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ABSTRACT

"Cycle skipping" is a severe issue in Full-Waveform Inversion. One option to overcome it is to extend the search space to allow for data comparisons beyond the "point-to-point" subtraction. A matching filter can be computed by deconvolving the measured data from the predicted ones. If the model is correct, the resulting matching filter would be a Dirac delta function in which the energy is focused at zero lag. An optimization problem can be formulated by penalizing this matching filter departure from a Dirac delta function. As the matching filter replaces the local, sample-by-sample, comparison with a global one using deconvolution, it can reduce the "cycle skipping" problem. As the matching filter is computed using the whole trace of the measured and predicted data, it is prone to unwanted cross-talks of different events. We propose to perform the deconvolution in the Radon domain to reduce those kinds of cross-talks and improve the inversion. We first transform both the measured and the predicted data into the $\tau - p$ domain using the local Radon transform. We then perform deconvolution for the trace indexed by the same slope value.

The main objective of the proposal is to use the slope information embedded in the Radon transform representation to separate the events and reduce the cross-talks in the deconvolution step. As a result, the objective function tends to be more convex and stabilizes the inversion process. The result obtained for the modified Marmousi model demonstrates the proposed Radon-domain matching-filter approach can converge to a meaningful model given data without the low frequencies below 3 Hz and a $v(z)$ initial model. Compared to the conventional time-space matching-filter approach, the Radon-domain approach shows fewer artifacts in the model and better fitting of the measured data. The result corresponding to the Chevron 2014 benchmark dataset also shows the good performance of the proposed approach.

INTRODUCTION

Full-Waveform Inversion (FWI) (Virieux and Operto, 2009) aims to build high-resolution subsurface models by exploiting the full information of prestack seismic dataset, i.e., reflections, refractions, diving waves, P- and S- waves, surface waves. A successful implementation of FWI can invert for the subsurface properties such as P and S wave velocities, density, attenuation, and anisotropy parameters with high resolution in the limit of half of the shortest propagated wavelength. Lailly (1983) and Tarantola (1984) proposed the basic concept and the theory of FWI in the time domain. There are variations of the FWI formulation such as frequency-domain FWI (Pratt, 1999; Sirgue and Pratt, 2004), Laplace domain and Laplace-Fourier domain FWI (Shin and Cha, 2008, 2009). Due to the relatively high computational cost of the FWI, and courtesy of the significant developments in high-performance computing, FWI was applied to real data and numerous successful applications to 3D and real data has been reported (Plessix and Perkins, 2010; Sirgue et al., 2010; Warner et al., 2013; Vigh et al., 2014; Górszczyk et al., 2017; Alkhalifah et al., 2018; Shen et al., 2018; Zhang et al., 2018).

FWI is a nonlinear inversion process, and thus, we iteratively update the subsurface model to reduce the mismatch between the predicted and the measured seismic data. Mathematically, we design a misfit function to characterize such mismatch. Designing a misfit function that tolerates the expected poor quality and limited-band field data is highly important. A well-behaved misfit function also would relax the requirement for a good initial velocity model and usable low-frequency signals in the data, and thus, resolve the so-called "cycle skipping" issue. Over the past decade, the least-square L2-norm was widely used as a misfit function for its simplicity and high resolution potential. It, however, suffers

from the well-known cycle-skipping problem. Currently, new and more advanced misfit functions were proposed; those include, but not limited to, the matching-filter misfit function (Van Leeuwen and Mulder, 2008, 2010; Luo and Sava, 2011; Warner and Guasch, 2016; Debens et al., 2017; Huang et al., 2017; Sun and Alkhalifah, 2019a,b), the optimal-transport misfit function (Engquist and Froese, 2014; Engquist et al., 2016; Métivier et al., 2016a,b, 2018; Qiu et al., 2017; Yang et al., 2018; Yang and Engquist, 2018; Sun and Alkhalifah, 2018a, 2019c,d), the instantaneous travel-time misfit function (Alkhalifah and Choi, 2014; Sun and Alkhalifah, 2018b), etc. Unlike the conventional L2-norm based misfit function, those newly proposed methods transform sample-by-sample comparisons to global, trace-by-trace comparisons. As expected, the resulting misfit function shows convex behavior and can reduce the cycle-skipping problem.

We will mainly focus on the application of a Radon-domain misfit function in the category of the matching-filter based methods. However, the Radon-domain representation of the data can also be implemented with other kinds of misfit functions like the optimal transport or the normalized global crosscorrelation as long as those misfit functions compare the predicted and the measured data in a global way in which cross-talks may occur. Luo and Sava (2011) described an approach for designing a misfit function, which seeks to minimize the penalized deconvolution coefficients between measured and predicted data. In their proposal, the misfit function uses deconvolution instead of crosscorrelation (Van Leeuwen and Mulder, 2008, 2010). The resulting objective function would be less sensitive to a band-limited or non-impulsive source function which may result in a nonzero gradient of the objective function even when the inverted velocity model matches the true model. Warner and Guasch (2016) introduced an extra normalization term, and in their study, the normalization term would provide better convexity and accelerate the convergences. In those

approaches, the matching filter is designed for each trace using deconvolution of the measured data and the predicted ones. Clearly, such a deconvolution-based filter extends the data comparison along the whole trace. However, though needed to avoid cycle skipping, the global comparison can induce cross-talks between events in the measured and predicted data that are not related. Thus, such a feature of the global comparison introduced a drawback in the application. As a result, the application of the global comparison methods, like the matching-filter and the optimal-transport methods, require some preprocessing and conditioning. For example, Huang et al. (2017) reported that the matching-filter approach deals mainly with the diving waves and suffers from strong multiples (Debens et al., 2017). A scheme based on deconvolution of the windowed signal may have the potential to partially resolve the problem and improve the robustness of the inversion. However, the determination of the length of the window is not straightforward and needs considerable human interaction in the inversion process. Thus, we propose a workflow for FWI in general that includes a Radon transform of the data to separate the data partially based on their slopes before computing the matching filter.

Radon transform is a mathematical technique that has been widely used in seismic data processing (Stoffa et al., 1981; Thorson and Claerbout, 1985; Sacchi and Ulrych, 1995; Luo et al., 2009; Gholami and Zand, 2018). A number of researchers have worked on the plane-wave- and the Radon-domain related inversion methods. Diebold and Stoffa (1981) suggested a traveltimes inversion in the $\tau - p$ domain. Shaw and Orcutt (1985) utilized the linearity relation between the representation in the $\tau - p$ domain and the model to facilitate an inversion. Pan et al. (1988) applied FWI in the $\tau - p$ domain for a stratified acoustic media while Kormendi and Dietrich (1991) investigated the $\tau - p$ domain FWI for a stratified elastic media. Minkoff and Symes (1997) presented the $\tau - p$ domain FWI inversion result of

a marine dataset from Gulf of Mexico based on Differential Semblance Optimization (DSO). Vigh and Starr (2008) utilized an efficient time-domain, plane-wave implementation of the 3D FWI. Tao and Sen (2013) studied the plane-wave frequency-domain FWI. Pan et al. (2015) proposed an efficient pseudo-Gauss-Newton FWI in the $\tau - p$ domain. Kwon et al. (2016) used the $\tau - p$ transform to reduce the noise in an elastic FWI.

Here, we further expand the applications of the Radon transform to FWI in designing a robust misfit function by integrating it with a matching-filter approach. For the Radon-domain matching-filter FWI, instead of computing the matching filter for each trace directly, we first transform a common-shot gather into the $\tau - p$ domain using Radon transform and then compute the matching filter for each trace indexed by the same p-value. We use the inherent benefits of the Radon transform in capturing the energy of the slopes of the seismic events to achieve separations, and reduce the cross-talks in the following deconvolution stages. Thus, instead of designing the matching filter trace by trace in the time-space domain, we apply the matching on traces that have been stacked over slopes and represented in the $\tau - p$ domain, and consequently, we intuitively take the coherency of the events along the offset axis into consideration. The criterion for a successful convergence is still the same in the Radon domain, i.e., the energy of the matching filter focuses at zero lag.

