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3D Super-virtual Refraction Interferometry

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ABSTRACT

We present the theory and practice of 3D supervirtual interferometry (SVI) for enhancing the signal-to-noise ratio of refraction arrivals in 3D data. Unlike 2D SVI, 3D SVI requires an extra integration along the inline direction to compute the stationary source-receiver pairs for enhanced stacking of the refraction events. The result is a significant increase in the signal-to-noise ratio of first arrivals in the far-offset traces. Both 3D synthetic and field data examples are presented to demonstrate the effectiveness of the proposed method. For the synthetic data tests, SVI extended the source-receiver offset range of pickable traces from 11 km to 15 km. In the field data example, SVI extended the source-receiver offset of traces with pickable first-arrival traveltimes from 12 km to a maximum of 18 km, and the total number of reliable traveltimes picks was increased by 12%, which contributed to a deeper velocity update in the traveltimes tomogram.

INTRODUCTION

Traveltime tomography is a widely used tool to estimate the subsurface velocity distribution (Aki et al., 1977; Zelt and Smith, 1992; Sheriff and Geldart, 1995) by inverting the first-arrival traveltimes picked from the seismic data. Unfortunately, for many wide-offset refraction surveys, accurately picking the traveltimes of first arrivals in traces with extremely wide source-receiver offsets is almost impossible because of the low signal-to-noise ratio (SNR). To partly overcome this problem, Dong et al. (2006) developed the theory of refraction interferometry to create virtual refractions with higher SNRs and correct move-out patterns. However, the virtual refraction arrivals created by this method have unknown excitation times, so they suggested that the arrival times of visible mid-offset traces can be used to correct for the unknown traveltimes of refractions in the far-offset traces. Mallinson et al. (2011) and Bharadwaj et al. (2012) extended this method by introducing the theory of super-virtual refraction interferometry (SVI), in which the time shifts of refraction arrival times are automatically computed, and the SNRs of the far-offset first arrivals are significantly enhanced.

Figure 1 describes the conventional SVI procedures for 2D surveys. The far-field reciprocity equations of both correlation and convolution types are used to construct super-virtual refractions (Bharadwaj et al., 2012). The correlated traces associated with any post-critical source are characterized by refractions with the same traveltimes and phases as one another (Figure 1a), and the same property applies to the convolved traces in Figure b; this assumes that the refractions originate from the same refractor. After stacking these correlated/convolved traces the SNR of the virtual/super-virtual refractions is significantly increased, even at the far-offset locations.

Despite the success of SVI in improving the SNR of far-offset first-arrivals for both marine and land data (Hanafy et al., 2011; Bharadwaj et al., 2013; Vaezi and Meersman, 2014; Li et al., 2018) and its application to improve first break auto-picking (Al-Shuhail, 2015; An et al., 2015b) and residual statics estimation (Gao and Zhang, 2017), SVI applications have been until now mostly restricted to data recorded by 2D surveys. When SVI is applied to data recorded by 3D surveys, the source locations at $\mathbf{x}$ are not necessarily at the stationary positions (see Figure 2), which means that the correlated traces associated with different sources no longer share the same traveltimes and phases with one another, and summation of these correlated traces is insufficient to create a SNR-enhanced virtual trace with accurate phase information. The convolution step in SVI suffers from the same problem. To solve this problem, Lu et al. (2014) proposed to correlate the traces at two receivers and effectively sum the correlated traces for both the inline and crossline source coordinates. According to the stationary integration theory, the summed correlogram has the same phase with the one of correlating arrivals that shares the same raypath. Thus, source-receiver pair with the stationary phase property is automatically computed. The stacked result for a specified pair of receivers gives a virtual refraction with an enhanced SNR. These virtual traces are then convolved with traces containing near-offset refractions. The sum of the convolved traces for inline and crossline receiver coordinates gives the final SVI trace. Later, Edigbue and Al-Shuhail (2016) implemented a 3D SVI method where prepicked traveltimes are used to align the correlated traces with one another, and then the aligned traces are stacked. A limitation with this method is that in situations where the SNR of the data is severely low, it is difficult to accurately pick traveltimes of the noisy correlograms. An et al. (2015a) also proposed to implement SVI with higher-order cumulant and multidimensional convolution.

