Multiparameter reflection waveform inversion for acoustic VTI media

Running head: **RWI for acoustic VTI media**

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ABSTRACT

Full waveform inversion (FWI) in transversely isotropic media with a vertical symmetry axis (VTI) provides an opportunity to better match the data at the near and far offsets. However, multiparameter FWI, in general, suffers from serious cycle-skipping and trade-off problems. Reflection waveform inversion (RWI) can help us recover a background model by projecting the residuals of reflections along the reflection wavepath. Thus, we extend RWI to acoustic VTI media utilizing the proper parameterization for reduced parameter tradeoff. From a radiation patterns analysis, an acoustic VTI medium is better described by a combination of the normal-moveout velocity v_n and the anisotropic parameters η and δ for RWI applications. We design a three-stage inversion strategy to construct the optimal VTI model. In the first stage, we only invert for the background v_n by matching the simulated reflections from the perturbations of v_n and δ with the observed reflections. In the second stage, the background v_n and η are optimized simultaneously and the far-offset reflections mainly contribute to their updates. We perform Born modelling to compute the reflections for the two stages of RWI. In the third stage, we perform FWI for the acoustic VTI medium to delineate the high-wavenumber structures. For this stage, the VTI medium is described by a combination of the horizontal velocity v_h , η and ϵ instead of v_n , η and δ . The acoustic VTI FWI utilizes the diving waves to improve the background, as well as utilizes the reflections for high-resolution information. Finally, we test our inversion algorithm on the modified VTI Sigsbee 2A model (a salt free part) and a 2D line from a 3D Ocean Bottom Cable dataset. The results demonstrate that the proposed VTI RWI approach can recover the background model for acoustic VTI media starting from an isotropic model. This background VTI model can mitigate the cycle skipping of FWI and help the inversion recover higher-resolution structures.

Keywords: Anisotropy, full-waveform inversion, reflection waveform inversion.

INTRODUCTION

Full waveform inversion (FWI) aims to build a high-resolution subsurface model by minimizing the misfits between the simulated and observed data (Tarantola 1984). For most field real wide-aperture seismic data, near and far offsets cannot be reproduced simultaneously without accounting for anisotropy (Prioux *et al.* 2011). Therefore, anisotropic data often yield poor results when the isotropic approximation is used to perform an acoustic FWI. As high-performance computing and data acquisition (like large aperture) improve, anisotropic FWI is drawing considerable attention (Cheng *et al.* 2014; Operto *et al.* 2015; Wu and Alkhalifah 2016; Cheng *et al.* 2016; Guitton and Alkhalifah 2017; Masmoudi and Alkhalifah 2018; Zhang *et al.* 2018; Djebbi and Alkhalifah 2019).

Even with a simple transversely isotropic with a vertical symmetry axis (VTI) assumption, it is very challenging to estimate the anisotropic medium properties from pressure wavefields (Prioux *et al.* 2011; Warner *et al.* 2013; Wang *et al.* 2012). An acoustic VTI FWI tends to stagnate at an unreasonable model because of the cycle skipping and trade-off issues, especially when the starting model can not describe the kinematics of the true model within half a cycle. Lots of work have been devoted to overcome these two limitations. In view of the important role that low frequencies play in FWI in building the long-wavelength structure information, a number of approaches to generate low-frequency information from the seismic data helped to guide the inversion to a global minimum with an enlarged attraction basin (Wu, Luo and Wu 2014; Li and Demanet 2016; Li *et al.* 2018). Besides, we can extend the search space by introducing additional degrees of freedom to improve