In the following, we first review the matching-filter based FWI approach and then we describe the method in the Radon domain. After that, we provide a detailed analysis of the proposed Radon-domain matching-filter approach using a modified Marmousi model. We also present the result for Chevron 2014 FWI benchmark dataset. At the end, we discuss the potential features and extensions of the approach.

FWI BY MATCHING FILTER

The Radon transform, with its ability to highlight coherency in the data, can potentially benefit nonlinear misfit functions including the normalized global crosscorrelation, envelope, optimal transport, and matching-filter approach. However, in this study, we will demonstrate its features using the matching-filter based FWI. Thus, we start with a brief review of the matching-filter approach.

Within the framework of FWI, we use an initial or available velocity model to simulate the predicted data $p(t, \mathbf{x}_r; \mathbf{x}_s)$ to compare with the measured data $d(t, \mathbf{x}_r; \mathbf{x}_s)$. The fitting criteria forms the optimization problem, in which we minimize, for example, the L2-norm objective function:

$$J = \frac{1}{2} \iiint [p(t, \mathbf{x}_r; \mathbf{x}_s) - d(t, \mathbf{x}_r; \mathbf{x}_s)]^2 dt d\mathbf{x}_r d\mathbf{x}_s, \quad (1)$$

where $\mathbf{x}_s$ and $\mathbf{x}_r$ are the coordinates for the source and receiver positions, respectively. It is obvious that the L2-norm misfit function measures the mismatch between two traces through a sample-by-sample comparison. As a result, it is prone to cycle skipping if the samples we are comparing do not share the same half cycle of the signal. In practice, the application of the L2-norm misfit function either requires a kinematically accurate initial model or very low frequencies in the data so that the half-cycle can tolerate poor initial models and thus, converge to the objective, which is referred to as the global minimum.

A strong remedy to such FWI limitations is to compute a matching filter that describes the difference between the measured and the predicted data over the whole trace. Thus, a matching filter, $w(t, \mathbf{x}_r; \mathbf{x}_s)$, satisfies:

$$w(t, \mathbf{x}_r; \mathbf{x}_s) * d(t, \mathbf{x}_r; \mathbf{x}_s) = p(t, \mathbf{x}_r; \mathbf{x}_s), \quad (2)$$

where $*$ denotes the convolution operation. Equation 2 is a linear equation, and thus, the computation of the matching filter is a well-resolved problem in signal processing (Claerbout, 1985). It can be computed either in the time domain or in the frequency domain as:

$$w(t, x_r; x_s) = \mathcal{F}^{-} \left[\frac{\mathcal{F}[p(t, \mathbf{x}_r; \mathbf{x}_s)] \overline{\mathcal{F}[d(t, \mathbf{x}_r; \mathbf{x}_s)]}}{\mathcal{F}[d(t, \mathbf{x}_r; \mathbf{x}_s)] \overline{\mathcal{F}[d(t, \mathbf{x}_r; \mathbf{x}_s)]} + \epsilon} \right], \quad (3)$$

where $\mathcal{F}$ and $\mathcal{F}^{-}$ denote the temporal Fourier transform and its inverse, respectively, and overline denotes for the complex conjugate. The parameter ϵ is a small value to avoid dividing by zero. For our tested examples in the paper, we find that choosing the parameter $\epsilon = 0.1 * \max(\mathcal{F}[d(t)] \overline{\mathcal{F}[d(t)]})$ can lead to stable results. As we perform the deconvolution in the frequency domain, we can guarantee accurate computation of the deconvolution for all frequencies without residuals compared to the time domain iterative least-squares method.

When the velocity model is correct, the energy in the matching filter focuses at zero lag, more like a Dirac delta function. Thus, we can formulate an optimization problem that penalizes energy in the matching filter beyond zero lag. As a result, the objective function has the following form:

$$J = \frac{1}{2} \iiint [P(t)w(t, \mathbf{x}_r; \mathbf{x}_s)]^2 dt d\mathbf{x}_r d\mathbf{x}_s, \quad (4)$$

where $P(t)$ is the penalty function, e.g., $P(t) = |t|$ will enhance energy beyond zero lag, and the objective is to minimize such enhancement. The gradient descent method for solving this optimization problem yields an adjoint source, through the adjoint-state method, that we will propagate back into the model. The adjoint source for the matching-filter misfit function of equation 4 is given by:

$$\delta s(t, \mathbf{x}_r, \mathbf{x}_s) = \frac{\partial J}{\partial p} = \frac{\partial w}{\partial p} [P(t)^2 w(t, \mathbf{x}_r; \mathbf{x}_s)]. \quad (5)$$

The term $\partial w / \partial p$ denotes the partial derivative of the matching filter with respect to the computed data; we can evaluate its action on the variable a using the adjoint analysis of

equation 3:

$$\frac{\partial w}{\partial p}[a] = \mathcal{F}^{-1} \left[\frac{\mathcal{F}[d(t, \mathbf{x}_r; \mathbf{x}_s)] \mathcal{F}[a]}{\mathcal{F}[d(t, \mathbf{x}_r; \mathbf{x}_s)] \mathcal{F}[d(t, \mathbf{x}_r; \mathbf{x}_s)] + \epsilon} \right]. \quad (6)$$

The matching filter computed per trace as previously discussed is prone to cross-talks of different events. In Figure 1a, we show a real data common-shot gather. If we computed the matching filter per trace, it is obvious that the events denoted by the blue arrow will have cross-talks and it causes problems as the event pointed by the first blue arrow is the direct arrival while the second is a reflection: Suppose your predicted data can provide a correct direct arrival and a time-shifted reflections, the deconvolution will introduce cross-talks between the direct arrivals in the measured data and the reflections in the predicted data. Figure 1b is the Radon transform of the record. We can see that those events would be separated by their distinguishable slope values. Similarly, in Figure 1a, the first white arrow points to events corresponding to noise while the second is pointing to useful refracted or diving waves. Those events have significant slope difference (actually with opposite sign) and they will be well separated in the Radon domain and this makes the resulting matching filter more accurate and the resulting inversion more robust. In next section, we will discuss those robust FWI based on computing of the matching filter in the Radon domain.

THE MATCHING FILTER IN THE RADON DOMAIN

The matching filter computed using equations 2 and 3 takes the whole trace of the measured and predicted data as input and would suffer from the cross-talks between different events. We use the Radon transform to separate the events based on their space-time coherency, i.e., the slope information. This would lead to a more robust computation of the matching filter.

For a common-shot gather $q(t, \mathbf{x}_r, \mathbf{x}_s)$, the Radon transform $\mathcal{R}$, a linear operator which maps the data from the $t - x$ domain to the $\tau - p$ domain, is defined as (Durrani and Bisset, 1984; Wang et al., 2010)

$$\tilde{q}(\tau, \mathbf{p}, \mathbf{x}_r, \mathbf{x}_s) = \mathcal{R}[q(t, \mathbf{x}_r, \mathbf{x}_s)] = \sum_{\mathbf{h}} q(\tau + \mathbf{p} \cdot (\mathbf{x}_r + \mathbf{h}), \mathbf{x}_r + \mathbf{h}, \mathbf{x}_s), \quad (7)$$

where $\mathbf{p}$ is the slope and $\mathbf{h}$ is the local spatial coordinate and for the 2D problem they would have one component and for the 3D problem, there will be two components. Here, we apply the Radon transform locally, so that we would have a constraint for the component $|h_i| < h_{max}$.

