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Extraction of the tomography mode with non-stationary smoothing for full-waveform inversion

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Running Head: Extraction of tomography mode for FWI

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ABSTRACT

Full-waveform inversion (FWI) includes both migration and tomography modes. The tomographic component of the gradient from reflection data is usually much weaker than the migration component. In order to use the tomography mode to fix background velocity errors, it is necessary to extract the tomographic component from the gradient. Otherwise, the inversion will be dominated by the migration mode. We propose a method based on non-stationary smoothing to extract the tomographic component from the raw gradient. By analyzing the characteristics of the scattering angle filtering, the wavenumber of the tomographic component at a given frequency is seen to be smaller than that of the migration component. Therefore, low-wavenumber-pass filtering can be applied to extract the tomographic component. The low-wavenumber-pass smoothing filters are designed with Gaussian filters that are determined by the frequency of inversion, the model velocity, and the minimum scattering angle. Thus, this filtering is non-stationary smoothing in the space domain. Since this filtering is carried out frequency by frequency, it works naturally and efficiently for FWI based on frequency-domain modeling. Furthermore, as the maximum opening angle of the reflections in a typical acquisition geometry is much smaller than the minimum scattering angle for the tomographic component, which is generally set to 160° , there is a relatively large gap between the wavenumbers of the tomographic and migration components. In other words, the non-stationary smoothing can be applied once to a group of frequencies for time-domain FWI without leaking the migration component into the tomographic component. Analyses and numerical tests show that two frequency groups are generally sufficient to extract the tomographic component for the typical frequency range of time-domain FWI. The numerical tests also demonstrate that the non-stationary smoothing method is effective and efficient at extracting the tomographic component for reflection waveform inversion.

INTRODUCTION

Full-waveform inversion (FWI) is a promising technique for recovering subsurface parameters, including the P-wave and S-wave velocities, in both exploration geophysics (e.g., Virieux and Operto, 2009; Warner et al., 2013; Cheng et al., 2016; da Silva et al., 2016, 2019; Cheng et al., 2017; Yao et al., 2019) and global seismology (e.g., Zhu et al., 2012; Chen et al., 2015; Tao et al., 2017). FWI includes both migration and tomography modes (Mora, 1989). The migration mode moves and stacks reflection events at their place of origin in order to recover the impedance perturbation, which is missing from the starting model. This migration mode helps FWI to achieve its maximum Rayleigh resolution, i.e., a quarter of the wavelength of the maximum frequency in the record (Kallweit and Wood, 1982). This quarter wavelength resolution means that the minimum wavelength of the image from FWI is half the minimum wavelength of the wavefields (Pratt, 1999; Virieux and Operto, 2009). The tomography mode fixes the background velocity errors along the wave paths inside the first Fresnel zone, the width of which defines the maximum resolution of the tomography mode (Williamson, 1991). Although the tomography mode has a much lower resolution than the resolution limit of the migration mode, it fixes the background velocity errors by improving the focus of the migration mode. The gap between the resolution limits of the two modes can be filled by inverting wide-angle reflection and broadband data (Virieux and Operto, 2009; Baeten et al, 2013).

Refraction data, or diving waves, contribute to the tomography mode, whereas reflection data contribute to both modes. The tomographic component in the gradient from reflection data is typically much weaker than the migration component. Therefore, it is necessary to separate them, especially for reflection waveform inversion (RWI), which aims to recover the background velocity using reflection data. Without emphasizing the tomography mode, RWI is dominated by the migration mode and acts like a least-squares reverse-time migration

(LSRTM), which recovers the interfaces rather than the background velocity. A more detailed analysis can be found in Figure 1 of Yao and Wu (2017).

Several methods exist for the separation of the two modes. All these methods separate the modes by splitting the raw gradient into its tomographic component and its migration component.

Born modeling decomposes the full wave equation into two wave equations, one which generates the incident wavefield and one which produces the scattered wavefield. As a result, the gradient is obtained by pairwise correlation of four wavefields, namely, the incident source wavefield, the scattered source wavefield, the incident residual wavefield, and the scattered residual wavefield. The tomographic component is formed by cross-correlating the scattered wavefields with the incident wavefields; the migration component is generated by cross-correlating the incident wavefields from the source and residual wavefields (Xu et al., 2012; Yao et al., 2014; Zhou et al. 2015; Sun et al., 2016; Audebert and Ortiz-Rubio, 2018). The scattered wavefield with the split wave equations contains both back-scattered and forward-scattered waves, which can be observed clearly with the increase of propagation time. Consequently, the extracted tomographic component contains some leakage from the migration component.

