Accuracy Limits of Embedded Smart Device Accelerometer Sensors

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Abstract—Smartphones are an indispensable tool in modern day-to-day life. Their widespread use has spawned numerous applications targeting diverse domains such as bio-medical, environment sensing and infrastructure monitoring. In such applications, the accuracy of the sensors at the core of the system is still questionable, since these devices are not originally designed for high accuracy sensing purposes. In this work, we investigate the accuracy limits of one of the commonly used sensors, namely, smartphone accelerometer. We focus on the efficacy of a smartphone as an acceleration measuring device, rather than focusing only on the accuracy of its internal accelerometer chip. This holistic approach includes additional errors that arise from the device operating system such as sampling time uncertainty. Hence, we propose a novel smart device accelerometer error model that includes the traditional additive noise as well as sampling time uncertainty errors represented by a white Gaussian process. The model is validated experimentally using shake table experiments, and maximum likelihood estimation (MLE) is used to estimate the sampling time uncertainty standard deviation. Moreover, we derive the Cramer-Rao lower bound (CRLB) of acceleration estimation based on the proposed model.

Keywords— smartphone; accelerometer; error model; estimation; MLE.

I. INTRODUCTION

With the advent of the smart devices era, numerous applications (*apps*) have emerged that monitor a wide range of phenomena, ranging from human activities and vitals to environment sensing and infrastructure monitoring. The driving factor behind these innovative apps is the availability of sensors such as inertial (accelerometers and gyroscopes), pressure, temperature, humidity, light and proximity sensors.

One of the commonly used smartphone sensors is the inertial sensing module which contains tri-axial accelerometers and gyroscopes. Accelerometers in general are capable of measuring the force accompanying motion and according to Newton's second law of mechanics that force is proportional to acceleration. Several apps utilize smartphone tri-axial accelerometers in bio-medical applications to detect or monitor diseases. In [1], a smartphone-based app that is used to assess upper limb tremor in Parkinson's disease (PD) patients has been developed. That app assists physicians remotely to evaluate the patient's condition. Depending on a smartphone in such

situations outperforms any other biomedical device in terms of cost, since no special hardware is needed. In [2], PD patients' actions have been identified by analyzing their speech and movement patterns. Smartphones in their pocket at different times of the day capture movement through the accelerometer chip. Several machine learning classification algorithms have been applied to classify patients as being Parkinson positive or negative. In [3], atrial fibrillation (AFib) detection has been proposed. A smartphone is placed on the chest of the patient and performs a noninvasive recording. No external sensors are needed yielding a very low cost approach. Using this method, it is possible to detect paroxysmal (or "silent") AFib. In [4], regular activities are detected and recorded as a main indicator of health. Data is captured by smartphones inertial sensors, then an online classifier differentiates between five common activities: cycling, running, sitting, standing, and walking. Monitoring these activities can effectively prevent and manage chronic diseases. Moreover, in [5], [6] an approach for the monitoring of elders and people with neurological pathologies (e.g., Alzheimer) has been proposed. Multi-sensor approaches are used to recognize critical events, such as falls or prolonged inactivity, that can be used to alert caregivers. Algorithms have been developed and applied on data provided by inertial sensors embedded in the user device. Other than alerts, the information provided by the proposed system can be used to track the evolution of diseases, and to track rehabilitation progress. In [7], smart watch accelerometers have been used to capture workout data which can assist users and physicians in monitoring health status and rehabilitation tasks. The proposed framework MiLift, automatically performs exercise identification, in addition to start/stop detection to remove user burdens of manual data entry.

On the other hand, smartphone apps also target environment and public safety monitoring. For instance, a crowdsensing framework for monitoring bridge vibrations using moving smartphones was presented in [8]. It was shown that acceleration data collected using smartphones in moving vehicles contain information about damage indicators such as the first three modal frequencies of the bridge under test. In addition, in [9], [10], [11], [12] structural health monitoring using a network of smartphones was proposed. A mobile app was developed that utilizes the phones internal accelerometers to sense earthquake vibration and assess building damage status. An early earthquake warning system (MyShake) was developed using only mobile sensors in [13]. MyShake successfully provided a 20 seconds warning for a magnitude 5 (M5) earthquake. Furthermore, community seismic network (CSN), another earthquake detection system used phone accelerometers to capture the s-wave in addition to the much harder to

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detect p-wave [14]. In [15], a crowd sensing application to estimate road conditions (CRATER) was presented. The app uses accelerometer data to detect speed bumps and potholes, and reports the locations to a cloud-based application.

