Frequency Domain Generalized Internal Multiple Imaging with Energy norm

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Seismic data often includes scattered energy of all types including those corresponding to multi scattering. Traditional imaging techniques are focused on the primary reflections, and thus, may fail to image crucial structures, like faults, which sometimes are only illuminated by the higher-order scattering. A process, referred to as generalized internal multiple imaging (GIMI), offers an opportunity to image higher order scattering by incorporating an interferometric cross-correlation step of the back propagated wavefield with the recorded data into the standard imaging process. The interferometric step is performed on a large portion of the data, which leads to a considerable increase in the cost of imaging, especially in the time domain. Therefore, we formulate a frequency domain GIMI to replace the cross correlation with a simple multiplication, which makes the implementation more practical. In addition, we apply the energy norm imaging condition to isolate the scattering component or its tomographic term. We show these features on numerical experiments.

Introduction

Seismic imaging is a crucial task in the conventional processing chain. The most popular seismic migration methods, like reverse time migration (RTM), are focused to image only primary reflections. The implementation of RTM can be generally described in two steps. We first forward extrapolate the source wavefield and back propagate the recorded data. We then apply the imaging condition using the two wavefields to extract points of wavefield continuation, referred to as reflectivity. Though this mechanism, RTM can only image single scattering reflections between the two wavefields. Multiple reflections are usually mapped in the wrong place, and are considered artefacts (references for RTM, and its limitations). However, multiple scattering data could contain valuable information, which can provide additional illumination of the subsurface (Malcolm et al., 2009; Zuberi and Alkhalifah, 2014a and 2014b).

A lot of interest has been devoted to imaging the Earth surface related multiples (Liu et al., 2010), whereas for the more complicated internal multiples the solutions are limited (Malcolm et al., 2009). Behura et al., (2012) suggested a Newton-Marchenko-Rose Imaging based on exact inverse scattering theory, but the procedure does not allow for separate imaging of different orders of multiples. Zuberi and Alkhalifah (2014) proposed a three steps approach based on the interferometric that migrates any given order internal multiples separately utilizing the Born series, which is referred to as generalized internal multiple imaging (GIMI). This approach transforms multiples into the leading-order scattering term by performing additional correlations between the back propagated wavefield at image points and the data. The number of cross-correlations required is one less than the order of the term in the Born scattering series (or equal to the order of internal multiple) we intend to migrate. The different orders of multiples can then be imaged using conventional migration methods. Alkhalifah and Guo (2018) formulated the adjoint operation of the GIMI process and used it to further suppress crosstalk artifacts resulting from the conventional GIMI using a least-squares optimization. They also argue for the use of the leading order term of the scattering series to extract wavepath updates in velocity model building. The additional computational cost caused by the interferometric step, however, would limit the application of GIMI to in 3D cases.

In this abstract, we implement GIMI in the frequency domain in which the cost of interferometric step is greatly reduced. This should help the convergence rate of a least-squares optimization of GIMI. In addition, we apply the energy imaging condition (Rocha et al., 2016) to isolate the reflectivity component of the desired order of scattering we intend to image and the corresponding wavepath long wavelength energy. We implement the frequency domain GIMI on numerical experiments to verify the effectiveness of our approach.

Generalized Internal Multiple Imaging Theory

In this section, we briefly introduce the frequency domain generalized internal multiple imaging (GIMI) process and highlight some of its advantages compared to the time domain implementation. As described by Zuberi and Alkhalifah (2014), Alkhalifah and Guo (2018), the GIMI process includes three steps:

1. Back propagate the recorded data from the surface to the image domain to obtain the mainly upgoing wavefield (superscript -) from the source, s , to the point, x , G_{xs}^- . In mathematical terms, we can write this step as follows

$$G_{xs}^- = G_{rx}^b R_{rs}^{-*}, \quad (1)$$

where G_{rx}^b is the background Green's function from x to r , and the R_{rs}^{-*} is the complex conjugate (superscript *) of the surface record. This step is the same as the back-propagation process in RTM.

2. We effectively transform the leading order scattering term of the Born series to the wavepath term and the second order to be the leading order scattering term. This is achieved using an interferometric cross-correlation step of the back propagated wavefield G_{xs}^- with the recorded data R_{rs}^- over sources to obtain, $G_{rx}^- = G_{xs}^{-*} R_{rs}^-$. To improve resolution, we employ here a deconvolution, which in the frequency domain and with utilizing the proper damping is given by

$$G_{rx}^- = G_{xs}^{-*} R_{rs}^- / (G_{xs}^{-*} G_{xs}^- + \varepsilon), \quad (2)$$

where ε is a small positive value.

3. Finally, we employ the zero-lag cross-correlation imaging condition for second order scattering (first order internal multiples),

$$I = (G_{xr}^b)^{+*} G_{rx}^-, \quad (3)$$

which includes a summation over receivers. $(G_{rx}^b)^{+*}$ is the corresponding background down going wavefield. The image I contains reflectivity corresponding to second-order scattering and the long wavelength wavepath energy from the primary (first order) reflections. These low-wavenumber components of the image can be utilized to update the background velocity model in waveform inversion. Alternatively, we utilize the energy norm imaging condition to isolate the reflectivity from long wavelength information.