the convexity of the objective function (Symes 2008). FWI using a reconstructed wavefield (van Leeuwen and Herrmann 2013; Alkhalifah and Song 2019; Lin *et al.* 2018) or a matching filter (Luo and Sava 2011; Warner and Guasch 2016; Sun and Alkhalifah 2019; Li and Alkhalifah 2019a) belong to this family. Considering that the deeper part, including the reservoir level, is mainly illuminated by reflection energy, Xu *et al.* (2012) developed reflection waveform inversion (RWI) to update the background model along the reflection wavepath by matching the reflections generated from migration/demigration to the observed data. Wang *et al.* (2013) observed that low-frequency data can mitigate the cycle-skipping of RWI and implemented the frequency-domain RWI in a multiscale way. The background updates in RWI requires true amplitude migration that can be used to predict the reflections with enough accuracy. Wu and Alkhalifah (2015) developed a new optimization problem to update the background and perturbation components simultaneously by matching both the wavefields from the source and the Born scattering to the observed data. Wu and Alkhalifah (2016) extended this approach to invert for the background component of VTI media using a parameterization given by the horizontal velocity v_h , and the anisotropy parameters η and ϵ . For elastic media, we can use this approach to invert for the background P- and S-wave velocities simultaneously to produce multi-component reflection data (Guo and Alkhalifah 2017; Li *et al.* 2019b). Pattnaik *et al.* (2016) used the reflections to estimate the VTI medium properties in terms of the normal-moveout (NMO) velocity, anisotropy parameters η and δ using a two-stage inversion algorithm. Li *et al.* (2019a) developed an image-domain tomographic algorithm for acoustic VTI media by optimizing the image quality and utilizing the well information to alleviate the depth deviation caused by an inaccurate δ field.

The analysis on the trade-off between the multiple parameters is necessary for designing an optimal parameterization for a specific acquisition scenario. Such tradeoff studies have been investigated to

choose a suitable parameterization by analyzing the radiation patterns according to the Born approximation (Gholami *et al.* 2013; Alkhalifah and Plessix 2014; Alkhalifah 2017; Kamath and Tsvankin 2016; Kazei and Alkhalifah 2018). Plessix and Cao (2011) applied an eigenvector decomposition analysis of the Hessian of the least-squares misfit functional to study the parameterization of FWI in an acoustic VTI medium. Alkhalifah and Plessix (2014) analyzed the perturbation radiation patterns for several parameterizations especially with a horizontal reflector and suggested different parameterization scenarios for different inversion settings. The parameterization in terms of the NMO velocity v_n , the anelliptic parameter η and Thomsen's parameter δ (Thomsen 1986) has limited trade-off and shows more potential for an inversion that includes reflections. Besides, a combination of the horizontal velocity v_h and the parameters η and ε can generate the optimal inversion results especially when we start from the diving waves in FWI. Alkhalifah, Fomel and Biondi (2001) showed that the kinematical information of P-wave data only relies on v_n and η , if δ does not change laterally. Alkhalifah (2017) shared the transmission radiation patterns for scattering from a horizontal reflector in the vertical time domain, which corresponds to an RWI scenario; it also shows that δ has no transmission influence on the scattering. Therefore, the background component of the δ field is hard to recover without additional information, such as well logs. We use the parameterization v_n , η and δ to describe the acoustic VTI model, where the background component of δ is not our objective but the perturbation of δ is used to fit the amplitudes of reflections at near offsets (Li and Alkhalifah 2019b). Besides, we assume that the density of the acoustic VTI model is homogeneous. δ and density have the same radiation pattern for a horizontal reflector, therefore the trade-off between δ and density cannot be resolved. Alkhalifah and Plessix (2014) showed that δ can play the role that density plays in improving the data fit for an imperfect physical model, yet providing us with more practical wave equations than those that

include variable density. This means we can use constant-density wave equations for simplicity. Pattnaik *et al.* (2016) and Li *et al.* (2017) also employed the parameterization given by v_n , η and δ to represent the VTI model in their inversion algorithm using reflections. Given the optimal parameterization, the trade-offs between multiple parameters should be further suppressed by designing a reasonable inversion workflow. Pattnaik *et al.* (2016) and Li *et al.* (2017) invert for the background v_n and η using a two-stage scheme. Liu *et al.* (2015) applied a matrix decomposition algorithm to compute the gradient directions for FWI and inverted for the VTI model parameters using a triple-round strategy. A multi-stage strategy is usually recommended for the multi-parameter anisotropic inversion to decouple the parameter influence (Oh and Alkhalifah 2019).

In this paper, we design an acoustic VTI RWI algorithm to retrieve the background components of the VTI model (focusing on the parameters that matter in this case) using the reflections in the pressure wavefield. We invert for the background v_n and η using the RWI method, which provides a starting model for the VTI FWI. We then perform FWI using a parameterization given by the horizontal velocity v_h , η and ϵ , which is optimal for FWI. We test this procedure on a portion of a modified VTI Sigsbee 2A model. Numerical tests on a 2D line extracted from the 3D Volve OBC field data demonstrates the feasibility of updating the anisotropic model using the presented inversion scheme.