In addition, smartphones have been used in intelligent transportation applications. For instance, in [16] movement detection is achieved based on a mobile device accelerometer found in a vehicle. Accelerometer is also used to detect motion events and trigger other sensors to capture data. Hence, a driving risk score is calculated based on the sensors data. Feedback notification is generated when the calculated risk score is above a predetermined threshold. Similarly, in [17] smart device accelerometer is used to estimate, acceleration, orientation, shock, vibration, velocity, speed, and position of vehicles. The accelerometer is part of telematics data acquisition device, which is configured to collect telematics data of a vehicle as an indication of vehicle safety. Then, a higher layer algorithm calculates insurance premium quote based on at least one of the received health telematics data.

Smart device sensors are designed for a consumer electronics market, not as a specialized sensing system, thus it is natural to expect that the specifications are lower than standalone sensors. In this work, we study the accuracy limits of a smartphone accelerometer, and the same approach is applicable to other smart devices such as wearables and tablets. Inertial sensors error is modeled in previous work [18], [19] as: 1) Scale factor error, 2) constant bias, 3) non-linearity and 4) additive random noise. While the first three error types are deterministic and can be compensated by calibration, the random noise cannot be calibrated and can only be characterized as mentioned in [20], [21]. However, using the above traditional error model for a smart device accelerometer (or other sensors as well) does not capture the operating system (OS) timing uncertainty. In [22], [23], it was shown that Android has some limitations to be considered as a real time OS, and consequently when a mobile app polls the internal accelerometer chip every constant time period, the execution is expected to have some uncertainty that strongly depends on the coding style. Hence, the main contributions of this work can be summarized as follows:

- 1) Proposing a novel and experimentally validated error model for smart device accelerometer that includes sampling time uncertainty.
- 2) Proposing a maximum likelihood estimation (MLE) methodology to estimate the model parameters.
- 3) Showing that linear estimation accuracy is close to the best achievable estimation accuracy determined by the Cramer-Rao lower bound (CRLB).

The rest of the paper is organized as follows. In section II, the proposed error model is illustrated. Section III describes the experimental procedure used to validate the proposed model and shows the results. The CRLB of the proposed model is derived in section IV. Finally, the conclusions are drawn in section V.

II. SMART DEVICE ACCELEROMETER PROPOSED NOISE MODEL

In this section smart device accelerometer error is modeled. We propose an error model that takes into account sampling time uncertainty as well as additive noise. Since, the additive noise parameters can be well estimated using power spectral density or Allan variance techniques illustrated in [20], [21], in this work we focus on estimating the sampling time uncertainty standard deviation (STD). In other words, we assume additive noise parameters are known and given. MLE is adopted to estimate the sampling time uncertainty STD.

A. Notation

Throughout the paper, vectors are denoted by boldface lowercase letters, e.g. $\mathbf{x}$, and matrices by boldface uppercase letters, e.g. $\mathbf{A}$. We use $(\cdot)^T$ for transpose of a vector or a matrix. Vector elements are denoted by lowercase normal font letters with subscript that indicates the element index, e.g. x_i represents the i th element in vector $\mathbf{x}$. In addition, the i th column of the matrix $\mathbf{A}$ is given by $\mathbf{A}[:, i]$. We also use $\mathbb{E}[\cdot]$ as an expectation operator. The probability density function (PDF) of single variable y_i and a vector $\mathbf{y}$ are denoted as $p(y_i; \mathbf{x})$ and $p(\mathbf{y}; \mathbf{x})$ respectively, where $\mathbf{x}$ is a nonrandom parameter vector. We use $\odot$ to denote the Hadamard multiplication operator. Identity matrix of K elements is denoted by I_K . Finally, $\text{diag}(\mathbf{x})$ is a diagonal matrix whose diagonal entries correspond to the elements in the vector $\mathbf{x}$:

$$\text{diag}(\mathbf{x})[i, j] = \begin{cases} x_i & i = j \\ 0 & i \neq j \end{cases}.$$

