**Poynting and polarization vectors based wavefield
decomposition and their application on elastic reverse time
migration in 2D transversely isotropic media**

Yongming Lu^{1,2}, Qiancheng Liu⁴, Jianfeng Zhang^{1,3*}, Kai Yang¹, Hui Sun^{1,3}

¹*Key Laboratory of Petroleum Resources Research, Institute of Geology and Geophysics, Chinese
Academy of Sciences, Beijing 100029, China.*

²*Department of Earth and Space Sciences, Southern University of Science and Technology,
Shenzhen 518055, China.*

³*University of Chinese Academy of Sciences, Beijing 100049, China.*

⁴*King Abdullah University of Science and Technology, Thuwal, 23955, Saudi Arabia.*

**Corresponding author*

Email: zhangjf@mail.iggcas.ac.cn

(February 26, 2019)

Running head: **ERTM in anisotropic media**

ABSTRACT

With the progress in computational power and seismic acquisition, elastic reverse time migration is becoming increasingly feasible and helpful in characterizing the physical properties of subsurface structures. To achieve high-resolution seismic imaging using elastic reverse time migration, it is necessary to separate the compressional (P-wave) and shear (S-wave) waves for both isotropic and anisotropic media. In elastic isotropic media, the conventional method for wave-mode separation is to use the divergence and

curl operators. However, in anisotropic media, the polarization direction of P waves is

This article has been accepted for publication and undergone full peer review but has not been through the copyediting, typesetting, pagination and proofreading process, which may lead to differences between this version and the Version of Record. Please cite this article as doi: 10.1111/1365-2478.12777

not exactly parallel to the direction of wave propagation. Also, the polarization direction of S waves is not totally perpendicular to the direction of wave propagation. For this reason, the conventional divergence and curl operators show poor performance in anisotropic media. Moreover, conventional methods only perform well in the space domain of regular grids, and they are not suitable for elastic numerical simulation algorithms based on non-regular grids. Besides, these methods distort the original wavefield by taking spatial derivatives. In this case, a new anisotropic wave-mode separation scheme is developed using Poynting vectors. This scheme can be performed in the angle domain by constructing the relationship between group and polarization angles of different wave modes. Also, it is performed pointwise, independent of adjacent space points, suitable for parallel computing. Moreover, there is no need to correct the changes in phase and amplitude caused by the derivative operators. By using this scheme, the anisotropic elastic reverse time migration is more efficiently performed on the unstructured mesh. The effectiveness of our scheme is verified by several numerical examples.

Keywords: Unstructured mesh, Elastic reverse time migration, Anisotropic wave-mode separation, Poynting vectors,

INTRODUCTION

Elastic reverse time migration (ERTM) has aroused increasing attention from geophysicists because of the development of multi-component acquisition techniques and high-performance computing. Compared with the conventional single-component imaging and approximate anisotropic acoustic wave equations (Alkhalifah 1998), ERTM can provide more physical information about the characterization of wave propagation and amplitude (Yan and Sava 2008) for isotropic and anisotropic media. In recent years, much research has been focused on multicomponent isotropic ERTM (Chang and McMechan 1987; Yan and Sava 2008; Wang and McMechan 2015; Du et al. 2012, 2017). However, anisotropic ERTM has been rarely discussed.

Prestack ERTM can efficiently image complex geologic structures. However, early attempts in imaging employ Cartesian coordinates for ERTM (Chang and McMechan 1987, 1994). These migrated images contain errors and noise due to P- and S-waves that are mixed in horizontal and vertical components (Yan and Sava 2008). To reduce crosstalk artifacts and provide more accurate images with clear physical meaning, an alternative scheme for ERTM imaging employs pure wave modes. Therefore, it is necessary to separate P- and S-wave modes before performing the imaging condition.

In elastic isotropic media, the wave-mode separation can be achieved by computing the divergence and curl of elastic vector wavefields following Helmholtz theory (Aki and Richards 1980). In anisotropic media, however, the two spatial derivative operators do not work well since the polarization and wave vectors directions are normally non-orthogonal at each grid point. As a result, the P- and S-waves cannot

be entirely separated. To address this problem, Dellinger and Etgen (1990) generalized the divergence and curl operators in the wavenumber domain, and according to Helmholtz decomposition, they achieved the separation in anisotropic media by projecting the whole wavefields onto the qP- and qS-waves polarization directions, respectively. However, their method is only suitable for a homogeneous model. To separate the qP- and qS-wave modes for VTI and TTI media, Yan and Sava (2009) proposed quasi-derivative wave-mode separation operators, whereas the computational cost is high. To improve efficiency, Yan and Sava (2011) performed the anisotropic wavefield separation in a mixed domain by means of the PSPI (phase-shift plus interpolation) using the one-way wave equation. Then, Zhang and McMechan (2010) separated the wave-modes for isotropic and VTI media using vector decomposition in the wavenumber domain. However, when a model consists of continuously varying parameters, it will become difficult to perform the scheme proposed by Zhang and McMechan (2010). Cheng and Fomel (2014) constructed a fast algorithm by rewriting the space-wavenumber domain operations as the Fourier integral operators and then numerically solving the operators based on the low-rank approximation. The low-rank approximation reduces the cost to a certain extent. To improve computational efficiency, Cheng et al. (2016) developed an efficient method to propagate and decouple the elastic waves using the low-rank approximations for anisotropic media. Zhou and Wang (2017) introduced efficient anisotropic separation operators that are performed by locally rotating wave vectors to the qP-wave polarization directions. They first calculate the deviation angles between these two vectors, which are estimated using the Poynting vectors. Then, they rotate the divergence and curl operators to

