Graphene Nanoelectromagnetics: From Radio-Frequency, Terahertz to Mid-Infrared

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Graphene nanoelectromagnetics has recently attracted tremendous research interest, as it merges two vibrant fields of study: plasmonics and nanoelectronics. In the relatively unexplored terahertz (THz) to mid-infrared (MIR) region, the collective oscillation of massless Dirac fermions in graphene can excite the propagating surface charge density waves (surface plasmon polaritons or SPP) tightly confined to the graphene surface. Graphene is the only known material whose equilibrium (non-equilibrium) conductivity can be tuned over a broad range, as a function of its Fermi (quasi-Fermi) level. Hence, the propagation, radiation and scattering properties of a graphene monolayer or graphene-based nanostructures can be dynamically tuned by chemical doping, electrostatic gating, or photopumping. Such tunable plasmonic properties open tremendous new possibilities in novel THz and infrared (IR) optoelectronic devices with compact size, ultrahigh speed, and low power consumption. In the visible (VIS) region, graphene has high optical transparency and good electrical conductivity, in addition to flexibility and robustness, thereby becoming a very promising material to be used as a transparent electrode in the next-generation flexible displays and solar panels. In the radio-frequency (RF) region, the graphene transistor is a good candidate for power amplifiers and frequency multipliers, due to its high carrier mobility and its unique ambipolar transport characteristics. In this chapter, we will review several of the most recent advances and potential applications of graphene nanoelectromagnetics in different spectral range.

2.1 Introduction

Graphene is a two-dimensional (2D) version of the three-dimensional (3D) crystalline graphite: a single layer of sp^2 -bonded carbon atoms arranged in a hexagonal structure [Fig. 2.1a]. Ever since such a “2D material” has been discovered in 2004 [1], it has attracted considerable attention, because of its exotic electronic, thermal, mechanical, chemical and optical properties. These include ultrahigh carrier mobility (exceeding 20,000 cm²/Vs and Fermi velocity (v_F) of 10⁸ cm/s), quantum Hall effect, high opacity (absorption of $\approx 2.3\%$ of incoming light in the visible spectrum), and mechanical flexibility, to name a few [1]-[5]. Since graphene exhibits a linear energy-momentum (E - k) dispersion relation in the low energy region [Fig. 2.1b], electrons in graphene behave as massless relativistic particles with extremely high mobility (Dirac fermions) and an energy-independent velocity, representing feasible perspective toward ballistic transport devices [6]. Graphene’s linear and gapless band structure results in the exotic ambipolar electric field effect and quantum-capacitance-limited devices [1]-[5], of which the charge carriers can be tuned continuously between electrons and holes in concentrations as high as 10¹³ cm⁻² [2] by the chemical or the electrostatic gating effect. Recently, it became possible to fabricate large-area mono-, bi- or few-layer graphene by the chemical vapor deposition (CVD) [7]-[9], which further makes graphene a very promising material candidate for post-silicon nano-electronic devices [1]-[5] and transparent electrodes for solar cells and flexible displays [9].

Recently, graphene has been shown to exhibit plasmonic properties in the THz-to-MIR spectral ranges [10]-[26]. Graphene emerges as a non-noble-metal plasmonic material with many favorable electromagnetic properties. In general, plasmonics exploits the mass inertia of conduction band free electrons to generate propagating charge density waves (e.g., the metal-dielectric interface [27]-[29]). The extreme light-matter interaction at the nanoscale is followed by a significant wavelength shortening and dramatically enhanced local fields, opening up new avenues for overcoming the diffraction limit and building a synergistic bridge between nanoelectronics and nanophotonics by enabling subwavelength optical signal transmission and processing. Plasmonics also enriches the

fields of optical nanoantennas, metasurfaces and metamaterials, used for tailoring and enhancing the scattering and absorption properties of light at the nanoscale. However, conventional metal-based plasmonics still face several challenges remaining to be addressed. For instance, plasma frequencies of noble metals, which are hardly tunable, usually fall into the visible (VIS) and ultraviolet (UV) regions. In addition, noble metals have large Ohmic losses, which, in turn, reduce the lifetime of SPP excitation and deteriorate the electric field confinement effect. It has been reported that the large-area, low-defect and crystalline graphene may raise the surface plasmon lifetime ideally up to hundreds of optical cycles, thus circumventing a major challenge of noble-metal-based plasmonics [17],[20]. Besides, thanks to the tunable and finite carrier density, the plasma frequency of graphene can be tuned, over a wide range, in the THz-to-MIR range, which has growing demand for sensing, imaging, security, and communications. Such interesting properties enable the gate-tunable graphene plasmon and the controllable electromagnetic wave transitions. The dynamic tuning of graphene's plasmonic characteristic may be possible by adjusting the Fermi level, related to carrier densities in graphene, via the external electric, magnetic, optical, and electrochemical forces [10]-[18]. Thanks to the gapless energy dispersion of graphene [1],[2], population inversion can be achieved in an optically pumped graphene monolayer, leading to the amplification of THz radiation. In some recent experiments, graphene has been demonstrated to support the surface wave in the long-wavelength region (e.g. THz, FIR and FIR frequencies), with moderate loss and strong field localization. Graphene plasmonics [10]-[26] emerges as a rapidly growing field of study in recent years, with very promising applications for tunable optoelectronic devices [32], metasurfaces and metamaterials [31],[34]-[35], polarizers [19], invisibility cloaks [18],[36], nano-antennas [22],[37]-[40], phase shifters [37], and oscillators (with proper optical pumping) [15]. In general, for those applications, the ultrafast, low-power and broadband modulations for amplitude, phase and polarization of electromagnetic radiation are potentially possible in the THz-to-MIR region. In this chapter, we will briefly review the recent developments of graphene-based nanoelectromagnetics, nano-electronics and plasmonics, with veracious applications found in different spectral ranges.

2.2 Conductivity of graphene and frequency dispersion

The one-atom-thick graphene monolayer has a complex-valued sheet conductivity $\sigma_s(\omega) = \sigma_s' + j\sigma_s''$ [10]-[13]; here we assume an $e^{j\omega t}$ time dependence throughout chapter 2. In the absence of external magnetic field, graphene's conductivity is given by:

$$\begin{aligned} \sigma_s(\omega, E_F, \tau, T) &= \sigma_{intra} + \sigma_{inter} \\ &= \frac{jq^2(\omega - j\tau^{-1})}{\pi\hbar^2} \left[\int_{-\infty}^{+\infty} \frac{|\varepsilon|}{(\omega - j\tau^{-1})^2} \frac{\partial f_d(\varepsilon)}{\partial \varepsilon} d\varepsilon - \int_0^{+\infty} \frac{\partial f_d(-\varepsilon) - \partial f_d(\varepsilon)}{(\omega - j\tau^{-1})^2 - 4(\varepsilon/\hbar)^2} d\varepsilon \right], \end{aligned} \quad (1)$$

where Fermi-Dirac distribution $f_d = 1/(1 + \exp[(\varepsilon - E_F)/k_B T])$, ε is the energy, E_F is the Fermi energy, T is the temperature, q is the electron charge, $\hbar$ is the reduced Planck's constant, and τ is the momentum relaxation time of charge carrier in graphene. The first and second terms in (1) account for the intraband and interband contributions, respectively. For $\hbar\omega, |E_F| \gg K_B T$, σ_{inter} is approximately given by [11]-[13]:

$$\sigma_{inter} \approx \frac{q^2}{4\hbar} \left(\theta(\hbar\omega - 2E_F) - j \frac{1}{\pi} \ln \left[\frac{2|E_F| - \hbar(\omega - j\tau^{-1})}{2|E_F| + \hbar(\omega - j\tau^{-1})} \right] \right), \quad (2)$$

where $\theta(\cdot)$ represents the step function. For a doped graphene with $|E_F| \gg k_B T$ and for frequencies far below the interband transition threshold (i.e., photon energy $\hbar\omega < 2|E_F|$), σ_{intra} dominates over σ_{inter} and (1) can be conveniently expressed using a Drude-type dispersion:

$$\sigma_{intra} \approx -j \frac{D}{\pi(\omega - j\tau^{-1})}, \quad (3)$$

where the Drude weight $D = q^2 E_F / \hbar^2$. The intrinsic relaxation time can be expressed as $\tau = \mu E_F / q v_F^2$, where v_F is the Fermi velocity and μ is the measured dc mobility. For instance, with a mobility of 10,000 cm²/V s, a conservative value $\tau \approx 10^{-13}$ s is measured from [20]. In the THz-to-MIR spectral range, graphene's conductivity is a function of

Fermi energy that can be tuned by electrostatic gating [24] (which yields an isotropic scalar conductivity), chemical doping [41], or static magnetic bias (which yields an anisotropic conductivity tensor) [10]-[13]. Fig. 2.1c reports the complex-valued conductivity of graphene with different values of Fermi energy. For simplicity, unless otherwise mentioned, $\tau = 1 \text{ ps}$ is assumed in this chapter. This value is consistent with the ballistic transport features of graphene, whose mean free path was measured to be up to 500 nm at room temperature and larger than 4 μm at low temperatures [2]. It is seen from Fig. 2.1c that the imaginary part of sheet conductivity (corresponding to the real part of the permittivity) is tunable by varying the Fermi energy of graphene.

