A Local Coupling Multi-Trace Domain Decomposition Method for Electromagnetic Scattering from Multilayered Dielectric Objects

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Abstract—In this paper, a local coupling multi-trace domain decomposition method (LCMT-DDM) based on surface integral equation (SIE) formulations is proposed to analyze electromagnetic scattering from multilayered dielectric objects. Different from the traditional SIE-DDM, where the interactions between sub-domains are accounted for using global radiation coupling, LCMT-DDM uses a local coupling scheme. The original multilayered object is decomposed into several independent domains, i.e. the exterior region (free space) and many homogeneous interior regions (dielectrics). The boundaries of sub-domains are all touching-faces, where only the Robin transmission conditions (RTCs) are enforced to ensure the field continuity. Hence, each sub-domain only couples with its neighboring regions, which makes the DDM system a highly sparse matrix especially when the number of sub-domains is large. In each sub-domain, the electric field integral equation (EFIE) and the magnetic field integral equation (MFIE) for dielectrics are used as the governing equations. By imposing RTCs, well-conditioned equations are formed in each sub-domain without invoking the combined field integral equation (CFIE), which usually causes accuracy issues in dielectric modeling. Since the sub-domain matrices are diagonally dominant, the flexible generalized minimal residual (FGMRES) technique is used to accelerate the iterative solution of the whole DDM system. Moreover, an effective preconditioner that can be recursively constructed is proposed.

Index Terms—Electromagnetic scattering, local coupling, domain decomposition, multi-trace, multilayered, dielectric.

I. INTRODUCTION

ACCURATE solution of electromagnetic (EM) scattering from multilayered dielectric objects is of vital importance

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in various applications, such as target identification, antenna design, and microwave circuit analysis, etc. The full-wave method of moments (MoM) is one of the most widely used methods due to its robustness, efficiency, and accuracy [1], [2], [3]. When the volume integral equation (VIE) is used, a large number of unknowns are inevitably generated because of volumetric discretization, especially for large-scale problems [4], [5], [6]. For homogeneous or piecewise homogeneous objects, surface integral equation (SIE), which requires only the surfaces to be discretized is usually a better choice [7], [8].

However, when SIE is used to model piecewise homogeneous or multilayered dielectric objects, careful treatment of complex junction basis functions is required to ensure the boundary conditions [9]. To overcome this difficulty, the contact-region modeling method (CRM) [10] and domain decomposition method (DDM) [11], [13], [14] have been developed. In CRM, the interface is divided into two contact surfaces to ensure the boundary condition between different regions. Although CRM can automatically guarantee the continuity of electric and magnetic fields at the interface, it always leads to an ill-conditioned matrix that can not be solved efficiently using an iterative solver. Similar to CRM, DDM also decomposes the original piecewise homogeneous dielectric objects into different sub-domains. Nevertheless, the boundary conditions in DDM are ensured by imposing the transmission conditions (TCs). Moreover, the resultant DDM matrix is usually well conditioned.

In the framework of DDM, the combined field integral equation (CFIE), such as the electric-magnetic field CFIE (JMCIE) [15], [16] and generalized-CFIE (GCFIE) [12], are commonly used as the governing equations on each sub-domain. Compared to the traditional Poggio-Miller-Chang-Harrington-Wu-Tsai (PMCHWT) equation [2], the iterative convergence of CFIE is much faster. This plays an important role in DDM, since the well-conditioned sub-domain matrices can be used as an preconditioner for the entire system. However, this favorable iterative convergence is achieved at the expense of compromised accuracy inherited from CFIE [17]. Moreover, due to the global radiation coupling among different sub-domains, full matrices are generated in the conventional SIE-DDM system.

To address the accuracy issue of the CFIE based DDM, a novel local coupling multi-trace domain decomposition

method (LCMT-DDM) is proposed in this work. This method first decomposed the original object into two sub-domains, the exterior region and the interior one. Then, the interior region is further decomposed, where each dielectric layer is defined as one sub-domain. By doing this, the boundaries of each sub-domain are always the touching faces. Since each sub-domain is meshed independently, there are always a pair of meshes at the sub-domain interfaces, which is referred to as multi-traces [19], [20]. It is noted that in traditional SIE-DDM and CRM, the interactions between sub-domains are accounted for using global radiation coupling. On the other hand, the proposed LCMT-DDM, couples each sub-domain only to its neighbors using the Robin transmission conditions (RTCs). After discretization, this local coupling scheme leads to a sparse DDM matrix and reduces the memory consumption, especially for multilayered dielectric objects.

In each sub-domain, EFIE and MFIE are simultaneously used as the governing equations to ensure the accuracy and to eliminate the interior resonance problem. However, neither EFIE nor MFIE is well-conditioned [21]. Fortunately, when they are combined with RTCs, the resultant matrix can be made diagonally dominant and well-conditioned, which is very similar to the structure of CFIE. Hence, the sub-domain matrices are used as the preconditioner in the FGMRES solver, which is based on an inner-outer iteration scheme [22], [23]. It is noted that this preconditioner does not require extra memory, in contrast with the traditional sparse approximate inverse (SAI) preconditioner, or the incomplete lower-upper factorization preconditioner [24], [25]. Moreover, FGMRES is compatible with these conventional preconditioners since they can also be applied during the inner iteration. In this work, the multilevel fast multiple algorithm (MLFMA) is applied in FGMRES to accelerate the computation of matrix-vector products required by the FGMRES solver[26].

The rest of this paper is organized as follows. In Section II, the detailed derivation of the LCMT-DDM equations are provided. Then the discretization, matrix system formulation and preconditioning techniques are discussed. In Section III, numerical examples are presented to demonstrate the performance of LCMT-DDM. Conclusions are summarized and future research directions are provided in Section IV.