We now review the theory of 3D SVI initially proposed by Lu et al. (2014), and present the results of applying 3D SVI to a whole 3D marine dataset, which lead to significant increase of the number of far-offset traveltimes picks that can contribute to the improvement for the 3D tomogram. We also discuss practical tips and issues when 3D SVI is applied to industrial-scale datasets, including the end-fire zone concept, the wavelet problem, the stability of the method and its limitations. This work is organized into five sections where, following this introduction, we present the theory of 3D SVI and explain how stationary phase integration extends SVI from 2D to 3D data. The next section presents numerical examples and field data applications, where SVI significantly improves the SNR of far-offset head-wave events for 3D surveys. We then present some practical tips for implementing 3D SVI.

THEORY

Stationary Phase Integration

We apply stationary phase integration to SVI for inline¹ sources and receivers in 3D survey.

The stationary phase analysis (Bleistein, 1984) is introduced here:

$$f(\omega) = \int_{-\infty}^{\infty} g(\mathbf{x}) e^{i\omega\phi(\mathbf{x})} d\mathbf{x} \sim \alpha g(\mathbf{x}^*) e^{i\omega\phi(\mathbf{x}^*)}, \quad (1)$$

where $\phi(\mathbf{x})$ is real and a well-behaved phase function with at most one simple stationary point, ω is the asymptotic high frequency, and $g(\mathbf{x})$ is a relatively slowly varying function, $\alpha = e^{i\pi/4} \sqrt{2\pi/(\omega|\phi(\mathbf{x}^*)|)''}$ is an asymptotic coefficient, and $\mathbf{x} = \mathbf{x}^*$ is a stationary point.

¹In this paper, inline direction refers to the source shooting direction (marine survey) or receiver cable direction (land survey), and crossline direction is perpendicular to the inline direction.

Equation 1 indicates that, under the far-field approximation, the function $g(\mathbf{x}^*)e^{i\omega\phi(\mathbf{x}^*)}$ with $\mathbf{x}^*$ at the stationary position is kinematically approximated by an integration of this function over all the positions $\mathbf{x}$, if the amplitude is ignored. The stationary position $\mathbf{x}^*$ does not have to be known.

Similar to the application of stationary phase integration to interferometric redatuming (Snieder, 2004), the virtual and super-virtual refractions in 3D SVI can be calculated by integrating the cross-correlated/convolved traces along a survey line, without the knowledge of the stationary source/receiver locations. The following two subsections describe how stationary phase integration is used in the correlation and convolution steps of 3D SVI, respectively.

Correlation Step

Figure 3a shows a diagram of refractions associated with two receivers located at $\mathbf{A}$ and $\mathbf{B}$, and sources located at $\mathbf{S}_j$ ($j = 1, 2, \dots, n$) along the inline direction for a 3D survey. The pair of traces containing refraction arrivals with raypaths $\mathbf{AS}_j$ and $\mathbf{BS}_j$ ($j = 1, 2, \dots, n$) are correlated with one another to give the cross-correlogram $\Phi(\mathbf{A}, \mathbf{B}, \delta\tau)_{\mathbf{S}_j}$ shown in Figure 3b, where the virtual refractions are out of alignment with one another; here $\delta\tau$ is the traveltime difference of the two correlated traces. The trace with wavelet at the trough of the virtual refraction event in the cross-correlogram is associated with the source at the stationary position, which is marked as $\mathbf{S}^*$. At the stationary source position $\mathbf{S}^*$, the raypaths $\mathbf{AS}^*$ and $\mathbf{BS}^*$ are partly coincident as illustrated in Figure 2b. Thus, the correlated trace $\Phi(\mathbf{A}, \mathbf{B}, \delta\tau)_{\mathbf{S}^*}$ contains the virtual refraction with the same physical meaning as that defined in 2D SVI. Theoretically, if the data are less noisy, such virtual refractions can be

obtained by either picking the trace with the stationary source or stacking all the traces in the cross-correlograms after they are aligned to the trace with the stationary source. However, this approach is impractical because SVI is designed for data with considerably low SNR where the virtual refraction event in the cross-correlogram can be unrecognizable.