In the near-infrared (NIR), VIS and UV spectral ranges, where the photon energy $\hbar\omega \gg E_F, K_B T$, the interband contribution in (1) dominates and graphene's conductivity can be derived as:

$$\sigma = \frac{q^2}{4\hbar} \tanh\left(\frac{\hbar\omega - 2E_F}{4k_B T}\right) \approx \frac{q^2}{4\hbar}, \quad (4)$$

which is constant over a wide spectral range. Therefore, the optical transmittance of a suspended graphene remains constant in the VIS and UV spectra, which is given by:

$$T_s = \left| 1 - \frac{\eta_0 / 2}{\eta_0 / 2 + \sigma^{-1}} \right|^2, \quad (5)$$

where η_0 is the free-space impedance and $\sigma = q^2 / 4\hbar$ here. According to (5), a pure graphene monolayer appears transparent to the human eye, with an optical transmittance of 97.7%. For an N -layer graphene, the sheet conductivity can be approximately expressed as $\sigma_{N\text{-layer}} = N\sigma$, which is valid for at least 10 layers [42]. Fig. 2.1d shows the photographs of flexible and “semi-transparent” RF monopole antennas [43] consisting of multiple-layer graphene patches ($N = 3, 5, 6$ and 8) on the polymer substrate. Fig. 2.1e shows their calculated (dashed) and measured (solid) transmittance spectra, which show good agreement even for eight stacked graphene monolayers. It is clearly seen from Figs. 2.1d and 2.1e that increasing the number of stacked graphene layers may reduce the

optical transparency, as the conductivity of an N -layer graphene is proportional to the number of graphene layers N . We note that the dc and low-frequency conductivity, however, increases with increasing the number of graphene layers, which may help reducing the power dissipation in the graphene-based RF components. Although the opacity is sacrificed when a multilayered graphene is used, a satisfactory transparency can still be obtained. In the RF and microwave region where $\omega \ll \tau^{-1}$, the dynamic conductivity of a multilayered graphene can be derived from (3) as:

$$\sigma(\omega) = -j \frac{Nq^2 E_F}{\pi \hbar^2 (\omega - j\tau^{-1})} \approx \frac{Nq^2 E_F}{\pi \hbar^2 \tau^{-1}}. \quad (6)$$

In this frequency range, the imaginary part of graphene's conductivity (i.e., kinetic inductance) is ignorable, and graphene's conductivity is almost frequency-independent, which is, in some sense, analogous to a resistive sheet with a finite conductance. The Fermi energy of graphene can be tuned over a wide range by an externally applied electrical field (i.e. electrostatic gating), as the Fermi energy in graphene is related to the carrier concentration n_s by [6]:

$$n_s = \frac{2}{\pi(\hbar v_F)^2} \int_0^\infty \varepsilon [F(\varepsilon - E_F) - F(\varepsilon + E_F)] d\varepsilon. \quad (7)$$

The Drude weight of graphene $D \propto E_F \propto \sqrt{n_s}$ can be tuned by varying the carrier density of graphene. For example, by placing a p^+/n^+ -doped polysilicon or metal gate behind a thin insulating layer that supports the graphene monolayer, the carrier density in graphene and thus its dynamic conductivity can be tuned through an electrical bias. Due to the electron-hole symmetry in the electronic band structure of graphene, both negative and positive signs of Fermi energy provide the same value of complex conductivity.

Graphene's conductivity can also be tuned by applying an external static magnetic field. In the local Hall-effect regime, with a magnetostatic bias and possibly electrostatic bias, the surface conductivity is a tensor that can be explicitly expressed as [10]-[11],[13]:

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} = \begin{bmatrix} \sigma_d(E_F(E_0), B_0) & \sigma_o(E_F(E_0), B_0) \\ -\sigma_o(E_F(E_0), B_0) & \sigma_d(E_F(E_0), B_0) \end{bmatrix}, \quad (8)$$

where

$$\begin{aligned} \sigma_d(E_F(E_0), B_0) = & -\frac{q^2 v_F^2 |eB_0| (\omega - j\tau^{-1}) \hbar}{j\pi} \\ & \sum_{n=0}^{\infty} \left\{ \frac{f_d(M_n) - f_d(M_{n+1}) + f_d(-M_{n+1}) - f_d(-M_n)}{(M_{n+1} - M_n)^2 - (\omega - j\tau^{-1})^2 \hbar^2} \times \left(1 - \frac{\Delta^2}{M_n M_{n+1}} \right) \frac{1}{M_{n+1} - M_n} + \right. \\ & \left. + \frac{(f_d(-M_n) - f_d(M_{n+1}) + f_d(-M_{n+1}) - f_d(M_n))}{(M_{n+1} + M_n)^2 - (\omega - j\tau^{-1})^2 \hbar^2} \times \left(1 + \frac{\Delta^2}{M_n M_{n+1}} \right) \frac{1}{M_{n+1} + M_n} \right\} \end{aligned} \quad (9a)$$

$$\begin{aligned} \sigma_o(E_F(E_0), B_0) = & -\frac{q^2 v_F^2 eB_0}{\pi} \sum_{n=0}^{\infty} \left\{ [f_d(M_n) - f_d(M_{n+1}) - f_d(-M_{n+1}) + f_d(-M_n)] \times \right. \\ & \times \left[\left(1 - \frac{\Delta^2}{M_n M_{n+1}} \right) \frac{1}{(M_{n+1} - M_n)^2 - (\omega - j\tau^{-1})^2 \hbar^2} + \right. \\ & \left. \left. + \left(1 + \frac{\Delta^2}{M_n M_{n+1}} \right) \frac{1}{(M_{n+1} + M_n)^2 - (\omega - j\tau^{-1})^2 \hbar^2} \right] \right\} \end{aligned} \quad (9b)$$

and $M_n = \sqrt{\Delta^2 + 2nv_F^2 |qB_0| \hbar}$, E_0 and B_0 are the external static electric and magnetic fields, respectively.

2.3 Photopumping effect on graphene's conductivity

Photopumped optical gain in graphene have been theoretically and experimentally studied by Rana [15], Rhyzi and Otsuji et al.[44]-[48], and recently detailed by Garcia-Vidal and Hess *et al.* [49][45], who have conducted more rigorous calculations considering realistic conditions (e.g., collision losses and temperature profiles). Under this non-equilibrium condition, graphene's THz properties become particularly interesting because the population inversion and thus negative dynamic conductivity (which implies a THz gain) can take place in a photopumped graphene, as a result of the cascaded optical-phonon emission and interband transitions around the Dirac point [45]-[48], as illustrated in Fig. 2.2a. Under sufficiently strong optical excitation (e.g. NIR and VIS pump lasers), the interband emission of photons may prevail over the intraband Drude absorption, and thus the real part of the dynamic conductivity of graphene can be

made negative, particularly for pristine graphene at low temperatures. Therefore, a photopumped graphene monolayer could be seen as an active distributed component with negative sheet conductivity. The photoexcited electron-hole pairs in graphene splits the Fermi level into two quasi-Fermi levels $E_{Fn}, E_{Fp} = \pm \varepsilon_F$ that define the electron and hole concentrations, respectively. Since the relaxation time for intraband transitions is of the order of $\tau \approx \text{ps}$, which is faster than the recombination time $\tau \approx \text{ns}$ for electron-hole pairs [50], the population inversion can be achieved. Under sufficiently strong optical excitation, the interband emission of photons may prevail over the intraband Drude absorption and, therefore, the real part of the dynamic conductivity of graphene $\text{Re}[\sigma]$ becomes negative. Due to the unique gapless energy spectra of electrons and holes in graphene, the negative $\text{Re}[\sigma]$ can occur at relatively low frequencies (e.g. THz frequency band where $\hbar\omega < 2\varepsilon_F$). The conductivity of graphene $\sigma = \sigma_{\text{intra}} + \sigma_{\text{inter}}$ under the non-equilibrium condition can be written as:

$$\sigma_{\text{intra}} = j \frac{q^2}{\pi \hbar^2} \frac{1}{\omega - j\tau^{-1}} \left[\int_0^\infty \varepsilon \left(\frac{\partial F_1(\varepsilon)}{\partial \varepsilon} - \frac{\partial F_2(-\varepsilon)}{\partial \varepsilon} \right) d\varepsilon \right] \quad (10a)$$

$$\sigma_{\text{inter}} = -j \frac{q^2}{\pi \hbar^2} (\omega - j\tau^{-1}) \int_0^\infty \frac{F_2(-\varepsilon) - F_1(\varepsilon)}{(\omega - j\tau^{-1})^2 - 4\varepsilon^2 / \hbar^2} d\varepsilon, \quad (10b)$$

where $F_1(\varepsilon) = [1 + e^{(\varepsilon - E_{Fn})/K_B T}]^{-1}$ and $F_2(\varepsilon) = [1 + e^{(\varepsilon - E_{Fp})/K_B T}]^{-1}$. By solving the integral in (10a), σ_{intra} can be explicitly written as:

$$\sigma_{\text{intra}} = -j \left(\frac{q^2}{4\hbar} \right) \left(\frac{8k_B T}{\pi \hbar} \ln \left[1 + e^{\varepsilon_F / k_B T} \right] \right) \left(\frac{\tau}{1 + \omega^2 \tau^2} - j \frac{\omega}{\omega^2 + \tau^{-2}} \right). \quad (11)$$