II. FORMULATION

A. LCMT-DDM Formulation

Consider the EM scattering problem as shown in Fig. 1, where a piecewise homogeneous structure Ω_d is illuminated by an incident plane wave $\{\mathbf{E}^{inc}, \mathbf{H}^{inc}\}$. The regions Ω_d is a combination of Ω_{d1} and Ω_{d2} , satisfying $\Omega_d = \Omega_{d1} \cup \Omega_{d2}$, $\Omega_{d1} \cap \Omega_{d2} = \emptyset$, where Ω_{d1} and Ω_{d2} are two homogeneous regions with relative permittivity, permeability $\varepsilon_{r1}, \mu_{r1}$ and $\varepsilon_{r2}, \mu_{r2}$ respectively. The background is the free space with permittivity, permeability ε_0, μ_0 . The scattered fields are denoted as $\{\mathbf{E}^{sca}, \mathbf{H}^{sca}\}$, and the equivalent electric current $\mathbf{J}$ and the scaled magnetic current $\mathbf{M}$ satisfy $\mathbf{J} = \hat{\mathbf{n}} \times (\mathbf{H}^{inc} + \mathbf{H}^{sca})$ and $\mathbf{M} = \frac{1}{\eta_0}(\mathbf{E}^{inc} + \mathbf{E}^{sca}) \times \hat{\mathbf{n}}$, where $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ is the intrinsic impedance of free space. As shown in Fig. 2, the original computational domain is decomposed into three

Fig. 1. EM scattering from a piecewise homogeneous object illuminated by a plane wave.

Fig. 2. The notations used in LCMT-DDM.

regions, the exterior region Ω_0 with boundary $\partial\Omega_0$, and the interior regions Ω_1 and Ω_2 with boundaries $\partial\Omega_1$ and $\partial\Omega_2$, respectively. The outward unit normal of $\partial\Omega_m$ is $\hat{\mathbf{n}}_m$ ($m = 0, 1, 2$). The exterior boundary satisfies $\partial\Omega_0 = \Gamma_{01} \cup \Gamma_{02}$, where $\Gamma_{01} = \partial\Omega_0 \cap \partial\Omega_1$, $\Gamma_{02} = \partial\Omega_0 \cap \partial\Omega_2$. The interior boundaries satisfy $\partial\Omega_1 = \Gamma_{10} \cup \Gamma_{12}$, $\partial\Omega_2 = \Gamma_{20} \cup \Gamma_{21}$, where $\Gamma_{10} = \partial\Omega_1 \cap \partial\Omega_0$, $\Gamma_{12} = \partial\Omega_1 \cap \partial\Omega_2$, $\Gamma_{20} = \partial\Omega_2 \cap \partial\Omega_0$, $\Gamma_{21} = \partial\Omega_2 \cap \partial\Omega_1$.

According to the Stratton-Chu representation formula, the scattering fields can be expressed as,

$$\begin{bmatrix} \mathbf{E}_m^{sca} \\ \eta_m \mathbf{H}_m^{sca} \end{bmatrix} = \begin{bmatrix} \eta_m \mathcal{L}_m & -\eta_0 \mathcal{K}_m \\ \eta_m \mathcal{K}_m & \eta_0 \mathcal{L}_m \end{bmatrix} \begin{bmatrix} \mathbf{J}_m \\ \mathbf{M}_m \end{bmatrix}. \quad (1)$$

where the operators $\mathcal{L}_m$ and $\mathcal{K}_m$ in Ω_m can be defined as

$$\begin{aligned} \mathcal{L}_m(\mathbf{X}(\mathbf{r}'); \partial\Omega_m) &= -jk_m \int_{\partial\Omega_m} \left[\mathbf{I} + \frac{1}{k_m^2} \nabla \nabla \cdot \right] \mathbf{X}(\mathbf{r}') G_m(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \\ \mathcal{K}_m(\mathbf{X}(\mathbf{r}'); \partial\Omega_m) &= \int_{\partial\Omega_m} \nabla G_m(\mathbf{r}, \mathbf{r}') \times \mathbf{X}(\mathbf{r}') d\mathbf{r}'. \end{aligned} \quad (2)$$

Here, the time convention is assumed as $e^{j\omega t}$, so that the Green's function for Ω_m can be written as $G_m(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk_m |\mathbf{r} - \mathbf{r}'|}}{4\pi |\mathbf{r} - \mathbf{r}'|}$, where $k_m = k_0 \sqrt{\mu_{rm} \varepsilon_{rm}}$ is the wavenumber in Ω_m , $k_0 = \omega \sqrt{\mu_0 \varepsilon_0}$ is the free-space wavenumber, $\omega = 2\pi f$

is the angular frequency, and the principle value of $\mathcal{K}_m$ is defined as $\bar{\mathcal{K}}_m$. The surface tangential trace operator $\pi_\tau^{(m)}$ and the twisted surface tangential trace operator $\pi_\times^{(m)}$ on the boundary $\partial\Omega_m$ are introduced as following,

$$\begin{aligned}\pi_\tau^{(m)}(\mathbf{u}) &:= \hat{\mathbf{n}}_m \times (\mathbf{u} \times \hat{\mathbf{n}}_m)|_{\partial\Omega_m} \\ \pi_\times^{(m)}(\mathbf{u}) &:= \hat{\mathbf{n}}_m \times \mathbf{u}|_{\partial\Omega_m}\end{aligned}\quad (3)$$