B. Model

Most prior work modeled the non-deterministic inertial sensor error as additive noise and characterised it using either power spectral density or Allan variance techniques as mentioned in [20], [21]. Sampling time uncertainty was neglected due to the fact that using a stable clock and proper sampling, results in negligible sampling time jitter error. While this is true in a general case, it is not true in smartphone systems, since reading acceleration data using a smart device is done through the OS. As mentioned in section I, smart device OS such as Android has limitations to act as a real time OS. Hence, it is expected to have sampling time uncertainty while polling the sensors. Therefore, we propose an error model for smart device accelerometer that includes additive noise as well as sampling time jitter error.

As mentioned in [24], [25], let the measured acceleration at any sampling instant i be calculated by (1).

$$a_i = \alpha x(iT + z_i) + w_i \quad (1)$$

where $x(t)$, T , z_i and w_i are the true acceleration, sampling period, sampling time jitter and additive noise at time sample i respectively, and α is the scale factor that is ideally equal to one. Without loss of generality, we use normalized sampling frequency, i.e. let $T = 1$. Error due to sampling time jitter was introduced in [24] by using the first order terms of Taylor series expansion of (1) and ignoring higher order terms.

However, we found that this approach leads to erroneous values at low values of $|\dot{x}|$, as will be illustrated later in this paper.

Taylor expansion of (1) is given by (2), where $\frac{d^n}{dt^n}(\cdot)$ is the n th derivative operator. Both noise sources z and w are modeled as zero mean Gaussian noise with variance σ_z^2 and σ_w^2 respectively [24]. Modeling timing error in Android has been studied in several previous work such as [23], and it was shown that there are inaccurate timestamps in the Android environment, that negatively affected program correctness and reliability. In addition, quantitative analysis of jitter performance on Android mobile devices was shown. However, previous work in this field focused on assessment of uncertainty from a quantitative perspective rather than probabilistic modeling. On the other hand, sampling time uncertainty has been modeled as white Gaussian process in other domains such as analog to digital converters (ADCs) as illustrated in [24], [25]. Thus, for mathematical tractability we adopt white Gaussian model for sampling time uncertainty in smartphones.

$$a_i = \alpha x_i + w_i + \alpha \sum_{n=1}^{\infty} \frac{1}{n!} \frac{d^n x(t)}{dt^n} \Big|_{t=i} z_i^n \quad (2)$$

For simplicity, let $\frac{dx}{dt}$, and $\frac{d^2x}{dt^2}$ be denoted as $\dot{x}$ and $\ddot{x}$ respectively. Based on the negligible terms of (2) we investigate the following two models:

- 1) **M1** First order: neglect $n > 1$ terms in (2).
- 2) **M2** Second order: neglect $n > 2$ terms in (2).

M1: In this model, higher order terms ($n > 1$) of (2) are neglected, hence measured acceleration is expressed by (3), and the vector form is given by (4), where $\mathbf{Z} = \text{diag}(\mathbf{z})$. The joint PDF of $\mathbf{a}$ is given by (5), where σ_i^2 is the error variance at sample i expressed by (6). The log likelihood (LL) function is given by (7).

$$a_i = \alpha x_i + w_i + \alpha \dot{x}_i z_i \quad (3)$$

$$\mathbf{a} = \alpha \mathbf{x} + \mathbf{w} + \alpha \mathbf{Z} \dot{\mathbf{x}} \quad (4)$$

$$p(\mathbf{a}; \mathbf{x}, \dot{\mathbf{x}}, \sigma_w, \sigma_z) \Big|_{M1} = \prod_{i=0}^{K-1} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(a_i - \alpha x_i)^2}{2\sigma_i^2}\right) \quad (5)$$

$$\sigma_i^2 = \alpha^2 \dot{x}_i^2 \sigma_z^2 + \sigma_w^2 \quad (6)$$

$$LL(\mathbf{a}; \mathbf{x}, \dot{\mathbf{x}}, \sigma_w, \sigma_z) \Big|_{M1} = \log(p(\mathbf{a}; \mathbf{x}, \dot{\mathbf{x}}, \sigma_w, \sigma_z)) = -\frac{1}{2} \sum_{i=0}^{K-1} \left(\log(2\pi\sigma_i^2) + \frac{(a_i - \alpha x_i)^2}{\sigma_i^2} \right) \quad (7)$$