While $\text{Re}[\sigma_{\text{intra}}]$ is always positive, $\text{Re}[\sigma_{\text{inter}}]$ in (10b) may be negative such that the real part of the net conductivity, $\text{Re}[\sigma]$, may be negative. In the long-wavelength region, $\text{Re}[\sigma_{\text{inter}}]$ can be approximately expressed as:

$$\text{Re}[\sigma_{\text{inter}}] \approx \frac{q^2}{4\hbar} \tanh\left(\frac{\hbar\omega - 2\varepsilon_F}{4k_B T}\right), \quad (12)$$

which is negative if $\hbar\omega < 2\varepsilon_F$. Both equilibrium and non-equilibrium conductivities of graphene would converge to a similar form in the very-low-frequency region (e.g. RF and microwave) and the very-high-frequency region (e.g. NIR, VIS, and UV). In the VIS and NIR spectral ranges, where the photon energy $\hbar\Omega_0 \gg \varepsilon_F, k_B T$, the optical conductivity has the same expression as (4). As known from (11) and (12), the magnitude of $\text{Re}[\sigma]$ is bounded by the physical limit of $\pm q^2 / 4\hbar$.

Fig. 2.2b shows the real part of the graphene's conductivity, $\text{Re}[\sigma]$, against the frequency under photopumping. It can be seen that the negative conductivity and therefore THz gain can be obtained over a broad frequency range. Combining these aspects with enhanced light-matter interactions of THz plasmons in graphene may open a plethora of novel applications, such as loss-compensated metamaterials [51], as well as chip-scale lasing and oscillating plasmonic devices [15]. Moreover, such an active plasmonic material may provide an ideal platform for studying THz parity-time (PT) symmetric systems, which exhibit exotic reciprocal and unidirectional scattering properties [52],[53], technologically viable with spatially distributed gain and loss components. The PT-symmetric THz devices may be feasible with the active graphene system, which, when combined with graphene's biosensing functions, may allow singularity-enhanced sensing and probing for gas and (bio-)chemical agents.

Some recent works have reported that for a poor-quality graphene monolayer, having a short phenomenological relaxation time (impurity-limited lifetime), the transient optical gain and population inversion could only last for a short period of time ($\sim$ ps scale). However, there are many time-resolved THz microscopic techniques and applications that use a short-pulsed THz (picosecond/femtosecond) wave for the study of dynamics on extremely short time, instead of continuous-wave excitations. The measured time-domain signals can be processed by a Fourier transform to recover the frequency domain spectral information. There have been several experimental and theoretical works studying the

transient population inversion and THz plasmon gain in graphene and their possible gain effect in a time-domain analysis [45]-[49].

Under equilibrium conditions, the electron density n_0 and hole density p_0 in graphene, as a function of Fermi energy E_F , can be obtained from the integration of density of states weighted by the Fermi-Dirac distribution as:

$$n_0(E_F) = -\frac{2}{\pi} \left(\frac{K_B T}{\hbar v_F} \right)^2 Li_2 \left[-e^{-qE_F/(K_B T)} \right], \quad (13a)$$

$$p_0(E_F) = -\frac{2}{\pi} \left(\frac{K_B T}{\hbar v_F} \right)^2 Li_2 \left[-e^{qE_F/(K_B T)} \right], \quad (13b)$$

where the dilogarithm Li_2 is a special case of the polylogarithm $Li_n(\cdot)$ for $n = 2$. For a pristine graphene with $E_F = 0$, the carrier densities are given by:

$$n_0(0) = p_0(0) = \frac{\pi}{6} \left(\frac{k_B T}{\hbar v_F} \right)^2, \quad (14)$$

where v_F is the Fermi velocity in graphene. The carrier densities for the n- or p-doped graphene with $|E_F| \gg k_B T$ can be derived as:

$$n_0(E_F) = p_0(-E_F) = \frac{|E_F|^2}{\pi \hbar^2 v_F^2}. \quad (15)$$

Under NIR and VIS illuminations, the excess electrons and holes are generated in pairs ($\delta n = \delta p$), and the total carrier densities become $n = n_0 + \delta n$ and $p = p_0 + \delta p$. The density of photoexcited carriers are given by:

$$\delta n = \delta p = \frac{\alpha I_{\Omega_0} \tau_r}{\hbar \Omega_0}, \quad (16)$$

where α is the optical absorptance, I_{Ω_0} and $\hbar \Omega_0$ are the intensity and photon energy of the pump laser, and τ_r is the recombination lifetime for electron-hole pairs

(typically $\tau_r \sim \text{ns}$). Under a weak optical excitation, i.e. $\varepsilon_F \ll k_B T$, the normalized densities of excess carriers are derived as:

$$\frac{\delta n}{n_0} = \frac{\delta p}{p_0} \approx \frac{12 \ln 2}{\pi^2} \frac{\varepsilon_F}{k_B T}. \quad (17)$$

From (16) and (17), the quasi-Fermi levels $E_{Fn}, E_{Fp} = \pm \varepsilon_F$, as a function of intensity of pump light, are given by:

$$\frac{\varepsilon_F}{k_B T} = \frac{\pi}{2 \ln 2} \left(\frac{\hbar \nu_F}{k_B T} \right)^2 \frac{\alpha I_{\Omega_0} \tau_r}{\hbar \Omega_0}. \quad (18)$$

On the other hand, under an intense optical excitation, i.e. $\varepsilon_F \gg K_B T$, the normalized densities of excess carriers are derived as:

$$\frac{\delta n}{n_0} = \frac{\delta p}{p_0} \approx \frac{6}{\pi^2} \left(\frac{\varepsilon_F}{k_B T} \right)^2 - 1. \quad (19)$$

As a result, ε_F as a function of intensity of pump light, is given by:

$$\frac{\varepsilon_F}{k_B T} = \sqrt{\frac{\pi^2}{6} \left(1 + \frac{\delta n}{n_0} \right)} \approx \sqrt{\left(\frac{\pi^2}{6} + \pi \left(\frac{\hbar \nu_F}{k_B T} \right)^2 \frac{\tau_r \alpha I_{\Omega_0}}{\hbar \Omega_0} \right)}. \quad (20)$$

For example, assuming a pump source at $\Omega_0 / 2\pi = 193 \text{ THz}$ and a recombination time $\tau_r = 1 \text{ ns}$, an energy splitting of $\varepsilon_F = 15 \text{ meV}$ at $T = 3 \text{ K}$ requires the pump intensity $I_{\Omega_0} = 146 \text{ mW / mm}^2$. Such intensity is fairly reasonable for telecommunication lasers.