The method of up-down wavefield separation shares the same principle as the Born modeling method, but uses either the Fourier transform or the Hilbert transform to separate the scattered wavefield from the incident wavefield (Wang et al., 2013; Irabor and Warner, 2016; Wu and Alkhalifah, 2016; Lian et al., 2018). One limitation is that this method cannot distinguish the scattered waves from the incident waves when both waves propagate along the same direction. A schematic illustration of this can be found in Figure 12 of Liu et al. (2011).

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Consequently, strong leakage of the migration component into the tomographic component occurs. The leakage may lead to incorrect updates in RWI (Wang et al. 2018).

Source and residual waves, which contribute to the tomographic component, have an opening angle of around 180° . By contrast, the migration component is formed from waves that have much smaller opening angles. A local slant stack can be used to measure the opening angles in order to distinguish the two components (Xie, 2015). Similarly, scattering angle filtering is an effective and versatile method of separating the two types of components from the raw gradient (Wu and Alkhalifah, 2015, 2017; Kazei et al., 2016; Yao et al., 2018). However, scattering angle filtering significantly increases the cost of the inversion.

The inverse scattering and energy norm (Rocha et al., 2016) imaging conditions can be considered as special cases of scattering angle filtering, in which the filter has a cosine-square transition band from 0° to 180° . These conditions are sufficient for extracting the migration component (Whitmore and Crawley, 2012; Fang et al., 2018; Rocha et al., 2018; Yang and Zhang, 2018). However, due to the wide transition band, modifications such as adaptive weighting are required to achieve an effective extraction of the tomographic component (Ramos-Martinez et al., 2016).

Here, we analyze the characteristics of scattering angle filtering and propose a method of non-stationary smoothing with Gaussian filters to extract the tomographic component from the raw gradient. Compared to a conventional FWI (Tarantola, 1984; Pratt, 1999) implementation, the computational cost of the non-stationary smoothing is trivial, though it needs one extra backpropagation for time-domain FWI in a typical frequency range (i.e., <20 Hz). Using numerical examples, we demonstrate that this method is effective and efficient for the extraction of the tomographic component and for applications to RWI.

METHOD

The objective function of conventional FWI reads

$$\phi = \frac{1}{2}(\mathbf{d} - \mathbf{d}_0)^T (\mathbf{d} - \mathbf{d}_0) = \frac{1}{2} \delta \mathbf{d}^T \delta \mathbf{d}, \quad (1)$$

where $\mathbf{d}$ and $\mathbf{d}_0$ represent the predicted and observed data, respectively, and $\delta \mathbf{d}$ is the residual.

If the P-wave velocity is the inversion parameter, then the gradient is expressed as

$$\frac{\partial \phi}{\partial v} = g(\mathbf{x}) = -\frac{2}{v^3} \sum_t \dot{p}_s(\mathbf{x}, t) \dot{p}_r(\mathbf{x}, t), \quad (2)$$

where p_s and p_r represent the source wavefield and the backpropagated residual wavefield, respectively, v denotes the P-wave velocity, and ‘ $\dot{\cdot}$ ’ means the first-order time derivative. For a constant velocity model, the wavenumber-domain counterpart of the gradient expressed in equation 2 can be written as

$$\tilde{g}(\mathbf{k}) = -\frac{2}{v^3} \sum_t \tilde{p}_s(\mathbf{k}_s, t) * \tilde{p}_r(\mathbf{k}_r, t), \quad (3)$$

where ‘ $\sim$ ’ denotes the counterpart of wavefields or gradient in the wavenumber domain, $\mathbf{k}_s$, $\mathbf{k}_r$, and $\mathbf{k}$ are the wavenumber vectors of the source wavefield, the residual wavefield, and the gradient, respectively, and ‘ $*$ ’ represents a multi-dimensional wavenumber convolution. The wavenumber vectors of the gradient satisfy the relationship

$$\mathbf{k} = \mathbf{k}_s + \mathbf{k}_r. \quad (4)$$

The detailed derivation of equation 4 can be found around equation 7 of Yao et al. (2018). In acoustic isotropic media, for a frequency, ω , and a velocity, v ,

$$|\mathbf{k}_s| = |\mathbf{k}_r| = \frac{\omega}{v}. \quad (5)$$

According to equations 4 and 5, the magnitude of the gradient's wavenumber can be calculated using

$$|\mathbf{k}| = \frac{2\omega}{v} \cos \frac{\alpha}{2}, \quad (6)$$

where α is the opening angle of the source and receiver wavenumber vectors.