THEORY

We use the 2D pseudo-acoustic VTI wave equation presented by Duveneck *et al.* (2008) to describe the propagation of waves in VTI media. The coupled second-order differential equations are given as follows:

$$\begin{aligned} \frac{1}{v_n^2} \frac{\partial^2 p}{\partial t^2} - \frac{1}{\sqrt{1+2\delta}} \frac{\partial^2 q}{\partial x^2} - \frac{1}{1+2\delta} \frac{\partial^2 p}{\partial z^2} &= f_p \\ \frac{1}{v_n^2} \frac{\partial^2 q}{\partial t^2} - (1+2\eta) \frac{\partial^2 q}{\partial x^2} - \frac{1}{\sqrt{1+2\delta}} \frac{\partial^2 p}{\partial z^2} &= f_q \end{aligned} \quad (1)$$

where v_n is the NMO velocity, δ is a Thomsen parameter that relates the NMO velocity to the vertical velocity, η is the anellipticity parameter, f_p and f_q are the source functions, and the pressure wavefield is expressed as a combination of the wavefield components p and q , i.e.,

$$u = \frac{1}{2}(p + q).$$

In RWI, we split the model (m_t) into the background model (m) and model perturbations (m_d), i.e., $m_t = m + m_d$, where m is the background model determining the kinematic characteristics of reflections and m_d is the model perturbations used to excite reflections. RWI aims to find the background model that best predicts the kinematics of surface recorded reflections. That is to say, the reflections generated from the perturbations match the observed reflections well in terms of phase information. We simulate the reflections using Born modelling. It is not essential to perturb all the parameters v_n , δ and η to generate the reflections. Based on the radiation pattern analysis (Alkhalifah and Plessix 2014), δ is sensitive to the small scattering angles and can be used to match near-offset

reflection amplitudes. On the other hand, the parameter v_n with an isotropic radiation pattern can also serve as the perturbation to generate reflections required by RWI at the far offsets as well. However, η is mostly sensitive to large scattering angles. Therefore, the perturbations in δ and v_n are used to produce the reflections with the Born modelling operator. To derive the Born modelling operator corresponding to perturbations in δ and v_n , we split the total parameter model v_{nt} , δ_t and the corresponding wavefields p_t and q_t into two parts, i.e., the background and perturbation components:

$$\begin{aligned} v_{nt} &= v_n + v_{nd}, & \delta_t &= \delta + \delta_d, \\ p_t &= p + p_d, & q_t &= q + q_d. \end{aligned} \quad (2)$$

where the subscript d denotes the perturbation component. The exact expression of the perturbed wavefields can be obtained by subtracting the wave equation for the background wavefields from that for the total wavefields. Using the first-order Born approximation, the Born expression of the perturbed wavefields generated from the perturbation of δ and v_n is given by:

$$\begin{aligned} \frac{1}{v_n^2} \frac{\partial^2 p_d}{\partial t^2} - \frac{1}{\sqrt{1+2\delta}} \frac{\partial^2 q_d}{\partial x^2} - \frac{1}{1+2\delta} \frac{\partial^2 p_d}{\partial z^2} &= \frac{2}{v_n^3} v_{nd} \frac{\partial^2 p}{\partial t^2} \\ &\quad - (1+2\delta)^{-3/2} \delta_d \frac{\partial^2 q}{\partial x^2} - 2(1+2\delta)^{-2} \delta_d \frac{\partial^2 p}{\partial z^2} \\ \frac{1}{v_n^2} \frac{\partial^2 q_d}{\partial t^2} - (1+2\eta) \frac{\partial^2 q_d}{\partial x^2} - \frac{1}{\sqrt{1+2\delta}} \frac{\partial^2 p_d}{\partial z^2} &= \frac{2}{v_n^3} v_{nd} \frac{\partial^2 q}{\partial t^2} \\ &\quad - (1+2\delta)^{-3/2} \delta_d \frac{\partial^2 p}{\partial z^2} \end{aligned} \quad (3)$$

The perturbed wavefields are excited by a source given by a combination of the background wavefield and the model perturbations. Besides, the propagation property of the perturbed wavefields depends on the background model. Therefore, the background and perturbation models jointly determine the features of the perturbed wavefields.