2.4 Excitation of surface wave in graphene

It is known that the surface wave mode may or may not lie on the proper Riemann sheet, depending on the value of complex conductivity of graphene and types of SPP, i.e., transverse electric (TE) and transverse magnetic (TM) modes. Here we assume there is no external magnetic field and the conductivity is therefore isotropic without the Hall

conductivity. Consider a graphene monolayer located at the air-dielectrics interface [Fig. 2.3a], the fields in each region can be obtained by solving the source-free Maxwell's equations. By matching the surface impedance boundary conditions at the graphene surface, $\hat{\mathbf{n}} \times (\mathbf{H}^+ - \mathbf{H}^-) = \mathbf{J}_s = \sigma_s \mathbf{E}$ and $\hat{\mathbf{n}} \times (\mathbf{E}^+ - \mathbf{E}^-) = 0$, the dispersion equation for the TM surface mode (i.e., y-directed magnetic field) can be expressed as:

$$\frac{\varepsilon_2}{\varepsilon_1} \sqrt{\beta^2 - k_1^2} + \sqrt{\beta^2 - k_2^2} + \frac{\sigma_s}{j\omega\varepsilon_1} \sqrt{\beta^2 - k_1^2} \sqrt{\beta^2 - k_2^2} = 0, \quad (21)$$

whereas for transverse electric (TE), the dispersion equation becomes:

$$\frac{\mu_2}{\mu_1} \sqrt{\beta^2 - k_1^2} + \sqrt{\beta^2 - k_2^2} + j\omega\mu_2\sigma_s = 0, \quad (22)$$

where β is the complex phase constant of the surface wave, k_i , ε_i , and μ_i are the wave number, permittivity, and permeability inside the i -th medium, respectively. The excitation of surface wave mode depends on both signs and value of imaginary part of graphene's conductivity. The choice of the sign of real and imaginary parts of β are related to active or passive conductivity of graphene, which ensures no violation of passivity and causality. In general, only modes on the proper Riemann sheet may provide meaningful physical wave phenomena, and leaky modes on the improper sheet is used to approximate parts of the spectrum in restricted spatial region and to explain certain radiation phenomena [11],[13],[54]. Consider a free-standing graphene in vacuum ($\varepsilon_1 = \varepsilon_2 = \varepsilon_0$, and $\mu_1 = \mu_2 = \mu_0$), the eigenmodal solutions for (11) can be explicitly written as:

$$\beta = \sqrt{k_0^2 - \left(\frac{2\omega\varepsilon_0}{\sigma_s} \right)^2}, \quad (23)$$

whereas the similar explicit solution for (22) is:

$$\beta = \sqrt{k_0^2 - \left(\frac{\omega \mu_0 \sigma_s}{2} \right)^2}. \quad (24)$$

It is interesting to note that the dispersion of the complex conductivity results in several distinct regimes of modes. From (23) and (24), we notice that if $\text{Im}[\sigma_s] < 0$ and $\text{Re}[\sigma_s]$ is small (possible with a high-quality graphene at low temperatures), a TM-mode surface wave can exist. In the THz and far-infrared (FIR) region, where the intraband conductivity dominates over interband one, the excited surface wave can be a slow wave ($\text{Re}[\beta] \gg k_0$) tightly confined to the graphene surface, i.e., energy is concentrated in the near-field of graphene surface, as can be seen in the frequency dispersion shown in Fig. 2.3a. However, in the RF and microwave region where $\omega \ll \tau^{-1}$, the TM surface wave is poorly confined to the graphene surface and relatively fast ($\text{Re}[\beta] \cong k_0$). On the other hand, around the interband transition threshold ($\hbar\omega = 2|E_F|$), a moderate change in Fermi energy can dramatically alter the sign of imaginary conductivity from negative to positive. If $\text{Im}[\sigma_s] > 0$, the TE-mode surface wave is excited, which, unlike strongly localized TM-mode surface wave, is poorly confined at the surface (i.e. $\text{Re}[\beta/k_0] \approx 1$) and is basically nondispersive, as can be seen in Fig. 2.3a. The TM slow-wave mode is particularly interesting, as its drastic frequency dispersion can be a function of the Fermi energy, as can be straightforwardly understood from (23), thus providing a compelling platform for guiding, gate-tuning and modulating the THz and IR radiation by electrical gating or chemical doping. In contrast, the poorly-confined TE mode is essentially non-dispersive such that the tunability is rather poor.

The same conclusions hold for a photopumped, population-inverted graphene monolayer. Despite the sign of $\text{Re}[\sigma_s]$ is flipped by optical pumping, the TM (TE) surface wave exists if $\text{Im}[\sigma_s] > 0$ ($\text{Im}[\sigma_s] < 0$). In the negative-conductivity band of the population-inverted graphene (Fig. 2.2b), the amplitude of TM surface wave can be greatly amplified within a short propagation distance (i.e, a few effective wavelengths), as $\text{Re}[\beta] \gg k_0$ and $\text{Im}[\beta] > 0$, as seen in Fig. 2.3b. On the other hand, the TE surface wave is poorly

confined to the graphene surface and is lightly amplified or damped, depending on the sign of $\text{Im}[\beta]$, as seen in Fig. 2.2b.

Since the TE and TM surface waves do not co-exist in a graphene monolayer, such property may allow for making a broadband graphene-based polarizer, which supports only the TE surface wave propagation for frequencies above the interband transition threshold [19]. In this context, Vakil and Engheta have theoretically shown that by designing and manipulating spatially-inhomogeneous and non-uniform conductivity patterns across a graphene sheet, which is possible with a corrugated gate electrode [17]. This material can be used as a flatland platform for THz and IR transformation optics devices. In Ref. [17], Vakil and Engheta present several numerical examples demonstrating a variety of IR modulation and transformation functions. In Ref. [31], a Fourier optics lens, transforming a broadcasting surface wave from source into a perfect planar wavefront on a single sheet of graphene, has been numerically studied in the MIR region [17].

A subwavelength parallel-plate plasmonic waveguide (e.g. metal-insulator-metal (MIM) heterostructure) has been extensively used to confine and control electromagnetic fields. A low-loss, low-signal-dispersion THz interconnect will soon be required by the insatiable demand for high-speed devices and wideband communication. Fig. 2.4a illustrates a graphene-based THz interconnect, constructed with two graphene sheets with width much larger than separation distance d (a parallel-plate waveguide configuration with the propagation axis oriented along the z -axis). In many senses, the propagation properties of this graphene waveguide are analogous to the ones of the MIM optical waveguide supporting similarly confined electromagnetic modes, but suitable for THz and FIR frequencies. For TM_z waves with magnetic field $H_x(y)\exp[j(\omega t - \beta z)]$, the complex eigenmodal phase constant β can be evaluated by solving the dispersion equation:

$$\tanh[\sqrt{\beta^2 - \omega^2 \mu_0 \epsilon_2} d / 2] \frac{\sqrt{\beta^2 - \omega^2 \mu_0 \epsilon_2}}{\sqrt{\beta^2 - \omega^2 \mu_0 \epsilon_1}} = - \frac{\epsilon_2}{\epsilon_1 - j \sqrt{\beta^2 - \omega^2 \mu_0 \epsilon_1} \frac{\sigma_s}{\omega}}, \quad (25)$$

where ε_2 and ε_1 are the material permittivity inside and outside the waveguide, respectively [see Fig. 2.4a]. When the separation between the graphene monolayers is reduced to a deep-subwavelength scale $d \ll \lambda_0$, such graphene waveguide can support a quasi-TEM mode [12]-[37]. Under the long-wavelength approximation $d \ll \min\left(2\pi/\left|\omega\sqrt{\varepsilon_2\mu_0}\right|, 2\pi/\left|\omega\sqrt{\varepsilon_1\mu_0}\right|\right)$ and $\sigma_s\sqrt{\beta^2 - \omega^2\mu_0\varepsilon_2}/\omega\varepsilon_2$, $\sigma_s\sqrt{\beta^2 - \omega^2\mu_0\varepsilon_{\text{out}}}/\omega\varepsilon_{\text{out}} \gg 1$, a simple explicit dispersion relation can be obtained as:

$$\beta/k_0 \cong \sqrt{\frac{\varepsilon_2}{\varepsilon_0} \left(1 - j \frac{2}{\mu_0\omega\sigma_s d}\right)}. \quad (26)$$

As expected, for sufficiently large conductivity, the permittivity of the outer cladding layer has a negligible effect on the complex phase constant β , since the mode is tightly confined between the graphene layers. For the quasi-TEM mode that, in principle, has no cutoff frequency, the longitudinal field E_z is nonzero but very small compared to the uniform transverse field E_x , provided that the waveguide height d is very small. Since the quasi-TEM mode has the symmetric field distribution, the graphene parallel-plate waveguide in Fig. 2.4a can be replaced by a half-size waveguide Fig. 2.4b, of which a single graphene sheet is separated from a conducting ground plane by a half-thickness dielectric layer. Fig. 2.4c presents the normalized phase constant β/k_1 for a graphene waveguide in Fig. 2.4b, filled with a 50 nm-thick SiO_2 ($\varepsilon_2 = 4\varepsilon_0$ and $d/2 = 50$ nm) in the air background ($\varepsilon_1 = \varepsilon_0$). It is clearly seen that in the long-wavelength region, the quasi-TEM mode is relatively nondispersive, supporting a slow-wave propagation with strongly confined electric fields inside the waveguide. The guided wavelength can be much smaller than the free-space wavelength $\lambda_g \ll \lambda_0$, thanks to the large kinetic inductance of graphene. In the RF and microwave region, the value of attenuation constant corresponding to $\text{Im}[\beta]$ is quite large and the real phase constant is dispersive, due to the significant plasmon losses. As a result, the group velocity $\partial\omega/\partial\beta$ is frequency dependent, which usually cause the undesired signal dispersion. In the low-frequency range, the

graphene waveguide is basically a lossy RC transmission line (where the R and C are mainly contributed from the intrinsic loss of graphene and electrostatic capacitance yielded by the geometry, respectively). In some recent experiments, graphene-based transmission line has been demonstrated to be useful for making RF and microwave attenuators [55]. For frequencies above THz, the attenuation constant becomes small and the group velocity is almost constant without dispersion. In this case, a low-loss LC transmission line with minimized signal attenuation and dispersion is obtained, where the inductance is mainly contributed by graphene's large kinetic inductance resulted from the collective electron-inertia effect [6]. From Fig. 2.4c, we note that the phase constant of graphene waveguide is tunable by shifting the Fermi level, enabling a tunable transmission line at sub-THz and THz frequencies.