M2: In this model, higher order terms ($n > 2$) of (2) are neglected, hence measured acceleration is expressed by (8), and the vector form is given by (9). The joint PDF of $\mathbf{a}$ is given by (10). $p(a_i; x_i, \dot{x}_i, \ddot{x}_i, \sigma_w, \sigma_z)$ is calculated by (11) where $*$ is the convolution operator, $\mathcal{N}(0, \sigma_w^2)$ is a zero mean normal

distribution with variance σ_w^2 and y_i is a random variable defined by (12). Since the PDF of z is $\mathcal{N}(0, \sigma_z^2)$ and y is a function of z then $p(y_i; \dot{x}_i, \ddot{x}_i, \sigma_z)$ is calculated numerically as described in [26]. The LL function of **M2** is given by (13).

$$a_i = \alpha x_i + w_i + \alpha \dot{x}_i z + \alpha \frac{\ddot{x}_i}{2} z_i^2 \quad (8)$$

$$\mathbf{a} = \alpha \mathbf{x} + \mathbf{w} + \alpha \mathbf{Z} \dot{\mathbf{x}} + \frac{1}{2} \alpha \mathbf{Z} \mathbf{Z} \ddot{\mathbf{x}} \quad (9)$$

$$p(\mathbf{a}; \mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}, \sigma_w, \sigma_z) = \prod_{i=0}^{K-1} p(a_i; x_i, \dot{x}_i, \ddot{x}_i, \sigma_w, \sigma_z) \quad (10)$$

$$p(a_i; x_i, \dot{x}_i, \ddot{x}_i, \sigma_w, \sigma_z) = \mathcal{N}(0, \sigma_w^2) * p(y_i; \dot{x}_i, \ddot{x}_i, \sigma_z) \quad (11)$$

$$y_i = \alpha \dot{x}_i z_i + \alpha \frac{\ddot{x}_i}{2} z_i^2 \quad (12)$$

$$LL(\mathbf{a}; \mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}, \sigma_w, \sigma_z) \Big|_{M2} = \sum_{i=0}^{K-1} \log(p(a_i; x_i, \dot{x}_i, \ddot{x}_i, \sigma_w, \sigma_z)) \quad (13)$$

As mentioned in [24], MLE is applied to estimate the model parameters, i.e. σ_w and σ_z . However, since σ_w can be estimated while the device is at rest as described in [20], [21], hence, we assume σ_w is known.

For **M1** and **M2** models, σ_z is estimated using (14). Without assuming negligible additive noise, i.e. $\sigma_w \neq 0$, equation (14) has no closed form solution, hence we used a numerical approach as explained in section II-C.

$$\hat{\sigma}_z = \underset{\sigma_z}{\operatorname{argmax}} LL(\mathbf{a}; \mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}, \sigma_w, \sigma_z) \quad (14)$$

C. Simulation Results

In this section, we evaluate the efficacy of the models proposed in section II-B. A sinusoidal signal is used as the ground truth acceleration $x(t)$ whose normalized frequency is 0.04 and amplitude 10^4 mm/s^2 . Let the sampling clock have jitter of Gaussian distribution with zero mean and $\sigma_z = 0.2$. For smartphone accelerometers, according to [20], [27], the additive noise typically has $\sigma_w = 10 \text{ mm/s}^2$.

The simulation is repeated 1000 times to calculate the estimation accuracy of the proposed models. At $\alpha = 1$, the exact error, **M1** and **M2** errors are calculated by (15-17) respectively. Figure 1 compares between the root mean squared (RMS) of the exact error, **M1** and **M2** RMS errors. It is clear that:

- 1) The RMS error is greater than the additive noise level represented by σ_w .
- 2) Error is maximum at the points of highest $|\dot{x}|$ values, i.e. the points that have the fastest transitions.
- 3) **M1** and **M2** match the exact error RMS at high values of $|\dot{x}|$, however at low values, **M1** underestimates the error.

Fig. 1: Exact, **M1** and **M2** RMS (left axis). True first derivative of acceleration signal $\dot{x}$ (right axis).