the directions of the polarization vectors based on the deviation angles. Obviously, the separation results have the same form as that of applying divergence and curl operators, and the waveform changes are inevitable. Hence, additional calculations are required to correct the phase and amplitude of the input wavefields. In anisotropic media, Oh et al. (2018) developed an efficient approach using the divergence and curl with the excitation method. They computed the excitation time of the divergence of wavefields to choose the excitation time of the P-wave. Compared with conventional mode-separated anisotropic FWI, the P-wave ExA approach is more efficient.

In brief, these above methods are based on the structured mesh, and cannot be well used on the unstructured mesh, such as the spectral-element method (Komatitsch and Vilotte 1998) and the grid method (Zhang and Liu 1999). To address these problems, we propose a method to separate the wavefields for anisotropic media by establishing the numerical relationship between the group and polarization angles and then perform the separation according to the polarization angles. This procedure can be used in both the structured and unstructured mesh with high computing efficiency. Moreover, the proposed method preserves the amplitude and phase information compared with conventional pseudo-derivative methods with the correction of the waveform changes (Zhou and Wang 2017).

The layout of this paper is as follows: First, we explain how to efficiently construct the numerical relationship between the group and polarization angles. Then, several numerical examples are presented to verify the validity and limitations of the proposed scheme. Finally, the proposed scheme is applied in anisotropic ERTM to prove that it can achieve high-quality imaging even for complicated anisotropic media.

METHODS

Separation method

In anisotropic media, the elastic wave equation is given by

$$\begin{aligned}\frac{\partial u_i}{\partial t} &= v_i, \\ \rho \frac{\partial v_i}{\partial t} &= \frac{\partial \tau_{ij}}{\partial x_j}, \\ \frac{\partial \tau_{ij}}{\partial t} &= \frac{1}{2} c_{ijkl} \left(\frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k} \right), i, j, k, l = 1, \dots, 3,\end{aligned}\tag{1}$$

where ρ represents the density, t represents the time, $\mathbf{v}$ represents the particle velocity, $\mathbf{u}$ represents the particle displacement, τ_{ij} represents the stress tensor, $\mathbf{x}$ represents the space coordinate system and c_{ijkl} represents the stiffness tensor. To solve equation (1), we use the grid method to integrate both sides of equation (1) as shown in Fig. 1, which is an irregular, unstructured mesh-based numerical algorithm developed by Zhang and Liu (1999, 2010). The grid method with a double-mesh implementation (Yang et al. 2017) is adopted to improve the computational efficiency in this study.

In elastic isotropic media, the P- and S-wave modes can be obtained by applying the divergence and curl operators to vector wavefield (Yan and Sava 2009):

$$P(\mathbf{x}, t) = \nabla \cdot \mathbf{u}(\mathbf{x}, t), \mathbf{S}(\mathbf{x}, t) = \nabla \times \mathbf{u}(\mathbf{x}, t),\tag{2}$$

where $\mathbf{x} = (x, y, z)$ is the Cartesian coordinate system, $\mathbf{u}$ is the particle displacement, t is the time and $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$. The P and $\mathbf{S}$ denote the scalar and vector potential wavefields, respectively. In the wavenumber-domain, equation (2) can be rewritten as follows:

$$\tilde{P}(\mathbf{k}) = i\mathbf{k} \cdot \tilde{\mathbf{u}}(\mathbf{k}), \tilde{\mathbf{S}}(\mathbf{k}) = i\mathbf{k} \times \tilde{\mathbf{u}}(\mathbf{k}),\tag{3}$$

where i represents the imaginary unit, $\mathbf{k} = (k_x, k_y, k_z)$ denotes the wavenumber vector. Here, $i\mathbf{k}$ can be replaced by the $\frac{iw}{v} \frac{\mathbf{k}}{|\mathbf{k}|}$ (Duan and Sava 2015), v represents the migration velocity, and w represents the angular frequency. As we can see, the original amplitude and phase characteristics of $\mathbf{u}(\mathbf{x}, t)$ are destroyed by the derivative operator. Note that the key factor to determine the separation of the wavefields is the unit vector $\frac{\mathbf{k}}{|\mathbf{k}|}$. In 2D media, the unit vector can be expressed as $(\cos \theta, \sin \theta)$, where θ is the phase angle. In elastic isotropic media, the directions of phase and group velocities coincide with each other. Hence, the phase angle θ can be replaced by the group angle ψ , and we perform the wave-mode separation in the angle domain as follows

$$\begin{aligned} U_P &= u_x \cos \psi + u_z \sin \psi, \\ U_{SV} &= -u_x \sin \psi + u_z \cos \psi, \end{aligned} \tag{4}$$