We know that if the source wavefield propagates opposite the residual wavefield, then the cross-correlation of the two wavefields contributes to the tomographic component of the gradient. In other words, the tomographic component is formed by the source and residual wavefields, which have opening angles close to 180° ; other wavefields with smaller angles contribute to the migration component of the gradient. One effective method of separating the two components is scattering angle filtering based on equation 6 (Alkhalifah, 2015; Wu and Alkhalifah, 2015, 2017). As the tomographic component comes from greater scattering angles than the migration component, equation 6 implies that the tomographic component has a smaller wavenumber than the migration component at a given frequency and velocity (Figure 1). In the figure, the wavenumbers of the migration component sit in the light-gray outer ring, and the wavenumbers of the tomographic component are located inside the dark-gray inner circle. The transition zone lies between the two. Therefore, we extract the tomographic component by using low-wavenumber-pass filtering, which is equivalent to a process of smoothing in the space domain.

To extract the tomographic component from one element of the raw gradient at one frequency, we begin by computing the cut-off wavenumber from

$$k_{tomo_max} = \frac{2\omega}{v} \cos \frac{\alpha_{min}}{2}, \quad (7)$$

where ω is the frequency of inversion, v is the model velocity at this element, and α_{min} is the selected minimum opening angle. Next, we design a space-domain smoothing filter that is equivalent to having a preset damped amplitude at the cutoff wavenumber, k_{tomo_max} . Then, we convolve this filter with the raw gradient to extract its tomographic component at the element being processed for the frequency of inversion. Finally, we repeat this process for all the elements and frequencies. Thus, the smoothing process is non-stationary.

In this paper, we design the space-domain smoothing filter by using the Gaussian function, which is defined as

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}; \quad (8)$$

its Fourier domain expression is

$$G(k) = e^{-\frac{k^2}{2\sigma_k^2}}, \quad (9)$$

where σ and σ_k denote the standard deviations in the space domain and in the Fourier domain, respectively, and k is the wavenumber (units in 1/m). The standard deviations determine the shape of the Gaussian functions in each respective domain. The Fourier transform pair used for equations 8 and 9 is listed in APPENDIX A. The two standard deviations share the relationship of

$$\sigma\sigma_k = \frac{1}{2\pi}. \quad (10)$$

Thus, the cut-off wavenumber k_{tomo_max} can be calculated using equation 7 with the frequency of inversion, the velocity of the element being processed, and the minimum opening angle, α_{min} , which is set to 160° for the tests in this paper. To define the Fourier-domain Gaussian function, we set the amplitude value, β , to 0.3 for the cut-off wavenumber k_{tomo_max} (Figure 2a). According to equation 9, the standard deviation, σ_k , can be computed with

$$\sigma_k = \frac{k_{tomo_max}}{\sqrt{-2 \log \beta}}. \quad (11)$$

Then, the standard deviation, σ , can be determined from equations 10 and 11 as

$$\sigma = \sqrt{\frac{-\log(\beta)}{2\pi^2 k_{tomo_max}^2}}. \quad (12)$$

Finally, the Gaussian filter is obtained using equation 8. Since a Gaussian function has infinitely long tails, we truncate its tails by setting the truncation factor, γ , to 0.01 (Figure 2b). Consequently, the effective radius of the Gaussian filter is

$$l = \sqrt{-2\sigma^2 \log(\gamma)}, \quad (13)$$

which is illustrated in Figure 2b.

