Effects of solid circular cylinders on the dynamics of the Richtmyer-Meshkov instability

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Abstract We discuss the effect on the transient dynamics of the Richtmyer-Meshkov instability due to small-scale perturbations introduced by a set of solid circular cylinders located along the material interface. The arrangement of the cylinders mimics (in a two-dimensional domain) the presence of the solid supporting grid wires used in the formation of the material interface in an experimental setup. The numerical experiments are performed by utilizing an embedded boundary-Adaptive mesh refinement method. We quantify the effect of introducing the cylinders on the mixing layer growth and the mixedness level, and qualitatively demonstrate the effect of the perturbation introduced by the cylinders on the mixing layer structure. We empirically model the effect of the solid cylinders based on their diameter, and find that the effect of the cylinders on the mixing layer growth is a linear function of the cylinder diameter. The linear model allows us to predict the change in the mixing layer growth within a $\pm 10\%$ range, except for very small ratios of the initial perturbation amplitude of the density interface to the cylinder diameter.

Keywords Richtmyer-Meshkov instability · Density interface formation · Compressible Navier Stokes equations · Adaptive mesh refinement · Embedded boundary method

1 Introduction

The Richtmyer-Meshkov instability (RMI) occurs when a perturbed material interface, separating two fluids

Fig. 1 A schematic illustrating the physical domain's initial configuration; the shock wave at Mach number M_0 and perturbed interface with the left and right densities (ρ_1, ρ_2), the perturbation wavelength λ_0 and amplitude a_0 .

of different densities, is impulsively accelerated by a shock wave, (see Figure 1). The instability is driven by the baroclinic vorticity generation, i.e., the misalignment between the shock wave pressure gradient and the density gradient across the interface, that amplifies the interface perturbation [1] and [2]. Neglecting viscous effects, the compressible vorticity equation in two dimensions is written as:

$$\frac{D}{Dt} \left(\frac{\omega}{\rho} \right) = \frac{\nabla \rho \times \nabla p}{\rho^3}, \quad (1)$$

The right-hand side of the above equation is the so-called baroclinic torque ideally leading to a vorticity layer generated on an initially diffuse interface.

The Richtmyer-Meshkov instability plays an important role in applications [1]; such as the mixing of the capsule material and fuel in process of the inertial confinement fusion [3]. RMI also appears in deflagration-to-detonation transitions which depends on the shock-flame interaction [4]; and in fuel and oxidizer mixing in supersonic and hypersonic air-breathing engines [5]. In astrophysical contexts, RMI is essential in explaining the lack of the stratification of the product of supernovae [6].

The impulsive acceleration of the material interface was first considered by Markstein [7]. However, the first detailed comprehensive linear stability analysis of the RMI was the theoretical and numerical study of Richtmyer [8], which was later confirmed by the experimental work of Meshkov [9].

Experimentally, RMI investigations face one challenging task: this stems from the technical difficulties in forming a perturbed density interface. Several mechanisms have been attempted so far in order to initialize the interface perturbation. One technique involves using a thin membrane film supported by a wire mesh. However, this affects the temporal history of the interface perturbation growth [1]. Investigations of the membrane film effects on the RMI dynamics were performed by Abakumov et al. [10] and Erez et al. [11]. Fontaine et al. [12] proposed a method for reducing the membrane effects using a nitrocellulose thin film. Another alternative to producing a perturbed interface was by pulling a thin sliding plate that separates two fluids with different densities as developed by Brouillette and Sturtevant [13]. Yet another technique involves forming a stable interface in a vertical shock tube by a downward (upward) flow of a light (heavy) gas so that the gases meet at a lateral slot introduced for outflow: the interface is perturbed by oscillating the shock tube in the lateral direction [14] or by inclining the shock tube [15] (for additional examples on the inclined the shock tube technique please refer to McFarland et al. [16,17]).

Zou et al. [18] have carried out experimental studies in which the RMI is initiated by accelerating a flat interface by a perturbed shock wave that is diffracted over a small circular cylinder, and in their recent work they studied the dynamics of this RMI mechanism in a convergent shock configuration [19]. The effect of the circular cylinder perturbation is somewhat similar to the influence of the perturbation that is initiated by solid grid wires used to support the membrane film in the density interface formation mechanism mentioned above. A quantitative study of the effects of the wire mesh supporting the membrane on the RMI dynamics is warranted. We model the wire mesh in 2D by including solid cylinders in the vicinity of the interface. Our work may be considered complementary to previous studies on the role of the membrane on the RMI dynamics, see for examples the references [10,20,11,12]; and the recent work of Vandenboomgaerde et al. [21] in which they considered the effect of the supporting grid bars on the RM instability growth in a convergent geometry.

A brief outline of the paper is as follows. In section 2, we present the governing model, domain setup and the numerical methodology. In section 3, we discuss our computational results, by first validating our

results with experimental measurements followed by an analysis and modelling the effects of the solid cylinders on the RMI growth. We finally conclude in section 4.