In the RWI method, when the background and perturbation models can reproduce the observed reflections, the long-wavelength information in the background model not only leads to a high-quality perturbation image, but also provides an initial model for the conventional FWI to mitigate cycle-skipping issue. We extend the RWI method to acoustic VTI media. The objective function given by the L_2 norm of misfits between the simulated and observed reflections can be expressed as:

$$J(\mathbf{m}, \mathbf{m}_d) = \frac{1}{2} \|\mathbf{u}_d(\mathbf{m}, \mathbf{m}_d) - \mathbf{d}_{ref}\|_2^2, \quad (4)$$

where $\mathbf{d}_{ref}$ represents the observed reflections, $\mathbf{u}_d(\mathbf{m}, \mathbf{m}_d)$ is the perturbed wavefields controlled by equation 3, $\mathbf{m}$ and $\mathbf{m}_d$ denote the background and perturbation models, respectively. Then we can derive the gradients of the objective function with respect to the perturbation model v_{nd} and δ_d using the adjoint state method (Plessix and Cao 2011)

$$\begin{aligned} \frac{\partial J}{\partial v_{nd}} &= \frac{2}{v_n^3} \int dt \left(\lambda_q \frac{\partial^2 q}{\partial t^2} + \lambda_p \frac{\partial^2 p}{\partial t^2} \right) \\ \frac{\partial J}{\partial \delta_d} &= - \int dt (1 + 2\delta)^{-\frac{3}{2}} \left(\lambda_q \frac{\partial^2 p}{\partial z^2} + \lambda_p \frac{\partial^2 q}{\partial x^2} \right) + 2(1 + 2\delta)^{-2} \lambda_p \frac{\partial^2 p}{\partial z^2}, \end{aligned} \quad (5)$$

and the gradients with respect to the background v_n and η :

$$\begin{aligned}\frac{\partial J}{\partial v_n} &= \frac{2}{v_n^3} \left[\int dt \left(\lambda_{qd} \frac{\partial^2 q}{\partial t^2} + \lambda_{pd} \frac{\partial^2 p}{\partial t^2} \right) + \int dt \left(\lambda_q \frac{\partial^2 q_d}{\partial t^2} + \lambda_p \frac{\partial^2 p_d}{\partial t^2} \right) \right], \\ \frac{\partial J}{\partial \eta} &= 2 \left[\int dt \lambda_{qd} \frac{\partial^2 q}{\partial x^2} + \int dt \lambda_q \frac{\partial^2 q_d}{\partial x^2} \right]\end{aligned}\quad (6)$$

where λ refers to the adjoint wavefield computed using the adjoint operator corresponding to equations 1 and 3. In detail, the adjoint operator corresponding to equation 1, which controls the background adjoint wavefield λ_p and λ_q , is expressed as:

$$\begin{aligned}\frac{1}{v_n^2} \frac{\partial^2 \lambda_p}{\partial t^2} - \frac{1}{1+2\delta} \frac{\partial^2 \lambda_p}{\partial z^2} - \frac{1}{\sqrt{1+2\delta}} \frac{\partial^2 \lambda_q}{\partial z^2} &= r_p \\ \frac{1}{v_n^2} \frac{\partial^2 \lambda_q}{\partial t^2} - (1+2\eta) \frac{\partial^2 \lambda_q}{\partial x^2} - \frac{1}{\sqrt{1+2\delta}} \frac{\partial^2 \lambda_p}{\partial x^2} &= r_q\end{aligned}\quad (7)$$