If we assume there are virtual gaps between Ω_0 , Ω_1 , and Ω_2 , the original boundary value problem (BVP) of the whole model can be cast into a set of BVPs,

$$\nabla \times \nabla \times \mathbf{E}_0 - k_0^2 \mathbf{E}_0 = 0 \quad \text{in } \Omega_0 \quad (4)$$

$$\nabla \times \mu_{r1}^{-1} \nabla \times \mathbf{E}_1 - k_0^2 \varepsilon_{r1} \mathbf{E}_1 = 0 \quad \text{in } \Omega_1 \quad (5)$$

$$\nabla \times \mu_{r2}^{-1} \nabla \times \mathbf{E}_2 - k_0^2 \varepsilon_{r2} \mathbf{E}_2 = 0 \quad \text{in } \Omega_2 \quad (6)$$

$$\pi_\tau^{(0)}(\mathbf{E}_0) = \pi_\tau^{(1)}(\mathbf{E}_1) \quad \text{on } \Gamma_{01} \quad (7)$$

$$\pi_\tau^{(0)}(\mathbf{E}_0) = \pi_\tau^{(2)}(\mathbf{E}_2) \quad \text{on } \Gamma_{02} \quad (8)$$

$$\pi_\times^{(0)}(\nabla \times \mathbf{E}_0) = -\pi_\times^{(1)}\left(\frac{1}{\mu_{r1}} \nabla \times \mathbf{E}_1\right) \quad \text{on } \Gamma_{01} \quad (9)$$

$$\pi_\times^{(0)}(\nabla \times \mathbf{E}_0) = -\pi_\times^{(2)}\left(\frac{1}{\mu_{r2}} \nabla \times \mathbf{E}_2\right) \quad \text{on } \Gamma_{02} \quad (10)$$

$$\pi_\tau^{(1)}(\mathbf{E}_1) = \pi_\tau^{(2)}(\mathbf{E}_2) \quad \text{on } \Gamma_{12} \quad (11)$$

$$\pi_\times^{(1)}\left(\frac{1}{\mu_{r1}} \nabla \times \mathbf{E}_1\right) = -\pi_\times^{(2)}\left(\frac{1}{\mu_{r2}} \nabla \times \mathbf{E}_2\right) \quad \text{on } \Gamma_{12} \quad (12)$$

$$\lim_{|\mathbf{r}| \rightarrow \infty} |\mathbf{r}| (\nabla \times \mathbf{E}^{\text{sca}} - j k_0 \hat{\mathbf{r}} \times \mathbf{E}^{\text{sca}}) = 0 \quad (13)$$

where (13) is the Silver-Müller radiation condition for the scattered field in the exterior region Ω_0 . The total electric fields in Ω_0 , Ω_1 , and Ω_2 are denoted as $\mathbf{E}_0$, $\mathbf{E}_1$, and $\mathbf{E}_2$, which can be obtained by solving (4)-(6). By invoking the equivalence principle for each sub-region Ω_m , the electric field integral equation (EFIE_m) and magnetic field integral equation (MFIE_m) can be derived as

$$\begin{cases} \frac{\eta_0}{2} \pi_\times^{(0)} \mathbf{M}_0(\mathbf{r}) - \pi_\tau^{(0)} (\eta_0 \mathcal{L}_0(\mathbf{J}_0(\mathbf{r}'); \partial\Omega_0)) \\ + \pi_\tau^{(0)} (\eta_0 \bar{\mathcal{K}}_0(\mathbf{M}_0(\mathbf{r}'); \partial\Omega_0)) = \pi_\tau^{(0)} (\mathbf{E}^{\text{inc}}(\mathbf{r})) \\ - \frac{\eta_0}{2} \pi_\times^{(0)} \mathbf{J}_0(\mathbf{r}) - \pi_\tau^{(0)} (\eta_0 \mathcal{L}_0(\mathbf{M}_0(\mathbf{r}'); \partial\Omega_0)) \\ - \pi_\tau^{(0)} (\eta_0 \bar{\mathcal{K}}_0(\mathbf{J}_0(\mathbf{r}'); \partial\Omega_0)) = \pi_\tau^{(0)} (\eta_0 \mathbf{H}^{\text{inc}}(\mathbf{r})) \end{cases} \quad (14)$$

$$\begin{cases} \frac{\eta_0}{2} \pi_\times^{(m)} \mathbf{M}_m(\mathbf{r}) - \pi_\tau^{(m)} (\eta_m \mathcal{L}_m(\mathbf{J}_m(\mathbf{r}'); \partial\Omega_m)) \\ + \pi_\tau^{(m)} (\eta_0 \bar{\mathcal{K}}_m(\mathbf{M}_m(\mathbf{r}'); \partial\Omega_m)) = 0 \\ - \frac{\eta_m}{2} \pi_\times^{(m)} \mathbf{J}_m(\mathbf{r}) - \pi_\tau^{(m)} (\eta_0 \mathcal{L}_m(\mathbf{M}_m(\mathbf{r}'); \partial\Omega_m)) \\ - \pi_\tau^{(m)} (\eta_m \bar{\mathcal{K}}_m(\mathbf{J}_m(\mathbf{r}'); \partial\Omega_m)) = 0 \end{cases} \quad (15)$$

where (14) is for the exterior sub-domain and (15) is for the interior sub-domains ($m = 1, 2$). To make sure that the solution of the decomposed problem is equivalent to that of the original one, the boundary conditions in (7)-(12) should be properly enforced. The following RTCs, which are equivalent to these boundary conditions, are used in LCMT-DDM:

$$\begin{cases} \eta_{rm} \mathbf{J}_m - \hat{\mathbf{n}}_m \times \mathbf{M}_m + \eta_{rm} \mathbf{J}_n + \hat{\mathbf{n}}_n \times \mathbf{M}_n = 0 \\ \eta_{rm} \hat{\mathbf{n}}_m \times \mathbf{J}_m + \mathbf{M}_m - \eta_{rm} \hat{\mathbf{n}}_n \times \mathbf{J}_n + \mathbf{M}_n = 0 \end{cases}, \mathbf{r} \in \Gamma_{mn} \quad (16)$$

where $m, n = 0, 1, 2, m \neq n$, $\eta_{rm} = \eta_m/\eta_0$ and Γ_{mn} means the touching face between adjacent sub-domain m and sub-domain n .