2 Numerical Methodology

2.1 Governing Equations

In the present numerical experiments, the flow is governed by compressible Navier Stokes equations written below in strong conservative form:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x_j} = \frac{\partial \mathbf{G}}{\partial x_j}. \quad (2)$$

where the density ρ , momentum ρu_i , and the total energy per unit volume E are the elements of the solution vector $\mathbf{U}(x, y, t) = (\rho, \rho u_i, E, \rho \phi)$; and ϕ is the mass fraction variable that is introduced to track the time variation of the density interface that separates the heavy and light gas species. $\mathbf{F}$ and $\mathbf{G}$ are the inviscid and viscous fluxes in the non-dimensional form, respectively, as given below:

$$\mathbf{F} = \begin{pmatrix} \rho u_j \\ \rho u_i u_j + p \delta_{ij} \\ (E + p) u_j \\ \rho \phi u_j \end{pmatrix}, \quad (3)$$

$$\mathbf{G} = \begin{pmatrix} 0 \\ \frac{1}{Re} \tau_{ij} \\ \frac{1}{Re} \tau_{ij} u_i - \frac{\gamma}{(\gamma-1)PrRe} q_j \\ 0 \end{pmatrix}. \quad (4)$$

The pressure p depends on the total energy per unit volume E according to:

$$E = \frac{p}{\gamma - 1} + \frac{\rho}{2} u_i u_i, \quad (5)$$

and where γ is the ratio of the specific heats. The deviatoric stress tensor τ_{ij} is expressed as:

$$\tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right). \quad (6)$$

The heat flux vector is given by:

$$q_j = -\kappa \left(\frac{\partial T}{\partial x_j} \right), \quad (7)$$

where the temperature T is related to the pressure and density by the non-dimensional perfect gas equation of the state ($p = \rho T$). The Reynolds number ($Re = \rho_0 c_0 \lambda_0 / \mu_0$) is computed based on the initial perturbation wavelength λ_0 , density ρ_0 , sound speed c_0 and the viscosity μ_0 of the unshocked light gas initialized between the shock wave and the material interface. An

alternative definition of Reynolds number is one defined based on the initial growth rate of the instability, $\dot{a}_0$, i.e., $Re_{\dot{a}_0} = \rho_0 \dot{a}_0 \lambda_0 / \mu_0$. The Prandtl number ($Pr = \rho_0 \mu_0 C_p / \kappa_0$), is calculated using air viscosity μ_0 , specific heat at constant pressure C_p , thermal conductivity κ_0 and the initial unshocked light gas density ρ_0 . Both Reynolds and Prandtl number are kept constant for all the simulations at $Re = 2.17 \times 10^6$ and $Pr = 0.7$. For the specific case considered, estimating $\dot{a}_0$ by the impulse model, leads to $Re_{\dot{a}_0} = 1.17 \times 10^5$. Due to the large Re number, the viscous fluxes effects are almost negligible. In the present implementation, we have neglected binary mass diffusion for simplicity.

2.2 Numerical Scheme

We employ the recently developed embedded boundary Adaptive mesh refinement (AMR) technique for solving the compressible Navier Stokes equations [22], that utilizes a high order multi-stage reconstruction algorithm to improve the solution accuracy at the fluid-solid interface. The block-based AMR technique of Berger and Colella [23], implemented under the Chombo framework [24], improves the solution accuracy while maintaining a reasonable level of computational cost. The use of AMR is beneficial for computing these simulations in a reasonable amount of time, specifically for the present multi-scale numerical experiments and the need to adequately resolve the cylinders that mimic a wire mesh. As shown later, the ratio of the interface perturbation to the perturbation introduced by the circular cylinders is about two orders of magnitude.

2.3 Physical Domain and Simulation Setup

We first follow the experimental setup of Vandenboomgaerde et al. [25] in order to validate our numerical results with their experimental measurements. In the computational shock-tube domain, a shock wave at Mach number of $M = 1.45$ is initialized in the light gas ahead of a material interface of Atwood ratio $A = 0.668$ that separates the heavy and light gas. We utilize the air constant- γ for both light and heavy gases as it simplifies the numerical implementation. Based on a variety of numerical tests [26], we note that the difference in the results obtained by constant and variable- γ formulations is somewhat negligible; which is not surprising as the baroclinic vorticity generated by the variable- γ formulation differs only slightly from the constant- γ formulation (see Samtaney and Zabusky [2]). Moreover, as noted before [27], the variable- γ formulation may lead to nonphysical oscillations at the interface, causing a

Fig. 2 The initial setup showing the interface (red curve), the shock-wave (black line) and the solid cylinders (black circles). The perturbation is confined to within one wavelength above and below the tube center $y = 0$, as marked in the figure. Flat (unperturbed) interface segments are added to extend the interface to the upper and lower tube walls, at $|y| = [1, 1.25]$.

higher (spurious) mixing at the interface. Therefore, we conduct our numerical experiments and analyses by utilizing constant- γ (single-fluid) formulation. The RM instability is initiated by introducing a sinusoidal perturbation along the interface specified as:

$$f(y) = \begin{cases} a_0 (1 - \cos(2\pi y / \lambda_0)) & \text{for } -1 \leq y \leq 1, \\ 0 & \text{otherwise,} \end{cases} \quad (8)$$

where $a_0 = 3.06$ mm and λ_0 are, respectively the pre-shock amplitude and wavelength $\lambda_0 = 80$ mm. As illustrated in Figure 2, the perturbation is confined to within one wavelength above and below the tube center $y = 0$, with a flat interface segments extend the interface to the upper and lower tube walls; in order to exactly mimic the experimental conditions of [25].