We can see that the adjoint source is computed from the residuals of reflections $r_p = r_q = \frac{1}{2}(\mathbf{u}_d - \mathbf{d}_{ref})$. The adjoint wavefield can be obtained by backpropagating the adjoint source from the receiver positions. The adjoint operator corresponding to equation 3, which controls the perturbed adjoint wavefield λ_{pd} and λ_{qd} , is given by:

$$\begin{aligned}\frac{1}{v_n^2} \frac{\partial^2 \lambda_{pd}}{\partial t^2} - \frac{1}{1+2\delta} \frac{\partial^2 \lambda_{pd}}{\partial z^2} - \frac{1}{\sqrt{1+2\delta}} \frac{\partial^2 \lambda_{qd}}{\partial z^2} &= -2(1+2\delta)^{-2} \delta_d \frac{\partial^2 \lambda_p}{\partial z^2} \\ &\quad - (1+2\delta)^{-\frac{3}{2}} \delta_d \frac{\partial^2 \lambda_q}{\partial z^2} + \frac{2}{v_n^3} v_{nd} \frac{\partial^2 \lambda_p}{\partial t^2} \\ \frac{1}{v_n^2} \frac{\partial^2 \lambda_{qd}}{\partial t^2} - (1+2\eta) \frac{\partial^2 \lambda_{qd}}{\partial x^2} - \frac{1}{\sqrt{1+2\delta}} \frac{\partial^2 \lambda_{pd}}{\partial x^2} &= 2\eta_d \frac{\partial^2 \lambda_q}{\partial x^2} \\ &\quad - (1+2\delta)^{-\frac{3}{2}} \delta_d \frac{\partial^2 \lambda_p}{\partial x^2} + \frac{2}{v_n^3} v_{nd} \frac{\partial^2 \lambda_q}{\partial t^2}\end{aligned}\quad (8)$$

The perturbed adjoint wavefield $\lambda_{p,d}$ and $\lambda_{q,d}$ are generated by backpropagating the wavefields λ_p and λ_q through the model perturbations.

According to the gradient formula (6), we can compute the reflection wavepath (sensitivity kernel) by cross-correlating the forward-propagating Green's function from the source and the backward-propagating Green's function from the receiver. Figure 1 shows the reflection wavepath in a two-layer VTI model with a reflector at depth 0.8 km for a single source and receiver. The reflection wavepath with a source-receiver offset of 1 km for v_n and η are presented in Figures 1a and 1b, respectively. The reflection wavepath with a source-receiver offset of 2.6 km for v_n and η are shown in Figures 1c and 1d, respectively. We can see that the background update of η is more sensitive to the far offset data, and the far- and near-offset data show similar contributions to the background update of v_n . Such characters of the sensitivity kernel provide some insights for designing the inversion strategy.

The inverse problem in VTI media is an ill-posed problem. A carefully chosen strategy or workflow is crucial to the stability of the inversion. Alkhalifah (2017) presented the transmission radiation patterns from a reflector in the pseudo-depth domain for an acoustic VTI model parameterized by v_n , η and δ , shown here in Figure 2. The absence of the curve for δ reflects the insensitivity of the kinematics of the data to it, because the energy at all angles of its corresponding radiation pattern is zero. That means δ has no influence on the kinematics of the measured reflections in the pseudo-depth domain. This further explains why we can not retrieve the background δ using reflections. Besides, the background v_n and η can be constrained by moderate and long offsets, respectively.

Such analyses suggest the following inversion strategy:

- First, we invert for the background v_n using the reflections from the perturbation of v_n and δ , which are simultaneously optimized in a least square migration.
- Given the inverted background v_n , we update the background v_n and η simultaneously by minimizing the long-offset reflection residuals. The reflections are produced by the perturbation of v_n .
- At last, the inverted VTI model is described by a combination of horizontal velocity v_h , η and ϵ based on the following relations:

$$v_h = v_n \sqrt{1 + 2\eta}, \quad \epsilon = (1 + 2\delta)\eta + \delta. \quad (6)$$

After RWI, we supposedly recovered the background information of the acoustic VTI model that interprets the reflection kinematics well. A subsequent implementation of FWI is essential to improve the model resolution using the whole wavefield and recover the high wavenumber components. A combination of the horizontal velocity v_h , anisotropy parameters η and ϵ can generate the optimal inversion results especially when the diving waves are used in FWI (Alkhalifah and Plessix 2014). Thus, we conduct FWI for the acoustic VTI medium in the model space defined by a combination of v_h , η and ϵ . In detail, the parameter v_h is inverted first, and then we add the anisotropic parameters η and ϵ , successively. Considering that ϵ and density have the same radiation pattern, the trade-offs between them are also unresolvable. Thus, ϵ can handle the inadequate amplitude fitting due to ignoring the elastic nature of the Earth and thus replacing density in this task.