where U_P and U_{SV} are the separated wavefields of P-wave and S-wave, respectively. However, the directions of phase and group velocities are not consistent with each other in anisotropic media (Yan and Dickens 2016; McGarry and Qin 2013; Lu and Liu 2018). As shown in Fig. 2, the direction vectors of phase velocity are indicated by the red arrows, and the direction vectors of group velocity are indicated by the blue arrows. As a result, equation (4) is invalid for anisotropic media, and the angle-domain anisotropic wave-mode separation (Liu et al., 2017a) should be used, which is expressed as follows:

$$\begin{aligned} U_P &= u_x \cos v + u_z \sin v, \\ U_{SV} &= -u_x \sin v + u_z \cos v, \end{aligned} \tag{5}$$

where v is the polarization angle. Note that equation (5) is a pointwise separation independent of its neighbor points. This separation is much easier to perform on the

unstructured mesh and run in parallel compared to conventional pseudo-derivative methods. As we can see, in anisotropic media, the key to wave-mode separation is how to construct the relationship between phase and polarization angles. In the following subsections, the relationship will be explained in more details.

Calculation of Poynting vectors

In this section, we demonstrate how to obtain group angles by Poynting vectors. The Poynting vector is first used to estimate the local directions of the receiver and source wavefield propagations by Yoon and Marfurt (2006). Afterwards, Dickens and Winbow (2011), and Yoon et al. (2011) used the Poynting vector to generate angle-domain common-imaging-gathers (ADCIGs) in RTM. Du et al. (2012) used the Poynting vector to derive a polarity reversal correction based method. Wang et al. (2013) employed the Poynting vector in illumination analysis. Xie (2015) introduced the Poynting vector in full-waveform inversion. The Poynting vector has been widely used as it is easy to compute during wavefield extrapolation. However, this method can not handle overlapping events (Tang et al. 2016; Zhou and Wang 2017; Lu and Liu 2018), which we will demonstrate in the following numerical experiment. The elastic Poynting vectors can be expressed as (Červený 2000)

$$s_i = -\tau_{ij}v_j, \quad (6)$$

where i and j represent the x - or z -component of the elastic Poynting vectors $\mathbf{s}$, respectively, τ_{ij} and v_j are the stress tensor and particle velocity, respectively.

To improve the stability of equation (6), a new energy flow density unit vector $\mathbf{n}$

is obtained from the local least squares

$$\mathbf{n} = \frac{\sum_{\Omega} |\mathbf{s}| \mathbf{s}}{\sum_{\Omega} |\mathbf{s}|^2}, \quad (7)$$

where $\sum_{\Omega}$ indicates the summation in the neighborhood of an image point. In ERTM, the Poynting vector represents the energy propagational direction, which is the group velocity direction. As a result, the Poynting vector (equation (7)) is used to estimate the local group angle ψ

$$\psi = \arctan(n_x/n_z). \quad (8)$$

The relationship between the group and phase angles

In anisotropic media, it is necessary to distinguish the phase and group velocities (or the polarization) directions (Tsvankin 2005), since reflection properties are determined by the phase velocity direction (Dickens and Winbow 2011; McGarry and Qin 2013). Here, we first discuss the relationship between the group and phase velocities. As demonstrated for the 2D case in Fig. 3, in homogeneous media, the group velocity vector is along the source-receiver direction, whereas the phase velocity vector is perpendicular to the wavefront.

In 2D VTI media, the group angle ψ can be expressed as follows (Berryman 1979):

$$\tan \psi = \frac{V \sin \theta + \frac{\partial V}{\partial \theta} \cos \theta}{V \cos \theta - \frac{\partial V}{\partial \theta} \sin \theta}, \quad (9)$$

where $V(\theta)$ is the phase velocity, which can be obtained by solving the Christoffel equation. For simplicity, equation (9) can be expressed as:

$$\psi_P = F_P(\theta_P), \psi_{SV} = F_{SV}(\theta_{SV}), \quad (10)$$

Note that equation (9) expresses the group angle as a function of the phase angle. In the proposed scheme, the group angle should be used to evaluate the phase angle. However, the calculation from group angle to phase angle is difficult to achieve (Sava and Alkhalifah 2013) and the traditional scheme requires approximations (Thomsen 1986), which can be expressed as:

$$\tan \psi_P = \tan \theta_P [1 + 2\delta + 4(\epsilon - \delta) \sin^2 \theta_P], \quad (11)$$

In the implementation here, we adopt the inverse functions, calculated using numerical methods as follows:

$$\theta_P = F_P^{-1}(\psi_P), \theta_{SV} = F_{SV}^{-1}(\psi_{SV}). \quad (12)$$

The pseudocode of this numerical scheme is displayed in Algorithm 1. Figure 4 shows the comparisons of two algorithms for computing the group angles of qP-wave in a homogeneous model. With the increase of anisotropy strength, the deviations calculated from equation (11) become obvious. However, the proposed scheme still preserves high accuracy.

The relationship between the group and polarization angles

To calculate the polarization angle, solving the Christoffel equation is required. The expression for the Christoffel equation reads

$$[\mathbf{G} - \rho V^2 \mathbf{I}] \mathbf{U} = 0, \quad (13)$$

where $\mathbf{G}$ denotes the Christoffel matrix with $G_{ij} = c_{ijkl} n_j n_l$, c_{ijkl} denotes the stiffness tensor, and n_j and n_l denote the unit wave vectors along the j - and l -directions, respectively, $i, j, k, l = 1, 2, 3$.