The adjoint operator of the Radon transform $\mathcal{R}^T$, will transform the data from the $\tau - p$ domain back to the $t - x$ domain :

$$q(t, \mathbf{x}_r, \mathbf{x}_s) = \mathcal{R}^T[\tilde{q}(\tau, \mathbf{p}, \mathbf{x}_r, \mathbf{x}_s)]. \quad (8)$$

For a $\tau - p$ domain implementation of the matching-filter based approach, we would first Radon transform both the measured and the predicted data into the $\tau - p$ domain using equation 7:

$$\tilde{p}(\tau, \mathbf{p}, \mathbf{x}_r, \mathbf{x}_s) = \sum_{\mathbf{h}} p(\tau + \mathbf{p} \cdot (\mathbf{x}_r + \mathbf{h}), \mathbf{x}_r + \mathbf{h}, \mathbf{x}_s), \quad (9)$$

$$\tilde{d}(\tau, \mathbf{p}, \mathbf{x}_r, \mathbf{x}_s) = \sum_{\mathbf{h}} d(\tau + \mathbf{p} \cdot (\mathbf{x}_r + \mathbf{h}), \mathbf{x}_r + \mathbf{h}, \mathbf{x}_s). \quad (10)$$

Then, we compute the matching filter in the $\tau - p$ domain $\tilde{w}(\tau, \mathbf{p}, \mathbf{x}_r, \mathbf{x}_s)$:

$$\tilde{w}(\tau, \mathbf{p}, \mathbf{x}_r, \mathbf{x}_s) * \tilde{d}(\tau, \mathbf{p}, \mathbf{x}_r, \mathbf{x}_s) = \tilde{p}(\tau, \mathbf{p}, \mathbf{x}_r, \mathbf{x}_s), \quad (11)$$

and it can be computed in the frequency domain similar to equation 3. In the $\tau - p$ domain, the misfit function would then be formulated as

$$J = \frac{1}{2} \iiint [P(\tau) \tilde{w}(\tau, \mathbf{p}, \mathbf{x}_r, \mathbf{x}_s)]^2 d\tau d\mathbf{p} d\mathbf{x}_r d\mathbf{x}_s, \quad (12)$$

where the penalty function $P(\tau)$ is the same as $P(t)$ in equation 4. Similarly, the adjoint source is given by:

$$\delta s(t, \mathbf{x}_r, \mathbf{x}_s) = \frac{\partial J}{\partial p} = \mathcal{R}^T \left[\frac{\partial \tilde{w}}{\partial \tilde{p}} [P(\tau)^2 \tilde{w}(\tau, \mathbf{p}, \mathbf{x}_r; \mathbf{x}_s)] \right]. \quad (13)$$

The action of $\partial \tilde{w} / \partial \tilde{p}$ on the function a can be derived similar to equation 6.

Convexity analysis

In this subsection, we analyze the convexity properties of the discussed misfit functions. We will compare the conventional time-space domain matching-filter approach and the proposed Radon-domain matching-filter approach. The benefits of the Radon-domain approach becomes evident only when multiple events are present. Thus, contrary to the conventional setting of using one shifted signal event, we consider shifting of two events with distinct slopes in our experiments and such setups will help to analysis the cross-talks issues between multiple events during the matching filter computation.

In the first setup, the measured data are shown in Figure 2a and their Radon transform is shown in Figure 2b. The predicted data are set to have a time shift related to the measured one (note although there are two events present in the data, they have the same time shift in the current setup). Figures 2c and 2d show one example of such predicted data and their Radon transform. (Here, the predicted data have been advanced 0.3 s.). We overlap the measured data and the predicted ones and their Radon transforms in Figures 2e and 2f, respectively. (We use the black curves to indicate the measured data and the red curves to denote the predicted data, respectively). In Figure 2e, we can clearly see a constant global time shift between the predicted and the measured data. In the Radon domain of Figure 2f, the events are well separated by their slopes, and the time shift for the events are well

preserved in the Radon domain: two events show the correct time shift value of 0.3 s in the Radon domain. Figure 3a shows the resulting matching filter computed in the time-space domain, i.e., by deconvolution of each trace indexed by the same distance value shown in Figures 2a and 2c. Figure 3b shows the resulting matching filter computed in the Radon domain, i.e., by the deconvolution of each trace indexed by the same slope value shown in Figures 2b and 2d. The resulting matching filter can capture the time shift correctly but in the Radon domain, the matching filter shows fewer artifacts as the deconvolution is performed on separated events. By scanning a range of time shifts from -0.3 s to 0.3 s, we compute the misfit value corresponding to the time-space domain and the Radon-domain matching filter. (Here and below, for the computation of the Radon-domain misfit function, in order to accommodate the energy distribution in the Radon domain, we weight the misfit by the L2 norm of each trace indexed by the same slope value) We normalize the resulting misfit value to be in the range of $[0, 1]$ and show the result as curves in Figure 4. In this setup, both the time-space domain and the Radon-domain approaches show good convexity with respect to time shifts. The Radon-domain approach shows subtle improved convexity, which is due to the fact that the matching filter computed in the Radon domain has less artifacts as shown in Figure 3b.

In the second setup, the measured data and their Radon transform as shown in Figures 5a and 5b are the same as before, and it has two events with distinct slopes, i.e., $d(t, x) = d_A(t, x) + d_B(t, x)$. We keep event A fixed while we shift event B only, and the resulting predicted data becomes $p(t, x) = d_A(t, x) + d_B(t - \tau, x)$. Figures 5c and 5d show one such shifted predicted data in the time-space domain and the Radon-domain, respectively (Here, $\tau = 0.3$ s). The setting proposed here is realistic, and in a real application, we can consider event A as a representative of the arrivals from the shallow part which is already accurate,

i.e., the direct arrivals and the shifting of the event B is mimicking the later arrivals that may include errors. We overlap the measured and the predicted data in Figure 5e and their Radon transforms in Figure 5f. In Figure 5f, due to the separation feature of the Radon transform, events A and B are separated, and the shifting of the event B only happens at its corresponding slope value. Figures 6a and 6b show the computed matching filter in the time-space and the Radon domains, respectively. The resulting matching filters in the time-space domain are rather complex while in the Radon domain, they become reasonably simple. It is evident that the matching filter in the Radon domain clear captures that the event A has zero time shift while the event B has a shift about 0.3 s in Figure 6b. Similarly, by scanning the shifting value τ from -0.3 s to 0.3 s, we obtain the resulting misfit curves shown in Figure 7. Now, the benefits of the Radon-domain approach become evident and the conventional time-space matching-filter approach has obvious local minimal. The good convexity of the Radon-domain approach is due to that the separation of the events is achieved in the Radon domain so that the resulting matching filter can capture the time shift of the events with similar slope values more effectively. In real applications, the seismic record is far more complex than including only two events, e.g., the Marmousi dataset shown below. However, the separation of the events in the Radon domain can partially reduce the nonlinearity of the dataset and make the corresponding matching filter capture the misfit more effectively. In the next section, we use the Marmousi model to demonstrate these features in detail.

EXAMPLES

We will test the proposed approach to the modified Marmousi model first. We create two datasets for inversion: one with an absorbing-boundary condition at the surface; the other

with a free-surface boundary condition. The second example is the Chevron 2014 FWI benchmark dataset; we will focus on the potential of the proposed method to converge to a high-resolution model.

Marmousi model

We use the modified Marmousi model to demonstrate the effectiveness of the proposed method. As shown in Figure 8a, the true velocity v_{true} model extends 2 km in depth and 8 km laterally. The initial velocity $v_{\text{ini}} = v(z)$ is shown in Figure 8b. The initial velocity we used is similar to the one used in Warner and Guasch (2016). The 1D initial model has a correct water depth of 40 m as in the true model and increases linearly from the water depth to 2550 m/s at 1000 m depth and then reaches a velocity of 3250 m/s at the bottom of the model. The dataset is modeled using 80 shots with a source interval of 100 m and 400 receivers with an interval of 20 m. The source wavelet is a Ricker wavelet with a 10 Hz peak frequency. In the inversion, we mute data below 3 Hz to verify that our proposed method is capable of overcoming the "cycle skipping" problem without the low frequencies.

Absorbing boundary condition at the surface

In the first setup, we intend to invert for the dataset created with an absorbing boundary condition at the surface. Figure 9a shows the common shot gather at a distance 2 km computed from the true model. Figure 9b shows the overlay of the measured data (the red curves) and the modeled data by the initial model (the black curves). It is obvious that it is cycle-skipped at far offsets for the current frequency band.