By linearly combining the IEs and RTCs as (14) + $\frac{1}{2}\eta_0(16)$ and (15) + $\frac{1}{2}\eta_0(16)$, the resultant equations can be expressed as

$$\begin{aligned} \frac{\eta_0}{2} \mathbf{J}_0(\mathbf{r}) - \eta_0 \pi_\tau^{(0)} (\mathcal{L}_0(\mathbf{J}_0(\mathbf{r}'); \partial\Omega_0) - \bar{\mathcal{K}}_0(\mathbf{M}_0(\mathbf{r}'); \partial\Omega_0)) \\ + \frac{\eta_0}{2} \{\mathbf{J}_n + \hat{\mathbf{n}}_n \times \mathbf{M}_n\} = \pi_\tau^{(0)} (\mathbf{E}^{\text{inc}}(\mathbf{r})), \mathbf{r} \in \Gamma_{0n} \\ \frac{\eta_0}{2} \mathbf{M}_0(\mathbf{r}) - \eta_0 \pi_\tau^{(0)} (\mathcal{L}_0(\mathbf{M}_0(\mathbf{r}'); \partial\Omega_0) + \bar{\mathcal{K}}_0(\mathbf{J}_0(\mathbf{r}'); \partial\Omega_0)) \\ + \frac{\eta_0}{2} \{\mathbf{M}_n - \hat{\mathbf{n}}_n \times \mathbf{J}_n\} = \eta_0 \pi_\tau^{(0)} (\mathbf{H}^{\text{inc}}(\mathbf{r})), \mathbf{r} \in \Gamma_{0n} \end{aligned} \quad (17)$$

$$\begin{aligned} - \pi_\tau^{(m)} (\eta_m \mathcal{L}_m(\mathbf{J}_m(\mathbf{r}'); \partial\Omega_m) - \eta_0 \bar{\mathcal{K}}_m(\mathbf{M}_m(\mathbf{r}'); \partial\Omega_m)) \\ + \frac{\eta_m}{2} \mathbf{J}_m(\mathbf{r}) + \frac{1}{2} \{\eta_m \mathbf{J}_n + \eta_0 \hat{\mathbf{n}}_n \times \mathbf{M}_n\} = 0, \mathbf{r} \in \Gamma_{mn} \\ - \pi_\tau^{(m)} (\eta_0 \mathcal{L}_m(\mathbf{M}_m(\mathbf{r}'); \partial\Omega_m) + \eta_m \bar{\mathcal{K}}_m(\mathbf{J}_m(\mathbf{r}'); \partial\Omega_m)) \\ + \frac{\eta_0}{2} \mathbf{M}_m(\mathbf{r}) + \frac{1}{2} \{\eta_0 \mathbf{M}_n - \eta_m \hat{\mathbf{n}}_n \times \mathbf{J}_n\} = 0, \mathbf{r} \in \Gamma_{mn} \end{aligned} \quad (18)$$

where $n = 1, 2$ in (17), and m, n satisfy $\{m = 1, 2\} \cap \{n = 0, 1, 2\} \cap \{m \neq n\}$ in (18).

B. Matrices Formulation

Equations (17) and (18) can be discretized by using the Rao-Wilton-Glisson (RWG) basis function and Galerkin test [27]. The resultant linear system can then be written as:

$$\begin{bmatrix} \mathbf{A}_0 & \mathbf{M}_{01} & \mathbf{M}_{02} \\ \mathbf{M}_{10} & \mathbf{A}_1 & \mathbf{M}_{12} \\ \mathbf{M}_{20} & \mathbf{M}_{21} & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_0 \\ \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{V}^0 \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (19)$$

where $\mathbf{A}_n$ is either the self-interaction matrix for the exterior sub-domain ($n = 0$) or the interior sub-domains ($n = 1, 2$), and $\mathbf{M}_{mn}$ is the mutual-interaction matrix between any two sub-domains. Due to the local coupling nature of LCMT-DDM, $\mathbf{M}_{mn}$ is either a sparse matrix for touching sub-domains or a zero matrix for non-touching sub-domains. Moreover, $\mathbf{I}_n$ and $\mathbf{V}^0$ can be expressed as $\mathbf{I}_n = (\mathbf{J}_n, \mathbf{M}_n)$ and $\mathbf{V}^0 = (\mathbf{V}_j^0, \mathbf{V}_m^0)$, and $\mathbf{A}_n$ can further be written as:

$$\mathbf{A}_n = \begin{bmatrix} \mathbf{A}_{jj}^n & \mathbf{A}_{jm}^n \\ \mathbf{A}_{mj}^n & \mathbf{A}_{mm}^n \end{bmatrix}. \quad (20)$$

The detailed expressions of these sub-matrices can be derived as:

$$\begin{aligned} \mathbf{A}_{jj}^n &= \left\langle \mathbf{f}_n(\mathbf{r}), \frac{\eta_n}{2} \mathbf{f}_n(\mathbf{r}) - \eta_n \pi_\tau^{(n)} \mathcal{L}_n(\mathbf{f}_n(\mathbf{r}'); \partial\Omega_n) \right\rangle_{\partial\Omega_n} \\ \mathbf{A}_{jm}^n &= \left\langle \mathbf{f}_n(\mathbf{r}), \eta_0 \pi_\tau^{(n)} \bar{\mathcal{K}}_n(\mathbf{m}_m(\mathbf{r}'); \partial\Omega_n) \right\rangle_{\partial\Omega_n} \\ \mathbf{A}_{mj}^n &= \left\langle \mathbf{m}_m(\mathbf{r}), -\eta_n \pi_\tau^{(n)} \bar{\mathcal{K}}_n(\mathbf{f}_n(\mathbf{r}'); \partial\Omega_n) \right\rangle_{\partial\Omega_n} \\ \mathbf{A}_{mm}^n &= \left\langle \mathbf{m}_m(\mathbf{r}), \frac{\eta_0}{2} \mathbf{m}_m(\mathbf{r}) - \eta_0 \pi_\tau^{(n)} \mathcal{L}_n(\mathbf{m}_m(\mathbf{r}'); \partial\Omega_n) \right\rangle_{\partial\Omega_n} \end{aligned} \quad (21)$$

where $\mathbf{f}_n$ and $\mathbf{m}_n$ denote as the basis function for the electric and magnetic currents $\mathbf{J}_n$ and $\mathbf{M}_n$, respectively, and the inner

product of two complex-valued vector functions $\mathbf{u}, \mathbf{v}$ is defined as

$$\langle \mathbf{u}, \mathbf{v} \rangle_{\Gamma} = \int_{\Gamma} (\mathbf{u} \cdot \mathbf{v}) d\Gamma. \quad (22)$$