An alternative approach for calculating the virtual refraction is to integrate the traces in the correlogram based on stationary-phase integration theory. Only considering the phase term, we approximate the normalized head-wave traces in the frequency domain as $G^{head}(\mathbf{A}|\mathbf{S}_j) = e^{i\omega\tau_{\mathbf{S}_j\mathbf{A}}}$ and $G^{head}(\mathbf{B}|\mathbf{S}_j) = e^{i\omega\tau_{\mathbf{S}_j\mathbf{B}}}$, where $G^{head}(\mathbf{A}|\mathbf{S}_j)$ and $G^{head}(\mathbf{B}|\mathbf{S}_j)$ denote the head-wave response from source $\mathbf{S}_j$ to receivers $\mathbf{A}$ and $\mathbf{B}$, respectively. Also $\tau_{\mathbf{S}_j\mathbf{A}}$ and $\tau_{\mathbf{S}_j\mathbf{B}}$ represent the traveltimes along refraction raypaths $\mathbf{S}_j\mathbf{A}$ and $\mathbf{S}_j\mathbf{B}$. The virtual refraction associated with the stationary source $\mathbf{S}^*$ can be expressed as $G^{virt}(\mathbf{A}|\mathbf{B}) = e^{i\omega(\tau_{\mathbf{S}^*\mathbf{A}} - \tau_{\mathbf{S}^*\mathbf{B}})}$.

Substituting $g(\mathbf{x})e^{i\omega\phi(\mathbf{x})}$ in equation 1 by $G^{head}(\mathbf{A}|\mathbf{S}_j)\bar{G}^{head}(\mathbf{B}|\mathbf{S}_j)$ gives

$$\begin{aligned} \int_{\mathbf{S}_1}^{\mathbf{S}_n} G^{head}(\mathbf{A}|\mathbf{S}_j)\bar{G}^{head}(\mathbf{B}|\mathbf{S}_j)d\mathbf{S}_j &= \int_{\mathbf{S}_1}^{\mathbf{S}_n} e^{i\omega(\tau_{\mathbf{S}_j\mathbf{A}} - \tau_{\mathbf{S}_j\mathbf{B}})}d\mathbf{S}_j, \\ &\approx \alpha_1 e^{i\omega(\tau_{\mathbf{S}^*\mathbf{A}} - \tau_{\mathbf{S}^*\mathbf{B}})}, \\ &= \alpha_1 G^{virt}(\mathbf{A}|\mathbf{B}), \end{aligned} \quad (2)$$

where $\bar{G}$ represents the complex conjugate of G , and α_1 is the coefficient described in equation 1. As in the 2D example, this assumes that the refraction events propagate along the same refraction interface. This equation demonstrates that integrating the cross-correlated traces $G(\mathbf{A}|\mathbf{S}_j)\bar{G}(\mathbf{B}|\mathbf{S}_j)$ over the source line along the inline direction results in a trace with the arrival that has the same phase as that of the stationary virtual refraction.

In this way, we require no knowledge of the stationary source positions.

Figure 3c shows the comparison between the integrated trace $\int_{\mathbf{S}_1}^{\mathbf{S}_n} G^{head}(\mathbf{A}|\mathbf{S}_j)\bar{G}^{head}(\mathbf{B}|\mathbf{S}_j)dS_j$ and the stationary virtual-refraction trace $G^{virt}(\mathbf{A}|\mathbf{B})$. The left-hand-side figure compares the two traces after normalization, where the phases mostly agree with one another, though the wavelet is distorted due to the aperture cutoff in the integration and the spectrum amplitude change in equation 2. The right-hand-side of Figure 3c compares the amplitudes of the two traces. The stacked trace is much stronger than the virtual-refraction arrival, indicating that the amplitude is also increased by the stationary-phase integration, which contributes to the SNR enhancement.

So far, we have successfully created a trace with a virtual refraction arrival associated with a pair of receivers in a 3D survey using one source line along the inline direction. A similar stacking procedure as 2D SVI is then conducted (see Figure 4a). Equation 2 shows that the virtual refraction can be created regardless of which source line is selected. Therefore, all available inline source lines can be used to generate virtual refraction traces that have the same phase. Stacking these traces results in one with a highly improved SNR.