NUMERICAL EXAMPLES

We apply the inversion scheme to a synthetic example and a 2D line from the Volve OBC field data to demonstrate the performance of our inversion method for acoustic VTI model. In the first example, we use a small part of the Sigsbee2A model, and generate the corresponding anisotropic parameter field with a nonlinear relation. In the second example, a 2D line from the Volve OBC dataset contains clear reflections, which are good for the RWI approach.

Sigsbee2A model

We test the inversion algorithm using a part of the modified Sigsbee 2A model. The true NMO velocity v_n is given in Figure 3a. The true $\eta = \delta$ is shown in Figure 3b. The top layer is purely isotropic to avoid the shear wave artifacts caused by the pseudo-acoustic approximation (Duveneck *et al.* 2008). We use 67 shots to generate the seismic data with the maximum offset of 7.4 km. The receivers are located at every grid point on the surface. The inversion result of η , a parameter that thrives on the large-offset information, is influenced by the edge effects because of the limited offset at the edge. Therefore, the model information is limited to mainly the middle part. An 8-Hz Ricker wavelet without frequencies below 3Hz is used as the source wavelet. In multi-parameter inversion, we utilize the pseudo-Hessian matrix (Shin, Jang and Min 2001) to precondition the gradient direction. To scale the multi-parameter updates, we normalize the gradients and employ the backtracking line-search approach to estimate the step length of each parameter, which satisfies the Wolfe condition (Wolfe 1969). We initialize the search direction using a fixed

empirical step length. The initial step lengths of velocity and anisotropy parameters are 40m/s and 0.01, respectively. We also apply a bound constraint to the inversion scheme. The lower and upper bounds for v_n and η are [1.6: 4.0 km/s] and [0: 0.15], respectively.

We start from an isotropic model where the NMO velocity is linearly increasing with depth as shown in Figure 4. We first update the perturbation of v_n and δ simultaneously to produce the reflections for the update of the background v_n . Figure 5 shows the inverted perturbation images using the initial background model, where the reflectors obviously deviate from their accurate spatial position and the diffraction energy is not focused. After 30 outer iterations, the long-wavelength components of the NMO velocity v_n are recovered well (Figure 6). The corresponding perturbation image is obviously improved (Figure 7). In detail, the reflectors are shifted up to some extent, but still deviate slightly from their exact positions. The depth deviation is mainly caused by the inaccurate δ , which can not be retrieved from surface seismic data. Besides, the diffraction energy is not fully focused. Next, we update the background v_n and η simultaneously to reduce the reflection residuals at far offsets. During the inversion, the gradient of η is considerably smoothed to mitigate possible tradeoff with the NMO velocity. Figure 8 shows the inverted background v_n and η after 20 outer iterations. The quality of the corresponding perturbation image is further improved (Figure 9), especially near the diffraction points. Meanwhile, Figure 8b illustrates that the long-wavelength information of η is recovered except the deep and near the edges, in which they were not illuminated well by the limited aperture reflections. Besides, the accuracy of

the inverted η is also influenced by the inverted v_n , whose accuracy degrades in the deep and marginal areas.

At last, we conduct acoustic VTI FWI to introduce high-wavenumber components. Given the inverted v_n and η (Figure 8), we re-describe the VTI model with a combination of the horizontal velocity v_h , η and ε . We first invert for v_h , and then include the anisotropic parameters η and ε , successively. Figure 10 shows the final inversion results. For comparison, we also apply the above VTI FWI algorithm to the isotropic initial model (Figure 4). Obviously, it stagnates at a local minimum and fails to get a reasonable result, especially for the anisotropic parameters (Figure 11). For a detailed comparison, the vertical profiles at $x = 3.584$ km of v_h , η and ε are shown in Figure 12. Due to the trade-offs between the depth and the δ field, the inverted model has a depth deviation relative to the true model. To ignore the influences of the inaccurate δ field, we convert the depth axis to vertical time using the inverted model, and the corresponding vertical time profiles are shown in Figure 13. The inverted v_h is very close to the true horizontal velocity in the vertical time domain. The background components of η can also be recovered using the proposed inversion algorithm. This test also points out the ambiguity between δ and depth which is hard to resolve without additional information. We can see from these Figures that the inverted VTI model interprets the kinematics of the reflections successfully.