According to equation 7, we extract the tomographic component by smoothing the raw gradient frequency by frequency. This approach is convenient for FWI based on frequency-domain modeling. However, if we directly apply this approach to FWI based on time-domain modeling, we need to Fourier transform the wavefields in the whole model from the time domain into the frequency domain. This transform leads to a significant increase in storage and computational cost. As illustrated in Figure 1, there is a transition zone between the

tomographic component zone and the migration component zone in the wavenumber domain because the maximum opening angle for the migration component is smaller than the minimum opening angle for the tomographic component in a typical acquisition geometry. The transition zone implies that the smoothing process based on one reference frequency can be applied on the raw gradient generated by a group of frequencies as long as the minimum wavenumber of the migration component of the lowest frequency is bigger than the maximum wavenumber of the tomographic component of the highest frequency. We give additional details about the strategy of the frequency grouping in the next section.

EXAMPLES

One-Source-One-Receiver Geometry

The first example is on a two-layer model (Figure 3a). In this test, we used one-source-one-receiver geometry. The source and the receiver are located at a distance of 2 km and 6 km, respectively, and a depth of 2.5 km. More detailed information about the model and the geometry can be found in the caption of Figure 3. The source wavelet is a bandlimited spike, which has a flat spectrum from 3 Hz to 20 Hz and is shown in Figure 3b. This spectrum covers the typical frequency range of current FWI applications. In this test, we only used reflections to form the gradient. The raw gradient of the first iteration is shown in Figure 3a. The migration component is represented by the ellipse in the figure; the remainder of the gradient is the tomographic component, which contains the information required for the background update. The wavenumber spectrum of the raw gradient is shown in Figures 3c-d, where the spectrum of the tomographic component appears as four narrow ellipses symmetrically located around the origin; the remaining circular shapes are the migration component. As can be seen, the two components are mixed in the wavenumber domain.

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To extract the tomographic component, we apply non-stationary smoothing to the raw gradient. The cutoff angle is set to 160° . The value of the Gaussian function of the cutoff wavenumber (equation 9) is 0.3 in the Fourier domain. Figure 4a shows that the tomographic component is successfully extracted frequency by frequency. The wavenumber spectrum of the extracted tomographic component is shown in Figure 4b. However, smoothing frequency by frequency requires a Fourier transform on the wavefields; therefore, the computational and storage costs are prohibitive for time-domain FWI.

As the discussion at the end of the previous section and the illustration of Figure 1 show, there exists a transition zone between the tomographic component zone and the migration component zone in the wavenumber domain because the maximum opening angle of reflection events is smaller than the cutoff angle. In this case, we set the cutoff angle to 160° . Consequently, if the opening angle of the reflection events is smaller than 140° , which satisfies most real applications, the maximum wavenumber of the tomographic component will be less than half of the minimum wavenumber of the migration component. As a result, we can apply the non-stationary smoothing once on a group of frequencies ranging from half to double the reference frequency.

To test this strategy, we define four frequency groups with reference frequencies of 1.5 Hz, 3 Hz, 6 Hz, and 12 Hz, which cover the whole frequency range of the data. The reasons for choosing the four reference frequencies in this example are as follows: (1) half of one reference frequency is equal to the immediately lower reference frequency, while double the reference frequency is equal to the immediately higher reference frequency; and (2) the frequency range from half of the smallest reference frequency to double the highest reference frequency covers the frequency bandwidth of the backpropagated residual wavefields. In order to select these frequencies, we filter the residual using the four filters shown in Figure 5a. The transition bands

of the four filters are formed by cosine-square functions, so the summation of the four filtered residuals is identical to the original residual. We do not need to filter the source wavelet or the source wavefield because the zero-lag cross-correlation used to form the gradient removes the frequencies of the source wavefield that do not exist in the filtered residual wavefield. The four residuals yield four sub-gradients. The tomographic components of the sub-gradients were extracted separately by using non-stationary smoothing with the four reference frequencies. Figure 5c shows the stack of four tomographic components, the wavenumber spectrum of which is shown in Figure 5d. From Figures 5c and 5d, we see that the strategy of using frequency groups is as effective as the means of filtering frequency by frequency.

However, in practice, we have to backpropagate the filtered residuals four times, which is computationally expensive. To reduce the computational cost, we reduce the reference frequencies to only 6 Hz and 12 Hz since the wavelet has very weak energy for the frequencies lower than 3 Hz and higher than 24 Hz. Thus, only one extra backpropagation is sufficient in practice. The two sub-gradients are shown in Figures 5e and 5f, and their wavenumber spectra are depicted in Figures 5h and 5i. The tomographic component is clearly separated in the wavenumber domain. In other words, the tomographic component is extracted successfully with just two frequency groups (Figure 5g). In comparison, using just two frequency groups does not noticeably decrease the quality, except for some weak artifacts between the two ‘rabbit ears’ indicated by the triangle in Figure 5g. Fortunately, these weak artifacts (about 5% in this example) do not noticeably affect the inversion.