Looking at the expressions of the diagonal blocks $\mathbf{A}_{jj}^n$ and $\mathbf{A}_{mn}^n$ in (21), one can see that their entries consist of tested contributions from EFIE and the residue term of MFIE (same structure as CFIE). This makes $\mathbf{A}_n$ a diagonally dominant matrix and, therefore, its nearfield component (which also includes the diagonal entries) is used to precondition the original linear system matrix as described in Section II-C. The sparse matrix $\mathbf{M}_{mn}$ and the right-hand-side (RHS) vectors $\mathbf{V}_j^0, \mathbf{V}_m^0$ can further be expressed as

$$\mathbf{M}_{mn} = \begin{bmatrix} \frac{\eta_m}{2} \langle \mathbf{f}_m, \mathbf{f}_n \rangle_{\Gamma_{mn}} & \frac{\eta_0}{2} \langle \mathbf{f}_m, \hat{\mathbf{n}}_n \times \mathbf{m}_n \rangle_{\Gamma_{mn}} \\ -\frac{\eta_0}{2} \langle \mathbf{m}_m, \hat{\mathbf{n}}_n \times \mathbf{f}_n \rangle_{\Gamma_{mn}} & \frac{\eta_m}{2} \langle \mathbf{m}_m, \mathbf{m}_n \rangle_{\Gamma_{mn}} \end{bmatrix} \quad (23)$$

and

$$\begin{aligned} \mathbf{V}_j^0 &= \left\langle \mathbf{f}_j^o(\mathbf{r}), \pi_{\tau}^{(0)} (\mathbf{E}^{inc}(\mathbf{r})) \right\rangle_{\partial\Omega_0} \\ \mathbf{V}_m^0 &= \left\langle \mathbf{m}_d^o(\mathbf{r}), \eta_0 \pi_{\tau}^{(0)} (\mathbf{H}^{inc}(\mathbf{r})) \right\rangle_{\partial\Omega_0} \end{aligned} \quad (24)$$

To further show the local coupling nature and the resultant sparse matrix of LCMT-DDM, we take the problem in Fig. 3 for example. In Fig. 3, the original multilayered dielectric object is decomposed into 5 sub-domains, and the system matrix can be written in the form of (25). In (25), $\mathbf{A}_{ii}$ ($i = 0, 1, \dots, 4$) is the dense self-coupling matrix for each sub-domain, $\mathbf{M}_{ij}$ is the sparse local coupling matrix. From Fig 3, it can be observed that Ω_1 does not touch Ω_3, Ω_4 and Ω_2 does not touch Ω_4 , therefore the mutual coupling matrices for these sub-domains are all zero, as shown in (25). Obviously, for a multilayered structure, the system matrix is even more sparse.

Fig. 3. Domain decomposition notation of five sub-regions. Arrows stand for the coupling between different sub-regions, which only exist between neighboring regions.

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{00} & \mathbf{M}_{01} & \mathbf{M}_{02} & \mathbf{M}_{03} & \mathbf{M}_{04} \\ \mathbf{M}_{10} & \mathbf{A}_{11} & \mathbf{M}_{12} & 0 & 0 \\ \mathbf{M}_{20} & \mathbf{M}_{21} & \mathbf{A}_{22} & \mathbf{A}_{23} & 0 \\ \mathbf{M}_{30} & 0 & \mathbf{A}_{32} & \mathbf{A}_{33} & \mathbf{M}_{34} \\ \mathbf{M}_{40} & 0 & 0 & \mathbf{M}_{43} & \mathbf{A}_{44} \end{bmatrix} \quad (25)$$

C. Preconditioning Technique

In the proposed LCMT-DDM, the FGMRES method, which is an inner-outer iteration scheme, is used to solve the preconditioned system,

$$\mathcal{A} \mathcal{P}^{-1} \mathcal{P} \mathbf{x} = \mathbf{b} \quad (26)$$

where $\mathcal{A}$ is the matrix of the linear system shown in (19), and $\mathcal{P}^{-1}$ is the preconditioner.

In this work, $\mathcal{P}$ is constructed as:

$$\mathcal{P} = \begin{bmatrix} \mathbf{A}_{nea0} & \mathbf{0} & \mathbf{0} \\ \mathbf{M}_{10} & \mathbf{A}_{nea1} & \mathbf{0} \\ \mathbf{M}_{20} & \mathbf{M}_{12} & \mathbf{A}_{nea2} \end{bmatrix} \quad (27)$$

where $\mathbf{A}_{neai}$ ($i = 0, 1, 2$) is the near-group coupling matrix in MLMFA for each sub-domain. Then the inverse of $\mathcal{P}$ has the form of

$$\mathcal{P}^{-1} = \begin{bmatrix} \mathbf{A}_{nea0}^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{B} & \mathbf{A}_{nea1}^{-1} & \mathbf{0} \\ \mathbf{D} & \mathbf{C} & \mathbf{A}_{nea2}^{-1} \end{bmatrix} \quad (28)$$

where $\mathbf{B}, \mathbf{C}, \mathbf{D}$ can be explicitly calculated as

$$\mathbf{B} = -\mathbf{A}_{nea1}^{-1} \mathbf{M}_{10} \mathbf{A}_{nea0}^{-1} \quad (29)$$

$$\mathbf{C} = -\mathbf{A}_{nea2}^{-1} \mathbf{M}_{21} \mathbf{A}_{nea1}^{-1} \quad (30)$$

$$\mathbf{D} = -\mathbf{A}_{nea2}^{-1} (\mathbf{M}_{20} - \mathbf{M}_{21} \mathbf{A}_{nea1}^{-1} \mathbf{M}_{10}) \mathbf{A}_{nea0}^{-1} \quad (31)$$