Volve OBC data

We apply our acoustic VTI RWI method to a 2D line extracted from the OBC Volve field data set to further verify its feasibility. A chalk layer is located in the inversion area which is characterized by a small dome-shaped structure (Szydluk *et al.* 2007). For our inversion, the 2D line contains 240 receivers with 25 m interval. The receivers are located at the water bottom with a depth of about 92 m. 121 shot gathers are selected as the data set for our inversion, with a distance of 100 m between the shots, at a depth of 6 m. We were provided with the pre-processed PP and PS data. We only use the PP data as our observed data. Therefore, the acoustic wave equation in VTI media should be a good enough modelling operator, since it can simulate the kinematic information of PP waves accurately. In the observed data, the direct waves are limited to near offsets and all postcritical arrivals were muted out. However, the reflections are clear in the shot gathers with a maximum offset of about 5 km. The data are not friendly to a conventional VTI FWI method, which relies heavily on the diving waves to retrieve the long-wavelength structure. We can take advantage of the observed reflections to recover the background model with our VTI RWI approach.

To ensure the stability of the inversion and reduce the computational cost, we band-pass the data with Butterworth filters to generate 3 data sets with bandwidths of 2-4, 2-6 and 2-8 Hz, respectively. The corresponding shot gathers are shown in Figure 14. The strong reflection event near $t = 2.5$ s is generated from the top of chalk layer. We are also provided with a tomography model (Figure 15) as reference. The model dimensions are 12.3 km and 4.5 km in horizontal and vertical directions, respectively. The starting model is a simple isotropic model ($\eta = \delta = 0$), whose velocity is linearly increasing from the bottom of the water layer (Figure 16). The density is fixed at 2000 kg/m^3 during the inversion. The source wavelet is estimated by matching the near-offset direct

arrivals of the simulated and observed data. The initial step lengths for velocity and anisotropy parameters are 50m/s and 0.01, respectively. The lower and upper bounds for v_n and η are [1.4: 5.0 km/s] and [0: 0.15], respectively.

The inversion is conducted in 3 stages. In the first stage, we only invert for the background NMO velocity (v_n) using the multiscale strategy, which can mitigate the cycle-skipping issue by including progressively the high-frequency information (Bunks *et al.* 1995; Sirgue and Pratt 2004). Starting from the frequency band of 2-4Hz, the perturbation images v_{nd} and δ_d are inverted by matching the near-offset reflections (<1.0 km) in a least-squares reverse time migration (LSRTM) fashion. Given the perturbation image, the gradient for background v_n can be obtained using equation 6. To suppress the effects from the anisotropy parameter η , the reflections with offsets less than 3.0 km are utilized. We can see from Figure 17a that the low-wavenumber information is extracted effectively, especially the area above the chalk layer which is penetrated by strong reflections. We apply the pseudo-Hessian matrix (Shin *et al.* 2001) and a Gaussian smoothing to improve the original gradient. We update the background v_n for 10 iterations within every frequency band, and the final inverted background v_n is shown in Figure 18a, which includes the obvious large-scale updates. In the second stage, we only use the frequency band of 2-8 Hz to recover the background η considering that the background v_n can predict the kinematics of reflections more accurately. The first gradient of η is shown in Figure 17b. After 10 iterations, we can recover the background η well (Figure 18b). The values between depths 1.3 to 2.5 km are similar to the tomography model. As a quality control, Figure 19 shows the perturbation images using the initial and inverted background model. In the inverted perturbation image (Figure 19b), the strong reflector is shifted up and a more obvious dome-shaped structure is depicted as marked by the yellow arrow. Besides, Figures 20a and 20b show the angle-domain common-image gathers (ADCIGs)

for the initial and inverted background models, respectively (Sava and Fomel 2003). The ADCIGs for the inverted model become much flatter, especially the events near the depth of 2.5 km as marked by the yellow box. However, the events in the shallow are not flattened well because of the underestimated updates in the shallow. In the last stage, we perform acoustic VTI FWI to recover high-wavenumber features. Here, we define the VTI model with a combination of v_n , η and ϵ instead of v_n , η and δ . The initial values for the new parameters are extracted from the inverted values for the previous parameterization. Figure 21 shows the final FWI result using the multiscale strategy, and it contains some high-resolution structures. The inverted model reproduces reasonably the reflections in the observed data (Figure 22). Thanks to the reasonable background VTI model, the reflections are matched well at near and also far offsets.