The introduced solid circular cylinders are aligned to the interface such that the interface is tangent to their windward side. The cylinder diameter is $D = 1$ mm and the vertical spacing is $L = 10$ mm. Two cylinders at the top and bottom boundary were removed to avoid any potential boundary complications. Based on the boundary considerations, the value of the vertical spacing L and the domain height, the resulting configuration is that we have 19 solid circular cylinders uniformly distributed along the interface. Figure 2, illustrates the initial setup of the shock-wave, density interface and the supporting grid cylinders.

The Reynolds number is computed based on these parameters which are evaluated as follows: $\lambda_0 = 0.08$

m, $c_0 = 345$ m/s, $\rho_0 = 1.19$ Kg/m³ and $\mu_0 = 1.511 \times 10^{-5}$ Kg/(ms). Thus, the resulting Reynolds number is $Re = 2.17 \times 10^6$. At this Reynolds number, one expects three-dimensional effects to become important, and the flow to become turbulent. Nevertheless, consistent with previous studies (see for example [28,29]), we neglect the three-dimensional effects, leaving that for further study in the future.

The shock-tube numerical experiments were performed in a two-dimensional domain of size $[15\lambda_0 \times 2.5\lambda_0]$ discretized with a base mesh of resolution $[480 \times 80]$. Four refinement levels are introduced with the refinement ratio set $r = (2, 4, 4, 4)$ (i.e., where r is set of multiplication factors by which the mesh resolution is increased from the current to the next refinement level). Thus, the resulting mesh spacing at the finest level is $h = 1/4096 = 2.44 \times 10^{-4}$. In other words, the interface perturbation wavelength is resolved by 32 mesh points on the base mesh, and by 4096 mesh points at the finest level. The effective mesh resolution at the finest level is $[61440 \times 10240]$. The high mesh resolution is needed to resolve the perturbations that are introduced by the relatively small cylinders of a diameter of $D = \lambda_0/80$. The importance of utilizing the AMR technique is somewhat obvious, and stems from the large difference between the prescribed perturbation on the interface and that imposed by the cylinders. Such a multi-scale problem is computationally very expensive, perhaps prohibitively so, on a uniform static mesh.

Presently, we compare the dynamics of the RMI in the presence and absence of the cylinders. These two cases are denoted by **case W** (with the presence of the cylinders) and **case N** (no cylinders in the computational domain).

A reflecting free-slip boundary condition is imposed on the upper, lower and the right-side boundaries of the computational domain. Since the computational domain is large, imposing a constant inflow condition on the left boundary of the domain is acceptable as any reflected waves from left boundary do not reach the interface during the simulation time. A no-slip boundary condition is imposed on the solid cylinders surface.

Owing to the very large Reynolds number, our viscous results indicate that the mesh-dependent numerical viscosity (intrinsically present in the upwind method) is somewhat larger than the physical viscosity, and hence grid scale perturbation cannot be completely eliminated by mesh refinement in the present results. These grid scale perturbations also initiate a Kelvin-Helmholtz instability that results in fine-scales appearing atop the usual spike-bubble pattern, and leads to complex structure for the mixing layer. A diffuse interface may mitigate the growth of the grid scale secondary perturba-

Fig. 3 The spike-bubble morphology that evolves from the initial sinusoidal interface which is impulsively accelerated by a shock wave. An illustration of the mixing layer width w and the spike bubble amplitudes (a_s, a_b) are shown as well.

tions, and preserves the RMI spike-bubble structure, see for example [30], [25] and [27]. To initialize a diffuse interface we define a new parameter δ , that characterizes the interface thickness. Then, a diffusively mixed layer of 2δ thickness, is initialized according:

$$\rho = \begin{cases} \rho_1 & \text{if } x_c \leq f - \delta, \\ \rho_1 + (\rho_2 - \rho_1)S(x - f) & \text{if } f - \delta < x \leq f + \delta, \\ \rho_2 & \text{if } x > f + \delta, \end{cases} \quad (9)$$

where f is defined in equation 8, the weight parameter $S(x)$ is designed to map $x \in [-\delta, \delta]$ to $S \in [0, 1]$, according to the scaled hyperbolic tangent function:

$$S = \frac{\tanh\left(\pi \frac{x-f}{\delta}\right) + \tanh(\pi)}{2 \tanh(\pi)}. \quad (10)$$

The diffuse interface allows for smooth transition from the left to right gas species. Based on trial and error, we choose the interface thickness of $2\delta = 9$ mm that results in the standard or well-accepted interface morphology while maintaining acceptable growth predictions.

2.4 Analysis and Post-Processing

To quantify the effect of the cylinders on the RM instability dynamics, the variation of the mixing layer width (i.e., the growth of the perturbation amplitude) over time is examined. A typical initially sinusoidal perturbation profile evolves to a spike-bubble structure, (see

Figure 3). In our case, where the shock refracts at the interface from the light to heavy gas, the spike is a mushroom-like pattern formed by the penetration of the heavy gas region into the light gas side, whereas the bubble structure is formed by the growth of the light gas region into the heavy gas side. The perturbation amplitude a is defined as half of the mixing layer width w , which is the distance between the spike and bubble peaks:

$$a = \frac{w}{2} = \frac{(a_b + a_s)}{2}, \quad (11)$$

where (a_b, a_s) are, respectively the distance from the bubble and spike peaks to the unperturbed interface.