We apply time-domain inversions by using a one-time low-pass filter to the data up to 10

Hz. Therefore, we do not sequentially increase the frequency as often implemented. Instead, we perform a full-band inversion with a frequency range from 3 Hz to 10 Hz. Lacking low frequency is a challenge for the conventional L2-norm misfit function as for this frequency band we face the "cycle skipping" issue. We run the inversion over 200 iterations using a nonlinear conjugate gradient method. In each iteration, we normalize the gradient by its maximum value and perform backtracking line search with a maximum step length of 100 m/s allowing for a maximum of five iterations for the line search with a minimum step length of 15 m/s. For the local Radon transform, the half local window is selected to be 600 m. The slope range used for computation is between -0.67 s/km and 0.67 s/km ($1/1.5 \approx 0.67$ and 1.5 km/s is the water velocity).

The inverted result for the L2-norm misfit function is shown in Figure 10a. Due to the cycle skipping, the result has very strong artifacts in the left part of the model and it is generally far from the true model. The Radon-domain L2-norm result is shown in Figure 10b, and it is similar to the result of Figure 10a using the L2 norm. These results indicate that the Radon-domain approach will not help the conventional L2 norm in the aspect of reducing cycle skipping. We will discuss this feature again in the Discussion section. Figure 10d is the result of the proposed Radon-domain matching-filter approach. Compared to the true model, the inverted model recovers well both of the low-wavenumber parts and the fine-scale parts of the true model although there are some artifacts and reduced accuracy at the boundary due to the limited illumination. Compared to the conventional matching-filter result of Figure 10c, the Radon-domain approach shows fewer artifacts and higher accuracy especially in the deeper part of the model. We compare the resulting shot records for a source at a distance of 2 km in Figure 11. Due to cycle skipping, the L2 norm (Figure 11a) and the Radon-domain L2-norm approach (Figure 11b) can hardly fit the data. The

Radon-domain matching-filter approach (Figure 11d) shows better fitting of the data than the conventional matching-filter approach (Figure 11c). The high accuracy of the inverted model by the Radon-domain approach is well demonstrated with the good alignment of the events between the true record and the inverted one, especially at far offsets.

Figure 12 shows the intermediate results of the inversion process using the Radon-domain matching-filter approach. We can see that the inversion recovers the areas where the data provides better coverage, specifically from inwards to outwards, and shallow to deep as we gradually update the model. As a counterpart, we show the intermediate results by the conventional matching-filter approach in Figure 13. From the results, we can observe that the conventional matching-filter approach leads to relatively strong artifacts, e.g., at the shallow part and the left edge, and slow convergence. The Radon-domain approach tends to converge faster and leads to a relatively well-inverted model after 50 iterations (see Figure 12d).

Both of the matching-filter based approaches aim at improving the focusing of the matching filter during iterations. To measure the focusing of the resulting matching filter, we define the function of a matching-filter distribution to characterize such focusing property. Given a matching filter $w(t)$, the matching-filter distribution is defined as the normalized squares:

$$w'(t) = \frac{w^2(t)}{\int w^2(t)dt}. \quad (14)$$

With this definition, the matching-filter distribution measures the focusing of the matching filter around the zero time-lag free of amplitude effects.

Figures 14 shows the evolution of the resulting matching-filter distribution for a shot at a distance 2 km using the proposed Radon-domain matching-filter approach. Note, as shown

in equation 11, the Radon-domain matching filter for a common shot would be a three-dimensional array because of the newly introduced slope information. Thus, here we collect the matching filters every 500 m in the distance and reshape them into a two-dimensional array for simplification of the demonstration. The matching filter becomes more and more focused around zero time-lag over iterations. The final matching-filter distribution shown in Figure 14f can be a good approximation to the band-limited Dirac delta function, and this further proves the high accuracy of the inverted model and its good fitting to the record data. As a comparison, we show the evolution of the resulting matching-filter distribution for the same shot in Figures 15 for the conventional matching-filter approach. Similarly, the matching filter tries to focus per iteration; however, compared to Figures 14, the focusing speed tends to be slow, and the final resulting matching-filter distribution (see Figure 14f) has relatively large residuals at non-zero lags indicating a less reliable fitting to the data compared to the Radon-domain matching-filter approach.

Free-surface boundary condition

In the second setup, we modeled the data with a free-surface boundary condition. This adds more nonlinearity to the dataset and makes the inversion more challenging. Figure 16a shows the common shot gather at a distance 2 km computed from the true model. Figure 16b shows the overlay of the measured data (the red curves) and the modeled data by the initial model (the black curves). Compared with the data modeled with an absorbing boundary condition of Figure 16, the data by free-surface boundary condition shows much more complexity.

Using the same setup for inversion as before, we get the inversion result in Figure 17.

As expected, the L2-norm result of Figure 17a badly recovered the true model due to cycle skipping. The result of Figure 17c by Radon-domain matching-filter approach shows fewer artifacts and higher accuracy compared to the result of Figure 17b by the conventional matching-filter approach. The good performance of the Radon-domain approach in this challenging situation indicated that performing Radon transform and computing the misfit function in the $\tau - p$ domain can reduce the nonlinearity of the misfit function and as a result, lead to a more robust inversion. We compare the resulting shot records for a source at a distance of 2 km in Figure 18. Due to cycle skipping, the L2 norm (Figure 18a) shows the worst fitting of the data. The Radon-domain matching-filter approach (Figure 18c) shows better fitting of the data than the conventional matching-filter approach (Figure 18b), e.g., in the area denoted by the blue rectangle. Figures 19 and 20 show the evolution of the resulting matching-filter distribution for a shot at a distance 2 km for the proposed Radon-domain matching-filter approach and the conventional matching-filter approach respectively. It is clear that the focusing of the matching filter distribution by the Radon-domain approach improves significantly over iterations and the final matching filter distribution shows better-focusing property than that by the conventional matching-filter approach. This observation again demonstrates the potential of the proposed Radon-domain matching-filter approach.

Chevron 2014 FWI benchmark

The second example is the Chevron 2014 FWI benchmark dataset. It is a synthetic dataset representing marine acquisition and modeled by the elastic wave equation with free-surface multiples. The data are noisy, and the minimum usable frequency is about 3 Hz. The total dataset has 1600 shots with 25 m shot intervals at depth 15 m. Each shot gather has 321

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2
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4 receivers with 25 m intervals at a depth of 15 m. The initial model is provided by Chevron
5
6 as shown in Figure 21a. We perform the inversion with frequency bands of 0-3 Hz, 0-6
7
8 Hz, 0-10 Hz, 0-25 Hz, and 0-45 Hz, sequentially. We use the Radon-domain matching-filter
9
10 misfit function first and then use L2-norm misfit function to further refine the inversion
11
12 result for higher resolution information. When we finish one round of inversion, we slightly
13
14 smooth the obtained model to have another run to further improve the inverted result. We
15
16 use a nonlinear conjugate gradient method, and the line search strategy is the same as the
17
18 last example. For the local Radon transform, the half local window is chosen to be 600 m.
19
20 The slope range is between -0.67 s/km and 0.67 s/km. We smooth the gradient horizontally
21
22 in each step with a relatively small window in the shallow part and a wider window in the
23
24 deeper part. We also smooth the model after finishing one frequency-band inversion.
25
26

27
28 The inverted model is shown in Figure 21b. The inverted result captures the low-velocity
29
30 zone and gas cloud around depth 2.5 km. Note the relatively low velocity marked by the
31
32 black ellipse and cycle. The well log comparison is shown in Figure 22, and we can see that
33
34 the initial velocity is relatively high while the inverted result shows good fitting with the well
35
36 from 1000 m to 2500 m. We select one shot at the middle of the model, and in Figure 23 we
37
38 compare the observed record and the modeled record using the inverted model. It is clear
39
40 that the modeled and observed data resemble each other well. Figure 24a and 24b shows the
41
42 RTM image with the initial model and the inverted model, respectively. The inverted image
43
44 shows more continuous structures and is better focused specifically in places marked by the
45
46 yellow ellipse in Figure 24a. It is due to the accurate representation of the low-velocity zone
47
48 around depth 2.5 km in the inverted model. Further, we compare angle-domain common-
49
50 image gathers in Figure 25. From the gathers with the initial velocity shown in Figure 25a,
51
52 specifically the misalignment of the deep part indicates that the velocity of the overburden
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is higher while the inverted velocity correctly flattens those deeper gathers in Figure 25b by finding the low-velocity zone around depth 2.5 km.