If we do not apply the frequency group strategy, the quality of the extracted tomographic component is reduced significantly. For example, when we choose 6 Hz as the only reference frequency, the non-stationary smoothing removes some high wavenumbers from the tomographic component (Figures 6a and 6c). If the reference frequency is 12 Hz, some low

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wavenumbers from the migration component leak into the tomographic component (Figures 6b and 6d).

From the colorbars in this example, the smoothing process noticeably shrinks the amplitude of the tomographic component but keeps its shape. This correct shape is essential for a successful inversion, while the amplitude loss can be compensated by the step length.

Application to Reflection Waveform Inversion

In the second example, we apply the two-frequency-group strategy to the Marmousi model (Figure 7a) for RWI (Yao et al., 2014; Yao and Wu, 2017). We generate 128 shots using the acoustic wave equation. The source wavelet is a bandlimited spike, which is shown in Figure 8a. It has a flat spectrum between 3 Hz and 12 Hz (Figure 8b). The source of the first shot is located at a distance of 187.5 m. The shot interval is 100 m. The maximum offset of each shot is 3.75 km. All the sources and receivers are fixed at a depth of 25 m. An absorbing boundary condition is applied around the whole model. Consequently, the synthesized record simulates the seismic data after multiple removal and deghosting. However, the nonstationary smoothing method also works in the presence of multiples. The one shot record is shown in Figure 9a. The direct arrivals and refractions are removed in the inversion. The muted record corresponding to the shot gather in Figure 9a is depicted in Figure 9b.

We alternated between a migration-like step and a tomography-like step to iteratively update the background velocity model with reflection-only data (Yao et al., 2014; Yao and Wu, 2017). The migration-like step uses the short-offset data to build a temporary model of reflectors, which produce predicted reflection events that match the recorded reflection events inside the short offsets in a least-squares sense. However, the rest of the offsets have relatively

large residuals, which contain information for the background velocity update. Then, the tomography-like step uses all the offsets to update the background.

A multiscale strategy (Bunks et al., 1995) is applied in the inversion. The inversion is started from 4 Hz and increases to 10 Hz in increments of 1 Hz. Each frequency is extracted by bandpass filtering on the residual centered at this frequency. The filter damps the frequencies lower than the central frequency, fully stopping at 0 Hz; it also damps the frequencies higher than the central frequency, coming to a full stop at 2.5 times the central frequency. Each frequency is inverted for five iterations; therefore, there are 35 iterations in total.

The gradients of the tomography-like step from the first iteration are shown in Figure 10. The data from the first iteration are extracted using a band-passed filter centered at 4 Hz. The two reference frequencies used to extract the tomographic component are $2.67 \text{ Hz} (= \frac{2f}{3})$, where f is the central frequency, i.e. 4 Hz) and $5.33 \text{ Hz} (= \frac{4f}{3})$, and their corresponding raw gradients are shown in Figures 10a and 10c, respectively. Their stack is depicted in Figure 10e; it is dominated by the high-wavenumber migration component, which shows the geological structure of Marmousi. In contrast, the tomographic component is much weaker.

To complete the tomography-like step, we apply non-stationary smoothing with the two-frequency-group strategy to the raw gradients. The output is shown in Figures 10b, 10d, and 10f. The extracted tomographic component is very smooth and matches the background trend that faintly appears in the raw gradients, so we conclude that it is very useful for the background velocity update. The colors indicate that the velocity in the left top region should decrease while the velocity in the right part should increase. This coincides with the background trend of the errors in the initial model. The successful extraction can be attributed to the clear

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separation of the migration and tomographic components in the wavenumber domain. An illustration is shown in Figure 11.

From the spectra shown in Figure 11, we see that in this example, unlike the one-source-one-receiver example shown in Figures 4-6, non-stationary smoothing does not reduce the amplitude of the tomographic component significantly. This difference between the two examples is mainly due to the high wavenumbers of the tomographic component (i.e. the rabbit ears) in the one-source-one-receiver example being generated by the oscillations of the sidebands of the rabbit ears, which do not include useful information about the background update. These oscillatory sidebands are cancelled out in the Marmousi example by the stack of multi-shots and multi-receivers.