Fig. 2: **M1** and **M2** estimation bias due to scale factor error.

$$e_i = a_i - x_i \quad (15)$$

$$e_i \Big|_{M1} = w_i + \dot{x}_i z_i \quad (16)$$

$$e_i \Big|_{M2} = w_i + \dot{x}_i z_i + \frac{1}{2} \ddot{x}_i z_i^2 \quad (17)$$

In order to estimate σ_z using MLE, equation (14) is solved numerically by sweeping on a range of values to get the σ_z value that maximizes the LL function. That is repeated for a range of α values to consider the scale factor error that exists even at nominal temperature, and is reported by the sensor vendor. Figure 2 shows σ_z estimation for **M1** and **M2** in case a slight scale factor error is introduced. It is clear that **M2** is less sensitive to scale factor errors than **M1**. This is due to the fact that **M2** is much more accurate at the low $|\dot{x}|$ values as mentioned earlier, and these points are the maximum values of x , i.e. the points that have the maximum error in case there is a scale factor error.

III. EXPERIMENTS

In order to validate the proposed models, shake table experiments were performed. In this section, the experiments are illustrated including the testing flow and the experimental setup. The main steps of the experiments are: 1) estimation of σ_z using the proposed models **M1** and **M2**, 2) calculation of the RMS error based on these models and 3) comparing the calculated RMS to the actual measured error. Four different smartphones are used in the experiments: SMG900T, SMT350 and two different SMG920V devices. SMG900T and SMG920V are equipped with triaxial accelerometer chip manufactured by Invensense [27] and SMT350 includes STMicroelectronics accelerometer chip. The operating system running

Fig. 3: Shake table experiment setup.

on the four phones is Android and the versions are summarized in table I. A seismic grade accelerometer PCB Piezotronics Model393C [28] is used as a reference to measure the ground truth acceleration.

Five experiments are performed, where in each one the table is excited for 25 seconds by a sinusoidal signal of one of the following frequencies: 4, 5, 6, 7 and 8 Hz. The four smart devices are mounted to the shake table to avoid sliding. The

TABLE I: Characteristics of smart devices used in the experiments.

	SMG900T	SMG920V	SMT350
OS	Android V5.1.1	Android V7	Android V6
Accelerometer Chip manufacturer	Invensense	Invensense	STMicroelectronics
Commercial Name	Samsung Galaxy S5	Samsung Galaxy S6	Samsung Galaxy Tab A

Fig. 4: Smartphone accelerometer error calculation flow chart. Phone acceleration is up-sampled to match the reference sampling rate. Cross correlation is used to align the phone data with the reference one. Reference is down-sampled to the phone nominal rate after alignment. Error is calculated by subtraction.

smart devices are sampling at 100 *samples/s* rate, i.e. for each 25-sec experiment 2500 acceleration samples are stored. Figure 3 shows the experiment setup. We selected this specific range of frequencies for the experiments based on the region of interest of the target applications. For instance frequency of tremors of a Parkinson's disease (or similar diseases) patients are between 5-8 Hz [29]. Human activities frequency spectrum is between 0 and 20 Hz, with about 98% of the Fourier transform amplitude is below 10 Hz [30], and 95% of spectrum amplitude of gait is below 10 Hz [31]. For structural health monitoring, several previous work reported buildings natural frequency to be below 10 Hz [32], [33], [34].

Figure 4 shows the procedure followed to calculate the error of each smartphone. A nominal sampling rate of 100 *samples/s* is selected for the smartphones accelerometers, whereas the reference is sampled at 20 *ksample/s*. First,

TABLE II: Summary of experimental results. Five experiments are performed to estimate the proposed model **M1** parameter σ_z for four different phones. Each experiment is a sinusoidal wave with certain frequency. The average σ_z value is the square root of the average variance σ_z^2 of all the experiments.

	σ_{z1} (s)			
	SMG900T	SMG920V	SMG920V (2nd)	SMT350
Sine 4Hz	0.213	0.21	0.141	0.243
Sine 5Hz	0.3	0.25	0.1160	0.3
Sine 6Hz	0.239	0.187	0.12	0.272
Sine 7Hz	0.156	0.216	0.131	0.235
Sine 8Hz	0.214	0.208	0.131	0.235
Average	0.2292	0.2166	0.1248	0.2714

TABLE III: Summary of experimental results. Five experiments are performed to estimate the proposed model **M2** parameter σ_z for four different phones. Each experiment is a sinusoidal wave with certain frequency. The average σ_z value is the square root of the average variance σ_z^2 of all the experiments.

