, which has no dependency on spatial coordinates. By comparing equations 6 and 8,

we find that these stress tensors have the same expression as the back-propagated wavefield (back-propagation of the recorded data from the receiver locations) replaced by $[w_{ij}]$, which means $\frac{\partial u_x}{\partial x}$ is replaced by w_{11} , $\frac{\partial u_z}{\partial z}$ is replaced by w_{22} , and $\frac{\partial u_x}{\partial z}$ and $\frac{\partial u_z}{\partial x}$ are replaced by w_{12} and w_{21} , respectively. Note that in ERWI, $\delta\alpha$ and $\delta\beta$ represent the model perturbations (or the scatter image), and the back-propagated wavefield $\mathbf{u}$ is needed at each model point. In the microseismic scenario, $\delta\alpha$ and $\delta\beta$ represent the actual microseismic sources distribution, which may also be considered as the perturbations of P- and S-wave velocities, respectively, but they only have values at the source position and $[w_{ij}]$ represent the source time function at the source location. Later we will estimate $[w_{ij}]$ by back-propagating the wavefields to this source position. Thus, the source position information $\mathbf{x}^s$ and the source mechanism description $\mathbf{M}$ are embedded into the same terms, source images.

Gradients of the source images and the source function

Considering the observed data $\mathbf{d}(x_r, t)$ from the passive events, we estimate the sources by solving an optimization problem with the misfit given by

$$E(m) = \frac{1}{2} \int \|\mathbf{u}(x_r, t; m) - \mathbf{d}(x_r, t)\|_2^2 dt, \quad (9)$$

with respect to the source images ($\delta\alpha$ and $\delta\beta$), and the source function ($[w_{ij}]$), where $\mathbf{u}(x_r, t; m)$ is the predicted data determined by the current parameters, including $\delta\alpha$, $\delta\beta$, w_{ij} and c_{ijkl} , and recorded by the receivers at x_r . $\delta\alpha$ and $\delta\beta$ can be obtained iteratively using the corresponding gradients:

$$\nabla_{\delta\alpha} E = - \int_t dt (w_{11} + w_{22}) \left(\frac{\partial \hat{u}_x}{\partial x} + \frac{\partial \hat{u}_z}{\partial z} \right), \quad (10)$$

$$\nabla_{\delta\beta} E = - \int_t dt 2w_{11} \frac{\partial \hat{u}_x}{\partial x} + 2w_{22} \frac{\partial \hat{u}_z}{\partial z} + (w_{12} + w_{21}) \left(\frac{\partial \hat{u}_x}{\partial z} + \frac{\partial \hat{u}_z}{\partial x} \right), \quad (11)$$

where $\hat{\mathbf{u}}$ is the adjoint wavefield with respect to the corresponding passive source. Here, $\delta\alpha$ and $\delta\beta$ are the source images representing the source mechanism in a more compact way, in comparison to the moment tensor.

Although we can update the velocity and source function independently, we prefer to apply a simultaneous inversion to obtain a more accurate estimate:

$$\begin{aligned} \nabla E_{w_{11}} &= \int_x \int_z dx dz \delta\alpha \left(\frac{\partial \hat{u}_x}{\partial x} + \frac{\partial \hat{u}_z}{\partial z} \right) - 2\delta\beta \frac{\partial \hat{u}_z}{\partial z}; \\ \nabla E_{w_{22}} &= \int_x \int_z dx dz \delta\alpha \left(\frac{\partial \hat{u}_x}{\partial x} + \frac{\partial \hat{u}_z}{\partial z} \right) - 2\delta\beta \frac{\partial \hat{u}_x}{\partial x}; \\ \nabla E_{w_{12/21}} &= \int_x \int_z dx dz \delta\beta \left(\frac{\partial \hat{u}_x}{\partial z} + \frac{\partial \hat{u}_z}{\partial x} \right). \end{aligned} \quad (12)$$

More details on the gradient derivations are provided in Appendix A.

REGULARIZED PASSIVE SOURCE IMAGES INVERSION

Equations 10 and 11 show the gradients of the source images, but using the source images directly to update the velocity could be a problem, since the source images are often noisy in passive seismic cases. Ideally, the energy should focus at the true source location. However, because of the inaccurate velocities and incorrect wavelet, the source images are not focused. This will further lead to errors in the updates. These errors are caused by the misfocused energy in the source images as they act as sources and introduce artifacts into the modeled data as we iterate. Since we do not have accurate enough initial velocities and source wavelets, it is impossible to focus all the energy to the true source location. To mitigate this issue, we suppress the unwanted energy and constrain the distribution of energy to the

source location, where the source location is determined by the maximum energy point in the source image.