The final formula of the correlation and summation in 3D SVI is given as

$$\begin{aligned} G_{stack}^{virt}(\mathbf{A}|\mathbf{B}) &= \sum_i G^{virt}(\mathbf{A}|\mathbf{B})_i, \\ &\approx \sum_i \left[\int_{\mathbf{S}_{i1}}^{\mathbf{S}_{in}} G^{head}(\mathbf{A}|\mathbf{S}_{ij})\bar{G}^{head}(\mathbf{B}|\mathbf{S}_{ij})d\mathbf{S}_{ij} \right], \end{aligned} \quad (3)$$

where $G_{stack}^{virt}(\mathbf{A}|\mathbf{B})$ is the enhanced virtual trace associated with the receivers at $\mathbf{A}$ and $\mathbf{B}$, $G^{virt}(\mathbf{A}|\mathbf{B})_i$ is the virtual-refraction trace created using the i th inline source line, and $\mathbf{S}_{ij}$ is the j th source in the i th source line.

Convolution Step

In Figure 4b, the far-offset refraction to be reconstructed has the receiver and the source located at $\mathbf{A}$ and $\mathbf{S}$, respectively, and the receiver of the short-offset refraction has optional positions at $\mathbf{B}_j$ ($j = 1, 2, \dots, n$) along the inline direction. Let $G_{stack}^{virt}(\mathbf{A}|\mathbf{B}_j) = e^{i\omega\delta\tau_{\mathbf{AB}_j}}$ denotes the normalized virtual refraction created in the previous correlation step, where $\delta\tau_{\mathbf{AB}_j}$ is the traveltime of the virtual refraction arrival associated with the receivers located at $\mathbf{A}$ and $\mathbf{B}_j$. Applying equation 1 to traces along the receiver line with inline direction gives

$$\begin{aligned} \int_{\mathbf{B}_1}^{\mathbf{B}_n} G^{head}(\mathbf{B}_j|\mathbf{S}) G_{stack}^{virt}(\mathbf{A}|\mathbf{B}_j) d\mathbf{B} &= \int_{\mathbf{B}_1}^{\mathbf{B}_n} e^{i\omega(\tau_{\mathbf{SB}_j} + \delta\tau_{\mathbf{AB}_j})} d\mathbf{B}_j, \\ &\approx \alpha_2 e^{i\omega(\tau_{\mathbf{SB}^*} + \delta\tau_{\mathbf{AB}^*})}, \end{aligned} \quad (4)$$

where α_2 is the stationary phase integration coefficient, and $\mathbf{B}^*$ is the stationary receiver location associated with the given source $\mathbf{S}$ and receiver $\mathbf{A}$. Noticing the fact that the given source at $\mathbf{S}$ is also the stationary source associated with the pair of receivers at $\mathbf{A}$ and $\mathbf{B}^{*2}$, $\delta\tau_{\mathbf{AB}^*}$ can be expressed as $\delta\tau_{\mathbf{AB}^*} = \tau_{\mathbf{SA}} - \tau_{\mathbf{SB}^*}$. Thus, equation 4 can be recast as

$$\begin{aligned} \int_{\mathbf{B}_1}^{\mathbf{B}_n} G^{head}(\mathbf{B}_j|\mathbf{S}) G_{stack}^{virt}(\mathbf{A}, \mathbf{B}_j) d\mathbf{B}_j &\approx \alpha_2 e^{i\omega[\tau_{\mathbf{SB}^*} + (\tau_{\mathbf{SA}} - \tau_{\mathbf{SB}^*})]}, \\ &= \alpha_2 e^{i\omega\tau_{\mathbf{SA}}}, \\ &= \alpha_2 G(\mathbf{A}|\mathbf{S})^{sv}, \end{aligned} \quad (5)$$

where $G(\mathbf{A}|\mathbf{S})^{sv}$ is the reconstructed far-offset super-virtual refraction.