The transmission line model and transfer matrix method can be used to calculate the propagation characteristics of integrated graphene-based waveguide circuits, which are potentially frequency-reconfigurable. The characteristic impedance of the graphene waveguide in Fig. 2.4 can be defined as $Z_c = -E_y/H_x = \beta/\omega\epsilon_2$. The phase constant and characteristic impedance of this graphene-based transmission line may be varied from high (e.g. unbiased graphene with a small E_F) to low (e.g. electrically biased or doped graphene with large E_F). By combining a parallel-plate graphene waveguide with a double gate (or the graphene waveguide in Fig. 2.4b with a single back-gate, as shown in Fig. 2.4d), the Fermi energy of graphene can be controlled by the applied gate voltage such that the propagation constant, phase velocity, and local impedance may be dynamically tuned. This is arguably the most significant advantage of graphene over conventional noble-metal plasmonic waveguides, providing an exciting venue to realize electronically-programmable THz and IR tunable transmission lines, in analogy to a microwave tunable transmission loaded with varactors, diodes or transistors [56]. The gated graphene waveguides, with tunable and strongly-confined guiding mode, may enable a number of chip-scale passive and active THz nanocircuit components, including switching, matching, coupling, power dividing or combining, and filtering devices. It can be envisioned that the integrally gated graphene transmission line may serve as a practical paradigm and platform for THz and IR nano-communication, biosensors, and information processing

systems on wafer, with real-time electro-optically tuning, phasing and modulating functions [57]-[60]. The integration of several active graphene-based nanocircuit components into a single entity (chip) will present a fundamental step towards design architectures and protocols for innovative communications, biomedicine, sensing and actuation [57]-[60] using THz and IR light.

2.5 Reflection, transmission, and scattering properties of graphene

The ballistic transport characteristic and ultrahigh electron mobility in a suspended graphene [2] could provide a large value of $|\text{Im}[\sigma_s]/\text{Re}[\sigma_s]|$ such that graphene can be seen equivalently as a reactive sheet in the THz-to-MIR spectral range, effectively playing a role similar to a low-loss reactive surface (e.g. frequency selective surfaces or metasurfaces). This section will discuss with the scattering properties of a graphene sheet that effectively acts as an impedance surface. From the macroscopic electromagnetic viewpoint, the surface conductivity of graphene corresponding to a surface impedance $Z_s = 1/\sigma_s$, which behaves like an inductive surface with moderate losses ($\text{Re}[Z_s] > 0$ and $\text{Im}[Z_s] > 0$) in the THz-to-MIR region. Since a graphene monolayer can be described by a dispersive complex sheet conductivity, the reflection coefficient R and transmission coefficient T of a plane-wave incident on a large-area graphene can be calculated by applying the two-sided impedance boundary conditions at the graphene-dielectric interface, namely [61]: $\mathbf{H}_{\text{tan}}|_{r=a^+} - \mathbf{H}_{\text{tan}}|_{r=a^-} = \hat{\mathbf{r}} \times \mathbf{E}_{\text{tan}}|_{r=a}/Z_s$. Fig. 2.5a shows the reflectance ($|R|^2$) and transmittance ($|T|^2$) for a graphene sheet placed on top of a silicon dioxide (SiO_2) substrate with permittivity $\epsilon_{\text{SiO}_2} = 4\epsilon_0$; here the Fermi energy of graphene is varied. The inset of Fig. 2.5a shows the equivalent transmission line model, where graphene is described by a shunt surface impedance $Z_s = \sigma_s^{-1}$ (or a shunt admittance $Y_s = \sigma_s$). It is seen that a graphene sheet is highly reflective on the air-silicon dioxide interface at the low THz frequencies, in some sense similar to inductive metallic partially reflective surface (PRS) (e.g., metallic grid arrays at microwaves [62]-[64]). As seen in Fig. 2.5a, a homogenous graphene sheet effectively acts like a “high-pass filter,”

whose transparency depends on the variable Fermi level in graphene [24]. Such property may be utilized to make the tunable graphene filters, cavities, and bandgap structures, as well as reconfigurable artificial medium (e.g. hyperbolic metamaterials constituted by multiple graphene-dielectric layers [65]).

Fig.s 2.5b show the transmittance ($|T|^2$), reflectance ($|R|^2$), and absorptance ($A=1-|R|^2-|T|^2$) for a photopumped, population-inverted graphene monolayer with different quasi-Fermi levels. It is seen from Fig. 2.5a that magnitudes of absorption and scattering coefficients of an unpumped graphene (with $\text{Re}[\sigma_s] > 0$ over the whole spectrum) cannot be greater than unity. However, the transmittance and absorption of a photopumped graphene monolayer can exceed one at certain frequencies (colored area in Fig. 2.5b), provided that $\text{Re}[\sigma_s] < 0$ that leads to negative absorptance associated with optical amplification. We should note that the absorption of photopumped graphene can be further boosted by using suitably designed graphene nano-structures, such as metasurfaces or frequency selective surfaces that make the equivalent surface impedance $Z_s \approx -Z_0/2$. In this scenario, the reflection and transmission coefficients may approach infinity, if the optical amplification effect is not saturated by nonlinearities in graphene. The gapless graphene under population inversion may be of great interest for creating amplification and oscillation of THz and FIR radiations, as a broadband gain effect can be achieved with a strong optical pumping. As seen in Fig. 2.2b, the bandwidth of negative conductivity increases with increasing the quasi-Fermi level of graphene, which is proportional to the intensity of pump light. The active graphene monolayer has also been proposed to compensate material losses in the THz metamaterials [51], by loading the micro-resonators with an active graphene layer in each metamaterial unit cell.

Recently, specific interest has been devoted to novel material platforms, such as plasmonic materials and metamaterials, which may scatter and re-radiate light in anomalous and exotic ways, providing new phenomena and applications. One of the most exciting applications of these materials is to achieve exotic transparency effects, namely “hiding” an opaque object and making it effectively invisible to the electromagnetic

radiation [66]-[70]. This passive camouflaging technique has the iconic term of *invisibility cloaking*. In addition to camouflaging and mirage applications, the cloaking technique may improve the performance of near-field sensors and detectors, as their presence may cause less disturbance to the environment in complex sensing networks and measurement systems [66]-[70]. This “cloaking a sensor” technique may be useful in many setups, e.g., a near-field scanning optical microscope (NSOM) probe or a receiving antenna [67]-[68]. Since most sub-diffractive measurements are usually performed in the very near-field of the details to be detected, their accuracy is essentially limited by the disturbance introduced in the close proximity of the sensing instrument, i.e., a sensing probe that may also perturb the near-field field distributions and influence the measurement results [67]-[68]. As a result, a “cloaked sensor” with low scattering cross-section is expected to significantly improve sensing, imaging and communication systems that require low levels of interference and noise [67]-[68].

Plasmonic cloaking is a scattering cancellation technique based on a homogeneous layer with low- or negative- permittivity/permeability of plasmonic materials or metamaterials, which may produce a local polarization vector that is in “anti-phase” with respect to that of the object to be cloaked. A cancellation between the scattering of the object and of a properly designed plasmonic cover may restore the incident wavefront in the near- and far-field, which is independent of the polarization and incidence angle of the impinging wave and the position of the observer. A single homogeneous plasmonic layer, such as noble metal thin-film, has been shown to considerably suppress the electromagnetic scattering from a few multipolar scattering orders, allowing for effectively cloaking objects on the scale of the wavelength of operation. In the THz and FIR spectrums, the graphene plasmonics would be best suited to manipulate the light scattering properties of graphene-covered objects. The physical principle behind such graphene cloak is similar to a bulky plasmonic cloak, for which the induced surface current on graphene surface can be properly tailored to radiate anti-phase scattered fields to achieve the scattering cancellation effect [18],[36]. The tunable sheet reactance of graphene, however, offers the unique advantage of being frequency-reconfigurable.