After each iteration, we add a focusing function as regularization to the source images to suppress the misfocused energy. We minimize the following objective function J with respect to image $\mathbf{I}$, which is the desired focused source image:

$$J = \|I - I_0\|^2 + \kappa \left\| \sum_x |\mathbf{x} - \mathbf{x}_s|^2 I \right\|^2, \quad (13)$$

where I_0 is the source image (either $\delta\alpha$ or $\delta\beta$) at the current iteration, I is the source image after regularization, κ is a weighting factor for the focusing function, and it is dependent on the artifact level. Obviously a larger κ (e.g, 5 to 10) will give higher weight to the second term and a smaller κ (e.g, 0.1 to 0.5) will emphasize the first term. We use higher κ in the first few iterations and decrease κ values with iterations as we start to trust the source images more in the later iterations. $\mathbf{x}$ is the spatial coordinates in the model space, and $\mathbf{x}_s$ is the spatial coordinates of the focusing position given by predicted source location for the current iteration, which can be calculated as:

$$\mathbf{x}_s = \frac{\sum_x I_0(\mathbf{x})^2 \mathbf{x}}{\sum_x I_0(\mathbf{x})^2}. \quad (14)$$

The above objective function is constructed for $\delta\alpha$ or $\delta\beta$ separately. More commonly, $\delta\alpha$ and $\delta\beta$ share a similar spatial distribution and if they show any mismatch in space, we define as the average of the two $\mathbf{x}_s$ as the source location for the current iteration. Let the derivative of the objective function J be zero, we then obtain the analytic solution for

equation 13,

$$\begin{aligned}\frac{\partial J}{\partial I} &= 2(I - I_0) + 2\kappa |\mathbf{x} - \mathbf{x}_s|^2 I = 0 \\ \Rightarrow I &= \frac{I_0}{1 + \kappa |\mathbf{x} - \mathbf{x}_s|^2}.\end{aligned}\tag{15}$$

We use a split approach to focus the source images. At each iteration, we calculate the regularized source images I , and use it as the input source image for the next iteration. For comparison, the $\delta\beta$ of an example event with and without the focusing function are shown in Figures 1a and 1b.

These images are inverted from the field monitoring microseismic data of a poorly covered acquisition system, with only 15 receivers on the left boundary. We can clearly see the difference between the two images in which the misfocused energy is greatly suppressed by the focusing function and mainly the energy at the source location is kept. The misfocused energy will lead to erratic updates of velocities and source functions. With the help of the focusing function, we obtain clearer and more stable updates. Note that this focusing penalty function is useful when the source image is imperfect. In such situations, we tend to emphasize the source location rather than the source type. The focusing function will also suppress some useful information around the source location that can be used to describe the source type. Due to the poor illumination provided by the acquisition system, we are not able to recover the source mechanism from the source images. Sacrificing such information in favor of making the algorithm more stable seem to be a good choice to this particular problem. If the acquisition is good enough to recover the source mechanism, we may decrease the weighting factor κ or simply remove this regularization term at later iterations.

VELOCITY UPDATE WITH THE SOURCE-INDEPENDENT INVERSION

In conventional FWI, we update the velocity models by minimizing the L2-norm data misfit function, shown in equation 9. The gradients for α and β , in this case, can be written, respectively as:

$$\nabla_{\alpha} E = - \int_t dt \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) \left(\frac{\partial \hat{u}_x}{\partial x} + \frac{\partial \hat{u}_z}{\partial z} \right); \quad (16)$$

$$\nabla_{\beta} E = - \int_t dt \left(2 \frac{\partial u_x}{\partial x} \frac{\partial \hat{u}_x}{\partial x} \right) + \left(2 \frac{\partial u_z}{\partial z} \frac{\partial \hat{u}_z}{\partial z} \right) + \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) \left(\frac{\partial \hat{u}_x}{\partial z} + \frac{\partial \hat{u}_z}{\partial x} \right). \quad (17)$$

For passive seismic inversion, the initial estimate of the source terms usually is far from the actual ones, for both the spatial and temporal components, and especially the unknown origin time. Directly using the initial source function in the inversion is not acceptable because a small shift in the origin time leads to critical cycle skipping in the data. To avoid this potential problem, we utilize advanced objective functions (Sun and Alkhalifah, 2019). Here, we use a convolution objective function (Choi and Alkhalifah, 2011), which is reasonably independent of the source function by eliminating the effect of the unknown source time. In this case, the objective function, along with the total variation term as a regularization, is given by:

$$E_{indp}(\mathbf{m}) = \frac{1}{2} \int \|\mathbf{u} * \mathbf{d}_{ref} - \mathbf{d} * \mathbf{u}_{ref}\|_2^2 dt + \lambda \sqrt{\frac{\partial \mathbf{m}^2}{\partial x} + \frac{\partial \mathbf{m}^2}{\partial z} + \epsilon^2}, \quad (18)$$

where $\mathbf{d}_{ref}$ and $\mathbf{u}_{ref}$ represent reference traces in both observed and modeled data, respectively. λ is a weighting factor, and ϵ is a smoothing factor. The symbol $*$ represents the convolution operator. In practice, we choose a near-offset trace, which is the deepest receiver trace in our specific dataset, in both the observed and modeled data to be the reference

trace. The aim of adding the total variation regularization to the objective function is to smooth the velocity updates. Although the total variation term makes the gradients blocky, we still can get smoother updates with a larger ϵ . In our implementation, $\lambda = 0.25$ at the first iteration and we linearly decrease it with iterations to zero. The velocity model could be far from the true one in the early iterations so we use a larger weighting factor. As we update the models, the velocities are more reliable so we use a smaller weighting factor. ϵ is set to 1% of the model summation over space through the whole inversion since it is large enough to smooth the gradients. In this new objective function, the true source function is in both terms, equally; this means that the effect of the delay in the source time function $\mathbf{w}$ is mitigated. We no longer need to know the true source origin time. The gradient of the convolution objective function is calculated by taking the derivative of equation 18 with respect to the model parameters, $\mathbf{m}$. The gradient can then be written as:

$$\frac{\partial E_{indp}}{\partial \mathbf{m}} = \left[\left(\frac{\partial u_i}{\partial \mathbf{m}} * \mathbf{d}_{ref} \right) \cdot \mathbf{r}_i^{(1)} - \left(\mathbf{d}_i * \frac{\partial \mathbf{u}_{ref}}{\partial \mathbf{m}} \right) \cdot \mathbf{r}_i^{(2)} \right] + \lambda \nabla \cdot \left(\frac{\nabla \mathbf{m}}{\sqrt{\frac{\partial \mathbf{m}^2}{\partial x} + \frac{\partial \mathbf{m}^2}{\partial z} + \epsilon^2}} \right), \quad (19)$$

where i represent the i -th trace. Thus, the gradient involves calculating the adjoint wavefields using the following adjoint sources:

$$\mathbf{r}_i^{(1)} = \mathbf{d}_{ref} \otimes (\mathbf{u}_i(x_r, t) * \mathbf{d}_{ref} - \mathbf{d}_i * \mathbf{u}_{ref}(x_{ref}, t)), \quad (20)$$

$$\mathbf{r}_i^{(2)} = -\mathbf{d}_i \otimes (\mathbf{u}_i(x_r, t) * \mathbf{d}_{ref} - \mathbf{d}_i * \mathbf{u}_{ref}(x_{ref}, t)), \quad (21)$$

at the i^{th} receiver positions and the reference trace position, respectively, where $\otimes$ is the cross-correlation operator. The two cross-correlated seismograms are back-propagated at the same time in order to reduce the computational cost (equivalent to FWI).

More details on the gradient derivations are provided in Appendix *B*.

The inversion is implemented using a nested approach, i.e. (1) inverting for $\delta\alpha$ and $\delta\beta$ using equations 10 and 11 for five iterations; (2) inverting for α and β using equation 19 with the adjoint sources equations 20 and 21 for 10 iterations; (3) updating $\mathbf{w}$ using equation 12 for five iterations. We repeat the above steps until convergence is achieved. We start the inversion by estimating the source images using the TRI in the proposed method, because the first iteration of our inversion is equivalent to TRI with the initial velocity models.

It is in the nature of multi-parameter FWI methods to encounter cross-talk between the different parameters, which in the proposed method, are the source locations and the velocity models. To mitigate the cross-talk between different parameters, we need to carefully choose the optimization scheme.

A SYNTHETIC DATA EXAMPLE

We test our proposed method using a modified Marmousi model, which is shown in Figures 2a and 2b. The S-wave velocity is modified by adding a half-circled low velocity layer (representing the hydraulic injected area) pointed out by the magenta arrow in the figures. Five sources are separately triggered at a depth of 1.55km and located horizontally from 2.4 to 3.2km with a 200m interval. The model has $600 * 200$ grid points with a 10m spatial interval, in both X and Z directions. Each grid point on the top surface acts as a receiver (marked by magenta lines). 100 additional receivers are set in a vertical well from $(X = 1, Z = 0.5\text{km})$ to $(X = 1, Z = 1.5\text{km})$ with a 10m interval. The true source function is given by a Ricker wavelet with a 10Hz peak frequency. Note that the frequency band below 3Hz has been filtered out. The initial velocity model is obtained by strongly smoothing the true model and adding a 10% error (slower velocity), which makes it almost

linearly increasing with depth, as shown in Figures 2c and 2d. In some aspects, this initial velocity model, due to the 10% deviation is worse than what is often used as an initial linear increasing velocity to start the inversion. Note that the initial S-wave velocity does not contain the hydraulic injected area information. We record 5000 time samples, with 1ms temporal interval. A free surface condition is applied to the top boundary and absorbing boundary conditions are applied to the other three boundaries. No noise is added to the data. In this experiment, sources are triggered separately, which means we can separate the events from each other in the continuous record. This is closer to what we expect in the real world.

The inputs needed to start the inversion iterations are an initial velocity model, an initial source function and initial source images. The starting source images are zero everywhere ($\delta\alpha = \delta\beta = 0$). In other words, no prior information of the source spatial distribution is given. The initial source time function is obtained by TRI by simply back-propagating the complete record into the initial model, and subsequently extracting the wavefield along the time axis at the maximum energy point in the wavefield domain. With these initial conditions, we then start the inversion iteratively updating the source images, the velocity model, and the source function, until the data residual is sufficiently small.

Our inversion strategy has been described in the previous section, which includes five iterations of source image update, ten iterations of velocity update and five iterations of $[w_{ij}]$ update. The weighting factor λ of the total variation regularization term decreases with the outer loop iteration number, starting with 0.5 and decreasing linearly to 0.05 when reaching the last iteration. Larger λ in the beginning of the inversion process helps stabilize the inversion due to the unreliable nature of the initial velocity. Smaller λ at the end makes the gradient with respect to the velocity focus more on the high wavenumber

information instead of only the smooth background. The smoothing factor ϵ is set to 5% of the maximum value of the total variation term at current iteration, so that the output of the TV term is sufficiently smooth instead of it being blocky when ϵ is small. A fixed step length (0.05km/s for V_p and 0.03km/s for V_s) for updating velocities is applied through the whole inversion. We use a reasonably small step length so the inversion is more stable. The weighting factor κ of the source image regularization term is set to be 10 in the first iteration and decreases to 0.1 at the end.