²We note $\mathbf{S}'$ as the stationary source associated with the pair of receivers at $\mathbf{A}$ and $\mathbf{B}^*$, then the two refraction raypaths $\mathbf{S}'\mathbf{A}$ and $\mathbf{S}'\mathbf{B}^*$ are partly overlapping. On the other hand, receiver $\mathbf{B}^*$ is stationary, which means the two refraction raypaths $\mathbf{SA}$ and $\mathbf{SB}^*$ partly coincide as well. Assuming there is only one stationary position on the source line, the source location $\mathbf{S}$ should be at the same position with $\mathbf{S}'$.

Equation 5 shows that the 3D far-offset refraction is kinematically reconstructed by summing the convolutions between the refraction arrival $G^{head}(\mathbf{B}_j|\mathbf{S})$ and the pre-calculated virtual refraction data $G_{stack}^{virt}(\mathbf{A}|\mathbf{B}_j)$. Equation 5 is satisfied regardless of which receiver line is selected for integration, so we are able to produce super-virtual refractions $G(\mathbf{A}|\mathbf{B})^{sv}$ with different inline receiver lines and stack these super-virtual traces for one with a much higher SNR. The stacking formula is given as

$$G_{stack}^{sv}(\mathbf{A}|\mathbf{S}) = \sum_i G^{sv}(\mathbf{A}|\mathbf{S})_i, \\ \approx \sum_i \left[\int_{\mathbf{B}_{i1}}^{\mathbf{B}_{in}} G^{head}(\mathbf{B}_{ij}|\mathbf{S}) G_{stack}^{virt}(\mathbf{A}, \mathbf{B}_{ij}) d\mathbf{B}_{ij} \right], \quad (6)$$

where $G_{stack}^{sv}(\mathbf{A}|\mathbf{S})$ is the enhanced super-virtual refraction, i indicates the i th inline receiver line, and $\mathbf{B}_{ij}$ is the j th receiver in the i th receiver line.

Iterative SVI

For data with strong noise, the SNR of the enhanced SVI data might be still insufficient for accurate traveltimes picking. A solution to this problem is the iterative SVI (Al-Hagan et al., 2014), which reuses the SVI-enhanced traces as the input for one more SVI iteration. It is also applicable for 3D SVI. However, using too many iterations is not recommended because each SVI iteration amplifies artifacts and wavelet distortions. This phenomenon is explained in the discussion part. The rule of thumb is to use no more than about two iterations, otherwise it might deteriorate the data.

Workflow

The workflow for 3D super-virtual refraction interferometry is depicted in Figure 5 and summarized as follows:

1. Properly window the data around the target refraction³.
2. Generate a stationary virtual refraction trace by integrating the cross-correlated traces over a inline source line with equation 2.
3. Stack the virtual refraction traces associated with different inline source lines as shown in Figure 4a with equation 3.
4. Generate a stationary super-virtual refraction trace by integrating the convolved traces over a inline receiver line with equation 5.
5. Stack the super-virtual traces associated with different inline receiver lines as shown in Figure 4b with equation 6.
6. Use the output of step 5 as the input of step 2 for another iteration (optional if the SNR of the SVI data is not satisfactory).

NUMERICAL RESULTS

Synthetic Data Example

3D SVI is tested on synthetic data to demonstrate its effectiveness in significantly enhancing the SNR of noisy refractions. Figure 6 presents a 3D undulating two-layer model with

³The SVI theory assumes that we only manipulate the refractions refracted from the same interface. Applying SVI to the targeted refraction arrivals with other coherent events produces strong cross-talk artifacts. Therefore, it is necessary to temporally window the data around the targeted refractions at the beginning of SVI procedures.

homogeneous velocities of 1500 m/s and 3000 m/s for each layer, respectively. The survey has 11 lines on the surface with spacing of 100 m. There are 76 shots and 301 receivers distributed on each line, with the receiver spacing of 10 m and shot spacing of 40 m.

A common shot gather is displayed in Figure 7a, with the source located at the left end of Line 1, and the receivers at Line N . The first arrivals of the far-offset traces are impossible to pick after strong random noises are added to the synthetic data as shown in Figure 7b. 3D SVI is applied to the noisy data, and the result is displayed in Figure 7c, showing that the SNR is significantly enhanced and the refraction moveout pattern still agrees with that in the clean synthetic data. Iterative SVI with one more iteration is carried out so that the traveltimes of the first arrivals can be easily picked over the full-offset range in Figure 7d. Figure 8 presents the zoom view of the marked zone by the dashed red box in Figure 7, where wavelet distortions can be observed. This phenomenon is caused by the spectral sharpening in the correlation and convolution operations. The solution to this problem is discussed in the Discussion section.