The inset of Fig. 2.6a shows a graphene cloak used for reducing the electromagnetic scattering from a dielectric cylinder, when illuminated by a normally incident transverse magnetic (TM) plane wave. The graphene cloak can be, for example, a single-walled carbon microtube [71] made by wrapping a graphene monolayer around a seamless cylinder [2]. The scattering coefficients of the cloaked object can be obtained with the Lorentz-Mie scattering theory and impedance boundary conditions discussed earlier [18], which are given by [68]:

$$c_n^{\text{TM}} = -\frac{P_n^{\text{TM}}}{P_n^{\text{TM}} - jQ_n^{\text{TM}}}. \quad (27)$$

The expressions for the n -th order P_n^{TM} and Q_n^{TM} coefficients may be found in Ref. [68], as a function of the geometry and the surface impedance of graphene. For an isotropic surface with negligible cross-polarization coupling, such as a graphene microtube presented here, the total scattering width (SW), providing a quantitative measure of the overall visibility of the object at the frequency of interest, is given by [68]:

$$\sigma_s = \frac{4}{k_0} \sum_{n=-\infty}^{n=\infty} |c_n^{\text{TM}}|^2. \quad (28)$$

In the quasi-static limit, the closed-form cloaking condition for a dielectric cylinder under the TM-polarized illumination can be derived as $X_{\text{diel}} = 2/[\omega a \epsilon_0 (\epsilon_r - 1)]$, where a is the radius of cylinder. Obviously, an inductive surface will be required for efficiently cloaking a moderately-sized dielectric object. Fig. 2.6a presents the total SW for an infinite SiO_2 cylinder with diameter $2a = 40 \mu\text{m}$, which is covered by a graphene-wrapped microtube with different Fermi energies; here an uncloaked SiO_2 cylinder is also presented for a fair comparison (grey dashed line). It is clearly seen from Fig. 2.6a that the SW can be greatly reduced at specific frequencies in the THz spectrum, by using an atomically-thin and conformal graphene cloak. In addition, the frequency of operation (dip in the scattering spectra) can be tuned by varying the Fermi energy. At specific frequencies, it is possible to vary the total SW by over two orders of magnitude, by varying the Fermi energy of graphene. Figs 2.6b and 2.6c present, respectively, the near-field phase contours of the magnetic field on the E plane for a cloaked ($E_F = 0.25 \text{ eV}$)

and uncloaked SiO_2 cylinder at the operating frequency $f_0 = 1.5$ THz. In Figs. 2.6b and 2.6c, we consider an incident angle $\alpha = 60^\circ$ (the angle between the incident wave direction and the cylinder axis; see the inset of Fig. 2.6a). In both panels the plane-wave illuminates the cloaked dielectric object from the left side and contours are plotted on the same color scale. It is found that for around the cylinder, the significant scattering suppression and restoration of original phase fronts can be achieved, in contrast to the distorted phase fronts observed for the case of an uncloaked dielectric cylinder. It is also worth noting that, similar to those plasmonic cloaking technique [67]–[68], the THz wave can penetrate the graphene cloak and object, thereby enabling applications in cloaked sensors and non-invasive probing in the THz and IR spectrum. The multiband operation and cloaking larger objects may also be possible by using multiple graphene layers, which offer more degrees-of-freedom and allow suppressing a larger number of scattering orders. A theoretical discussion on the possibility of cloaking a dielectric sphere with multiband operation can be found in Ref. [36]. For a conducting cylinder with moderate cross-section covered by a thin spacer, the cloaking condition can be derived as: $X_{\text{cond}} \cong -\omega\mu_0 a \left[(\gamma^2 - 1) / (\gamma(\gamma^2 + 1)) \right]$ [69], where γ is the radius ratio between the graphene cloak and the conducting cylinder, and μ_0 is the permeability of background medium. As observed from this formula, an ultrathin cloak with capacitive surface reactance is required to effectively reduce the scattering from a conducting cylinder. A nano-structured graphene monolayer, such as a graphene nano-patch metasurface [69], can have a surface reactance tuned from inductive to capacitive, as a function of the graphene's Fermi energy and the geometry of metasurface inclusions. Such a dual-reactive property may allow for cloaking both dielectric and conducting objects. The graphene cloak may also be exploited to conceal a 3D dielectric or conducting object, such as a dielectric spherical particle [36],[72] or a more complicated geometry like 3D finite wedges [73].

2.6 Graphene-based metamaterials and metasurfaces

Metamaterials are artificially engineered composite materials, which have electromagnetic properties that may not be found in nature, such as negative refraction

and magnetism above the THz frequency. Metamaterials usually gain their properties from their deep-subwavelength constituent inclusions designed to tailor the electric, magnetic or both responses [74]-[75]. Plasmonic metamaterials composed of noble-metal nano-structures further exploit surface plasmons to achieve abnormal optical properties, such as sub-diffraction imaging [76] and transformation optics [77]. Recently, a new class of metamaterials based on micro/nano-structured graphene have been proposed to achieve exotic electromagnetic properties in the relatively unexplored THz, FIR and MIR frequencies, where the plasmonic effects of natural materials are generally weak. Metasurfaces (or metamaterial-surfaces) are 2D planarized versions of metamaterials, composed of subwavelength inclusions over an otherwise homogeneous host surface. Atomically-thin and flat graphene with relevant plasmonic characteristics is particularly suitable for making a metasurfaces.

The research on tailoring light scattering properties using graphene has initiated a substantial number of studies dedicated to graphene nanostructures, such as ribbons [34],[78],[79] and disks [20],[23], of which finite size effects and boundary conditions are important, yielding new electromagnetic phenomena and opening up the way of dynamically modulating the amplitude, phase, and polarization states of THz and IR radiation over an ultrathin impedance surface. In this section, we review the graphene-based tunable metamaterials and metasurfaces operating in the THz and IR region, with a number of applications, such as sub-diffraction lenses, efficient amplitude and phase modulators, switches, beamformers, and transformation-optics devices. Here, we note two main features of the graphene-based metamaterial and metasurface: (i) the large kinetic inductance yielded by subwavelength confinement of surface plasmons in the structured graphene may help scaling the size of metamaterial inclusions to deep subwavelength, thus enhancing the homogeneity and granularity of artificial materials; (ii) the field-effect-tuned surface impedance can be achieved, as the dynamic conductivity of graphene is controlled by its carrier concentration. Recent progress in the growth and lithographic patterning of large-area epitaxial graphene [33]-[35] presents great opportunities for graphene-based metamaterials and metasurfaces, readily integrated with silicon-based plasmonics and photonics systems.

Fig. 2.7a presents a metasurface formed by a collection of specific graphene surface inclusions (e.g. a lithographically patterned graphene). Each graphene surface inclusion, when illuminated by external radiation, may generate localized or mixed-type semi-localized SPPs, leading to the strong resonant scattering. The scattering fields ($\mathbf{E}_s$, $\mathbf{H}_s$) are responsible for the localized, field-dependent surface current $\mathbf{k}_s \sim \sigma \mathbf{E}_0^{\text{tan}}$ inside each graphene inclusion. The polarizability α and the equivalent electric dipole moment per unit length $\mathbf{p}$ (parallel to the surface) are given by $\mathbf{p} = (-j / \omega) \int_{S_{in}} \mathbf{k}_s ds = \alpha \mathbf{E}_0^{\text{tan}}$, where σ and S_{in} is the conductivity and the enclosed area of each graphene inclusion [80]. In the metasurface configuration [Fig. 2.7], the induced dipole moment of an inclusion, considering the inter-dipole interactions via the dyadic Green function [81] can be expressed as:

$$\mathbf{p} = p_0 \hat{\mathbf{x}} = \frac{\mathbf{E}_0^{\text{tan}}}{\mathcal{A}^{-1} - \sum_{(N_x, N_y) \neq (0,0)} \hat{\mathbf{a}} \mathbf{G}(\mathbf{r}_{N_x N_y}) \times \hat{\mathbf{x}} \times \hat{\mathbf{x}}}. \quad (29)$$

The equivalent surface impedance of the metasurface, defined as the ratio of local electric field to surface current density $\mathbf{J}_s = J_s \hat{\mathbf{x}} = j\omega \mathbf{p} / (d_x d_y)$, is given by $Z_s = E_0 / J_s - \eta_0 / 2$, where η_0 is the free-space impedance, and d_x , d_y are the lattice spacing of the metasurface. Fig. 2.7b shows the spectral absorption of a graphene metasurface depicted in Fig. 2.7a, considering the dipole-dipole mutual coupling between adjacent graphene inclusions. In spite of its ultrathin profile, the graphene metasurface provides a perfect IR absorption. Further, the peak frequency may be wideband tunable via the electrostatic gating, thus being of interest for a variety of optical applications, such as tunable filters and absorbers. We also note that in a symmetric environment (e.g. a graphene immersed in the uniform medium), a maximum absorption of 50 % can be achieved, which is consistent with what is predicted from the optical theorem [23]. The perfect absorption can be achieved by considering the asymmetric environment, e.g. a dissimilar dielectric interface with $\epsilon_1 \neq \epsilon_2$ or a patterned graphene on top of a metallic ground plane [78]-[79],[82]-[83].