The bubble and spike peak positions are estimated by averaging the mass fraction variable ϕ , over the transverse y -direction as:

$$\Phi(x) = \frac{1}{l_y} \int_{l_y} \phi(x, y) dy, \quad (12)$$

where l_y is the computational domain height. The resulting function $\Phi(x)$ can then be used to divide the computational domain over the stream-wise direction into three subregions. Two regions of segregated pure species, in which $\Phi(x)$ is constant, either equal to one for pure light gas or zero for pure heavy gas. In the third region where the gases mix, $\Phi(x)$ varies between zero and one. The size of this region yields a reasonable estimation for the mixing layer width. To retrieve this width, we define the mixing region interval $[x_s, x_b]$, such that $\forall x_i \in [x_s, x_b], 1 > \Phi(x_i) > 0$. The interval limits (x_s, x_b) are practically evaluated to satisfy: $\Phi(x_s) = 1 - \epsilon_m$ and $\Phi(x_b) = \epsilon_m$ where ϵ_m is a small parameter $\epsilon_m \ll 1$. The (x_s, x_b) limits can, in fact, be assumed to represent the spike and bubble peaks positions.

In an AMR context, it is sufficient to evaluate $\Phi(x)$ using only the solution on the finest mesh level, which in one way, simplifies the practical implementation of averaging the mass fraction by avoiding the multi-level integration. However, it imposes an additional restriction for the mesh refinement criteria, i.e., the mixing layer must be entirely covered by the finest level. Thus, the refinement method has to include an additional criterion based on the exact value of the mass fraction. Practically, we implement an algorithm to refine the region where the mass fraction varies from ϵ_f to $1 - \epsilon_f$, where $\epsilon_f \ll 1$ and $\epsilon_f < \epsilon_m$. In the current implementation, we set $\epsilon_m = 0.01$ and $\epsilon_f = 0.005$. Note that in the evaluation of equation 12, the width of extent over which the grids at the finest level, i.e., the $l_y(x)$ is a function of the x -coordinate. Comparing the mixing width in the cylinders absence obtained by averaging over the finest level and point-wise tracking of the

spike-bubble width, shows that the effect of $l_y(x)$ variation over x -direction is negligible. However, due to the dynamic mesh regridding, averaging over the grids at the finest level leads to a somewhat noisy estimation for the growth of the mixing layer width. Thus, we implement a two-stage smoothing technique by applying a moving average and low-pass filter. We suggest the interested readers to consult section 9.3.2 in reference [26] for detailed discussion about the effect of the computing equation 12 in the finest level.

3 Numerical Results

3.1 Convergence Study and Validation

Here, we discuss the convergence study results that we obtained to test the sufficiency of the mesh resolution, and understand its effects on the computational analysis. We used a base mesh resolution of $[960 \times 80]$ that was further refined by a number of AMR levels using the following refinement ratio sets: $r_A = (2, 4, 4)$, $r_B = (4, 4, 4)$ and $r_C = (2, 4, 4, 4)$, which were denoted by (A, B, C) starting from the coarsest (grid A) to the finest (grid C). The finest mesh spacing for the three different refinement grids are ($h_A = 1/1024$, $h_B = 1/2048$, $h_C = 1/4096$). The perturbation amplitude variation over time was obtained using the three grid resolutions for both cases: N and W; and compared with the experimental measurements of [25] in Figure 4(a). To simplify the comparison, we non-dimensionalized the amplitude results based on the perturbation wavelength (a/λ_0) and the sound speed of the unshocked light gas; thus $\tau = tc_0/\lambda_0$. First, in the absence of the cylinders (case N), we observe no differences between the computational results obtained by the three different grids. Moreover, a good agreement between the present computational results and the experimental results is observed. When the cylinders effects are considered in the case W, we notice an increase in the measured perturbation growth by increasing the mesh resolution from grid A to grid B. Then, we see a slight increase again as the mesh is further refined in grid C. We believe that this difference is a result of the growth rate fluctuation due the dynamic mesh regridding, and also due to the fact that grid A results with the cylinders are well resolved. For grid A, the cylinder itself is resolved by only about 12 mesh points for the diameter. Thus, we selected grid C resolution with the refinement ratio set $r_C = (2, 4, 4, 4)$ to perform the numerical tests, as we believe it provides a sufficient mesh resolution for the perturbation growth analysis.

Though, an important observation in Figure 4(a), is that the case N numerical solutions better match the

Fig. 4 (a) The time history of the perturbation amplitude obtained using three different mesh resolutions denoted by grid A, B and C, for both case N and W. The results are compared with the experimental result of Vandenboomgaerde et al. [25]. The time history of (a) the integral mixing width Ψ , (b) the molecular mixing fraction Θ and (c) the mixing parameter Ξ over the non-dimensional time τ in the absence (case N) and presence (case W) of the cylinders, obtained using two single mesh resolutions denoted by grid α and β .

experimental results in comparison with case W. This is somewhat counter intuitive as one expects that the case W results would be in better agreement with experimental results since the effect of the wire mesh supporting the membrane is now taken into account. This seeming paradox is perhaps explained by considering that a number of factors are still neglected: such as ignoring the effects of the membrane film which would absorb some of the flow energy. Another important consideration is that both the computational and experimental results are obtained based on some threshold selection that is mostly tuned qualitatively. Thus, the obtained growth results might be reduced/increased by tuning the threshold. Finally, neglecting the flow dynamics in the third dimension, in which additional cylinders would

need to be introduced with alignment with the third direction, may also lead to a noticeable discrepancy between the experimental and computational results.