Although the 2014 Chevron FWI dataset is a blind test, several groups in the FWI community have worked on it (Almomin and Biondi, 2015; Warner and Guasch, 2015; Métivier et al., 2016b; Vigh et al., 2016) and gained rich and profound experience related to this dataset. It has already been shown by those researchers that FWI using L2-norm misfit solely can not resolve the low-velocity zone and gas cloud very well due to cycle skipping. Remedy to conventional L2-norm misfit should be made for a robust inversion. For example, Warner and Guasch (2015) used the adaptive waveform inversion for reducing cycle skipping; Almomin and Biondi (2015) implemented the tomographic FWI for updating the background velocity; Métivier et al. (2016b) designed a new misfit function based on optimal transport theory; Vigh et al. (2016) tried the reflection waveform-inversion method using a robust misfit function which can capture the time shift information. Similarly, our proposed Radon-domain matching-filter approach successfully inverted for the challenging low-velocity zone and gas cloud. This demonstrates the robustness of the proposed Radon-domain matching-filter method and its ability to overcome cycle skipping.

DISCUSSION

Radon Transform Beyond Preprocessing

The Radon transform has been widely used for seismic data processing. In FWI, we can use Radon transform as a preprocessing tool (Cheng et al., 2016) to enhance the S/N of the seismic data. In this approach, the misfit function for FWI can be written as :

$$J = f(p, \mathcal{R}^{-1} \mathcal{W} \mathcal{R} d), \quad (15)$$

here f is a misfit function which measures the distance between the predicted data p and the preprocessed measured data $\mathcal{R}^{-1}\mathcal{W}\mathcal{R}d$. For example, we can use L2-norm misfit function $f(x, y) = \|x - y\|_2^2$ and the corresponding misfit function would be modified to:

$$J_{L2} = \|p - \mathcal{R}^{-1}\mathcal{W}\mathcal{R}d\|_2^2. \quad (16)$$

The symbol $\mathcal{W}$ denotes a processing strategy, e.g., muting for specific slope values. Using Radon transform as a preprocessing tool in FWI in this way requires that we recover reliable amplitudes, i.e., the operator $\mathcal{R}^{-1}$ should be a good approximation of the inverse operator of the forward Radon transform $\mathcal{R}$. Also, in this approach, the measured data will be transformed back to the time-space domain, and we measure the distance in $t - x$ domain. Besides, as the Radon transform is used as a preprocessing strategy, in real applications, the preprocessing step can be separated from the inversion step and involves only one time of the forward and inverse Radon transform.

In our approach, the Radon transform is coupled with the inversion algorithm. In general, the misfit function in the Radon domain can be formulated as

$$J = f(\mathcal{R}p, \mathcal{R}d). \quad (17)$$

Here, compared with equation 15, we transform both the predicted data and the measured data into the Radon domain, and this step corresponds to equations 9 and 10. Function f serves as the same aim of measuring the distance as that in equation 15. The difference is that in equation 15, the distance measurement is in the $t - x$ domain while here, in equation 17, it is in the $\tau - p$ domain. For the Radon-domain matching-filter approach, the distance measurement function is defined in equation 12, where we measure the focusing by applying a penalty to the resulting matching filter in the $\tau - p$ domain.

The events will be separated by their slopes in the Radon domain, and the matching filter is

computed using equation 11 for each trace indexed by the same slope value, and thus, can reduce the cross-talks between different events in the original $t-x$ domain. The effectiveness of the Radon-domain approach depends on the property of the misfit function f . When the misfit function f involves a nonlinear operation between the measured data p and the predicted data d , the difference between the Radon domain and the original time-space domain would be significant. Otherwise, for example, if we use a misfit function such as L2 norm which only includes a linear operation (subtraction), the difference would be minor. For example, considering f in equation 17 as the L2-norm misfit function, we have

$$J = \frac{1}{2} \|\mathcal{R}p - \mathcal{R}d\|_2^2. \quad (18)$$

We immediately get the adjoint source as

$$\delta s = \mathcal{R}^T \mathcal{R}(p - d). \quad (19)$$

Compared with the conventional time-domain adjoint source $p - d$, the Radon-domain adjoint source has a weight $\mathcal{R}^T \mathcal{R}$; it will not achieve the objective of measuring the misfit for separated events with a single p-value and will not add to the benefits of reducing cycle skipping. This explains why for the previous Marmousi example, the L2 norm and the Radon-domain L2 norm gave similar results. For our matching-filter approach, as the misfit function is nonlinear (deconvolution plus penalty), the Radon-domain approach as we demonstrated before does help in reducing the cross-talks between different events and makes the corresponding misfit function more robust. Other misfit functions like the normalized global crosscorrelation, envelope, and optimal transport misfit functions can also gain from incorporating the Radon-domain approach.

In the Radon-domain misfit-function design, we apply the Radon transform to both predicted and measured data at the same time. Thus, when we compute the adjoint source,

we just need to apply the adjoint Radon transform $\mathcal{R}^T$ as the adjoint-state analysis suggests. We do not need to apply an inverse Radon transform $\mathcal{R}^{-1}$. However, unlike using the Radon transform as a preprocessing tool, the computation of the Radon transform is needed in every iteration of the inversion process whenever we compute the misfit function and the adjoint source. Thus, we need an efficient implementation of the Radon transform and its adjoint.

Robust and efficient computation of the Radon transform

For an efficient and robust implementation of the Radon transform, we need to take a look at the time, space and slope sampling (Turner, 1990). Sampling in the τ direction is controlled by the frequency content of the data:

$$\Delta\tau \leq \frac{1}{2f_{\max}}, \quad (20)$$

where $\Delta\tau$ is the spacing of τ samples and $f_{\max}$ is the maximum frequency used in the inversion. For a maximum frequency of 10 Hz, $\Delta\tau = 1/(2 * 10) = 0.05$ s. This value could be one order of magnitude larger than the time sampling used in finite-difference modeling of the wave equation. Thus, acceleration of the Radon transform could be achieved by recognizing that fewer samples are needed at low frequencies, resulting in faster implementations. In order to avoid spatial aliasing, the spacing of the seismic traces Δx needs to satisfy

$$\Delta x \leq \frac{1}{p_r f_{\max}}, \quad (21)$$

where p_r is the range of p values. Considering we compute the slope value range from $-1/1500$ s/m to $1/1500$ s/m with maximum frequency $f_{\max} = 10$ Hz, the spacing $\Delta x = 1/(2/1500 * 10) = 75$ m. If the actual spacing of the seismic traces is larger than this value, an interpolation scheme must be applied first.

Similarly, to avoid aliasing in the p direction, the spacing of the p -value should adhere to

$$\Delta p < \frac{1}{x_h f_{\max}}, \quad (22)$$

where x_h is the size of the local window. Given $x_h = 600$ m and $f_{\max} = 10$ Hz, the spacing of p is given by : $\Delta p < 1/(600 * 10) = 1/6000$ s/m, thus we can choose $\Delta p = 1/12000$ s/m to be on the safe side. Suppose that we want to compute the range of p values from $-1/1500$ s/m to $1/1500$ s/m, the total number of p samples needs to be computed is given by $n_p = (2/1500)/(1/12000) = 16$. For a lower maximum frequency, the total number of p samples will be less.

Except for the sampling issues, a recursive algorithm can be employed for efficient computation of the local Radon transform defined in equation 7. Take the 2D problem as an example, suppose a common-shot record has n_r traces and n_t time samples, if we choose n_p slopes and the local window size n_h , a naive application of the local Radon transform of equation 7 would have a complexity $O(n_p n_h n_t n_r)$. Instead, we use a fast local Radon transform as demonstrated by Liu et al. (2016), and the computation cost is dramatically reduced. The basic principle is that the 2D version of the local Radon transform of equation 7 can be equivalently rewritten in the frequency domain as :

$$\tilde{Q}(\omega, p, x_r) = e^{-i\omega p x_r} \sum_{x'_r = x_r - \delta x_r}^{x_r + \delta x_r} Q(\omega, x'_r) e^{-i\omega p x'_r}, \quad (23)$$

where ω is the angular temporal frequency. The functions $\tilde{Q}$ and Q are the Fourier-transformed data of $\tilde{q}$ and q in equation 7, respectively. Figure 26 demonstrates how to compute $\tilde{Q}$ using equation 23 recursively. For two consecutive windows along the x_r direction, there are only two distinct unshared x locations. Thus, a recursive algorithm can be developed for computing equation 23: when a window with range $[x_r - \delta x_r, x_r + \delta x_r]$ moves

to the next sample, we only need to remove the contribution of the point $x_r - \delta x_r$, which is leaving the new window and add the contribution of the new point $x_r + \delta x_r + 1$. This recursive algorithm avoids the summation over local windows and improves the efficiency. For the 3D cases, we can apply the 2D local Radon transform in the inline direction only and this would be particularly ideal for the narrow azimuth datasets.