To construct the relationship between group and polarization angles, we firstly analyze the relationship between phase and polarization angles using the Christoffel equation. In 2D VTI media, the Christoffel equation can be expressed in the following form

$$\begin{bmatrix} c_{11} \cos^2 \theta + c_{55} \sin^2 \theta - \rho V^2 & (c_{13} + c_{55}) \cos \theta \sin \theta \\ (c_{13} + c_{55}) \cos \theta \sin \theta & c_{55} \cos^2 \theta + c_{33} \sin^2 \theta - \rho V^2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_3 \end{bmatrix} = 0, \quad (14)$$

where we represent the unit vector $\mathbf{n}$ using the phase angle θ , $n_x = \cos \theta$, $n_z = \sin \theta$.

The relationships between the Thomson parameters (Thomsen 1986; Tsvankin 2005) and the stiffness coefficients are given by

$$\begin{aligned} V_{P0} &= \sqrt{\frac{c_{33}}{\rho}}, V_{SV0} = \sqrt{\frac{c_{55}}{\rho}}, \epsilon = \frac{c_{11} - c_{33}}{2c_{33}}, \\ \delta &= \frac{(c_{13} + c_{55})^2 - (c_{33} - c_{55})^2}{2c_{33}(c_{33} - c_{55})}, f = 1 - \frac{V_{SV0}^2}{V_{P0}^2}, \end{aligned} \quad (15)$$

To simplify equation (15), we replace the stiffness coefficients with the Thomson parameters, the two solutions of equation (14) can be expressed compactly as

$$\begin{aligned} V_P(\theta) &= V_{P0} \sqrt{1 + \epsilon \cos^2 \theta + \frac{f}{2} \left(\sqrt{1 + \frac{4 \cos^2 \theta}{f} (2\delta \sin^2 \theta - \epsilon \sin 2\theta) + \frac{4\epsilon^2 \cos^4 \theta}{f^2}} - 1 \right)}, \\ V_{SV}(\theta) &= V_{SV0} \sqrt{1 + \epsilon \cos^2 \theta + \frac{f}{2} \left(-\sqrt{1 + \frac{4 \cos^2 \theta}{f} (2\delta \sin^2 \theta - \epsilon \sin 2\theta) + \frac{4\epsilon^2 \cos^4 \theta}{f^2}} - 1 \right)}, \end{aligned} \quad (16)$$

Then we substitute the phase angle θ and phase velocity $V(\theta)$ back into equation (14), the polarization angle can be expressed (Tsvankin 1996):

$$\tan v = \frac{\sin \theta \cos \theta (c_{13} + c_{55})}{\rho V^2 - c_{11} \cos^2 \theta - c_{55} \sin^2 \theta}, \quad (17)$$

After some numerical manipulation, equation (17) can be expressed compactly as follows:

$$v_P = f_P(\theta_P), v_{SV} = f_{SV}(\theta_{SV}), \quad (18)$$

where f denotes the relationship between phase and polarization angles, v_P and v_{SV} denote the P-wave and SV-wave polarization angles, respectively. In this case, we finally construct a new numerical relationship between the group and polarization angles via the phase angle θ

$$v_P = f_P(\psi_P), v_{SV} = f_{SV}(\psi_{SV}). \quad (19)$$

Note that although the polarization vectors of qP-wave and qSV-wave are orthogonal, the corresponding group vectors are not (Liu et al., 2017a). Our process is given as follows: first, the wave-mode separation is performed using the polarization angle v_P globally, if the angle at one space point is v_P , thus $U_{SV} = -u_x \sin v_P + u_z \cos v_P$ is close to 0, which can be judged by the ratio of $|U_{SV}|$ energy to the $\sqrt{u_x^2 + u_z^2}$ energy at this point. If the ratio is lower than a threshold, the angle at this point is determined by v_P , and the angle v_P is preserved. Finally, the angle v_{SV} is used to separate the anisotropic wavefield in the rest of the area.

Establishment of efficient numerical table

The construction of a numerical table involves the selection of the sampling intervals of the corresponding parameters. First, we analyze which parameters determine the relationship between group and polarization angles. From equations (9),(15),(16) and (17), we conclude that these parameters are f, ϵ, δ . We can construct a numerical table of these parameters and interpolate the appropriate operators at each spatial point when performing the wave-mode separation. To improve the computational efficiency, the sampling intervals of the corresponding parameters are critical while ensuring computational precision. Figure 5 shows the effect of various factors on

the relationship between group and polarization angles, results denote the fact that δ has its influence at low angles, meanwhile ϵ and even more so f , influence the large angles. For 2D VTI media, the numerical table is multi-dimensional: $\mathbf{m} = \{f, \epsilon, \delta, \psi\}$. We can obtain the ranges of parameters from the model and calculate the numerical table in advance. During the wave-mode separation, we only need to find the specified coefficient after obtaining group angle from the Poynting vectors, then the relation between the group and polarization angles in model space is calculated by interpolation. For example, assume that $V_P/V_S \in [1.5, 2.5]$, $\epsilon \in [0, 0.4]$, $\delta \in [0, 0.3]$ are known, one can sample the ϵ ratio at every 0.05, δ at every 0.05, and V_P/V_S at every 0.1. If a spatial point has the following anisotropic parameters: $\epsilon = 0.23$, $\delta = 0.11$, $V_P/V_S = 1.63$, we can bilinearly interpolate the polarization angle of this point using the $\epsilon = 0.2, 0.25$, $\delta = 0.1, 0.15$, $V_P/V_S = 1.6, 1.7$ in the numerical table.