For more insight on the grid convergence, additional study was performed for three mixing diagnostics that have been used frequently in the literature, see for example [31], [32] and [17]. The measures of mixing or “mixedness” are estimated using the volume fraction that is computed based on the mass fraction and molar mass. The volume fraction of the light (left) gas η can be calculated as:

$$\eta = \frac{\phi/M_1}{\phi/M_1 + (1 - \phi)/M_2}, \quad (13)$$

where M_1 and M_2 are, respectively the molar mass of the light and heavy gases. The volume fraction of the heavy gas is then $(1 - \eta)$.

The first mixedness measure is the integral mixing width defined as:

$$\Psi = \int_{l_x} \langle \eta \rangle \langle 1 - \eta \rangle dx. \quad (14)$$

where l_x is the stream-wise length of the computational domain, $\langle \eta \rangle$ is the averaged volume fraction over the transverse direction of the domain, computed similar to equation 12.

The second and third mixing measures are the molecular mixing fraction:

$$\Theta = \frac{\int_{l_x} \langle \eta(1 - \eta) \rangle dx}{\int_{l_x} \langle \eta \rangle \langle 1 - \eta \rangle dx}, \quad (15)$$

and the mixing parameter [32]:

$$\Xi = \frac{\int_{l_x} \langle \min(\eta, 1 - \eta) \rangle dx}{\int_{l_x} \langle \min(\eta, \langle 1 - \eta \rangle) \rangle dx}. \quad (16)$$

As the Θ and Ξ increase (approaching one), more mixing is present, and they approach zero when the mixing area vanishes. We perform a simple convergence study for the mixedness using two grids with resolution $[15370 \times 1280]$, and $[30720 \times 2560]$ which, respectively, are denoted as grid α and grid β ; where grid β is equivalent to the effective resolution of the adaptive grid A. We note that the convergence for case N results, Figure 4(b-d) is acceptable. Moreover, we notice that the case W results are also well converged especially during the early duration of the simulation although the differences between the two grids α and β gradually increase during the late stages of the simulation. The main results presented later pertain to the time history of the perturbation amplitude and its dependence on the wire mesh.

3.2 Verification Against Theoretical Models

3.2.1 Growth Rate

The computational growth rate result ($\dot{a} = da/dt$) was compared with the Richtmyer model [8,1], and the linear simulation results of [33].

Richtmyer approximated the shock wave influence as an impulse forcing exerted on the interface along with an incompressibility assumption. The growth rate is estimated by the linear theory of Taylor [34], as long the amplitude is much smaller than the wavelength: $a \ll \lambda$. The Richtmyer linear model can be stated as:

$$\dot{a} = k \Delta u A^+ a_0^+, \quad (17)$$

(a)

(b)

Fig. 5 (a) Growth rate in the absence of the cylinders compared with Richtmyer linear model [8] and linear compressible simulation result of Samtaney [33]. (b) Comparison of the computed circulation with the analytical model of Samtaney and Zabusky [2].

where $k = 2\pi/\lambda$ is the wavenumber, λ is the perturbation wavelength, Δu is the change of the interface velocity due to the impulse of the shock wave, A^+ and a_0^+ are, respectively, post-shock Atwood number and amplitude.

The amplitude growth was also obtained by solving the linearized Euler equations, using a numerical Eulerian approach combined with regularization technique for smoothing the contact discontinuity and shocks [33]. From Figure 5(a) we see that the numerical growth rate increases at early time, and is close to the Richtmyer linear model. Then, it gradually decreases over the time due to nonlinear effects. A good agreement between nonlinear and linear growth rates is observed as well.

3.2.2 Circulation Deposition

The baroclinic circulation deposited on the interface by the shock was computed in the absence of the cylinders according to:

$$\Gamma = \int_{\sigma} \omega dA, \quad (18)$$

where σ is extend over the x-interval $[-\infty, \infty]$, and y-interval $[0, \lambda/2]$. Figures 5(b) displays comparison of the present circulation results with the analytical third order model of Samtaney and Zabusky [2]. An acceptable agreement is observed between our numerical results and the analytical approximation.