The procedure of the inner-outer iteration in FGMRES is shown in Algorithm 1. Compared with the traditional GMRES method, the only difference in FGMRES is that the preconditioned vector $\mathbf{z}_j = \mathcal{P}^{-1} \mathbf{v}_j$ needs to be computed (see Algorithm 1). In order to obtain the preconditioner $\mathcal{P}^{-1}$ in (28), $\mathbf{A}_{neai}^{-1}$ ($i = 0, 1, 2$) should be calculated. In FGMRES, this can be accomplished by an inner iteration. Since $\mathbf{A}_{neai}$ in our method is diagonally dominant, the preconditioned vector $\mathbf{z}_j$ can be efficiently calculated in the inner iteration without extra memory consumption.

To better explain the procedure, the indices of the matrices will be started from 1 instead of 0. Moreover, we denote $\mathbf{P}_{1,1} = \mathbf{A}_{nea0}$, $\mathbf{P}_{2,2} = \mathbf{A}_{nea1}$, and $\mathbf{P}_{3,3} = \mathbf{A}_{nea2}$. Then $\mathcal{P}$ in (27) can be rewritten as

$$\mathcal{P} = \mathcal{P}_{3,3} = \begin{bmatrix} \mathbf{P}_{1,1} & 0 & 0 \\ \mathbf{M}_{2,1} & \mathbf{P}_{2,2} & 0 \\ \mathbf{M}_{3,1} & \mathbf{M}_{3,2} & \mathbf{P}_{3,3} \end{bmatrix}. \quad (32)$$

If we denote the upper-left 1×1 block as

$$\mathcal{P}_{1,1} = [\mathbf{P}_{1,1}], \quad (33)$$

then the upper-left 2×2 block can be denoted as

$$\mathcal{P}_{2,2} = \begin{bmatrix} \mathcal{P}_{1,1} & 0 \\ \mathbf{M}_{2,1} & \mathbf{P}_{2,2} \end{bmatrix}. \quad (34)$$

Similarly, $\mathcal{P}_{3,3}$ can be expressed as,

$$\mathcal{P}_{3,3} = \begin{bmatrix} \mathcal{P}_{2,2} & 0 \\ \mathbf{M}_{3,1 \rightarrow 2} & \mathbf{P}_{3,3} \end{bmatrix}. \quad (35)$$

where $\mathbf{M}_{3,1 \rightarrow 2}$ stands for $[\mathbf{M}_{3,1}, \mathbf{M}_{3,2}]$. Apparently, this notation leads to a recursive formulation. Generally, when the problem is decomposed into L regions, $\mathcal{P}$ can be written as

$$\mathcal{P} = \mathcal{P}_{L,L} = \begin{bmatrix} \mathcal{P}_{L-1,L-1} & 0 \\ \mathbf{M}_{L,1 \rightarrow L-1} & \mathbf{P}_{L,L} \end{bmatrix}. \quad (36)$$

Hence, the preconditioner $\mathcal{P}^{-1}$ can be calculated as

$$\mathcal{P}^{-1} = \mathcal{P}_{L,L}^{-1} = \begin{bmatrix} \mathcal{P}_{L-1,L-1}^{-1} & 0 \\ -\mathbf{P}_{L,L}^{-1} \mathbf{M}_{L,1 \rightarrow L-1} \mathcal{P}_{L-1,L-1}^{-1} & \mathbf{P}_{L,L}^{-1} \end{bmatrix}. \quad (37)$$

Consequently, the multiplication of $\mathcal{P}^{-1}$ and the vector $\mathbf{X}_L = (\mathbf{X}_{L-1}; \mathbf{x}_L^T)^T = (\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \cdots \mathbf{x}_L)^T$ can be expressed as:

$$\begin{aligned} \mathbf{Y}_L &= \mathcal{P}^{-1} \mathbf{X}_L = \mathcal{P}_{L,L}^{-1} \begin{bmatrix} \mathbf{X}_{L-1} \\ \mathbf{x}_L \end{bmatrix} \\ &= \begin{bmatrix} \mathcal{P}_{L-1,L-1}^{-1} & 0 \\ -\mathbf{P}_{L,L}^{-1} \mathbf{M}_{L,1 \rightarrow L-1} \mathcal{P}_{L-1,L-1}^{-1} & \mathbf{P}_{L,L}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{X}_{L-1} \\ \mathbf{x}_L \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{Y}_{L-1} \\ \mathbf{P}_{L,L}^{-1} (-[\mathbf{M}_{L,1} \mathbf{y}_1 + \cdots + \mathbf{M}_{L,L-1} \mathbf{y}_{L-1}] + \mathbf{x}_L) \end{bmatrix} \end{aligned} \quad (38)$$

where $\mathbf{Y}_L = (\mathbf{Y}_{L-1}; \mathbf{y}_L^T) = (\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3 \cdots \mathbf{y}_L)^T$. It is obvious that $\mathcal{P}^{-1} \mathbf{X}_L$ can be calculated recursively. In the computation, we only need to iteratively solve a sequence of linear systems as

$$\mathbf{P}_{l,l} \mathbf{y}_l = \mathbf{x}_l \quad (l = 1, \cdots, L) \quad (39)$$

to finally obtain the vector $\mathbf{Y}_L$. Because of the sparse nature and favorable iterative convergence of $\mathbf{P}_{l,l}$ (self- and near-group interaction of CFIE-like matrices in MLFMA), the preconditioner can be efficiently applied within the inner iterations of FGMRES (which is a GMRES process [18]). The details of the FGMRES procedure can be found in [23].