The numerical table for TTI media can be obtained based on the numerical table for VTI media by using the tilt angle (assuming the angle is β) rotation. For TTI media, first, we calculate the group angle produced by the Poynting vectors, then rotate the group angle by β . After rotation, we find the polarization angle with respect to the group angle from the numerical table for VTI media. Finally, we rotate the polarization angle of VTI media by $-\beta$ back to TTI media.

Work flow for anisotropic ERTM

The proposed wave-mode separation scheme for anisotropic ERTM, performed in the angle domain, can be summarized as follows:

- (i) Determine the ranges of model parameters: ϵ , δ , f . Give a group of discrete

phase angle values, such as those ranging from 0° to 90° with an interval of 1° , we substitute the discrete phase angle θ into the Christoffel equation (equation (14)) and obtain the phase velocity $V_P(\theta)$ (equation (16)). Then, substituting the phase angle θ , $V_P(\theta)$ and its derivative back into equation (9) generates $\psi_P = F_P(\theta_P)$, which shares the common variable θ_P with $v_P = f_P(\theta_P)$. We finally construct a numerical relationship between ψ_P and v_P through θ_P , and store the numerical table in memory in advance.

(ii) Forward propagate the source wavefield, and save it only on the boundary for all the time steps and recompute the wavefield once again by using the saved values as boundary. Then, the receiver and source wavefields are each backward propagated to produce an image. Before the wave-mode separation, the Poynting vector s can be computed using equations (6) and (7), then we obtain the group angle ψ using equation (8).

(iii) Obtain polarization angle v by looking up the established numerical table and interpolation. Apply equation (5) to obtain the separated qP-wave and qS-wave in 2D TI media.

(iv) During wavefield extrapolation, the Poynting vectors are used to perform the polarity correction following Du et al. (2012). Then, the modified cross-imaging condition is used (Liu et al. 2017b) to different wave modes, and the imaging angle is limited using the Poynting vectors. In this way, high-quality migrated images can be obtained. To gain more insights into the whole process for anisotropic ERTM, Fig. 6 is used to outline the process.

SYNTHETIC TESTS FOR ANISOTROPIC ERTM

In this section, we illustrate our proposed scheme using four synthetic examples. The first example is a homogeneous TI model. Then, the BP 2007 TTI model is used to demonstrate the validity of our scheme in complicated geologic structures. Finally, we test our proposed scheme for anisotropic ERTM on the 2D SEG Hess and Foothill models. In our ERTM scheme, to attenuate the artifacts caused by secondary reflections and overlapping events (Wang and McMechan 2015; Du et al. 2017), we smooth the true velocity models with a gaussian filter using a window length of 200m.

A homogeneous TI model

In this section, two TI models have the same P-wave velocity, S-wave velocity and density: $V_{P0} = 3000m/s$, $V_{S0} = 2000m/s$ and $\rho = 2.4g/cm^3$. The horizontal and vertical extent of the model are both 3 km with a space sampling interval of 10 m in the x - and z -directions. In the first case, the Thomsen parameters are as follows: $\epsilon = 0.3$, $\delta = 0.1$. The time snapshots at 600 ms for x - and z -components are illustrated in Figs. 6(a) and (b). The snapshots for qP-wave and qS-wave using the proposed wave-mode separation are shown in Figs. 7(c) and (d). Figures 7(e) and (f) present the Poynting vectors. The results verify the validity of the proposed scheme in a homogeneous model.

In the second case, we consider $\epsilon = 0.3$, $\delta = 0.1$, and the tilt angle $\beta = 60^\circ$. In this example, The time snapshots at 600 ms for x - and z -components are shown in Figs. 8(a) and (b). By performing the wave-mode separation scheme, the separated

qP-wave and qS-wave are shown in Figs. 8(c) and (d). The Poynting vectors are shown in Figs. 8(e) and (f). It can be seen that the qP-wave and qS-wave are well separated with no phase shift.

BP model

The BP 2007 TTI model is used to further prove the feasibility and validity of the proposed scheme. The model parameters are shown in Fig. 9. The vertical S-wave velocity is set to equal half the vertical P-wave velocity. The grid interval is 10 m in the x - and z -directions, the time step is 0.5 ms. An explosive source located at (35.625, 5.0) km of the model is used. A Ricker wavelet with peak-frequency of 25 Hz is used as a source. The time snapshots for x - and z -components of velocity wavefields at 950 ms are shown in Figs. 10(a) and (b). Figures 10(c) and (d) show the separated results of qP- and qS-waves using our proposed scheme on the input wavefields. We can see that although the wavefields are complex in this model, good separation results are obtained for different wave modes. Moreover, there is no need to correct the waveform changes caused by the derivative operators. The Poynting vectors are shown in Figs. 10(e) and (f).