If we design the Gaussian filter using a single reference frequency of 4 Hz or 12 Hz and a constant velocity of 2700 m/s, then the process becomes stationary Gaussian smoothing (e.g. Abdullah and Schuster, 2015; Kazei et al., 2016). The results are shown in Figures 10g and 10h. Compared with the nonstationary result shown in Figure 10f, the stationary smoothing either filters out some tomographic component (Figure 10g) or leaks some high-wavenumber migration component into the tomographic component (Figure 10h). Thus, stationary smoothing leads to a degradation of the inversion results.

The inverted model after 35 alternating iterations is shown in Figure 12. The RWI result with non-stationary smoothing is shown in Figure 12a. It is a good match to the background trend of the Marmousi velocity model, especially above the top boundary of the thick high-velocity salt layer, which is at a depth of around 2.5 km to 3 km. In contrast, RWI with stationary smoothing produces low-quality results. The reference frequency of 4 Hz produces a background model with lower resolution and more errors, for example, in the area at a distance of 8 km to 10 km (Figure 12b), because the stationary smoothing removes some high-

329 wavenumber tomographic component unnecessarily. The reference frequency of 12 Hz
 330 generates a background model full of high-wavenumber artifacts (Figure 12c). This is because
 331 the stationary smoothing leaks a significant amount of the migration component into the
 332 tomographic component. The leaked migration component prevents an effective background
 333 update.

334 Starting from the initial model and the RWI results (Figure 12), we carried out
 335 conventional FWI, as introduced by Tarantola (1984), with the reflection data (e.g. Figure 9b)
 336 from 4 Hz up to 10 Hz with an increment of 1 Hz (Bunks et al., 1995). Each frequency is
 337 inverted for five iterations; therefore, there are 35 iterations in total. We note that each
 338 ‘frequency’ means a frequency band centered at this frequency. The inverted models are
 339 depicted in Figure 13. From this comparison, we see the RWI model with nonstationary
 340 smoothing produces a better final model than the others. This is because the RWI model with
 341 nonstationary smoothing produces a good travel time match. This can be seen from the
 342 alternating display of a predicted shot gather and its corresponding recorded shot gather in
 343 Figure 14. The improved position of reflectors in the stacked migration profiles (Figure 15)
 344 and the flatness of two common-image-gathers (CIGs) (Figure 16) also suggest that the RWI
 345 model turned out to be kinematically correct.

DISCUSSION

347 We choose the Gaussian filter to extract the tomographic component of the gradient
 348 because the filter can be computed analytically, i.e., at a negligible computational cost.
 349 However, it also unnecessarily suppresses those wavenumbers smaller than the cutoff
 350 wavenumber, k_{tomo_max} (Figure 2a). To mitigate this undesired suppression, the cutoff angle
 351 α_{min} has to be chosen adequately and as small as possible, but without leakage from the
 352 migration component. As a rule of thumb, in this paper, we set the cutoff angle to 160°.

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Alternatively, we may find a filter that has a narrow transition band, unlike a Gaussian filter. Ideally, we expect this filter to have a simple analytical expression to avoid computing it numerically.

CONCLUSION

We analyzed the characteristics of scattering angle filtering and found that the wavenumber of the tomographic component of the raw gradient is smaller than that of its migration component. The tomographic component can, therefore, be extracted by low-wavenumber-pass filtering. We used Gaussian filters to achieve the low-wavenumber-pass filtering. As the Gaussian filters depend on the frequency of the inversion, the model velocity, and the minimum scattering angle, the filtering is a non-stationary smoothing process. This method can be directly applied to FWI based on frequency-domain modeling. As there is a relatively wide gap between the wavenumbers of the tomographic component and the migration component for reflection data, for the time-domain FWI, we divide the residual into several frequency groups. Then, we apply the non-stationary smoothing to each frequency group separately in order to prevent the migration component from leaking into the tomographic component. The two numerical tests demonstrated that the strategy of using just two frequency groups extracts the tomographic component successfully. With this strategy, only one extra backward propagation per iteration is necessary to extract the tomographic mode of the inversion within a typical frequency range.