Fig. 2.7c shows a graphene nano-patch metamaterial recently released by IBM Corporation [35]. A wafer-scale, large-area graphene metamaterials has been successfully fabricated and their properties have been characterized. Fig. 2.7d shows the measured rejection ratio (extinction in transmission) spectrum for the graphene metamaterial in Fig. 2.7c with different dimensions; here the transmission extinction is defined as $1 - T/T_0$, where T and T_0 are the transmission through the quartz substrate with and without the graphene-insulator stack, respectively. The measured peak transmission $\sim 80\%$ is achieved with a stacked device made of five graphene layers. It is seen that by changing the dimensions of the fabricated graphene inclusions and lattice constant, the resonance peak of the graphene metamaterial can be tuned. When the number of graphene layers is increased, the peak intensity increases and the resonance frequency upshifts. Moreover, the gate-tuning of surface plasmon waves on graphene ribbon has been experimentally demonstrated using the scatter-type scanning near-field microscopy [24]. The measurement is based on the feedback scheme taken from atomic force microscopy (AFM), of which the nano-manipulated nanotip is positioned and scanned in the immediate vicinity of graphene surface. As expected, the advancement of nanotechnology and nano-fabrication will benefit the rapidly expanding field of graphene metamaterials and plasmonic devices. The strong local-field confinement and gate-tuning standing-wave field patterns due to the interference in a finite-length graphene ribbon have been successfully characterized in Ref. [24]. The tunable electromagnetic response is one of the most exciting areas in current metamaterial research [84]-[88], since they may add a large degree of control and flexibility to the exotic right-/left-handed properties of metamaterials. In addition, extending these effects to the THz spectrum may provide new tools to push metamaterials in a frequency range that are more difficult to work in, compared to microwave or VIS regimes, where nonlinear tuning elements, such as diodes, transistors, and nonlinear optical materials are readily available.

Another metamaterial/metasurface geometry of interest is the lithographic graphene nanoribbon (GNR) arrays shown in Fig. 2.8a [34], where conduction currents are confined within the disjoint GNRs with strong reactive coupling between neighboring graphene inclusions, which have subwavelength width and period. We note that

according to the semi-empirical results [6], the bandgap of a graphene nanoribbon with width >100 nm is not open yet. Thus, the previously derived surface conductivity for a graphene monolayer is still valid. One can analytically solve the problem using the averaged boundary condition (ABC), for which a discontinuity on the tangential magnetic field on the metasurface is related to the averaged surface current by the equivalent surface impedance Z_s [81]:

$$\begin{aligned}\mathbf{E}_x^- &= j \frac{\eta_{\text{eff}}}{2} \alpha_{\text{ABC}} \left[\sigma_g \frac{p}{p-d} + \left(1 + \frac{1}{k_{\text{eff}}^2} \frac{\partial^2}{\partial x^2} \right) \right] (\mathbf{H}_y^- - \mathbf{H}_y^+) = Z_s^{\text{TM}} \mathbf{J}_x \\ \mathbf{E}_y^- &= j \frac{\eta_{\text{eff}}}{2} \alpha_{\text{ABC}} (\mathbf{H}_x^- - \mathbf{H}_x^+) = Z_s^{\text{TE}} \mathbf{J}_y,\end{aligned}\tag{30}$$

where α_{ABC} is called grid parameter, $\varepsilon_{\text{eff}}, \eta_{\text{eff}} = \sqrt{\mu_0 / \varepsilon_{\text{eff}} \varepsilon_0}$, and $k_{\text{eff}} = \omega \sqrt{\mu_0 \varepsilon_{\text{eff}} \varepsilon_0}$ are the relative permittivity, wave impedance, and wave number of the effective host medium, the superscripts TE and TM in (30) represent the TE and TM plane wave illuminations, respectively. Assuming perfectly-conducting strips of negligible thickness are aligned parallel to the magnetic field of the TM incident wave, the equivalent surface admittance can be explicitly written as [81]:

$$Z_s = -j Z_{\text{eff}} / 2\alpha,\tag{31a}$$

$$\begin{aligned}\alpha &= \frac{k_{\text{eff}} p}{\pi} \left[\ln \left(\csc \left(\frac{\pi g}{2p} \right) \right) + \frac{1}{2} \sum_{n=-\infty, n \neq 0}^{\infty} \left(\frac{2\pi}{\sqrt{(2\pi n)^2 - k_{\text{eff}}^2 p^2}} \frac{1}{|n|} \right) \right] \\ &\approx \frac{k_{\text{eff}} p}{\pi} \ln \left(\csc \left(\frac{\pi g}{2p} \right) \right) \quad \text{if } k_{\text{eff}} p \ll 2\pi,\end{aligned}\tag{31b}$$

Where p and g are the period and gap of the graphene metasurface in Fig. 2.8a, $Z_{\text{eff}} = \sqrt{\mu_0 / \varepsilon_{\text{eff}}}$, $k_{\text{eff}} = \omega \sqrt{\mu_0 \varepsilon_{\text{eff}}}$, $\varepsilon_{\text{eff}} = (\varepsilon + \varepsilon_0) / 2$. In (31), the higher-order Floquet-Bloch harmonics can be ignored if the period is subwavelength. Consider the case of oblique incidence with an incident angle θ , α for TM- and TE- incident waves can be expressed as $\alpha_{\text{TM}} = \alpha$ and $\alpha_{\text{TE}}(\theta) = \left[1 - \sin^2 \theta / (2\varepsilon_{\text{eff}}) \right] \alpha$, respectively. If one assume that the incident electric field is y-polarized, the incident planes for TM and TE incidence respectively correspond to the x - z and y - z planes. When a dispersive medium is used to

constitute the metasurface (e.g., GNRs with complex-valued sheet conductivity), the surface impedance is complex-valued and can be written as [62]-[64] :

$$Z_s = \frac{1}{\sigma_s(1 - g/p)} - j \frac{Z_{eff}}{2\alpha}. \quad (32)$$

The surface reactance of metasurface $\text{Im}[Z_s]$ may be tuned from inductive to capacitive, as a function of the tunable kinetic inductance of GNRs and the geometric capacitance yielded by gaps between neighboring GNRs [68]. It has been shown in Ref. [68] that the effective surface inductance/capacitance can be derived from (32), and the intriguing dual-reactive property on a single metasurface may provide many new possibilities for tailoring the electromagnetic scattering properties [68]. Different types of graphene nanostructures, including strip, patch, grid and Jerusalem-cross arrays, have been derived in Refs. [89]-[91]. The dual-reactive property of graphene metasurface also introduces an extra pole and zero in the Lorentzian dispersion of surface impedance, which may enhance extinction and scattering bandwidth [78] as well as the possibility of cloaking both dielectric and conducting objects using the same graphene metasurface with different Fermi energies of graphene [68].

Fig. 2.8b considers a GNR array separated from the metallic ground plane by a thin dielectric layer. Such structure exhibits interesting electromagnetic properties, equivalent to a grounded magnetic-metamaterial slab [83], in analogous to arrays of split-ring resonators with negative effective permeability,[92],[93]. The grounded dielectric slab with a subwavelength thickness offers a magnetic inductance [62] as a function of the height of dielectric slab, which may resonate with the geometric capacitance and the kinetic inductance of the GNR array. As a result, the frequency dispersion of this graphene metamaterial can be tailored by varying either the period/gap and carrier density of GNRs (which tunes the geometric capacitance and kinetic inductance) or the dielectric constant and thickness of the dielectric slab (which tunes the substrate-induced inductance). In addition, the metallic and dielectric layers may be used for the electrostatic back-gating, thus enabling a tunable THz and IR metamaterial. Consider a thin-grounded slab with thickness t and effective permeability $\mu = \mu' - j\mu''$, the input impedance, looking into the slab interface, is:

$$Z_{in} = Z_0 \sqrt{\mu' - j\mu''} \tanh\left(j\sqrt{\mu' - j\mu''} k_0 t\right). \quad (33)$$

By equating (33) to the input impedance $Z_{in} = R_{in} + jX_{in}$ at the graphene-dielectric interface and taking the quasistatic limit (i.e. $\tanh(x) \approx x$), the effective real and imaginary parts of permeability for a graphene metaferrite slab in the inset of Fig. 2.8b can be expressed as:

$$\begin{aligned} \mu'(\omega) &\approx X_{in}/Z_0 k_0 t \\ \mu''(\omega) &\approx R_{in}/Z_0 k_0 t. \end{aligned} \quad (34)$$