Results and Discussions

The inverted velocity models are shown in Figures 2e and 2f. The models generally exhibit low resolution, reflecting the peak frequency of the data and the limited illumination with only five nearby sources. The microseismic events are buried in the earth and the receivers are on the earth's surface and in a single well. Most of the energy in the observed data are transmission waves. In FWI, reflection waves update the high wavenumber parts of the model, whereas transmissions update the low wavenumber parts. Therefore, the gradients for the velocity updates are mostly focused on the smooth background. Unlike the conventional surface survey having enough sources covering the entire survey area, microseismic sources are close to each other, meaning that only certain areas can be illuminated properly by the records, especially when multi-scattering energy is weak in the records. This causes tremendous difficulties in updating the whole model area in a satisfactory manner. Although we can obtain much better updates by simply exciting sources in a wider area so that the data cover a larger region, we stick to this experiment setup because it is more realistic, as it reflects what happens in hydraulic fracturing. As a result, we can only obtain accurate (smooth) results for the kinematic components of the inversion. A quality control

(QC) of the inverted velocity models is performed by comparing the vertical V_p and V_s profiles at position $X = 3.0\text{km}$, as shown in Figures 3a and 3b. It is obvious that the initial velocities are in average lower than the true ones. For V_s , no hydraulic injected area is shown at a depth around 1.3km. After the inversion, we see that the velocity values are generally pulled upward, and the hydraulic injected area (low velocity zone) in V_s is also well inverted. Certainly, the inverted velocities are more kinematically accurate.

Although the inverted velocity models are imperfect, they are good enough for locating the microseismic sources in space, since they are still kinematically accurate. The inverted source images of one example source, which is an explosive point source, are shown in Figures 4a to 4b. We can extract the source location from the source images by simply choosing the maximum energy point in the source image. Since most of the energy is focused at that point, although there is some misfocused energy caused probably by the remaining data misfit, it is reasonable to consider the maximum energy point as the source location. For comparison, the source locations of the first iteration (equivalent to TRI locations) are represented by the red dots in the figures and the true source locations are represented by the blue dots. We find that both $\delta\alpha$ and $\delta\beta$ focus the energy of the source to the accurate location, while the initial (TRI) locations are wrong due to the velocity error.

Each components of the inverted source time function of the same source are shown in Figures 5a to 5c. We can find that the initial wavelets (blue) are shifted by more than one cycle from the true ones (magenta), which will lead to a cycle skipping problem in conventional FWI. By using the proposed method, we find that the inverted wavelets (red) are well recovered, although there are some small artifacts in w_{11} and w_{22} , caused by the remaining misfits. The more important fact is that the origin time of the micro-seismic source has been well inverted as the peaks of the inverted wavelets overlap the true ones.

The inverted amplitude of the w_{12} component is small enough compared to the other two components to ignore. The amplitudes of the source time functions were not well inverted due to the convolution operation in the source independent objective function.

The computational cost for each iteration of the proposed method is equivalent to a standard full waveform inversion iteration. Since we have three parameters to invert and tens of iterations of update, the total cost is still a challenging issue.

For the field data with low S/N ratio, we may need good pre-processing of the data to enhance the signal energy in order to obtain the kinematic features of the subsurface.

FIELD DATA EXAMPLE

We now focus our attention on the main contribution here, which is the application to field data. The real dataset is collected during a hydraulic fracture treatment. It was recorded in a vertical monitoring well with only 15 three-component geophones. The depth of the geophones starts from 2.443km to 2.673km. The distance between the geophones are uneven, either 10m or 20m. Figure 6 shows the survey geometry. In this study, we have been provided a total of 67 detected events from the same injection stage, including the ball-drop event for this stage whose ball-seat location is known (as pointed out in Figure 6). The corresponding ball-seat is 0.6736km laterally away from the recording well, and the depth of the ball-seat is 2.7016km, which is also the expected depth of the uncontrolled events. Note that the depth covered by the receivers is only about one-third of its distance from the treatment well, which means that the illumination aperture is quite limited in this experiment. The perforation shot is not used because it is fired in a third well before the hydraulic fracturing, so the velocity model inverted using this event cannot account for the velocity changes caused by the stimulation. Moreover, the S-wave arrivals of the

perforation shot are too weak to be observed due to the compressional source mechanism, thus we cannot use them to invert for the S-wave velocities. Tan and He (2016) are the first to work with the data, and their insights were helpful.

Ball-drop Event Inversion

To control the different stages of injection, the sleeves or fracports were activated by dropping or injecting different sized balls into the well. Balls fit into a corresponding sized seat at a target point in the well. Once seated, the balls block flow and increase back pressure that pushes open a sleeve/fracport, and at the same time, generate the ball-drop events. The ball-drop seismic signals are high in amplitude, low in frequency, with multiple discrete events with each event located at the fracport (Maxwell et al., 2011). The spatial locations of these events are known, which we can utilize as a quality control for the proposed method. Since it is a single monitoring well system, there is not enough information to directly invert for the $3D$ locations of the sources. Therefore we have to deal with the problem as a $2D$ one, which means we first need to locate the event direction from the well and then use a plane that crosses the monitoring well and the event to represent our $2D$ medium. Before we start the inversion, we define the $2D$ plane in space by estimating the events direction and rotate the data from north-east-depth (NEZ) coordinates to the corresponding plane coordinates. We first rotate the data and then define the true rotation angle whenever the P-wave energy in the R component reaches its maximum, because P-wave vibrates in the same direction as its propagation and its energy should be maximum along the vibration direction as well. Once we determine the rotation angle for one event, we collapse the $3D$ problem into a $2D$ one. In the $2D$ plane, we define only the horizontal distance to the receivers and the depth to determine the spatial location of the event.