Field Data Example

We apply 3D SVI to a 3D marine field survey in the Gulf of Mexico, the geometry of which is displayed in Figure 9 (Lu et al., 2014). The black dots represent OBS stations installed on the ocean bottom at the depth of 45 m. There are a total of 9 OBS lines, with 26 stations in each line with a spacing of approximately 400 m. The green lines represent 153 sail lines, with 360 shots excited along each sail line with a line interval of 50 m.

A common receiver gather associated with one sail line is extracted as an example. The location of the OBS station is at the red star and the sources are along the red line in

Figure 9. Figure 10a presents the raw data, and the red lines mark two targeted head-wave events at the far offsets. To conveniently pick the first arrivals, a 5-15 Hz band-pass filter is applied to the traces to suppress the high-frequency noise as shown in Figure 10b. To avoid artifacts produced by the interference between refraction events from different interfaces, we separately process these two refractions with SVI. The SVI result in Figure 10c has significantly improved SNR compared to the raw data. In the zoom-view comparison in Figure 11, the traces that are difficult to be picked in the raw data at the offsets of 12 km to 14 km and 16 km to 18 km, are now clearly pickable in the SVI data.

Figure 12 illustrates another example with weak far-offset first arrivals. The event pointed by the red arrow is the targeted refraction arrival. After SVI, the pickable offset range is extended from 12 km to 15 km, while the widest-offset first arrivals are still weak. Iterative SVI with one more iteration is applied to this data to further improve the SNR as shown in Figure 12, extending the pickable offset to as far as 18 km. A zoom-view comparison is shown in Figure 13.

3D SVI is now applied to the whole dataset, and the increased number of traveltime picks associated with each OBS station and sail line is shown in Figure 14. The data quality is acceptable for sources in the north, where SVI is not needed. The noisiest traces are for the data with sources in the south. After SVI processing the total number of usable traveltimes picks increased by 12% compared to the raw data.

3D traveltime tomograms are then inverted for the raw and SVI traveltimes (see Figure 15). With the contribution of the extra far-offset first-arrival picks, the SVI tomogram shows the reconstruction of velocities in the deeper part of the model. There is also extra velocity update at the south edge of the model in the SVI tomogram, which is consistent

with the fact that considerable improvement is made by SVI in the south as shown in Figure 14a.

In order to validate that the SVI tomogram benefits from the SVI far-offset pickings, rather than from other numerical effects, we conduct a checkerboard test using the same geometry as a sanity check. The background velocity in the checkerboard test is linearly increased along the vertical direction with the minimum velocity of 1.5 km/s and the maximum velocity of 4.5 km/s. We imposed $\pm 10\%$ velocity perturbations with dimensions of 1 km^3 onto the background velocity and the final model is shown in Figure 16a. Using the background velocity as the initial model, the tomogram is inverted and shown in Figure 16b. The velocity perturbations are well recovered to the depth of more than 3 km, which is the depth where the SVI tomogram shows significantly better resolution compared to the raw tomogram seen in Figure 15c.

DISCUSSIONS

In this section, we discuss practical tips for implementing 3D SVI, including the concept of end-fire zone, wavelet distortion problem of SVI and the stability and limitations of SVI.

End-fire Zone

For a large scale dataset, the number of the cross-correlations to be calculated in equations 3 and 6 can be huge with a significant computational cost. On the other hand, picking the first-arrival time windows for all the traces involved in the cross-correlations is also labour-intensive and time-consuming. To mitigate this problem, we introduce a concept called the end-fire zone, which is referred to by Campillo and Roux (2015) as the end-fire lobe

in surface-wave interferometry. It is defined as the area where the dominant contributions to the stationary phase integration come from. The phase of the integral does not change to first order with the sources located inside this zone. Limiting the integration range for sources only in the end-fire zone significantly reduces the cost of 3D SVI.