Algorithm 1 FGMRES Method

- 1: **Start** Choose an initial guess x_0 , m is the dimension of the Krylov subspaces. Setup a matrix $\bar{\mathbf{H}}_m$ with dimension $(m+1) \times m$.
 - 2: **procedure** ARNOLDI PROCESS:
 - 3: Compute $\mathbf{r}_0 = \mathbf{b} - \mathbf{A} \mathbf{x}_0$ and $\mathbf{v}_1 = \frac{\mathbf{r}_0}{\|\mathbf{r}_0\|_2}$
 - 4: **for** $j = 1, m$ **do**
 - 5: Compute $\mathbf{z}_j = \mathcal{P}^{-1} \mathbf{v}_j$
 - 6: Compute $\mathbf{w} = \mathbf{A} \mathbf{z}_j$
 - 7: **for** $i = 1, j$ **do**
 - 8: Compute $\begin{cases} h_{i,j} = \langle \mathbf{w}, \mathbf{v}_i \rangle \\ \mathbf{w} = \mathbf{w} - h_{i,j} \mathbf{v}_i \end{cases}$
 - 9: Compute $h_{j+1,j} = \|\mathbf{w}\|_2$ and $\mathbf{v}_{j+1} = \frac{\mathbf{w}}{\|\mathbf{w}\|_2}$.
 - 10: Define $\mathbf{Z}_m = [\mathbf{z}_1, \dots, \mathbf{z}_m]$.
 - 11: **Update the approximate solution:** Compute $x_m = x_0 + \mathbf{Z}_m \mathbf{y}_m$ where $\mathbf{y}_m = \arg \min \|\beta \mathbf{e}_1 - \bar{\mathbf{H}}_m \mathbf{y}\|_2$ and $\mathbf{e}_1 = [1, 0, \dots, 0]^T$.
 - 12: **Restart:** If convergence stop, else $\mathbf{x}_0 = \mathbf{x}_m$ goto 2.
-

III. NUMERICAL RESULTS

In this section, the accuracy and capability of the proposed LCMT-DDM are validated via several numerical examples. A homogeneous dielectric sphere and a two-layer sphere are first studied to show the high accuracy of this method. The effect of the inner-iteration tolerance on the outer iteration and the overall solution accuracy are also investigated. Then two multilayered dielectric objects are further studied to

demonstrate the superior performance of this method for more complicated objects, including accuracy, iterative convergence, and memory consumption.

A. Validation of LCMT-DDM

In the first numerical example, the EM scattering from a homogeneous dielectric sphere is investigated to demonstrate the accuracy of LCMT-DDM. The radius of the sphere is $0.5m$, and the relative permittivity, permeability are $\epsilon_{r1} = 2.0, \mu_{r1} = 1.0$. The plane wave is incident from the $-z$ axis at 0.3 GHz. In LCMT-DDM, the original object is decomposed into three regions, the exterior region and two hemi-sphere interior regions. The exterior region is discretized with 800 triangular patches, and the interior regions are discretized with 644 and 688 triangular patches, respectively. The V-V and H-H polarized bistatic radar cross section (RCS) of this dielectric sphere is calculated and shown in Fig. 4. For comparison, the analytical results from the Mie series are also shown. It can be seen that the results of the proposed method agree well with the Mie solution.

Fig. 4. The bistatic RCS of the dielectric sphere.

Next, we consider a two-layer dielectric sphere. It is composed of a dielectric sphere with radius of $0.5m$ and a dielectric coating layer with thickness of $0.1m$. The relative permittivity, permeability of the coating and sphere are ϵ_{r1}, μ_{r1} and ϵ_{r2}, μ_{r2} , respectively. To further demonstrate the accuracy of LCMT-DDM, the root mean square error (RMSE) of RCS is investigated, which is defined as

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N |\sigma_i - \sigma_i^{\text{Mie}}|^2}. \quad (40)$$

where N is the total number of observation points, σ is the RCS calculated from numerical methods, and σ_i^{Mie} is obtained by the Mie series. The incidence of the plane wave and the working frequency are set the same as the previous case. For comparison, the traditional CRM is also implemented. Two material settings are used, i.e. $\epsilon_{r1} = 9.0, \mu_{r1} = 1.0, \epsilon_{r2} = 8.0, \mu_{r2} = 1.0$ and $\epsilon_{r1} = 3.0, \mu_{r1} = 1.0, \epsilon_{r2} = 2.0, \mu_{r2} = 1.0$. In Fig. 5, RMSE of LCMT-DDM and CRM are computed and compared for different discretization densities. Here, four-point Gaussian quadrature rule is used in each triangle. It can

be seen that the two methods have almost the same accuracy under the same discretization, and the error decreases at the same speed when the discretization density increases.

Fig. 5. The RMSE of bistatic RCS of the two-layer dielectric sphere.

We also examine the accuracy and the outer iteration count affected by the convergence tolerance of the inner iteration. The convergence tolerance of the outer iteration is fixed as 0.001. As shown in Table I, the outer iteration count does not change with respect to the inner convergence tolerances (0.01, 0.001, 0.0001), and the variation of the RCS error is within 0.01 dB. It reveals that the accuracy is mainly governed by the tolerance of the outer iteration. The inner iteration does not have a significant impact on the outer iteration and the final results.

TABLE I

THE RMSE AND OUTER ITERATION COUNT WITH DIFFERENT INNER TOL.

Inner tol.	0.01	0.001	0.0001
RMSEs(dBsm)	0.1409	0.1373	0.1354
Outer Num.	29	29	29

B. EM Scattering from Multilayered Dielectric Plate

In the second numerical example, the EM scattering from a two-layer dielectric plate and a four-layer dielectric plate is investigated. The computation was performed on a personal computer with Intel Core i7-7700K @ 4.20GHz CPU. The geometry, dimension and material of the four-layer plate are shown in Fig. 6, and the two-layer model is just one half of the four-layer model. As shown in this figure, this multilayered plate consists of two alternately stacked slabs with different materials but same size.