CONCLUSION

We proposed to implement the matching-filter computation in the Radon domain in Full Waveform Inversion to reduce the cross-talks between different events in the deconvolution of the measured and predicted data. This features of the Radon domain approach helped to admit a more robust objective function. Using the Marmousi model, we tested the proposed Radon-domain matching-filter method and compared with the L2 norm and conventional matching-filter approach. The results show that the proposed Radon-domain approach can reduce the cycle-skipping effectively compared to the L2-norm approach and shows better performance than conventional matching-filter approach. The Chevron 2014 FWI benchmark dataset examples further demonstrate the effectiveness of the proposed Radon-domain matching-filter approach.

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LIST OF FIGURES

1 a) The common shot gather from a real dataset; b) its Radon transform. The blue
arrows point to the direct arrival and the reflection event which may introduce crosstalks
during the deconvolution operation. Those two events have different slopes and can be sep-
arated effectively in the $\tau - p$ domain; The white arrows point to two events with distinct
slopes: one of them can be noise and one is the signal, they can be separated in the $\tau - p$
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2 a) The measured data $d(t, x)$ and b) their Radon transform; c) the predicted data
 $p(t, x)$ and d) their Radon transform; overlap of the measured and predicted data e) in the
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consist of two events, in this setup, the predicted data shifts a constant time relative to the
measure data, i.e., $p(t, x) = d(t - \tau, x)$. Here the time shift $\tau = 0.3$ s.

3 Resulting matching filter by deconvolution of the measured data and the predicted
data : a) the matching filter computed in time-space domain, i.e., computed by deconvolu-
tion of Figures 2a and 2c per trace. b) the matching filter computed in the Radon domain,
i.e., computed by the deconvolution of Figure 2b and Figure 2d per trace.

4 The misfit function value for the time-domain matching-filter approach (doted
black line) and the Radon-domain matching-filter approach (solid red line). For the setup
of the experiment, please refer to Figure 2. In this step, the time-space domain and the
Radon-domain approach show similar convexity.

5 a) The measured data $d(t, x)$ and b) their Radon transform; c) the predicted data
 $p(t, x)$ and d) their Radon transform; overlap of the measured and the predicted data e) in
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with distinct slopes, i.e., $d(t, x) = d_A(t, x) + d_B(t, x)$. In this setup, the predicted data have

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6 Resulting matching filter by the deconvolution of the measured data and the predicted data : a) the matching filter computed in the time-space domain, i.e., computed by the deconvolution of Figures 5a and 5c per trace. b) the matching filter computed in the Radon domain, i.e., computed by the deconvolution of Figures 5b and 5d per trace.

7 The misfit-function value for the time-domain matching-filter approach (dotted black line) and the Radon-domain matching-filter approach (solid red line). For the setup of the experiment, please refer to Figure 5. In this setup, the Radon-domain matching-filter approach shows improved convexity than the conventional time-space domain approach.

8 a) The true velocity ; and b) the initial velocity for the modified Marmousi model. In the example, 80 shots and 400 receivers are distributed at depth 20 m with a source interval of 100 m and receiver interval of 20 m.

9 a) The measured data ; b) the overlay of the measured data and the modeled data by initial model for a shot at distance 2 km. The data is modeled with an absorbing boundary condition at the surface. The red curves indicate the measured data (the true model) while the black curves indicate the modeled data (initial model).

10 The inverted velocity using a) the L2 norm ; b) the Radon-domain L2 norm; c) the conventional matching-filter approach; d) the proposed Radon-domain matching-filter approach.

11 The overlay of the measured data (the red curves) for the true model and the modeled data (the black curves) for the model inverted by a) the L2 norm; b) the Radon-domain L2 norm; c) the conventional matching-filter approach; d) the proposed Radon domain matching-filter approach. Note the data is modeled with an absorbing boundary

condition at the surface.

12 The inverted models using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

13 The inverted models using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

14 The resulting matching-filter distribution using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

15 The resulting matching-filter distribution using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

16 a) The measured data ; b) the overlay of the measured data and the modeled data by initial model for a shot at distance 2 km. The data is modeled with a free-surface boundary condition. The red curves indicate the measured data (the true model) while the black curves indicate the modeled data (initial model).

17 The inverted velocity using a) the L2 norm ; b) the conventional matching-filter approach; c) the proposed Radon-domain matching-filter misfit function.

18 The overlay of the measured data (the red curves) for the true model and the modeled data (the black curves) for the model inverted by a) the L2 norm; b) the conventional matching-filter approach; c) the proposed Radon-domain matching-filter misfit function. Note the data is modeled with a free-surface boundary condition.

19 The resulting matching-filter distribution using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

20 The resulting matching-filter distribution using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

21 a) The initial model ; b) The inverted model for Chevron 2014 FWI Benchmark

dataset.

22 Well log comparison for Chevron 2014 FWI Benchmark dataset.

23 Comparison of the common shot record for the observed (left panel) and modeled using the inverted model (right panel).

24 RTM image of a) the initial model ; b) the inverted model.

25 Angle Domain Common Image Gathers for a) the initial model ; b) the inverted model.

26 Schematic diagram for recursive local Radon transform. Two consecutive windows shared the same black points where only two red points need to be handled, one subtracted out and one added in for the computation in window 2.

Figure 1 a) The common shot gather from a real dataset; b) its Radon transform. The blue arrows point to the direct arrival and the reflection event which may introduce crosstalks during the deconvolution operation. Those two events have different slopes and can be separated effectively in the τ - p domain; The white arrows point to two events with distinct slopes: one of them can be noise and one is the signal, they can be separated in the τ - p domain as well.

338x190mm (72 x 72 DPI)

Figure 2a a) The measured data $d(t, x)$ and b) their Radon transform; c) the predicted data $p(t, x)$ and d) their Radon transform; overlap of the measured and predicted data e) in the time-space domain and f) the Radon domain. Although the measured and predicted data consist of two events, in this setup, the predicted data shifts a constant time relative to the measure data, i.e., $p(t, x) = d(t - \tau, x)$. Here the time shift $\tau = 0.3$ s.

368x364mm (72 x 72 DPI)

Figure 2b a) The measured data $d(t, x)$ and b) their Radon transform; c) the predicted data $p(t, x)$ and d) their Radon transform; overlap of the measured and predicted data e) in the time-space domain and f) the Radon domain. Although the measured and predicted data consist of two events, in this setup, the predicted data shifts a constant time relative to the measure data, i.e., $p(t, x) = d(t - \tau, x)$. Here the time shift $\tau = 0.3$ s.

368x364mm (72 x 72 DPI)

Figure 2c a) The measured data $d(t, x)$ and b) their Radon transform; c) the predicted data $p(t, x)$ and d) their Radon transform; overlap of the measured and predicted data e) in the time-space domain and f) the Radon domain. Although the measured and predicted data consist of two events, in this setup, the predicted data shifts a constant time relative to the measure data, i.e., $p(t, x) = d(t - \tau, x)$. Here the time shift $\tau = 0.3$ s.

367x363mm (72 x 72 DPI)

Figure 2d a) The measured data $d(t, x)$ and b) their Radon transform; c) the predicted data $p(t, x)$ and d) their Radon transform; overlap of the measured and predicted data e) in the time-space domain and f) the Radon domain. Although the measured and predicted data consist of two events, in this setup, the predicted data shifts a constant time relative to the measure data, i.e., $p(t, x) = d(t - \tau, x)$. Here the time shift $\tau = 0.3$ s.