Figure 7 shows the raw data of the ball-drop event for the stage and the R and T components after rotation. The ball-seats are in a pipe, which is perpendicular to the receiver well, and this leads to no SV waves in the Z component, but only SH waves in the horizontal components. As a result, we have to use the R and T components instead of the R and Z components in the inversion, which means we treat SH waves as SV waves. This is possible since our focus is mainly transmission waves. The seismic radiation characteristics of ball-drop events are consistent with an axial displacement along the treatment well (Maxwell et al., 2011).

To implement the inversion, we start with homogeneous models $V_p = 3.3\text{km/s}$ and $V_s = 1.9\text{km/s}$ as the initial velocities. We have been provided a 1D V_p model from the previous study of this dataset (Tan et al., 2018), but we simply choose to start from a lower-accuracy initial model, the homogeneous background velocities, as a test to the robustness of our inversion. The model has 400-by-225 grid points with a 2m spatial interval in both the X and Z directions. The data have 3501 time samples with a 0.3ms sampling interval. The receivers are located at $X = 0, Z = 2443\text{m}$ to 2673m . The original data have reasonably high frequencies (Figure 8), ranging from 30Hz to 300Hz, which poses a cycle skipping and computational challenge for FWI. In our implementation, we use a multi-stage strategy, i.e., we start with lower frequencies and gradually add higher frequencies into the inversion in later iterations. Though the lowest frequency of the data is already very high (30Hz), using a multi-stage strategy may help further mitigate the cycle skipping problem. As a comparison, surface surveys usually have low frequencies down to 3Hz. An absorbing boundary condition is assumed on all the four boundaries. Attenuation may exist because of the higher frequency components of the data. However, since we are focused in the inversion on the phase component of the wave, we ignore attenuation. The monitoring well is not far

away from the injection area and the dispersion effect is small enough to be ignored.

The starting source images $\delta\alpha$ and $\delta\beta$ are set to zero everywhere. We treat it as a normal passive source inversion. In other words, no prior information of the source spatial distribution is used. Using the time-reversed method, the initial source function $\mathbf{w}$ is obtained by extracting the wavefield corresponding to the maximum energy point in space of the reversed wavefield. The reference trace corresponds to the deepest receiver trace since it is expected to be the closest to the source.

We stop the inversion after 38 nested iterations because the convergence curve, shown in Figure 9, simply does not decrease thereafter. The inverted velocities are shown in Figures 10a and 10b. The results have generally low resolution, because the updates are smeared over mainly the direct wavepath from the ball-drop source location to the receivers on the left boundary. With the help of the total variation regularization, we can clearly see the horizontal low velocity zones in both V_p and V_s at a depth around 2.7km, which indicate the injection zone. Because the injection well is a horizontal well at the same depth (around 2.7km), we can expect this low velocity feature. The inverted velocities show rather low resolution, mainly because the updated zone is dominated by transmission waves. Most of the energy in the observed data corresponds to transmissions. In FWI, reflections update the high wavenumber parts of the model, while transmissions update the low wavenumber parts (Alkhalifah, 2016). On the other hand, if the scattered waves had strong enough energy, we can invert for higher resolution information (Song et al., 2019b). In this paper, multi-scattering is beyond the scope so the gradients for the velocities are focused on the smooth background.

The inverted source images are shown in Figures 11a and 11b. With the help of the focusing regularization term, the energy is well focused at the ballseat location (overlapped

with the true source location). The inverted source coordinates $\mathbf{x}_s$, calculated using the focusing function, is (0.67km,2.71km), which is fairly close to the true ball-seat location (0.6736km,2.7016km). However, the ball-drop event location (0.53km,2.61km) at the first iteration (equivalent to TRI with the first iteration velocity models) is quite far away from the true ball-seat location.

The data comparison between the initial, the true and the modeled data is shown in Figure 12, with the R and T components given from left and right side, respectively. We may find that the events are matched in the time axis with almost the same moveout, while the initial data are shifted in time and different in shape, compared to the true data. The remaining data misfit corresponds mainly to amplitude differences. The convolution operation in the source independent objective function ignores the true amplitude information in the data.

Considering the lack of data and the extremely limited aperture (about 1 : 3 between receiver range and the source distance), the results show reasonably good resolution and accurate source position.

In real data tests, we do not know the true source origin time, which makes it impossible to judge the accuracy of the source time function. On the other hand, the source independent method makes the origin time information less significant because we do not rely on the wavelets to update the velocities. Thus, here we simply choose not to show the inverted source wavelets though we update the source wavelets for every single source in each iteration. Although we assume the source time functions are the same at every point on the estimated source image. In real cases, and specifically the extreme case when the source time function vary a lot as a function of location, we may face a problem. Ideally, the source images would focus to a single point like a delta function. In this case, the source function

is non-zero only at the source image point position and zero everywhere else. Under this assumption, we may use the same source time function throughout the whole source image.