	σ_{z2} (s)			
	SMG900T	SMG920V	SMG920V (2nd)	SMT350
Sine 4Hz	0.187	0.185	0.131	0.208
Sine 5Hz	0.178	0.174	0.108	0.208
Sine 6Hz	0.179	0.171	0.116	0.212
Sine 7Hz	0.132	0.18	0.126	0.201
Sine 8Hz	0.163	0.167	0.109	0.2
Average	0.1689	0.1755	0.1184	0.2059

the smartphone data is up-sampled by cubic interpolation to match the reference rate. Second, cross-correlation is used to estimate the time lag that maximizes the cross-correlation. The reference is shifted by the estimated lag and then down-sampled to the nominal phone sampling rate. Finally, the error is calculated by subtraction. The procedure is repeated for each of the four phones in the five experiments.

We used the proposed MLE **M1** and **M2** to estimate σ_z for each phone, and the results are summarized in tables II and III respectively. We denote **M1** and **M2** estimation results as σ_{z1} and σ_{z2} respectively. Based on the results, it is clear that the estimated σ_z values are not much varying across the experiments for a given device, which validates the assumption that this parameter is not signal dependent.

It is worth noting that the models described in section II have two use cases:

- 1) Characterization: where a known vibration signal is used to estimate σ_z .
- 2) Error estimation: where the model parameters σ_z and σ_w are known, i.e. already estimated in the characterization phase, and then the error level of measuring an unknown signal is to be estimated using the models.

Hence, for all the experiments, we compare the measurements

Fig. 5: TRMS error versus the frequency of shaking for $M1_{\sigma_{z1}}$, $M2_{\sigma_{z2}}$ and $M1_{\sigma_{z2}}$ compared to measurements of SMG900T.

to the following three techniques:

- 1) $M1_{\sigma_{z1}}$: σ_z is characterized by **M1** and error is estimated by **M1**
- 2) $M2_{\sigma_{z2}}$: σ_z is characterized by **M2** and error is estimated by **M2**
- 3) $M1_{\sigma_{z2}}$: σ_z is characterized by **M2** and error is estimated by **M1**

For the four phones, figures 5-8 compare the measured error represented by the time average root mean squared (TRMS) to $M1_{\sigma_{z1}}$, $M2_{\sigma_{z2}}$ and $M1_{\sigma_{z2}}$ models. We also added the traditional additive noise model to the figures which does not take into account sampling time jitter. It is clear that the closest models to the measurements are $M2_{\sigma_{z2}}$ $M1_{\sigma_{z2}}$, which indicates the effectiveness of using **M2** in the characterization phase. However, if σ_z is properly estimated or given, then estimating the error using **M1** and **M2** result in almost the same values. Since, **M1** is a less complex model, we believe $M1_{\sigma_{z2}}$ is the best technique.

As shown in the results, the proposed model and parameter estimation procedure is validated on devices running different operating system versions and different accelerometer chips. Since the proposed model has not assumed any special features of Android or the inertial module chip, we believe the same methodology of parameter estimation applies to other devices as well, however the jitter values might be different. For instance, devices running other operating systems such as iOS or Microsoft Windows might have another jitter performance. Nevertheless, characterizing that jitter can still be achieved using the methodology proposed in this work.

IV. CRLB FOR ACCELERATION ESTIMATION

In this section we derive the CRLB of acceleration estimation based on the proposed model. The CRLB evaluates

Fig. 6: TRMS error versus the frequency of shaking for $M1_{\sigma_{z1}}$, $M2_{\sigma_{z2}}$ and $M1_{\sigma_{z2}}$ compared to measurements of SMG920V.

Fig. 7: TRMS error versus the frequency of shaking for $M1_{\sigma_{z1}}$, $M2_{\sigma_{z2}}$ and $M1_{\sigma_{z2}}$ compared to measurements of the second SMG920V.