The plane wave is incident from the $-z$ direction at 3GHz. In LCMT-DDM, the two-layer and four-layer plates are decomposed into three and five regions respectively according to the material configuration. In the four-layer model, the number of triangles in each sub-regions of LCMT-DDM is, $Region_0$: 72,392, $Region_1$: 48,296, $Region_2$: 48,296, $Region_3$: 48,296, and $Region_4$: 48,296. The bistatic RCS results of the two models are shown in Fig. 7 and Fig. 8. For

Fig. 6. The geometry of the four-layer dielectric plate.

Fig. 7. The bistatic RCS of the two-layer dielectric plate.

comparison, the RCS calculated from CRM is also plotted. The discretization density of CRM is set as the same as that in LCMT-DDM for a fair comparison. It can be seen that the results of the two methods agree very well with each other.

The GMRES method utilized by CRM uses a tolerance of 0.001. The outer iteration convergence tolerance of FGMRES used by LCMT-DDM is also set to 0.001. The inner iteration convergence tolerance of FGMRES is set to 0.01. The consumption of computational resources is shown in Table II and Table III for the two models. It can be seen that LCMT-DDM has an apparent advantage over the traditional CRM for both cases. Enforcing local coupling between different domains reduces the memory requirement of LCMT-DDM (compared to the traditional CRM method) since it makes the coupling matrices sparse (as opposed to dense matrices in CRM). The CPU time required by LCMT-DDM is also reduced. This is because the flexible preconditioner used in the inner iterations of FGMRES significantly increases the convergence rate.

C. EM Scattering from a Multilayered Dielectric Radome

In the third numerical example, the EM scattering from a three-layer dielectric radome is investigated. This computation was performed on a workstation with two 64-bit Intel Xeon

Fig. 8. The bistatic RCS of the four-layer dielectric plate.

TABLE II
COMPUTATIONAL STATISTICS OF THE TWO-LAYER PLATE

	Time	Mem.	Iter. Num.
LCMT-DDM:	1,750 Sec.	4.7GB	83
CRM:	5,652 Sec.	7.0 GB	637

TABLE III
COMPUTATIONAL STATISTICS OF THE FOUR-LAYER PLATE

	Time	Mem.	Iter. Num.
LCMT-DDM:	3,976 Sec.	7.9GB	127
CRM:	15,308 Sec.	12.7GB	943

Gold 6148 @ 2.40GHz CPUs and run with twenty openMP parallel processes. The geometry and dimension of the radome are shown in Fig. 9. As shown in the figure, the radius of the air hemi-sphere is $r = 584mm$. The thickness of each layer is $d = 18mm$. The relative permittivity and relative permeability of the outermost layer and the innermost layer are both $\epsilon_{r1} = 3.0$, $\mu_{r1} = 1.0$, and those of the middle layer are $\epsilon_{r2} = 2.0$, $\mu_{r2} = 1.0$. The plane wave is incident from the $-z$ axis, and the frequency is 4.0 GHz. In LCMT-DDM, the radome is decomposed into four homogeneous sub-regions based on their material properties. The mesh size in each sub-region is set to be $7.5mm$, and the numbers of triangles in each sub-region are, $Region_0$: 170,708, $Region_1$: 173,720, $Region_2$: 163,714, and $Region_3$: 156,470. Again, CRM is utilized for comparison, in which the same discretization criterion is applied. As shown in Fig. 10, the H-H and V-V polarized Bistatic RCS results of LCMT-DDM and CRM agree well with each other. The electric and magnetic current distribution of LCMT-DDM and CRM are further shown in Fig. 11 and Fig. 12. It can be seen that the current distribution also agrees well with each other.

The computational statistics of the two methods are shown in Table IV. FGMRES is used in LCMT-DDM and GMRES is used in CRM. In FGMRES, the convergence tolerance of the inner and outer iterations is set to be 0.01 and 0.001, and in GMRES, the convergence tolerance is set to be 0.001, which is the same as the outer iteration tolerance in FGMRES. It can be seen from table IV that, with the same discretization, LCMT-DDM only needs one tenth of the iteration count required in

Fig. 9. The geometry and dimension of the three-layer dielectric radome.

Fig. 10. The bistatic RCS of the three-layer dielectric radome.

CRM. Just like the previous example, the use of local coupling and the flexible preconditioner reduce the memory requirement and the CPU time of LCMT-DDM to the three fourth and the three fifth of those of CRM, respectively. It can be seen clearly that for large and complex models, LCMT-DDM has an obvious advantage over CRM.

TABLE IV
COMPUTATIONAL STATISTICS OF THE THREE-LAYER RADOME

	Time	Mem.	Iter. Num.
LCMT-DDM:	12,523 Sec.	34.9GB	164
CRM:	50,280 Sec.	82.4GB	1755

IV. CONCLUSIONS

In this paper, a local coupling multi-trace domain decomposition method (LCMT-DDM) is presented to solve the EM scattering from multilayered dielectric objects. By a proper domain decomposition, the boundaries of all sub-domains can be made as touching-faces. Therefore, the interactions between different sub-domains are all local coupling, leading to a sparse system matrix. In each sub-domain, the accurate EFIE and MFIE for dielectrics are applied as the governing equations. By enforcing the Robin transmission conditions (RTCs) on the boundaries, a diagonally dominant CFIE-like equation can be formulated for each sub-domain. According to this feature, the FGMRES method based on an inner-outer iteration is applied to solve the system matrix. Moreover, a preconditioner

Fig. 11. The electric and magnetic currents of the three-layer dielectric radome from LCMT-DDM.

Fig. 12. The electric and magnetic currents of the three-layer dielectric radome from CRM.

that can be recursively constructed is proposed. Numerical results show that the proposed LCMT-DDM is superior over the existing CRM. With the same accuracy level, LCMT-DDM has a better iterative convergence, less memory and CPU time consumption. The advantage is expected to be more significant when structures with more layers are considered. As future work, a symmetric formulation and the corresponding preconditioning technique will be explored to further increase the iterative convergence rate and reduce the computational requirements.

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