In our case, for the fixed receivers located at $\mathbf{A}$ and $\mathbf{B}$, the end-fire zone in the cross-correlation step for the source location $\mathbf{S}$ is defined as

$$|\tau_{xcor}(\mathbf{S}, \mathbf{A}, \mathbf{B}) - \tau_{xcor}(\mathbf{S}^*, \mathbf{A}, \mathbf{B})| \leq T/2 \quad (7)$$

where $\tau_{xcor}(\mathbf{S}, \mathbf{A}, \mathbf{B})$ is the traveltime of the virtual refraction arrival associated with the receiver pair at $\mathbf{A}$ and $\mathbf{B}$, and $\mathbf{S}^*$ denotes the stationary source.

In the same way, for a fixed source located at $\mathbf{S}$ and a fixed receiver located at $\mathbf{A}$, the end-fire zone in the convolution step for the receiver location $\mathbf{B}$ is defined as

$$|\tau_{conv}(\mathbf{S}, \mathbf{A}, \mathbf{B}) - \tau_{conv}(\mathbf{S}, \mathbf{A}, \mathbf{B}^*)| \leq T/2 \quad (8)$$

where $\tau_{conv}(\mathbf{S}, \mathbf{A}, \mathbf{B})$ is the traveltime of the super-virtual refraction arrival associated with the source at $\mathbf{S}$ and the receiver at $\mathbf{A}$, and $\mathbf{B}^*$ denotes the stationary receiver position. Figure 17 represents an example of the Fresnel zones for both correlation and convolution steps with a simple two-layer model. In practice, we define the end-fire zone and only sum the correlograms associated with the end-fire sources.

Wavelet Distortion

Wavelet distortion is observed after the raw data are transformed to SVI data for both the synthetic and field data examples; in these cases, the SVI wavelet gets distorted because the correlation and convolution operations sharpens the spectrum as shown in Figure 18a. A possible solution to this problem is to correct the spectrum in the frequency domain, where the cross-correlation operation $G(\mathbf{A}|\mathbf{S})\bar{G}(\mathbf{B}|\mathbf{S})$ can be replaced by cross-coherence $\frac{G(\mathbf{A}|\mathbf{S})\bar{G}(\mathbf{B}|\mathbf{S})}{|G(\mathbf{A}|\mathbf{S})||G(\mathbf{B}|\mathbf{S})|}$ (Aki and Richards, 2002; Nakata et al., 2011), so that the result is theoretically a time delayed impulse with no wavelet distortion after the convolution step. Figure 18b is an example comparing the uncorrected traces next to the traces corrected by the cross-coherence operation.

In first-break picking, where the traveltimes is the only concern, the wavelet distortion brings errors to the picks of the SVI traces. Instead of correcting the wavelet by the cross-coherence operation, an alternative solution is to estimate this error between the picks of a raw trace and a SVI trace, and add a shift to the SVI picks as a correction; this assumes that the wavelet distortion causes a constant time shift for neighbouring traces. The benefit is that the cross-correlation operation is more robust than cross-coherence.

Stability and Limitations

The key idea of SVI is similar to the robust and widely adopted reflection processing method of obtaining SNR-enhanced zero-offset reflections by stacking reflection traces in CMP gathers after normal moveout (NMO) correction (Sheriff and Geldart, 1995; Yilmaz, 2001). In SVI, instead of flattening the event by NMO, refractions are manipulated by cross-correlation and convolution operations so that they have the same phase and arrival

times, and then are stacked for better SNR. Therefore, like the reflection stacking method, SVI shows its stability in many cases for both marine and land data (Hanafy et al., 2011; Bharadwaj et al., 2013; Lu et al., 2014; Vaezi and Meersman, 2014; Li et al., 2018). On the other hand, as a data driven method, which requires no subsurface structure information, SVI can be applied at the very beginning stages of data processing for improving the initial velocity model building.

The main limitation of SVI is that the reconstructed data can be inaccurate if the first arrivals are diving waves. In this case, the raypaths from a source to two neighboring receivers do not overlap one another (see Figure 19). This means that stacking correlated traces from different sources will often not lead to coherent summation. However, this inaccuracy can be controlled within an acceptable range by limiting the integration aperture and will be discussed in a separate paper. Another challenge for SVI is to adapt to acquisition geometries with sparse source and receiver distributions. In such cases, the improvement in SNR can be limited due to the insufficient number of sources and receivers available for stacking. There is also a risk that the stationary phase integration will fail. It may happen when the number of the sources/receivers are very few in the end-fire zones (Figure 17), or their distributions are heavily uneven, which do not happen in normal surveys.