Fig. 8: TRMS error versus the frequency of shaking for $M1_{\sigma_{z1}}$, $M2_{\sigma_{z2}}$ and $M1_{\sigma_{z2}}$ compared to measurements of SMT350.

the best achievable estimation accuracy of unbiased estimators [35]. We assume that since **M1** and **M2** are almost equivalent in the TRMS, then we evaluate the CRLB of **M1**. In addition, since we are concerned about the best achievable accuracy, let α be equal to one.

As mentioned in [35], for any parameter vector θ the CRLB for each element θ_i is defined by (18), where $\mathbf{I}(\theta)$ is the Fisher information matrix defined by (19).

$$\text{var}(\theta_i) \geq \mathbf{I}(\theta)^{-1}[i, i] \quad (18)$$

$$\mathbf{I}(\theta)[i, j] = -\mathbb{E}\left[\frac{\partial^2 \ln p(\mathbf{x}; \theta)}{\partial \theta_i \partial \theta_j}\right] \quad (19)$$

As mentioned in [25], n th derivative is approximately evaluated by (20), where $\text{sinc}^{(n)}$ is the n th derivative of the cardinal sine function defined by (21), and K is the number of time samples, refer to appendix A for more information. Consequently, (2) can be written as (22) and the vector form is expressed by (23), where $\mathbf{D}$ is a matrix whose rows are defined by (24).

$$\frac{d^n x(t)}{dt^n} \Big|_{t=i} \approx \sum_{k=0}^{K-1} x_k \text{sinc}^{(n)}(i-k) \quad (20)$$

$$\text{sinc}(t) = \frac{\sin \pi t}{\pi t} \quad (21)$$

$$a_i = x_i + w_i + \sum_{k=0}^{K-1} x_k \text{sinc}^{(1)}(i-k) z_i \quad (22)$$

$$\mathbf{a} = \mathbf{x} + \mathbf{w} + \mathbf{ZDx} \quad (23)$$

Hence, as described in [25], each element of the Fisher information matrix (FIM) is evaluated by (25), where the covariance matrix of $\mathbf{a}$ is given by (26) and its derivative with

Fig. 9: MSE of linear estimation of acceleration is plotted versus σ_z and compared to the CRLB, knowing that the true acceleration signal is a sinusoidal signal of 10^4 mm/s^2 amplitude and 0.04 Hz normalized frequency. The zero-dB level corresponds to the additive noise.

respect to x_i is given by (27). Then, according to the CRLB theorem, the minimum mean squared error (MSE) obtained by an unbiased estimator is given by (28).

$$\mathbf{D}[i, :] = \begin{bmatrix} \text{sinc}^{(1)}(-i) \\ \text{sinc}^{(1)}(1-i) \\ \text{sinc}^{(1)}(2-i) \\ \vdots \\ \text{sinc}^{(1)}((K-1)-i) \end{bmatrix}^T \quad (24)$$

$$\mathbf{FIM}[i, j] = \frac{1}{2} \text{tr} \left\{ \frac{\partial \Sigma_{\mathbf{a}}}{\partial x_i} \Sigma_{\mathbf{a}}^{-1} \frac{\partial \Sigma_{\mathbf{a}}}{\partial x_j} \Sigma_{\mathbf{a}}^{-1} \right\} + \Sigma_{\mathbf{a}}^{-1}[i, j] \quad (25)$$

$$\Sigma_{\mathbf{a}} = \sigma_z^2 I_K \odot (\mathbf{Dx} \mathbf{x}^T \mathbf{D}^T) + \sigma_w^2 I_K \quad (26)$$

$$\frac{\partial \Sigma_{\mathbf{a}}}{\partial x_i} = 2\sigma_z^2 \text{diag}\{\mathbf{D}[:, i] \odot (\mathbf{Dx})\} \quad (27)$$

$$MSE \Big|_{\min} = \mathbb{E}[(\hat{\mathbf{x}} - \mathbf{x})^T (\hat{\mathbf{x}} - \mathbf{x})] = \text{tr}\{\mathbf{FIM}^{-1}\} \quad (28)$$

The CRLB is evaluated numerically assuming ground truth acceleration is a sinusoidal signal of amplitude 10^4 mm/s^2 and normalized frequency of 0.04 Hz, $\sigma_w = 10 \text{ mm/s}^2$ and different σ_z values. Figure 9 compares the CRLB to the linear estimation accuracy. It is worth noting that a linear estimator as mentioned in [25], corresponds to $\hat{\mathbf{x}} = \mathbf{a}$, i.e. $MSE|_{\text{Linear}} = \text{tr}\{\Sigma_{\mathbf{a}}\}$ in case no over sampling is assumed.