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For Peer Review

APPENDIX A

FOURIER TRANSFORM PAIR

The continuous Fourier transform pair used for equations 8 and 9 is

$$f(k) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi x \cdot k} dx, \tag{A-1}$$

and

$$f(x) = \int_{-\infty}^{\infty} f(k) e^{j2\pi x \cdot k} dk, \tag{A-2}$$

where x denotes distance (unit: meter) and k represents wavenumber (unit: 1/m).

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For Peer Review

List of figure caption:

Figure 1. Schematic diagram for the relationship between opening angles and wavenumber distribution. α_{tomo_min} indicates the minimum opening angle for the tomographic component while α_{mig_max} indicates the maximum opening angle for the migration component. $\mathbf{k}_s$ and $\mathbf{k}_r$ are the wavenumber of the source and residual wavefields, respectively. They sit on the dashed circle. The tomographic component sits inside the inner dark-gray-filled circle while the migration component locates inside the outer light-gray ring. The transition zone is the middle-gray ring between them.

Figure 2. Sketch of a Gaussian filter in (a) the Fourier domain and (b) the space domain. In the Fourier domain, the Gaussian filter is determined by the cutoff wavenumber, $k_{tomo-max}$ and its corresponding amplitude value, β . γ is the truncation value to the space-domain Gaussian filter while l is the radius of the Gaussian filter corresponding to γ .

Figure 3. Gradients from the experiment of the one-source-one-receiver geometry. The true model is a two-layer model, the velocities of which are 2000 m/s for the top layer and 2200 m/s for the bottom layer. (a) The raw gradient. The blue and green crosses indicate the source and receiver position. The gray dash line in (a) represents the interface of the two layers. The initial model has the same geometry as the true model but the velocities are 1950 m/s and 2145 m/s for the top and bottom layers, respectively. (b) The source wavelet, the spectrum of which is flat between 3 Hz and 20 Hz shown as the dashed curve in Figures 5a and 5b. (c) The wavenumber spectrum of the raw gradient. (d) The enlarged view of the central part of (c). The solid blue and red circles represent the maximum wavenumber of the tomographic component, $k_{tomo-max}$, for 6 Hz and 12 Hz, respectively. The dash blue, red and cyan circles indicate the maximum wavenumber of the migration component for 6 Hz, 12 Hz and 24 Hz, respectively.

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As we invert pressure data for velocity, the unit of the gradient is $Pa^2 \cdot s/m$, where Pa stands for Pascal.

Figure 4. (a) The tomographic component of the gradient extracted with non-stationary smoothing frequency by frequency. (b) The wavenumber spectrum of (a).

Figure 5. Extraction of the tomographic component for the one-source-one-receiver geometry. (a) Four frequency groups used for frequency selection. The reference frequencies labelled on the horizontal axis are 1.5 Hz, 3 Hz, 6 Hz, and 12 Hz. The four color curves indicate the filters used for frequency selection. The dashed curve represents the spectrum of the wavelet shown in Figure 3b. (b) The same as (a) but with two frequency groups. (c) The tomographic component extracted with the four frequency groups shown in (a). (d) The wavenumber spectrum of the gradient shown in (c). (e) and (f) show the raw gradients selected by the two frequency groups. (e) corresponds to the lower frequency group while (f) is for the higher frequency group. (g) The tomographic component extracted with the two frequency groups shown in (b). The triangle indicates weak artifacts. (h-j) show the wavenumber spectra of the gradients in (e-g), respectively. The blue and red circles represent the maximum wavenumber of the tomographic component, $k_{tomo-max}$, for the reference frequencies, 6 Hz and 12 Hz, respectively.

Figure 6. Gaussian smoothing on the raw gradient shown in Figure 3a with the reference frequencies of (a) 6 Hz and (b) 12 Hz. (c) and (d) show the wavenumber spectra of (a) and (b), respectively. The blue and red circles represent the maximum wavenumber of the tomographic component, $k_{tomo-max}$, for the reference frequencies, 6 Hz and 12 Hz, respectively.

Figure 7. (a) The true Marmousi model. (b) The 1D initial velocity model.

Figure 8. (a) The source wavelet and (b) its frequency spectrum.

Figure 9. One shot gather of (a) the raw and (b) muted record.