3.3 The Effects of the Solid Cylinders on RMI

3.3.1 The Effects on the Perturbation Growth and Structure

The effect of the cylinders on the spike-bubble structure is illustrated by visualizing the density field and the vorticity for three different time instant in Figure 6. We observe from Figure 6(a) that the cylinders initiate a Λ shape perturbation, which is similar to the structure observed experimentally by Zou et al. [18]. These perturbations grow over the time to disturb the spike-bubble structure as seen in Figure 6(b,c). We also observe the existence of vortex projectiles. The concept of vortex projectile was introduced by Zabusky et al. [35] and Zabusky and Zeng [36], loosely defined as a “coherent structure” of a rotating vortex dipoles in a different orientation. In two-dimensional domains, the vortex projectiles are characterized by the mushroom pattern. As noticed in Figure 6(e,f), a number of the vortex projectiles emerge on the interface, and are advected away from the interface with a velocity that is different from the mean flow velocity. Moreover, the noticed early-time high growth rate is related to the rapid stretching of the Λ -structure perturbation because these structures stay attached (at the tail side) on the cylinders surface for a short time while the other side is propagated by the interface speed. Their detachment from the cylinders causes the rapid, albeit slight, decrease in the amplitude shown in Figure 7(a). The attachment of the Λ -structure perturbation on the cylinder surface is a consequence of the flow viscosity; based on which a no-slip boundary condition is imposed on the cylinder surfaces. Otherwise, the viscosity has a negligible effect on the instability dynamics during the simulation time [26].

We quantified the effect of the cylinders on the mixing layer structure and the amplitude growth computed

Fig. 6 Part of the computational domain illustrating the mixing layer by visualizing the (a,b,c) density ρ and (d,e,f) vorticity ω just after the shock wave past the interface at (a,d) $t = 0.01$ ms, and at two later time instants (b,e) $t = 1.35$ ms, (c,f) $t = 2.75$ ms. The upper row is the case N results, and the lower row is case W results. Examples for the vortex projectiles are outlined in (f).

based on equation 12. By comparing between the presented results in Figure 7 obtained in the absence (case N) and presence (case W) of the cylinders, we notice that the presence of the cylinders actually increases the perturbation amplitude growth over time; and initially leads to a high growth rate for a short time period, as just discussed above.

A time history of the integral width mixing parameter for each of the cases N and W is shown in Figures 4(b-d). We note that the presence of the cylinders has

Fig. 7 The time history of the perturbation amplitude growth.

no significant effect on the computed integral width Ψ . On the other hand, we note that the resulted mixing parameters Θ and Ξ show a dramatic decrease from initial high mixing level that occurs because of initialization with a diffuse material interface. However, the presence of the cylinders leads to an increase in the measured mixedness which is an expected result of the small scales perturbations introduced by the cylinders.

3.3.2 The Effect of Varying Geometrical Parameters

Presently, we discuss the effect of the cylinder diameter and vertical spacing between the cylinders on the instability growth. We also propose a rescaling model to account for the effect of the cylinders presence on the amplitude growth. In the next section, the validity of the suggested model will be tested over different perturbation amplitude, Mach number and Atwood number.

a) The Effects of Cylinder Diameter: We first examined the effect of the cylinder diameter by observing the influence of the cylinders' diameter on the growth for three different diameter values, (D_1, D_2, D_3) : $D_1 = 1$ mm, equals to the base diameter value used previously, which is doubled with $D_2 = 2$ mm and quadrupled with $D_3 = 4$ mm.

Two sets of cases are considered: in one set we have a single mode perturbation pre-shocked initial amplitude of $a_0 = 3.06$ mm. The second set examines the effect of introducing the cylinders on an initially unperturbed flat interface, with zero initial amplitude $a_u = 0$ mm. In this case, the mixing layer occurs as a result of the perturbations solely induced by the cylinders. In Figure 8(a), we compare the effect of the cylinders with different diameters introduced on an initially perturbed

Fig. 8 (a) The perturbation growth obtained for different cylinder diameters (D_1, D_2, D_3); with initially perturbed (denoted by p) and flat interface (denoted by u); in the absence (denoted by N) and presence (denoted by W) of the cylinders. (b) The circular symbols represent the time average of the amplitude growth of initially perturbed interface in the presence of the cylinders to initially perturbed interface in the absence of the cylinders for different cylinders' diameter; the solid line is the best linear fit to equation 20.

interface and initially flat interface. For smaller cylinder diameters the mixing layer width obtained for the initially flat interface is smaller compared with the mixing width of an initially perturbed interface. However, as we increase the cylinder diameter we find that the difference between a flat and perturbed interface are reduced. In other words, increasing the cylinder diameter makes the effect of the cylinders more significant than the initial interface perturbation. Nevertheless, we find that the mixing layer width result of the smallest cylinders introduced on the initially flat interface is close to the mixing width obtained by the initially perturbed interface in the absence of the cylinders.

Fig. 9 The obtained results by different cylinder diameters (D_1, D_2, D_3) for (a) computational perturbation growth (denoted by W) and the predicted growth by the fitted model (denoted by f); (b) the relative difference between the computational and predicted perturbation growth.

b) Modeling the Effect of the Cylinder Diameter: Now, we empirically modeled the cylinders effect on the perturbation growth, by computing the time average amplitude ratio $\langle a_W/a_N \rangle$ in the presence a_W and absence a_N of the cylinders. The amplitude ratio was averaged over a non-dimensional time interval $\tau = [2, 12]$, neglecting the early transient growth pattern; and obtained for three cylinder diameters considered in the previous section. In Figure 8(b) we observe that the calculated ratio varies linearly with the cylinder diameter normalized by the perturbation wavelength, i.e., D/λ_0 . Moreover, as the cylinder diameter approaches zero, the amplitude ratio $\langle a_W/a_N \rangle$ approaches one; basically, ($a_W = a_N$) when the cylinders vanish. On the other hand, we note that, from the error bars, that the errors increase with the cylinder diameter. In fact, this is related to the higher rate growth seen at early time. As we

can see from Figure 9, the larger the cylinder diameter, the longer the time period of the very early time rapid growth. For example, if the amplitude ratio is averaged over a non-dimensional time interval $\tau = [4, 12]$, the upper limit of the error at $D/\lambda_0 = 5 \times 10^{-2}$ will be reduced by about 50%, while the averaged ratio $\langle a_W/a_N \rangle$ will only changed by less than 2%.