As mentioned in section III, σ_z is approximately about 0.2, hence, it is clear from figure 9 that linear estimator accuracy is very close to the minimum theoretical bound. However, for higher jitter, other estimation techniques such as maximum likelihood estimation [25] can be adopted.

V. CONCLUSION

Smart devices are equipped with multiple sensors that measure most of the surrounding physical quantities. Lots of apps emerged that target biomedical, environment and infrastructure applications. In this work, we have investigated the accuracy limits of a smart device accelerometer which is considered one of the most used sensors in such applications. We proposed a novel error model that takes into account software sampling time error in addition to the additive noise model developed in previous work. The proposed model has been validated experimentally on four smart devices by several shake table experiments. It was shown that the proposed model estimates error TRMS that is much closer to the real measured error compared to the additive noise model which is about an order of magnitude less. In addition, with respect to acceleration estimation accuracy based on the proposed model, it was shown that the linear estimators are close to the CRLB at the current sampling time jitter value. Hence, using a less computationally expensive linear estimator will not affect much the achieved accuracy.

APPENDIX A

SIGNAL DERIVATIVE APPROXIMATION

Equation (20) is used to get an approximate value of the signal derivative as a function of the signal itself. Based on Nyquist sampling theorem, any band limited signal $x(t)$ of bandwidth B can be recovered from its sampled version x_i if the sampling frequency f_s is at least $2B$. The signal reconstruction is done using an ideal low pass filter of bandwidth B . Without the loss of generality, we use normalized frequency, i.e. $f_s = 1$. The low pass filter impulse response is defined by (29). Hence, $x(t)$ is recovered by (30). Then, the first derivative of $x(t)$ is calculated by (31), where $\text{sinc}^{(1)}(t)$ is the first derivative of $\text{sinc}(t)$. It is worth noting that since $x(t)$ is band limited then it is infinitely long in the time domain. In addition, $\text{sinc}(t)$ is also infinitely long. However, for practical implementation, both the signals are truncated which results in an approximation error. Let the approximated first derivative of $x(t)$ be denoted as $\tilde{x}(t)$ which is calculated by (32) and the discrete version is expressed by (33).

$$h(t) = \text{sinc}(t) \quad (29)$$

$$x(t) = \sum_{k=-\infty}^{\infty} x_k \text{sinc}(t - k) \quad (30)$$

$$\dot{x}(t) = \sum_{k=-\infty}^{\infty} x_k \text{sinc}^{(1)}(t - k) \quad (31)$$

$$\tilde{x}(t) = \sum_{k=0}^{K-1} x_k \text{sinc}^{(1)}(t - k) \quad (32)$$

$$\tilde{x}_i = \sum_{k=0}^{K-1} x_k \text{sinc}^{(1)}(i - k) \quad (33)$$

Since the sinc unction is decreasing, the approximation error mainly affects the data points at the boundaries, in other words, the middle points are not much affected by the truncated part of the functions. For the sake of demonstration,

Fig. 10: True and approximate first derivative example.

let $K = 200$, $\sigma_z = 0.2$ and $x_i = 10^4 \sin(2\pi 0.04i)$ then the true and approximate derivatives are expressed by (34) and (35) respectively. Figure 10 compares between the true and approximate first derivatives of the given example and it is clear that the error is larger at the boundaries.

For the same example, the **FIM** is evaluated using (25), and the CRLB is the diagonal of $\mathbf{FIM}^{-1}$. Figure 11 shows the CRLB at each sample time i . It is clear that at the boundaries the CRLB is different compared to the middle samples, which is expected due to the inaccurate signal derivative at the boundaries as mentioned earlier.

$$\dot{x}_i = 2\pi 0.04 \times 10^4 \cos(2\pi 0.04i) \quad (34)$$

$$\tilde{x}_i = \sum_{k=0}^{K-1} 10^4 \sin(2\pi 0.04k) \text{sinc}^{(1)}(i - k) \quad (35)$$

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Fig. 11: CRLB at each sample i of the given example.

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