It should be noted that minor inaccuracies exist in the superposition of qP-wave and qS-wave (as shown in Fig. 10(c)). In contrast to the accurate polarization angles, the separated results in these spatial points are inaccurate because of overlapping events. Many researchers have pointed out that the Poynting vectors (Du et al. 2012; Wang and McMechan 2015; Tang et al. 2016; Lu and Liu 2018) is easy to calculate. However, they are not clean everywhere, especially at the overlapped points. For

this reason, the precision of the polarization angles extracted from the group angles are degraded. Although the pseudo-derivative methods can handle the overlapping events, it is difficult to apply these methods on the unstructured mesh. However, our method on the unstructured mesh is easy to implement with low computational cost. Our numerical example on the BP model shows that the proposed method is effective even in a complex model.

Hess model

Furthermore, the anisotropic Hess model provided by the SEG is employed to show the validity of the proposed scheme. The vertical P-wave velocity model, and the Thomsen parameters are shown in Fig. 11. The vertical S-wave velocity is set to equal the vertical P-wave divided by 1.5. The model contains 3617×1500 grid points, and the grid spacing is 5 m in the x - and z -directions. The time step is 0.4 ms, the total record time is 6 s and the dominant frequency of the source wavelet is 25 Hz. It can be seen from the Figs. 12 (a) and (b) that the image of the fault location is accurate, the boundary of the high velocity salt dome is clear, and the fracture cavity and the anisotropic structure can also be well imaged. The anisotropy of earth media is widespread, and it is necessary to consider the influence of anisotropy. Figure 13 illustrates the migrated images for the anisotropic data using the isotropic ERTM, which ignores the anisotropy. Note that the reflectors with dip angles and anisotropy marked by the red circles are not presented clearly compared with the anisotropic ERTM (Fig. 12). This numerical example further validates the effectiveness of the proposed scheme.

Foothill model

The grid method is flexible in handling rugged topographies by naturally satisfying the free-surface boundary condition (Gao and Zhang 2010). In this section, we use the 2D SEG Foothill model (Fig.14(a)) to illustrate this feature, because the proposed scheme is based on the unstructured mesh. Therefore, it can accurately describe the surface topography and interfaces between high-velocity variation regions. We generate the unstructured mesh using the algorithm of CVDT with the P-wave velocity (Yang et al. 2017). One part of the mesh of the model is shown in Fig. 14(b).

As we can see, the rugged topography is associated with the steep-dip structures of high-velocity outcrops together with the low-velocity weathered structures near the surface. If a uniformly structured mesh is used, the spatial and time steps must both be sufficiently small enough to meet the sampling criteria, which leads to an over-sampling problem (Alkhalifah 2003). In contrast, the unstructured mesh sizes vary in line with the velocities, which results in significant reduction of the number of discretized nodes compared with the number of nodes in the conventional regular-grid method. As shown in Figs. 15 (a) and (b), the triangular grids in the high-velocity area are larger than those in the low-velocity area. As a result, the time sampling rate can be significantly reduced (Liu et al. 2017b). Accordingly, the unstructured mesh is used to perform the proposed method. By doing this, the implementation of ERTM can be easily achieved in regions of rugged topography, and high-velocity contrast.

The ratio of the S-velocity to the P-velocity is $\frac{2}{3}$. The grid interval is 15 m in the

x -direction and 10 m in the z -direction, the time step is 0.5 ms, and the total record time is 8 s. A Ricker wavelet with peak-frequency of 20 Hz is used as a source. There are 277 shots distributed along the rugged topography surface with a spacing interval of 90 m in the x -direction. Each shot is recorded by 480 receivers with a trace interval of 15 m. We consider the anisotropic parameters as: $\epsilon = 0.1$, $\delta = 0.1$, and the tilt angle $\beta = 10^\circ$. The migration results of the Foothill model are shown in Figs. 16 (a) and (b), presenting continuous steep-dip and clean thrust fault structures. Moreover, at the near-surface, the image is clear and has high resolution with less noise. This numerical example demonstrates that even with a large velocity contrast and irregular surface, the anisotropic ERTM based on the irregular, unstructured grid method can achieve high-resolution migrated images.

CONCLUSIONS

We have presented an anisotropic separation method for transversely isotropic media. The key to our method is to utilize the characteristic of the Poynting vectors that indicate the direction of the group velocity. A numerical table is precomputed and stored in advance by constructing the relationship between group and polarization angles. The storage and computational cost of this table are linearly related to model parameter ranges. Moreover, our proposed method is more suitable for parallel computing compared with conventional pseudo-derivative methods. Therefore, during wavefield extrapolation, the vector wavefields can be easily separated using the table-driven algorithm. In the meantime, the amplitude and phase characteristics of original wavefields are better preserved in the proposed method. In this way, the anisotropic

ERTM can be efficiently performed.

ACKNOWLEDGMENTS

We thank the National Natural Science Fund of China (under grant 41330316) and the National Major Project of China (under grant 2017ZX05008-007) for supporting this work. We also acknowledge the two reviewers and associate editor Tariq Alkhalifah for reviewing this manuscript and their constructive comments have greatly improved this manuscript. We are grateful to Dr Umair bin Waheed for his kind help. The research is a part of Collaboration Project of Seismic Imaging in Elastic Medium of the China National Petroleum Corporation.