367x363mm (72 x 72 DPI)

Figure 2e a) The measured data $d(t, x)$ and b) their Radon transform; c) the predicted data $p(t, x)$ and d) their Radon transform; overlap of the measured and predicted data e) in the time-space domain and f) the Radon domain. Although the measured and predicted data consist of two events, in this setup, the predicted data shifts a constant time relative to the measure data, i.e., $p(t, x) = d(t - \tau, x)$. Here the time shift $\tau = 0.3$ s.

367x363mm (72 x 72 DPI)

Figure 2f a) The measured data $d(t, x)$ and b) their Radon transform; c) the predicted data $p(t, x)$ and d) their Radon transform; overlap of the measured and predicted data e) in the time-space domain and f) the Radon domain. Although the measured and predicted data consist of two events, in this setup, the predicted data shifts a constant time relative to the measure data, i.e., $p(t, x) = d(t - \tau, x)$. Here the time shift $\tau = 0.3$ s.

367x363mm (72 x 72 DPI)

Figure 3a Resulting matching filter by deconvolution of the measured data and the predicted data : a) the matching filter computed in time-space domain, i.e., computed by deconvolu- tion of Figures 2a and 2c per trace. b) the matching filter computed in the Radon domain, i.e., computed by the deconvolution of Figure 2b and Figure 2d per trace.

367x363mm (72 x 72 DPI)

Figure 3b Resulting matching filter by deconvolution of the measured data and the predicted data : a) the matching filter computed in time-space domain, i.e., computed by deconvolution of Figures 2a and 2c per trace. b) the matching filter computed in the Radon domain, i.e., computed by the deconvolution of Figure 2b and Figure 2d per trace.

367x363mm (72 x 72 DPI)

Figure 4 The misfit function value for the time-domain matching-filter approach (doted black line) and the Radon-domain matching-filter approach (solid red line). For the setup of the experiment, please refer to Figure 2. In this step, the time-space domain and the Radon-domain approach show similar convexity.

474x260mm (72 x 72 DPI)

Figure 5a a) The measured data $d(t, x)$ and b) their Radon transform; c) the predicted data $p(t, x)$ and d) their Radon transform; overlap of the measured and the predicted data e) in the time-space domain and f) the Radon domain. The measured data consist of two events with distinct slopes, i.e., $d(t, x) = dA(t, x) + dB(t, x)$. In this setup, the predicted data have the same event A while event B has a constant time shift, i.e., $p(t, x) = dA(t, x) + dB(t - \tau, x)$. Here, the time shift $\tau = 0.3$ s.

368x364mm (72 x 72 DPI)

Figure 5b a) The measured data $d(t, x)$ and b) their Radon transform; c) the predicted data $p(t, x)$ and d) their Radon transform; overlap of the measured and the predicted data e) in the time-space domain and f) the Radon domain. The measured data consist of two events with distinct slopes, i.e., $d(t, x) = dA(t, x) + dB(t, x)$. In this setup, the predicted data have the same event A while event B has a constant time shift, i.e., $p(t, x) = dA(t, x) + dB(t - \tau, x)$. Here, the time shift $\tau = 0.3$ s.

368x364mm (72 x 72 DPI)

Figure 5c a) The measured data $d(t, x)$ and b) their Radon transform; c) the predicted data $p(t, x)$ and d) their Radon transform; overlap of the measured and the predicted data e) in the time-space domain and f) the Radon domain. The measured data consist of two events with distinct slopes, i.e., $d(t, x) = dA(t, x) + dB(t, x)$. In this setup, the predicted data have the same event A while event B has a constant time shift, i.e., $p(t, x) = dA(t, x) + dB(t - \tau, x)$. Here, the time shift $\tau = 0.3$ s.

367x363mm (72 x 72 DPI)

367x363mm (72 x 72 DPI)

Figure 5e a) The measured data $d(t, x)$ and b) their Radon transform; c) the predicted data $p(t, x)$ and d) their Radon transform; overlap of the measured and the predicted data e) in the time-space domain and f) the Radon domain. The measured data consist of two events with distinct slopes, i.e., $d(t, x) = dA(t, x) + dB(t, x)$. In this setup, the predicted data have the same event A while event B has a constant time shift, i.e., $p(t, x) = dA(t, x) + dB(t - \tau, x)$. Here, the time shift $\tau = 0.3$ s.

367x363mm (72 x 72 DPI)

Figure 5f a) The measured data $d(t, x)$ and b) their Radon transform; c) the predicted data $p(t, x)$ and d) their Radon transform; overlap of the measured and the predicted data e) in the time-space domain and f) the Radon domain. The measured data consist of two events with distinct slopes, i.e., $d(t,x) = dA(t,x)+dB(t,x)$. In this setup, the predicted data have the same event A while event B has a constant time shift, i.e., $p(t,x) = dA(t,x)+dB(t-\tau,x)$. Here, the time shift $\tau = 0.3$ s.

367x363mm (72 x 72 DPI)

Figure 6a Resulting matching filter by the deconvolution of the measured data and the predicted data : a) the matching filter computed in the time-space domain, i.e., computed by the deconvolution of Figures 5a and 5c per trace. b) the matching filter computed in the Radon domain, i.e., computed by the deconvolution of Figures 5b and 5d per trace.

367x363mm (72 x 72 DPI)

Figure 6b Resulting matching filter by the deconvolution of the measured data and the predicted data : a) the matching filter computed in the time-space domain, i.e., computed by the deconvolution of Figures 5a and 5c per trace. b) the matching filter computed in the Radon domain, i.e., computed by the deconvolution of Figures 5b and 5d per trace.

367x363mm (72 x 72 DPI)

Figure 7 The misfit-function value for the time-domain matching-filter approach (dotted black line) and the Radon-domain matching-filter approach (solid red line). For the setup of the experiment, please refer to Figure 5. In this setup, the Radon-domain matching-filter approach shows improved convexity than the conventional time-space domain approach.

463x260mm (72 x 72 DPI)

Figure 8a a) The true velocity ; and b) the initial velocity for the modified Marmousi model. In the example, 80 shots and 400 receivers are distributed at depth 20 m with a source interval of 100 m and receiver interval of 20 m.

1270x762mm (72 x 72 DPI)

Figure 8b a) The true velocity ; and b) the initial velocity for the modified Marmousi model. In the example, 80 shots and 400 receivers are distributed at depth 20 m with a source interval of 100 m and receiver interval of 20 m.

1270x762mm (72 x 72 DPI)

Figure 9a a) The measured data ; b) the overlay of the measured data and the modeled data by initial model for a shot at distance 2 km. The data is modeled with an absorbing boundary condition at the surface. The red curves indicate the measured data (the true model) while the black curves indicate the modeled data (initial model).

287x471mm (72 x 72 DPI)

Figure 9b a) The measured data ; b) the overlay of the measured data and the modeled data by initial model for a shot at distance 2 km. The data is modeled with an absorbing boundary condition at the surface. The red curves indicate the measured data (the true model) while the black curves indicate the modeled data (initial model).

472x471mm (72 x 72 DPI)

Figure 10a The inverted velocity using a) the L2 norm ; b) the Radon-domain L2 norm; c) the conventional matching-filter approach; d) the proposed Radon-domain matching-filter approach.

1270x762mm (72 x 72 DPI)

Figure 10b The inverted velocity using a) the L2 norm ; b) the Radon-domain L2 norm; c) the conventional matching-filter approach; d) the proposed Radon-domain matching-filter approach.

1270x762mm (72 x 72 DPI)

Figure 10c The inverted velocity using a) the L2 norm ; b) the Radon-domain L2 norm; c) the conventional matching-filter approach; d) the proposed Radon-domain matching-filter approach.

1270x762mm (72 x 72 DPI)

Figure 10d The inverted velocity using a) the L2 norm ; b) the Radon-domain L2 norm; c) the conventional matching-filter approach; d) the proposed Radon-domain matching-filter approach.

1270x762mm (72 x 72 DPI)

Figure 11a The overlay of the measured data (the red curves) for the true model and the modeled data (the black curves) for the model inverted by a) the L2 norm; b) the Radon- domain L2 norm; c) the conventional matching-filter approach; d) the proposed Radon domain matching-filter approach. Note the data is modeled with an absorbing boundary condition at the surface.