Figure 8 suggests that the presence of the cylinders rescales the amplitude growth by a constant factor, which depends linearly on the cylinder diameter. Thus, assuming that the perturbation growth in the absence of the cylinders a_N is known, then the growth, in the presence of the cylinders a_W , can be predicted by a linear rescaling (correction) factor, as follows:

$$\dot{a}_W = \dot{a}_N \left(\beta \frac{D}{\lambda_0} + 1 \right). \quad (19)$$

Hence, the amplitude ratio can be stated as:

$$\frac{a_W}{a_N} = \beta \frac{D}{\lambda_0} + 1, \quad (20)$$

Using the time average amplitude ratio $\langle a_W/a_N \rangle$ to perform data fitting to the proposed model, the slope for the best linear fit was evaluated as $\beta = 22.75$, Figure 8.(b).

The viability of the linear model to predict the change in the mixing layer growth is assessed in Figure 9.(a). The dotted black curve presents the computational result of the amplitude growth for case N, while the solid curves show the computational results of the amplitude growth for different cylinder diameters for case W. The dash-dotted curves are the obtained amplitude growth by the proposed linear model, equation 20. We observe good agreement for the amplitude prediction for the non-dimensional time $\tau > 2$, that is also demonstrated by the relative difference shown in Figure 9.(b), which is below 10% for the all considered diameters results obtained for $\tau > 2$.

c) The Effects of Vertical Spacing Between Cylinders:

Next, we examine the effect of vertical spacing between the cylinders by considering three different values chosen as: ($L_1 = 10, L_2 = 20, L_3 = 40$) mm, where L_1 is the base vertical spacing used in the earlier discussion (see Section 3.3.1). In Figure 10.(a) we observe that for the initially perturbed interface the amplitude growth is not significantly affected by varying the vertical spacing between the cylinder, except that for the L_2 case the amplitude growth is slightly smaller than the other cases. For the initially flat interface, increasing the vertical spacing from L_1 to L_2 leads to a reduction in the amplitude growth. However, a further increase to L_3 results in a slight increase in the amplitude growth. This

Fig. 10 The obtained perturbation growth by (a) different vertical spacing (L_1, L_2, L_3); (b) diameter/vertical spacing pairs ($(L_1, D_1), (L_2, D_2), (L_3, D_3)$).

behavior is somewhat counter-intuitive. Our attempt to understand this phenomenon starts by examining Figure 11. As can be observed, increasing the vertical spacing from L_1 to L_2 to L_3 reduces the mixing, and the number of the vortex projectiles. The complex vortex dynamics may imply a nonlinear competition between the perturbations introduced by the cylinders. However, for large cylinder spacing (Figure 11(c)), the simpler vortex results allow the Λ structure that induced by the cylinders to freely expand along the flow direction. We repeated these cases but by increasing the cylinder diameters in the same proportion as the increase in the spacing. Thus, the results were obtained for the pairs: $((L_1, D_1), (L_2, D_2), (L_3, D_3))$ where the cylinder diameters are the same as the ones used previously to study the diameter effect. Increasing the vertical spacing and the cylinder diameter simultaneously causes a rapid initial increase in the amplitude 10.(b). The rapid

Fig. 11 Part of the computational domain illustrating the mixing layer by visualizing the density field obtained in the presence of the supporting grid cylinders at initially flat interface; with different vertical spacing for the cylinders (a) L_1 , (b) L_2 , (c) L_3 .

Fig. 12 The obtained results by different initial perturbation amplitude (a_1, a_2, a_3) for (a) the perturbation growth in the absence of the cylinders (denoted by N), in the presence of the cylinders (denoted by W) and by the fitted linear model in the presence of the cylinders (denoted by f). The black dotted curve depicts the mixing growth on initially flat interface, in the presence of the cylinders. (b) Shows the relative difference between the computational growth and predicted growth by the fitted linear model.

increase is found to be connected to the cylinder diameter regardless the spacing between them, (Figures 8(a), 10.(b)). At a later stage, we observe the differences in the amplitude growth is reduced between the various cases.