Inversion of uncontrolled passive events

After we have verified the robustness of our method in locating the source using the ball-drop event, we apply the same inversion strategy to other uncontrolled passive events, with exactly the same inversion strategy as the ball-drop event since it passed our QC test. The events are separated from each other in time, which means we treat the events separately to avoid cross-talk. In addition, our source focusing step cannot handle multi-events, simultaneously. If more than one event overlap, the inverted sources may focus to the wrong location because in the equivalent source term, all source image points share the same source function. Different sources are expected to have different source functions due to their different source onsets but inverting the events together with the same source function is not practical. Most detected events in this experiment are distinguishable in time thus we may apply the proposed method to such data.

We start the inversion with the homogeneous initial model we used for the ball-drop event. The initial source image is again set to zero and the initial source function is obtained by the TRI technique, which is the wavefield reversal in time to the maximum amplitude position in the back-propagated wavefield.

Two examples of the inverted source images are shown in Figures 13a to 13d. The initial source positions are marked by the blue dots. We find that the inverted locations are quite different from the initial ones. The shift distances are more than 0.15km in average. After inverting for all events in the stage, we use the rotation angles and source images, which contain the horizontal distance and depth information for each event, to generate the 3D

locations of the passive sources. From the map of the microseismic event locations, shown in Figures 14a and 14b, we can see a clear distribution trend of the passive sources from northwest to southeast. This observation has been confirmed by the previous study of the local stress state of the principal horizontal stress in this area, which is oriented in the same direction, shown as the magenta arrow direction in Figure 14a. The results show that the injection treatment induced fractures and cracks along the existing stress direction. We also find that there are more events on the east side of the treatment well, which means there are more events detected in the region between the treatment well and the monitoring well. This is mainly because the monitoring well is on the same side as the treatment well. More events on east side can be detected.

We then build the 3D velocity models by interpolating all the inverted 2D velocity sections along angles, shown in Figure 15. After the interpolation, the 3D model is shown in Figures 16a and 16b. Note the low velocity zone in V_s , which indicates the injection area, matches the treatment well location.

Though the proposed method can produce the source locations, source time functions and the velocity model, a big limitation of the proposed method is the computational cost. We use an eight-core workstation for the real data test, and the computational time for each iteration with all the sources is about 1.5 hours. It may require higher computational power when more sources are included in the inversion. A much better result is expected if we have more illumination provided by wider aperture data. We may face a failure in the inversion if the initial values are too far from the true ones, which is a problem with most inversions. In applying the method for a 3D inversion, the biggest challenge will be the data coverage and the computational cost. Besides, high quality data with a reasonably high S/N ratio would often be needed. Additional study in the future will hopefully reveal the limits on the

initial values for the inversion, or in other words, the boundaries of the basin of attraction (Alkhalifah, 2014).

CONCLUSIONS

We developed a new equivalent seismic source for passive events, using a similar approach and framework to those used in reflection waveform inversion. We successfully separate the temporal and spatial components of a passive source and inverted both of them, along with the velocity model. In our method, the new equivalent source term describes the source mechanism through the source images, which, unfortunately, are sensitive to the acquisition aperture, and thus, may impact the quality of the inverted result. To mitigate the passive seismic limitation of an unknown source time, a source wavelet independent objective function is utilized by simply convolving the recorded and modeled data with a reference trace, through which the necessity of a source onset information is loosened. The inverted source images and velocities yield reasonably good results on one stage of a field monitoring dataset, in spite of the poor initial model. The proposed free-of-picking method is successful in finding the source location of the previously known ball-drop event, which is used to verify the accuracy of the method, as well as in building a reasonably accurate velocity model.

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APPENDIX A

GRADIENT CALCULATION FOR MODEL PARAMETERS WITH THE CONVENTIONAL OBJECTIVE FUNCTION

Here we derive the gradients of the objective function, equation 9, with respect to the model squared velocity $\mathbf{m}(\alpha, \beta)$, source images $\delta\mathbf{m}(\delta\alpha, \delta\beta)$ and source function $\mathbf{w}$:

$$\begin{aligned} dE(\mathbf{u}) &= \frac{\partial E}{\partial \mathbf{m}} d\mathbf{m} + \frac{\partial \mathbf{E}}{\partial \delta\mathbf{m}} \mathbf{d}\delta\mathbf{m} + \frac{\partial \mathbf{E}}{\partial [\mathbf{w}_{ij}]} \mathbf{d}\mathbf{w} \\ &= \int_t dt (\mathbf{u} - \mathbf{d}) \mathbf{d}\mathbf{u}, \end{aligned} \quad (\text{A-1})$$

The elastic displacement field for a constant density satisfies

$$\mathbf{u}_{\mathbf{t}\mathbf{t}} - \nabla \cdot (\mathbf{m} : \nabla \mathbf{u}) = \nabla \cdot (\delta\mathbf{m} : \mathbf{w}), \quad (\text{A-2})$$

where $:$ is the Frobenius inner product operator. The subscription $\mathbf{t}\mathbf{t}$ means the second-order derivative with respect to time. The total derivatives of $\mathbf{u}$ is given by:

$$\mathbf{d}\mathbf{u}_{\mathbf{t}\mathbf{t}} = \nabla \cdot \nabla \mathbf{d}\mathbf{u} : \mathbf{m} + \nabla \cdot \mathbf{d}\mathbf{m} : \nabla \mathbf{u} + \nabla \cdot \mathbf{d}\mathbf{w} : \delta\mathbf{m} + \nabla \mathbf{d}\delta\mathbf{m} : \mathbf{w}. \quad (\text{A-3})$$