CONCLUSIONS

This paper studies the extension of super-virtual interferometry to apply it to 3D seismic data. Unlike in 2D surveys, some of the source/receiver positions in the cross-correlation and convolution steps can be non-stationary in 3D surveys. A stationary virtual/super-virtual refraction trace is created by integrating the cross-correlated/convolved traces along the inline direction. Then these stationary virtual/super-virtual refraction traces associated

with different survey lines are stacked to enhance the SNR of the data. Iterative SVI, which takes the SVI data as the input for another SVI run, can achieve better SNR improvement when the raw data are very noisy. The synthetic and field data examples demonstrate that 3D SVI effectively improves the SNRs of the far-offset head waves. The SVI data can provide many more reliable traveltime picks at the far offsets that can be used to invert for the deeper portions of the velocity model.

SVI is now a practical method that can be used to invert 3D refraction data recorded by exploration surveys or earthquake networks. A practical approach to reduce the cost of SVI for large-scale data is to limit the integration areas to the end-fire zones. The wavelet distortion problem can be relieved by either applying cross-coherence or simply correcting the shifts in the traveltime picking. The robustness of the SVI method comes from two aspects: (1) it is data driven where no subsurface information is required; (2) the key idea is simple with few assumptions. The disadvantage of this method is that it can fail if the first arrivals are diving waves associated with strong velocity gradients rather than head waves. However, this issue can be partly corrected by limiting the integration aperture, and it is not much of a problem for the many data sets we have processed with SVI. Apparently, the vertical velocity gradients for the basins we have tested are mild enough for SVI to work well.

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LIST OF FIGURES

Figure 1: The steps for creating 2D super-virtual refraction arrivals (Bharadwaj et al., 2012). a). Correlation of the recorded trace at $\mathbf{A}$ with that at $\mathbf{B}$ for a source at $\mathbf{x}$ to give the trace $\phi_{\mathbf{x}}(\mathbf{A}, \mathbf{B}, t)$ with the virtual refraction having traveltime denoted by $\tau_{\mathbf{A}'\mathbf{B}} - \tau_{\mathbf{A}'\mathbf{A}}$. This arrival time will be the same for all post-critical source positions, so stacking $\sum_x \phi_x(\mathbf{A}, \mathbf{B}, t)$ will enhance the SNR of the virtual refraction by $\sqrt{N}$. b). Similar to that in a) except the virtual refraction traces are convolved with the actual refraction traces and stacked for different geophone positions to give the c) Super-virtual trace with a SNR enhanced by $\sqrt{N}$. Here, N denotes the number of coincident source and receiver positions that are at post-critical offset.

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Figure 1

216x167mm (300 x 300 DPI)

Figure 2

240x153mm (300 x 300 DPI)

Figure 3

255x182mm (300 x 300 DPI)

Figure 4

281x190mm (300 x 300 DPI)

Figure 5

234x176mm (300 x 300 DPI)

Figure 6

229x62mm (300 x 300 DPI)

Figure 7

212x198mm (300 x 300 DPI)

Figure 8

210x201mm (300 x 300 DPI)

Map view of the survey geometry

figure 9

176x141mm (300 x 300 DPI)

Figure 10

284x106mm (300 x 300 DPI)

Figure 11

258x137mm (300 x 300 DPI)

Figure 12

275x102mm (300 x 300 DPI)

Figure 13

272x102mm (300 x 300 DPI)

Figure 14

283x142mm (300 x 300 DPI)

Figure 15

144x307mm (300 x 300 DPI)

Figure 16

154x156mm (300 x 300 DPI)

Figure 17

268x120mm (300 x 300 DPI)

Figure 18

301x110mm (300 x 300 DPI)

Figure 19

202x179mm (300 x 300 DPI)

DATA AND MATERIALS AVAILABILITY

Data associated with this research are confidential and cannot be released.