472x471mm (72 x 72 DPI)

Figure 11b The overlay of the measured data (the red curves) for the true model and the modeled data (the black curves) for the model inverted by a) the L2 norm; b) the Radon-domain L2 norm; c) the conventional matching-filter approach; d) the proposed Radon domain matching-filter approach. Note the data is modeled with an absorbing boundary condition at the surface.

472x471mm (72 x 72 DPI)

Figure 11c The overlay of the measured data (the red curves) for the true model and the modeled data (the black curves) for the model inverted by a) the L2 norm; b) the Radon- domain L2 norm; c) the conventional matching-filter approach; d) the proposed Radon domain matching-filter approach. Note the data is modeled with an absorbing boundary condition at the surface.

472x471mm (72 x 72 DPI)

Figure 11d The overlay of the measured data (the red curves) for the true model and the modeled data (the black curves) for the model inverted by a) the L2 norm; b) the Radon- domain L2 norm; c) the conventional matching-filter approach; d) the proposed Radon domain matching-filter approach. Note the data is modeled with an absorbing boundary condition at the surface.

472x471mm (72 x 72 DPI)

Figure 12a The inverted models using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

1270x762mm (72 x 72 DPI)

Figure 12b The inverted models using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

1270x762mm (72 x 72 DPI)

Figure 12c The inverted models using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

1270x762mm (72 x 72 DPI)

Figure 12d The inverted models using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

1270x762mm (72 x 72 DPI)

Figure 12e The inverted models using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

1270x762mm (72 x 72 DPI)

Figure 12f The inverted models using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

1270x762mm (72 x 72 DPI)

Figure 13a The inverted models using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

1270x762mm (72 x 72 DPI)

Figure 13b The inverted models using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

1270x762mm (72 x 72 DPI)

Figure 13c The inverted models using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

1270x762mm (72 x 72 DPI)

Figure 13d The inverted models using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

1270x762mm (72 x 72 DPI)

Figure 13e The inverted models using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

1270x762mm (72 x 72 DPI)

Figure 13f The inverted models using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

1270x762mm (72 x 72 DPI)

Figure 14a The resulting matching-filter distribution using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x541mm (72 x 72 DPI)

Figure 14b The resulting matching-filter distribution using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x541mm (72 x 72 DPI)

Figure 14c The resulting matching-filter distribution using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x541mm (72 x 72 DPI)

Figure 14d The resulting matching-filter distribution using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x541mm (72 x 72 DPI)

Figure 14e The resulting matching-filter distribution using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x541mm (72 x 72 DPI)

Figure 14f The resulting matching-filter distribution using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x541mm (72 x 72 DPI)

Figure 15a The resulting matching-filter distribution using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x489mm (72 x 72 DPI)

Figure 15b The resulting matching-filter distribution using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x489mm (72 x 72 DPI)

Figure 15c The resulting matching-filter distribution using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x489mm (72 x 72 DPI)

Figure 15d The resulting matching-filter distribution using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x489mm (72 x 72 DPI)

Figure 15e The resulting matching-filter distribution using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x489mm (72 x 72 DPI)

Figure 15f The resulting matching-filter distribution using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x489mm (72 x 72 DPI)

Figure 16a a) The measured data ; b) the overlay of the measured data and the modeled data by initial model for a shot at distance 2 km. The data is modeled with a free-surface boundary condition. The red curves indicate the measured data (the true model) while the black curves indicate the modeled data (initial model).

287x471mm (72 x 72 DPI)

Figure 16b a) The measured data ; b) the overlay of the measured data and the modeled data by initial model for a shot at distance 2 km. The data is modeled with a free-surface boundary condition. The red curves indicate the measured data (the true model) while the black curves indicate the modeled data (initial model).

472x471mm (72 x 72 DPI)

Figure 17a The inverted velocity using a) the L2 norm ; b) the conventional matching-filter approach; c) the proposed Radon-domain matching-filter misfit function.

1270x762mm (72 x 72 DPI)

Figure 17b The inverted velocity using a) the L2 norm ; b) the conventional matching-filter approach; c) the proposed Radon-domain matching-filter misfit function.

1270x762mm (72 x 72 DPI)

Figure 17c The inverted velocity using a) the L2 norm ; b) the conventional matching-filter approach; c) the proposed Radon-domain matching-filter misfit function.

1270x762mm (72 x 72 DPI)

Figure 18a The overlay of the measured data (the red curves) for the true model and the modeled data (the black curves) for the model inverted by a) the L2 norm; b) the conventional matching-filter approach; c) the proposed Radon-domain matching-filter misfit function. Note the data is modeled with a free-surface boundary condition.

472x471mm (72 x 72 DPI)

Figure 18b The overlay of the measured data (the red curves) for the true model and the modeled data (the black curves) for the model inverted by a) the L2 norm; b) the conventional matching-filter approach; c) the proposed Radon-domain matching-filter misfit function. Note the data is modeled with a free-surface boundary condition.

472x471mm (72 x 72 DPI)

Figure 18c The overlay of the measured data (the red curves) for the true model and the modeled data (the black curves) for the model inverted by a) the L2 norm; b) the conventional matching-filter approach; c) the proposed Radon-domain matching-filter misfit function. Note the data is modeled with a free-surface boundary condition.

472x471mm (72 x 72 DPI)

Figure 19a The resulting matching-filter distribution using the proposed Radon-domain matching- filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x533mm (72 x 72 DPI)

Figure 19b The resulting matching-filter distribution using the proposed Radon-domain matching- filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x533mm (72 x 72 DPI)

Figure 19c The resulting matching-filter distribution using the proposed Radon-domain matching- filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x533mm (72 x 72 DPI)

Figure 19d The resulting matching-filter distribution using the proposed Radon-domain matching- filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x533mm (72 x 72 DPI)

Figure 19e The resulting matching-filter distribution using the proposed Radon-domain matching- filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x533mm (72 x 72 DPI)

Figure 19f The resulting matching-filter distribution using the proposed Radon-domain matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x533mm (72 x 72 DPI)

Figure 20a The resulting matching-filter distribution using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x489mm (72 x 72 DPI)

Figure 20b The resulting matching-filter distribution using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x489mm (72 x 72 DPI)

Figure 20c The resulting matching-filter distribution using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x489mm (72 x 72 DPI)

Figure 20d The resulting matching-filter distribution using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x489mm (72 x 72 DPI)

Figure 20e The resulting matching-filter distribution using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x489mm (72 x 72 DPI)

Figure 20f The resulting matching-filter distribution using the conventional matching-filter method at iteration a) 10; b) 20; c) 40; d) 50; e) 100; f) 200.

859x489mm (72 x 72 DPI)

Figure 21a a) The initial model ; b) The inverted model for Chevron 2014 FWI Benchmark dataset.

336x143mm (72 x 72 DPI)

Figure 21b a) The initial model ; b) The inverted model for Chevron 2014 FWI Benchmark dataset.

336x143mm (72 x 72 DPI)

Figure 22 Well log comparison for Chevron 2014 FWI Benchmark dataset.

59x128mm (72 x 72 DPI)

Figure 23 Comparison of the common shot record for the observed (left panel) and modeled using the inverted model (right panel).

254x190mm (72 x 72 DPI)

Figure 24a RTM image of a) the initial model ; b) the inverted model.

338x190mm (72 x 72 DPI)

Figure 24b RTM image of a) the initial model ; b) the inverted model.

338x190mm (72 x 72 DPI)

Figure 25a Angle Domain Common Image Gathers for a) the initial model ; b) the inverted model.

338x190mm (72 x 72 DPI)

Figure 25b Angle Domain Common Image Gathers for a) the initial model ; b) the inverted model.

338x190mm (72 x 72 DPI)

Figure 26 Schematic diagram for recursive local Radon transform. Two consecutive windows shared the same black points where only two red points need to be handled, one subtracted out and one added in for the computation in window 2.

360x99mm (72 x 72 DPI)

DATA AND MATERIALS AVAILABILITY

Data associated with this research are confidential and cannot be released.