REFERENCES

- Aki, K., and P. Richards, 1980, Quantitative seismology, theory and methods: W.H. Freeman and Co., New York.
- Alkhalifah, T., 1998, Acoustic approximations for processing in transversely isotropic media: *Geophysics*, **63**, 623–631.
- , 2003, Tau migration and velocity analysis: Theory and synthetic examples: *Geophysics*, **68**, 1331–1339.
- Berryman, J. G., 1979, Longwave elastic anisotropy in transversely isotropic media: *Geophysics*, **44**, 896–917.
- Červený, V., 2000, *Seismic ray method*: Cambridge University Press.
- Chang, W. F., and G. A. McMechan, 1987, Elastic reverse-time migration: *Geophysics*, **52**, 243–256.
- , 1994, 3-d elastic prestack, reverse-time depth migration: *Geophysics*, **59**, 597–609.
- Cheng, J., T. Alkhalifah, Z. Wu, P. Zou, and C. Wang, 2016, Simulating propagation of decoupled elastic waves using low-rank approximate mixed-domain integral operators for anisotropic media: *Geophysics*, **81**, T63–T77.
- Cheng, J., and S. Fomel, 2014, Fast algorithms for elastic-wave-mode separation and vector decomposition using low-rank approximation for anisotropic media: *Geophysics*, **79**, C97–C110.
- Dellinger, J., and J. Etgen, 1990, Wavefield separation in twodimensional anisotropic media: *Geophysics*, **55**, 914.
- Dickens, T. A., and G. A. Winbow, 2011, Rtm angle gathers using poynting vectors:

SEG Technical Program, Expanded Abstracts, 3109–3113.

Du, Q., C. F. Guo, Q. Zhao, X. Gong, C. Wang, and X. Y. Li, 2017, Vector-based elastic reverse time migration based on scalar imaging condition: *Geophysics*, **82**, S111–S127.

Du, Q., Y. Zhu, and J. Ba, 2012, Polarity reversal correction for elastic reverse time migration: *Geophysics*, **77**, 31.

Duan, Y., and P. Sava, 2015, Scalar imaging condition for elastic reverse time migration: *Geophysics*, **80**, S127–S136.

Gao, H., and J. Zhang, 2010, Parallel 3-d simulation of seismic wave propagation in heterogeneous anisotropic media: a grid method approach: *Geophysical Journal International*, **165**, 875–888.

Komatitsch, D., and J. P. Vilotte, 1998, The spectral element method: An efficient tool to simulate the seismic response of 2d and 3d geological structures: *Pacific Research Institute for Public Policy*.

Liu, Q., D. Peter, and Y. Lu, 2017a, A fast pointwise strategy for anisotropic wave-mode separation in ti media: *SEG Technical Program, Expanded Abstracts*, 447–451.

Liu, Q., J. Zhang, and H. Gao, 2017b, Reversetime migration from rugged topography using irregular, unstructured mesh: *Geophysical Prospecting*, **65**.

Lu, Y., and Q. Liu, 2018, Non-overlapped p- and s-wave poynting vectors and their solution by the grid method: *Journal of Geophysics and Engineering*, **15**, 788.

Mcgarry, R., and Y. Qin, 2013, Direction-vector-based angle gathers from anisotropic elastic rtm: *SEG Technical Program, Expanded Abstracts*, 3820–3824.

- Oh, J. W., M. Kalita, and T. Alkhalifah, 2018, 3d elastic full-waveform inversion using p-wave excitation amplitude: Application to ocean bottom cable field data: *Geophysics*, **83**, R129–R140.
- Sava, P., and T. Alkhalifah, 2013, Wide-azimuth angle gathers for anisotropic wave-equation migration: *Geophysical Prospecting*, **61**, 75–91.
- Tang, H., B. He, and H. Mou, 2016, P- and s-wave energy flux density vectors: *Geophysics*, **81**, T357–T368.
- Thomsen, L., 1986, Weak elastic anisotropy: *Geophysics*, **51**, 1954–1957.
- Tsvankin, I., 2005, *Seismic signatures and analysis of reflection data in anisotropic media* (2nd ed.): Elsevier.
- Wang, M. X., H. Yang, A. Osen, and D. H. Yang, 2013, Full-wave equation based illumination analysis by nad method: Presented at the 75th Annual Int. Conf. and Exhibition EAGE Extended Abstracts.
- Wang, W., and G. A. McMechan, 2015, Vector-based elastic reverse time migration: *Geophysics*, **80**, S245–S258.
- Xie, X. B., 2015, An angle-domain wavenumber filter for multi-scale full-waveform inversion: SEG Technical Program, Expanded Abstracts, 1132–1137.
- Yan, J., and T. A. Dickens, 2016, Reverse time migration angle gathers using poynting vectors: *Geophysics*, **81**, S511–S522.
- Yan, J., and P. Sava, 2008, Isotropic angle-domain elastic reverse-time migration: *Geophysics*, **73**, S229–S239.
- , 2009, Elastic wave-mode separation for vti media: *Geophysics*, **74**, WB19–WB32.