Energy Imaging Condition

We, next, utilize the spatial gradients and time derivatives of the wavefields to implement the energy norm imaging condition in our GIMI process (Rocha et al., 2016). A spatial and temporal Fourier transform of the wavefield renders it a function of the wavenumber vector $\mathbf{k} = [k_x, k_y, k_z]$ and frequency ω , in which we can apply the energy norm imaging condition in GIMI as,

$$I_E = W(G_{xr}^b)^{+*} \cdot W G_{rx}^- \quad (4)$$

where

$$W G_{rx}^- = (ivk_x, ivk_y, ivk_z, -i\omega) G_{rx}^- \quad (5)$$

$$W(G_{xr}^b)^{+*} = (ivk_x, ivk_y, ivk_z, +i\omega) (G_{xr}^b)^{+*} \quad (6)$$

Through further derivation, equation (4) leads to,

$$I_E = [-\omega^2 \cos 2\theta + \omega^2] (G_{xr}^b)^{+*} G_{rx}^- = +2\omega^2 \sin^2 \theta (G_{xr}^b)^{+*} G_{rx}^- \quad (7)$$

In equation (7), we can apply a scaling factor to the second term in brackets to control the contribution as a function of angle θ_c . To attenuate the low-wavenumber energy, which corresponds to having $\theta_c = 90^\circ$ reflection angle, we set the scaling factor to equal -1. Thus, the backscattering attenuation imaging condition is,

$$I_E^{mig} = [-\omega^2 \cos 2\theta - \omega^2] (G_{xr}^b)^{+*} G_{rx}^- = -2\omega^2 \cos^2 \theta (G_{xr}^b)^{+*} G_{rx}^- \quad (8)$$

Because these two imaging conditions (equation 7 and 8) are complementary with respect to a trigonometric function, the maximum value of one imaging condition is the minimum value of the other one, and vice versa. Therefore, we can image a narrow range of reflection angles near 90° , the tomographic term, by employing an exponential function associated with I_E^{mig} to I_E ,

$$I_E^{tomo} = I_E e^{-\alpha I_E^{mig}} \quad (9)$$

where factor $\alpha > 1$, and the I_E^{mig} is normalized, such that only the factor $\sin^2 \theta$ is accounted for in the exponential. In the example section, we use these symbols I_E (equation 7), I_E^{mig} (equation 8) and I_E^{tomo} (equation 9) to represent the conventional energy norm, backscattering attenuation and the tomographic term imaging conditions, respectively.

Examples

We test our frequency domain GIMI first by applying it to a simple salt flank-like model as displayed in Figure 1a. The model consists of two homogeneous layers (2400m/s and 2700m/s), and a vertical reflector located on the left side of the model, which doesn't produce primary reflections recorded in this limited-offset survey. Thus, the conventional RTM cannot image the vertical structure. The dimension of the model is $NX \times NZ = 201 \times 101$, and the grid interval is 15.24m in both the horizontal and vertical directions. 21 shots are evenly positioned on the surface with a spacing of 152.4m. We produce synthetic data for frequencies ranging from 15–25 Hz in a 2 Hz sampling interval using a unit Dirac source wavelet. Only the reflection data will be used for imaging; the migration model for the GIMI process is the first layer velocity (2400m/s). Figure 1b shows the result of the interferometric step for a receiver located at $x=1524$ m. The first-order scattering term is propagating downward (The triangle area), which will formulate the low-frequency wavepath components with the corresponding

background downgoing wavefield $(G_{rx}^b)^+$ in Figure 1c. On the other hand, the second order scattering is heading in the upward direction (Black arrows). Some of the energy in the second order term is covered by the dominating first order term, but it will not cause any interference because of the nearly opposite propagation directions, which will contribute to the second order reflectivity image. Figure 2 (b) and (c) show the two components of image. The migration term I_E^{mig} and the tomographic term I_E^{tomo} are well separated.

Figure 1 (a) The Salt flank-like velocity model; the functions G_{rx}^- (b) and $(G_{rx}^b)^+$ (c) for a frequency of 25 Hz.

Figure 2 (a) The image I_E ; (b) the image I_E^{mig} and (c) the image I_E^{tomo} .

Figure 3 (a) The modified Marmousi model, (b) a smoothed version; (c) A linearly-increasing model.

Figure 4 Images using the smooth velocity in Figure 3b. (a) I_E ; (b) I_E^{mig} and (c) I_E^{tomo} .

Now, we test our proposed approach on a more complicated modified Marmousi model that includes two “V” shaped reflectors (one side is given by faults) as shown in Figure 3a. In the Marmousi test, we use the same source wavelet and the frequency range with the salt flank model. The GIMI images are calculated using the different velocities are shown in Figures 4 and 5. As a result, the kinematically accurate smoothed model provided a more focussed double-scattered image (Figure 4b). The linearly-increasing model (Figure 5b) produces an image of the faults at the wrong locations. As indicated by the black line in Figure 3a, there is a large acoustic impedance contrast in the modified Marmousi model. Thus, wavepath energy of the smoothed model in Figure 4c only illuminates well above the fault. However, the wavepath energy in the linearly-increasing version (Figure 5c) penetrates this interface and shows the potential of GIMI for any wavepath based inversion updates when the velocity model is wrong.

Figure 5 Images using the linearly-increasing model in Figure 3c. (a) I_E ; (b) I_E^{mig} and (c) I_E^{tomo} .

Conclusions

We developed a frequency domain GIMI with the energy norm imaging condition to image clean double scattered reflections at a reduced cost. In the frequency domain, the interferometric step is evaluated by a deterministic deconvolution process, which provides much better resolution. The numerical examples demonstrate the effectiveness of the proposed method.

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