CONCLUSION

We designed an RWI algorithm to build the background NMO velocity (v_n) and η model for acoustic VTI media. With this parameterization we had limited trade-off. We first invert for the background v_n by matching the reflections from the perturbation of v_n and δ with those in the observed data. Then we optimize the background v_n and η simultaneously using the reflections generated from the perturbation of v_n . Least-squares reverse-time migration (LSRTM) plays an important role in providing a reasonable perturbation image for updating the background model. The numerical test on the modified VTI Sigsbee 2A model illustrates that the inversion algorithm can build a reasonable background model for VTI media, even starting from a simple isotropic model. The inversion result for v_n and η

significantly improved the perturbation image. The sequential VTI FWI result, especially the horizontal velocity (v_h), is very close to the true model in the vertical time domain. However, we still suffer from a slight depth deviation in the vertical depth domain resulting from the trade-offs between the depth and the δ field. The background δ model is not sensitive to the kinematics of the reflections, so it is impossible to build the background δ without additional information. The algorithm is also applied to a 2D line from an OBC dataset. The results illustrate that the inverted background VTI model interprets the kinematics of reflections successfully. The cycle skipping can not be avoided completely especially for the far offset data in RWI. It is better to perform the multi-scale strategy in the real case.

LIST OF FIGURES

1 The reflection wavepath for an acoustic VTI model with a horizontal reflector at a depth of 0.8 km for a single source-receiver pair of $x_s = 1\text{km}$ and $x_r = 2\text{km}$: (a) v_n , (b) η , and $x_s = 0.2\text{km}$ and $x_r = 2.8\text{km}$: (c) v_n , (d) η .

2 The transmission radiation patterns for scattering from a horizontal reflector in the pseudo-depth domain for an acoustic VTI model. (Alkhalifah, 2017)

3 The true NMO velocity (a) v_n and (b) $\eta = \delta$.

4 The initial NMO velocity v_n .

5 The perturbation images for (a) δ_d and (b) v_{nd} using the initial model.

6 The inverted background v_n in the first stage of VTI RWI method.

7 The perturbation images for (a) δ_d and (b) v_{nd} using the inverted v_n (Figure 6).

8 The inverted background (a) v_n and (b) η in the second stage of VTI RWI method.

9 The perturbation image for v_{nd} using the inverted model shown in Figure 8.

10 The VTI FWI results starting from the RWI background model (Figure 8): the inverted (a) v_h , (b) η and (c) ϵ .

11 The VTI FWI results starting from the initial isotropic model (Figure 4): the inverted (a) v_h , (b) η and (c) ϵ .

12 The vertical profiles at $x = 3.584$ km of (a) v_h , (b) η and (c) ϵ . (Cyan line: initial model, green line: conventional VTI FWI result, blue line: inverted model and red line: true model)

13 The vertical time profiles at $x = 3.584$ km of (a) v_h , (b) η and (c) ϵ .

14 Volve OBC data set. Shot gathers for the source located at $x=6.175$ km for the frequency bands (a) 2-4, (b) 2-6 and (c) 2-8 Hz.

15 The tomography (a) P-velocity, (b) η and (c) δ for the Volve data example.

16 The initial isotropic model ($\eta = \delta = 0$): NMO velocity v_n for the Volve data example.

17 The first gradient of (a) v_n and (b) η for the Volve data example.

18 The inverted background (a) v_n and (b) η for the Volve data example.

19 The (a) initial and (b) inverted model perturbations for the Volve data example.

20 The angle-domain image gathers using the (a) initial model and the (b) inverted model for the Volve data example.

21 The VTI FWI results starting from the RWI background model (Figure 18): the inverted (a) v_h , (b) η and (c) ϵ for the Volve data example.

22 Comparison of the shot gather ($x_s=6.175$ km) for the Volve data example. The simulated data is shown in the left panel, and the observed data is shown in the right panel.

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Data availability statement

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.