- Accepted Article
- , 2011, Improving the efficiency of elastic wave-mode separation for heterogeneous tilted transverse isotropic media: *Geophysics*, **76**, T65–T78.
- Yang, K., J. Zhang, and H. Gao, 2017, Unstructured mesh based elastic wave modelling on gpu: A double-mesh grid method: *Geophysical Journal International*, **211**, 741–750.
- Yoon, K., M. Guo, J. Cai, and B. Wang, 2011, 3d rtm angle gathers from source wave propagation direction and dip of reflector: *SEG Technical Program, Expanded Abstracts*, 3136–3140.
- Yoon, K., and K. J. Marfurt, 2006, Reverse-time migration using the poynting vector: *Exploration Geophysics*, **59**, 102–107.
- Zhang, J., and T. Liu, 1999, P-sv-wave propagation in heterogeneous media: grid method: *Geophysical Journal International*, **136**, 431–438.
- , 2010, Elastic wave modelling in 3d heterogeneous media: 3d grid method: *Geophysical Journal International*, **150**, 780–799.
- Zhang, Q., and G. A. McMechan, 2010, 2d and 3d elastic wavefield vector decomposition in the wavenumber domain for vti media: *Geophysics*, **75**, D13–D26.
- Zhou, Y., and H. Wang, 2017, Efficient wave-mode separation in vertical transversely isotropic media: *Geophysics*, **80**, C35–C47.

Algorithm 1 Pseudo code of inverse function of equation (10).

input: input parameter *ang_pg*.

output: output parameter *ang_gp*.

ang_pg denotes the group angle corresponding the phase angle from 0 to 90 degrees with interval of 1 degree.

for $i = 1$ to 90

$k=0$;

 for $j = k$ to 90

 if $((ang_pg[j]-i)*(ang_pg[j+1]-i) \leq 0)$

$k=j$;

 break;

 end if

end for

$a1=i-ang_pg[k]$; $b1=ang_pg[k+1]-i$;

$ang_gp[i]=(k+1)*a1+k*b1$;

$ang_gp[0]=0$; $ang_gp[90]=90$;

ang_gp denotes the phase angle corresponding the group angle from 0 to 90 degrees with interval of 1 degree.

end for

Figure 1: Local mesh for grid method. Integrating both sides of equation (1) over the domain enclosed by the dashed line 1-2-...-12-1. The triangles drawn by solid lines are basic cells in the grid method. The two velocity components are defined at the grid nodes, denoted by black dots; The three stress components are defined at the cell barycenters, denoted by empty dots. The dash lines link the centers of the grids and the edges.

Figure 2: TTI elastic wavefields with group angles at various points around the outer wavefront is indicated by green arrows, the red arrows in panel (a) show the direction vectors of the phase velocity, and the blue arrows in panel (b) show the direction vectors of the group velocity.

Figure 3: Schematic diagram of the phase and group velocities.

Figure 4: Comparison of qP-wave phase angle with exact and approximate group angle. (a) $\epsilon = 0.15$, $\delta = 0.05$, (b) $\epsilon = 0.3$, $\delta = 0.1$.

Figure 5: The difference between the group and polarization angles ($\psi - \nu$) (a) The deviation angle for parameter ϵ from 0.2 to 0.4, and $\delta = 0.1$, $V_P/V_S = 1.5$. (b) The deviation angle for parameter δ ranging from 0.1 to 0.3 and $\epsilon = 0.3$, $V_P/V_S = 1.5$. (c) The deviation angle for parameter V_P/V_S (equivalent to f) ranging from 1.5 to 2.5 and $\epsilon = 0.3$, $\delta = 0.1$.

Figure 6: Flow chart of the processes for anisotropic ERTM.

Figure 7: (a and b) Snapshots of the elastic wavefield in the x - and z -directions for a homogenous VTI model. (c and d) Separated qP - and qS -waves using the proposed method. (e and f) x - and z - components of the Poynting vectors.

Figure 8: (a and b) Snapshots of the elastic wavefield in the x - and z -directions for a homogenous TTI model. (c and d) Separated qP- and qS-waves using the proposed method. (e and f) x - and z - components of the Poynting vectors.

Figure 9: The BP 2007 TTI model with parameters: (a) vertical P-wave velocity, Thomsen coefficients (b) ϵ and (c) δ , and (d) tilt angles. This is a portion of the BP TTI model.

Figure 10: (a and b) Snapshots of the elastic wavefield in the x - and z -directions for the BP TTI model. (c and d) Separated qP- and qS-waves using the proposed method. (e and f) x - and z - components of the Poynting vectors.

Figure 11: The SEG Hess model with parameters of (a) vertical P-wave velocity, Thomsen coefficients (b) ϵ and (c) δ .

Figure 12: Stacked (a) PP and (b) PS images with the incident angles from -45° to 45° .

Figure 13: Stacked (a) PP and (b) PS images ignoring the anisotropy.

Figure 14: (a) The 2D SEG Foothill velocity model with rugged topography, (b) The local mesh in the area of the green box. The rugged surface topography are well described by the triangular grids.

Figure 15: Irregular, unstructured mesh generated by our method for the portion of the 2D SEG Foothill velocity model. Note that the mesh size varies according to the velocity.

Figure 16: Stacked images of SEG Foothill. (a) PP and (b) PS images. Not only the steep-dip seismic events, but also the negative thrust fault structures are imaged clearly and correctly.