Thus, $\mathbf{d}\mathbf{u}$ can be written as:

$$\mathbf{d}\mathbf{u} = \left(\frac{\partial^2}{\partial \mathbf{t}^2} - \nabla \cdot (\mathbf{m} : \nabla) \right)^{-1} (\nabla \cdot \mathbf{d}\mathbf{m} : \nabla \mathbf{u} + \nabla \cdot \mathbf{d}\mathbf{w} : \delta\mathbf{m} + \nabla \mathbf{d}\delta\mathbf{m} : \mathbf{w}). \quad (\text{A-4})$$

We have three terms for $\mathbf{d}\mathbf{m}$, $\mathbf{d}\mathbf{w}$ and $\mathbf{d}\delta\mathbf{m}$. The adjoint wavefield for back-propagating the data residual into the medium is given by:

$$\hat{\mathbf{u}} = \left(\frac{\partial^2}{\partial t^2} - \nabla \cdot (\mathbf{m} : \nabla) \right)^{-T} (\mathbf{u} - \mathbf{d}), \quad (\text{A-5})$$

where T is the adjoint operation.

To calculate the gradient for $\mathbf{m}(\alpha, \beta)$, $\delta\mathbf{m}$ or $\mathbf{w}$, we substitute $\mathbf{d}\mathbf{u}$ into equation A-1 to get

$$\begin{aligned} dE &= \int_t dt \left(\frac{\partial^2}{\partial t^2} - \nabla \cdot (\mathbf{m} : \nabla) \right)^{-1} (\mathbf{u} - \mathbf{d}) (\nabla \cdot \mathbf{d}\mathbf{m} : \nabla \mathbf{u} + \nabla \cdot \mathbf{d}\mathbf{w} : \delta\mathbf{m} + \nabla \mathbf{d}\delta\mathbf{m} : \mathbf{w}) \\ &= \int_t dt \left(\nabla \cdot \left(\frac{\partial^2}{\partial t^2} - \nabla \cdot (\mathbf{m} : \nabla) \right)^{-1} (\mathbf{u} - \mathbf{d}) : \nabla \mathbf{u} \mathbf{d}\mathbf{m} + \nabla \cdot \left(\frac{\partial^2}{\partial t^2} - \nabla \cdot (\mathbf{m} : \nabla) \right)^{-1} (\mathbf{u} - \mathbf{d}) : \delta\mathbf{m} \mathbf{d}\mathbf{w} \right. \\ &\quad \left. + \nabla \cdot \left(\frac{\partial^2}{\partial t^2} - \nabla \cdot (\mathbf{m} : \nabla) \right)^{-1} (\mathbf{u} - \mathbf{d}) : \mathbf{w} \mathbf{d}\delta\mathbf{m} \right). \end{aligned} \quad (\text{A-6})$$

To calculate the gradients for each parameter, we set the other two parameters' total derivatives to zero (stays fixed in the current iteration). For example, to calculate the gradient for $\mathbf{m}(\alpha, \beta)$, we set $\mathbf{d}\delta\mathbf{m} = \mathbf{0}$ and $\mathbf{d}\mathbf{w} = \mathbf{0}$, so we have

$$\nabla_{\mathbf{m}} E = \frac{dE}{d\mathbf{m}} = \int_t dt \nabla \cdot \left(\frac{\partial^2}{\partial t^2} - \nabla \cdot (\mathbf{m} : \nabla) \right)^{-1} (\mathbf{u} - \mathbf{d}) : \nabla \mathbf{u}; \quad (\text{A-7})$$

Similarly, by setting $\mathbf{d}\mathbf{m} = \mathbf{0}$ and $\mathbf{d}\mathbf{w} = \mathbf{0}$, we can obtain the gradient for $\delta\mathbf{m}$:

$$\nabla_{\delta\mathbf{m}} E = \frac{dE}{d\delta\mathbf{m}} = \int_t dt \nabla \cdot \left(\frac{\partial^2}{\partial t^2} - \nabla \cdot (\mathbf{m} : \nabla) \right)^{-1} (\mathbf{u} - \mathbf{d}) : \mathbf{w}, \quad (\text{A-8})$$

and finally we get the gradient for $\mathbf{w}$ by setting $\mathbf{d}\mathbf{m} = \mathbf{0}$ and $\mathbf{d}\delta\mathbf{m} = \mathbf{0}$:

$$\nabla_{\mathbf{w}} E = \frac{dE}{d\mathbf{w}} = \int_{\mathbf{x}} d\mathbf{x} \nabla \cdot \left(\frac{\partial^2}{\partial t^2} - \nabla \cdot (\mathbf{m} : \nabla) \right)^{-1} (\mathbf{u} - \mathbf{d}) : \delta\mathbf{m} \quad (\text{A-9})$$

APPENDIX B

SOURCE FUNCTION INDEPENDENT OBJECTIVE FUNCTION ADJOINT SOURCE AND GRADIENT CALCULATION

We derive the gradients for the source function independent objective function with respect to velocities c . In this case, the objective function without the optional total variation term is given by:

$$E_{ind} = \sum_i^{nr} \|r_i\|^2 = \sum_i^{nr} \left\| d_i^{pred} * d_{ref}^{obs} - d_i^{obs} * d_{ref}^{pred} \right\|^2, \quad (\text{B-1})$$