Fig. 2.8c shows the frequency dependency of effective permeability retrieved for the graphene metaferrite in Fig. 2.8b under normal incidence. The solid and dashed lines represent the real and imaginary parts of effective permeability, respectively. The geometry of GNRs are $g = 0.5 \mu\text{m}$ and $a = 4 \mu\text{m}$, and the dielectric slab is made of SiO_2 with thickness $t = 1 \mu\text{m}$. From Fig. 2.8c, it is seen that the Lorentzian resonance in the effective permeability can be tuned from low or negative to positive. A low reflection corresponding to a high absorption can be accomplished, if the equivalent permeability $m = \mathcal{M} - j\mathcal{M}\mathcal{I}$ is tailored to the optimum value for a ground-backed magnetic-film absorber: $\mu' \approx 0$ and $\mu'' \approx 1/k_0 t$, [78] where k_0 and t are the free-space wave number and the thickness of graphene metaferrite, respectively. Fig. 2.8d presents the absorption spectra for the graphene metaferrites in Fig. 2.8b, showing large and tunable absorption with respect to the doped Fermi energy of graphene. The GNR-based metamaterial may be of interest for a variety of long-wavelength IR applications, including the artificial magnetic conductor [93], ultracompact Salisbury absorbing films [79],[94], polarizing filters [96], and selective thermal emitters or absorber [96]-[97]. Recent experimental works have demonstrated that the THz extinction of a planar GNR array can be tailored by varying its geometry or by electrostatically gating the GNRs [35].

2.7 Graphene-based ambipolar electronics for RF and microwave applications

Due to the symmetric electronic structures of graphene, the ambipolar transport characteristic can be found in graphene-based transistors. The high carrier mobility and

ambipolar charge transports in graphene make the graphene-based transistors a compelling platform for radio-frequency (RF) and analog electronics. The symmetric “V-shape” drain current-gate voltage characteristic of a graphene field-effect transistors (GFET) can realize the simplest possible frequency multiplier, converting the input RF signal into its second harmonic [43],[44],[97]-[100]. This unique characteristic is not found in conventional semiconductor microelectronic devices. To date, high-frequency GFETs with the cut-off frequency up to 155 GHz [97] have been achieved by using wafer-scale graphene grown by chemical-vapor deposition (CVD) and standard nanolithography processes. In this context, it has been recently demonstrated that a graphene-based ambipolar transistor can realize frequency doubling in the Ku-band (12 - 18 GHz), without any amplification and filtering component [100].

2.7 Conclusions

In this chapter, we review graphene’s extraordinary electromagnetic properties and its applications spanning wide ranges of frequency. We first discuss the complex conductivity of graphene under equilibrium (passive), electrostatic gating, and population-inversion (active) conditions, showing how a rich variety of applications can be found in different spectral ranges due to graphene’s exotic frequency dispersion. A large body of research has shown that graphene exhibits a Drude-type conductivity with a tunable plasma frequency and is thus capable of supporting surface plasmon polaritons in the THz-to-MIR region. We have discussed how, by electrostatically modulating or lithographically patterning a graphene monolayer, one may be able to fully manipulate, enhance, and control the guided propagation, scattering, and radiation of long-wavelength light, similar to noble-metal-based plasmonics that deals with NIR and VIS light. We also mention the possibility of achieving the long-wavelength plasmon gain in an optically-pumped graphene, which may find potential applications in tunable THz lasers and oscillators. In the VIS and UV region, graphene with high optical transparency may be a candidate material for transparent electrodes used in flexible displays and solar panels. In the RF and microwave regions, high carrier mobility in graphene may enable high-speed field-effect transistors with intrinsic cut-off frequency up to hundreds of GHz. Besides,

its ambipolar transport characteristic, graphene offers a unique opportunity to make simple and efficient frequency multiplication and mixing devices.

Figure 2.1 (a) Schematic of graphene: an atomic-scale hexagonal lattice made of carbon atoms. (b) Energy dispersion relation for graphene around the Dirac point (K) in the Brillouin zone. (c) Frequency dependence of the real (solid line) and imaginary (dashed line) parts of graphene's dynamic conductivity, whose dual plasmonic ($\text{Im}[\sigma_s] < 0$) or dielectric ($\text{Im}[\sigma_s] > 0$) properties can be tuned by the Fermi energy of graphene. (d) Photographs and (e) measured optical transparency for the transparent and flexible RF antennas made of multi-layer graphene.

Figure 2.2 (a) Schematics of THz lasing effect in a pristine graphene ($E_F = 0$ eV) under optical pumping. (b) Frequency dispersion of real (solid lines) and imaginary (dashed lines) conductivity for a pristine graphene photoexcited to different quasi-Fermi energies at low temperatures ($T = 3$ K).

Figure 2.3 (a) Illustrative schematic of TM-mode surface wave propagating along a graphene sheet (left) and its complex wave number, which is tunable with respect to the Fermi energy of graphene. TM (TE) mode is excited when $\text{Im}[\sigma_s] < 0$ ($\text{Im}[\sigma_s] > 0$). (b) is similar to (a), but for a photopumped (active) graphene sheet. The plasmon amplification effect can be achieved if $\text{Im}[\beta] > 0$, yielded by $\text{Re}[\sigma_s] < 0$.

Figure 2.4 (a) Graphene parallel-plate waveguide and (b) its compact version inspired by the field symmetry of quasi-TEM mode. This graphene-based waveguide could be a low-loss (R)LC transmission line in the THz and infrared region, while being a lossy RC line in the RF and microwave region, as schematically shown in (b). (c) Complex phase constant of the graphene waveguide in (b), with thickness $d/2 = 50$ nm, supporting a quasi-TEM mode; here lines and symbols represent the eigenmodal solutions calculated by Eq. (12) and Eq. (13), respectively. (d) Illustrative schematics of a tunable and reconfigurable electronic-plasmonic (plasmo-electronic) device, comprising a graphene field-effect-transistor (GFET) and a graphene plasmonic waveguide (GPWG). Reprinted with permission from (Ref. [59]). Copyright (2014) American Chemical Society.

Figure 2.5 (a) Reflectance, transmittance, and absorptance spectra for a suspended graphene monolayer (schematically shown at the top). The scattering responses is a function of graphene's Fermi energy E_F . (b) Absorptance of a photopumped graphene monolayer (schematically shown at the top), under different pump intensities that lead to different quasi-Fermi energies ϵ_F . The gain bandwidth is increased by increasing ϵ_F or, effectively, intensity of pump light.

Figure 2.6 (a) Normalized total scattering width of a SiO_2 cylinder with diameter $2a=40\text{ }\mu\text{m}$ and permittivity $\varepsilon_d=3.9\text{ }\varepsilon_0$, without and with graphene cloak (i.e. graphene-wrapped microtube). Here the Fermi energy of graphene is varied to show its great tunability on the design frequency. The phase of magnetic fields on the E plane (b) with and (c) without the graphene cloak with Fermi energy $E_F = 0.25\text{ eV}$ are shown at the operating frequency $f_0=1.45\text{ THz}$. Reprinted with permission from (Ref. [18]). Copyright (2011) American Chemical Society.

Figure 2.7 (a) Schematics of a planar array of doped graphene nano-disks of diameter $D = 60$ nm and Fermi energy $E_F = 0.4$ eV, and (b) the corresponding absorption for (left) an asymmetric interface with $\epsilon_1 \neq \epsilon_2$ and (right) a symmetric interface with $\epsilon_1 = \epsilon_2$; the lower-index medium has relative permittivity $\epsilon_1 = 1$. Reprinted with permission from (Ref. [23]). Copyright (2012) American Physical Society. (c) Scanning electron microscopy (SEM) image for nano-fabricated infrared metamaterials made of multilayer circular graphene patches. (d) Measured electromagnetic wave rejection ratio of the graphene metamaterials in the mid-infrared and far-infrared spectrum. Reprinted with permission from (Ref. [35]). Copyright (2012) Nature Publishing Group.

Figure 2.8 (a) Graphene nanoribbon (GNR) array, as a plasmonic metasurface in the long-wavelength region. (b) Schematics of THz metaferrite composed of a planar array of GNRs on top of a SiO_2/Si slab backed by a metallic backgate, which can be equivalent to a ferrite with relative permeability $\mu = \mu' - j\mu''$ backed by a conducting plane, as shown in the bottom panel. (c) Frequency dispersion of effective permeability for a graphene metaferrite with different biased Fermi energy (solid line: real, dash line: imaginary). (d) Absorption spectra for the graphene metaferrite of (b). The thickness of SiO_2/Si is $25 \text{ nm}/2 \mu\text{m}$, and the dimensions of nano-patch are: period $d=1 \mu\text{m}$ and gap $d/p=0.25$. Reprinted with permission from (Ref. [83]). Copyright (2013) The Institute of Electrical and Electronics Engineers (IEEE).

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