3.3.3 Model Evaluation

We evaluated the accuracy of the model, equation 20, by predicting the effect on the perturbation growth for different Mach number, Atwood number and initial perturbation amplitude. We varied the initial perturbation amplitude to be three times larger or smaller than the base amplitude value (denoted here by $a_2 = a_0 = 3.06$

mm); denoting the higher initial amplitude by $a_3 = 3 \times a_2$ and the lower one by $a_1 = a_2/3$. Figure 12.(a) shows the mixing layer growth for different amplitude in the presence and absence of the cylinders, in addition to the growth prediction based on the model. The relative difference between the predicted growth and computational results is shown in Figure 12.(b). For the higher perturbation amplitude, the difference between the prediction and computational growth is around 20% in the early time stage. For later stages, this is reduced to around 10%. However, for the lower amplitude case, the difference between the model and the computational results exceeds 60% initially, before it reduces gradually to around 20% at the end of the simulation time. The large difference in the lower amplitude case can be interpreted from the comparison between the computed growth of the flat and initially lower amplitude interface results, (see Figure 8.(a)). We infer from this, that in the lowest initial amplitude case, the growth of the mixing layer width is dominated by the presence of the cylinders.

Next, the growth results were obtained for two sets of different Atwood numbers (A_1, A_2, A_3), and Mach numbers (M_1, M_2, M_3), where $A_2 = 0.668$ and $M_2 = 1.45$ are the base values of the Atwood and Mach numbers. The higher and lower values for the Atwood number are $A_3 = 0.75$ and $A_1 = 0.5$, while for the Mach number they are $M_3 = 1.85$ and $M_1 = 1.1$. Our proposed model produces good predictions for the amplitude growth at different Atwood and Mach numbers (Figures 13.(a) and 14.(a)); and as illustrated by the Figures 13.(b) and 14.(b), the relative differences are reduced to within 10% after the large initial difference due to the initial complex amplitude growth which was discussed before.

4 Conclusion

In this paper, we investigated the effects introduced by a supporting grid structure for the material interface in the experimental shock tube setup. We modeled the supporting grid wires in a two-dimensional computational domain by a set of small circular cylinders aligned along the interface such that the interface is tangent to the windward side of the cylinders.

We first validated our computational results with an experiment, and showed good agreement between our computational results and the experimental measurements. By qualitatively and quantitatively analyzing the effect of introducing the solid cylinders on the perturbation growth dynamics, we find that the small-scale perturbations introduced by the cylinders disrupts the usual spike-bubble structure on the mixing layer and

(a)

(a)

(b)

(b)

Fig. 13 The obtained results by different initial Atwood number (A_L, A_M, A_H) for (a) the perturbation growth in the absence of the cylinders (denoted by N), in the presence of the cylinders (denoted by W) and by the fitted linear model in the presence of the cylinders (denoted by f). (b) Shows the relative difference between the computational growth and predicted growth by the fitted linear model.

Fig. 14 The obtained results by different initial Mach number (a_L, a_M, a_H) for (a) the perturbation growth in the absence of the cylinders (denoted by N), in the presence of the cylinders (denoted by W) and by the fitted linear model in the presence of the cylinders (denoted by f). (b) Shows the relative difference between the computational growth and predicted growth by the fitted linear model.

lead to more chaotic patterns, increasing the mixing layer width growth and the level of the mixedness over time.

We also showed that the effect of the cylinders on the RMI is strengthened by increasing the cylinder diameter. For large cylinder diameters, there is little difference in the mixing layer growth obtained from the initially flat interface and those obtained from perturbed interface case as long as the perturbation amplitude was smaller than the cylinder diameter indicating that the mixing is driven mostly due to the shock perturbed by the cylinder. Moreover, the mixing layer width growth in the absence of the cylinders is of the same order as the growth obtained in the presence of the cylinders but with an initially flat interface. Thus, we conclude that

the effect of the cylinder increases with the ratio of their diameter to the initial interface amplitude. Moreover, we find that varying the vertical spacing between the cylinders does not show a significant effect on the mixing layer growth compared to the changes introduced by varying the diameter.

We note that introducing the cylinders rescales the mixing layer growth, by a constant factor which is directly proportional to the cylinder diameter; based on which, an empirical model is proposed. Equation 19, is a practical model to predict the effect of introducing the cylinders. The cylinder scale perturbation may not be modeled by any linear theory (e.g. Richtmyer impulse model) because the amplitude (radius of cylinder, $D/2$) to wavelength (D) ratio at the cylinder scale

is $2\pi(D/2)/D = \pi$ which does not satisfy the condition that this amplitude to wavelength ratio be $\ll 1$.

Our proposed model has limited applicability in the sense that it is useful for interface with a non-zero initial perturbation, from one side, Equation 19 requires that $a_N > 0$ which is only true when $a_0 > 0$. In fact, a_0 has to be large enough compared to D , or the model will fail in accurately predicting the change in the growth as seen in Figure 12(a). Reducing the ratio a_0/D , either by decreasing a_0 or increasing D , is nearly equivalent to results obtained by setting $a_0 = 0$ (i.e., introducing the cylinders on an initially flat interface). This basically tells that under a specific threshold for a_0/D , the initial interface perturbation become insignificant on the mixing width growth, such that the same mixing essentially arises due to the presence of cylinders rather than the perturbed density interface. As an assessment for the proposed model, it was utilized to estimate the growth variation for different Mach number, Atwood number, and initial amplitude. A good prediction for the variation on the mixing layer growth is obtained, except for the lowest initial amplitude case.

There are several potential studies left for the future: for examples, one may analyze results from three-dimensional simulations, introduce turbulence modeling, introduce the membrane film effect and/or come out with different physical or mathematical models to describe the dynamics with the same or different set of the parameters.

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