Taking the derivative of equation B-1 with respect to $c(x, z) = [V_p; V_s]$. The gradient can be written as:

$$\frac{\partial E_{ind}}{\partial c} = \sum_i^{nr} \left[\left(\frac{\partial d_i^{pred}}{\partial c} * d_{ref}^{obs} \right) \cdot r_i - \left(d_i^{obs} * \frac{\partial d_{ref}^{pred}}{\partial c} \right) \cdot r_i \right]. \quad (\text{B-2})$$

The first convolution term $\left(\frac{\partial d_i^{pred}}{\partial c} * d_{ref}^{obs} \right) \cdot r_i$ in equation B-2 can be rewritten in an integral form,

$$\left(\frac{\partial d_i^{pred}}{\partial c} * d_{ref}^{obs} \right) \cdot r_i = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\partial d_i^{pred}}{\partial c} (t - \tau) d_{ref}^{obs}(\tau) r_i(t) d\tau dt. \quad (\text{B-3})$$

Let $\eta = t - \tau$, thus, equation B-3 can be written as,

$$\left(\frac{\partial d_i^{pred}}{\partial c} * d_{ref}^{obs} \right) \cdot r_i = - \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\partial d_i^{pred}}{\partial c}(\eta) d_{ref}^{obs}(t - \eta) r_i(t) d\eta dt. \quad (\text{B-4})$$

$\frac{\partial d_i^{pred}}{\partial c}(\eta)$ is now independent on t so we move it outside of the integration of t , which gives,

$$\left(\frac{\partial d_i^{pred}}{\partial c} * d_{ref}^{obs} \right) \cdot r_i = - \int_{-\infty}^{+\infty} \frac{\partial d_i^{pred}}{\partial c}(\eta) \left[\int_{-\infty}^{+\infty} d_{ref}^{obs}(t - \eta) r_i(t) dt \right] d\eta. \quad (\text{B-5})$$

The partial derivative $\frac{\partial d_i^{pred}}{\partial c}(\eta)$ is calculated by the convolution between $U_{x,i}$ and S_x , where $U_{x,i}$ is the adjoint wavefield of the i th back-propagated residual, S_x is the source wavefield ignited at a virtual source location (since we do not know the true source location) and x represents the model space location. Using the convolution form to rebuild equation B-3, we obtain

$$\left(\frac{\partial d_i^{pred}}{\partial c} * d_{ref}^{obs} \right) \cdot r_i = - \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} S_x(\eta - \tau) U_{x,i}(\tau) d\tau \right] \left[\int_{-\infty}^{+\infty} d_{ref}^{obs}(t - \eta) r_i(t) dt \right] d\eta. \quad (\text{B-6})$$

Again let $\eta - \tau = t$, then equation B-6 becomes

$$\begin{aligned} \left(\frac{\partial d_i^{pred}}{\partial c} * d_{ref}^{obs} \right) \cdot r_i &= \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} S_x(t) U_{x,i}(\eta - t) dt \right] \left[\int_{-\infty}^{+\infty} d_{ref}^{obs}(t - \eta) r_i(t) dt \right] d\eta \\ &= \int_{-\infty}^{+\infty} S_x(t) \left[\int_{-\infty}^{+\infty} U_{x,i}(\eta - t) d_{ref}^{obs}(t - \eta) r_i(t) d\eta \right] dt. \end{aligned} \quad (\text{B-7})$$

Because of the reciprocity of $U_{x,i}$, the inner integration $\int_{-\infty}^{+\infty} U_{x,i}(\eta - t) d_{ref}^{obs}(t - \eta) r_i(t) d\eta$

can be considered as a back-propagation of the new data residual $R_i^{(1)} = \int_{-\infty}^{+\infty} d_{ref}^{obs}(t - \eta) r_i(t) dt$.

A similar derivation can be used for the second term of equation B-2, which is $-\left(d_i^{obs} * \frac{\partial d_{ref}^{pred}}{\partial c} \right) r_i$.

We finally end up with

$$-\left(d_i^{obs} * \frac{\partial d_{ref}^{pred}}{\partial c} \right) \cdot r_i = \int_{-\infty}^{+\infty} S_x(t) \left[\int_{-\infty}^{+\infty} U_{x,ref}(\eta - t) d_i^{obs}(t - \eta) r_i(t) d\eta \right] dt. \quad (\text{B-8})$$

In equation B-8, the second term of the gradient is nothing but the back-propagation of the

second residual $R_i^{(2)} = - \int_{-\infty}^{+\infty} d_i^{obs}(t - \eta) r_i(t) dt$.

Thus, the gradient involves calculating the adjoint wavefields with the adjoint sources:

$$R_i^{(1)} = d_{ref}^{obs} \otimes \left(d_i^{pred} * d_{ref}^{obs} - d_i^{obs} * d_{ref}^{pred} \right), \quad (\text{B-9})$$

at the i^{th} receiver position, and

$$R_i^{(2)} = -d_i^{obs} \otimes \left(d_i^{pred} * d_{ref}^{obs} - d_i^{obs} * d_{ref}^{pred} \right), \quad (\text{B-10})$$

at the reference traces position.

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