Figure 10. The gradients of the first iteration for the background update. A multi-scale strategy is applied in the inversion. The data for the first iteration are extracted by a band-passed filter centered at 4 Hz. The two reference frequencies used for extracting the tomographic component is 2.67 Hz and 5.33 Hz, the raw gradients of which are shown in (a) and (c), respectively. (e) The stack of (a) and (c). (b), (d) and (f) are the counterpart of (a), (c) and (e) after the non-stationary smoothing. (g) The tomographic component extracted using stationary smoothing with a reference frequency of 4 Hz, a reference velocity of 2700 m/s and a minimum opening angle, α_{min} , of 160° . (h) The same as (g) but for a reference frequency of 12 Hz.

Figure 11. The wavenumber spectra of the gradients shown in Figure 10. (a), (b), (e) and (f) correspond to panel (a), (c), (b) and (d) of Figure 10, respectively. (c) and (d) are an enlarged view of (a) and (b), respectively. The blue and red curves in (a), (c) and (e) represent the maximum wavenumber of the tomographic component, $k_{tomo-max}$, of the reference frequency of 2.67 Hz for the velocity of 1600 m/s and 4000 m/s, respectively. The blue and red curves in (b), (d) and (f) are the same as those in (a), (c) and (e) but for the reference frequency of 5.33 Hz.

Figure 12. Recovered Marmousi models from the 1D initial model using RWI with (a) non-stationary smoothing, stationary smoothing at the reference frequency of (b) 4 Hz and (c) 12 Hz, the reference velocity of 2700 m/s and the minimum opening angle, α_{min} , of 160° .

Figure 13. Recovered Marmousi models of FWI starting from (a) the initial 1D model, and starting from (b-d) the RWI inverted models depicted in Figures 12 (a-c), respectively.

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Figure 14. Alternating display of one shot gather, comparing the records with the corresponding predicted gathers from (a) the 1d initial model, (b) the recovered model of RWI with non-stationary smoothing shown in Figure 12a, and (c) the recovered model of further FWI shown in Figure 13b. From left to right in each panel, the recorded data are shown first followed by the predicted data, and them alternating them. The dashed lines indicate the mute position of the direct arrivals and refraction events.

Figure 15. RTM images with (a) the initial model, (b) the recovered model from RWI with non-stationary smoothing shown in Figure 12a, (c) the recovered model from further FWI shown in Figure 13b, and (d) the true model shown in Figure 7a.

Figure 16. Two common-image-gathers (CIGs) at a distance of (a-d) 2.5 km and (e-h) 8.75 km. (a) and (e) are from the initial model. (b) and (f) are from the RWI model with non-stationary smoothing shown in Figure 12a. (c) and (g) are from the further FWI model shown in Figure 13b. (d) and (h) are from the true model shown in Figure 7a.

Figure 1. Schematic diagram for the relationship between opening angles and wavenumber distribution. α_{tomo_min} indicates the minimum opening angle for the tomographic component while α_{mig_max} indicates the maximum opening angle for the migration component. $\mathbf{k}_s$ and $\mathbf{k}_r$ are the wavenumber of the source and residual wavefields, respectively. They sit on the dashed circle. The tomographic component sits inside the inner dark-gray-filled circle while the migration component locates inside the outer light-gray ring. The transition zone is the middle-gray ring between them.

112x112mm (300 x 300 DPI)

Figure 2. Sketch of a Gaussian filter in (a) the Fourier domain and (b) the space domain. In the Fourier domain, the Gaussian filter is determined by the cutoff wavenumber, $k_{tomo-max}$ and its corresponding amplitude value, β . γ is the truncation value to the space-domain Gaussian filter while l is the radius of the Gaussian filter corresponding to γ .

103x61mm (300 x 300 DPI)

Figure 3. Gradients from the experiment of the one-source-one-receiver geometry. The true model is a two-layer model, the velocities of which are 2000 m/s for the top layer and 2200 m/s for the bottom layer. (a) The raw gradient. The blue and green crosses indicate the source and receiver position. The gray dash line in (a) represents the interface of the two layers. The initial model has the same geometry as the true model but the velocities are 1950 m/s and 2145 m/s for the top and bottom layers, respectively. (b) The source wavelet, the spectrum of which is flat between 3 Hz and 20 Hz shown as the dashed curve in Figures 5a and 5b. (c) The wavenumber spectrum of the raw gradient. (d) The enlarged view of the central part of (c). The solid blue and red circles represent the maximum wavenumber of the tomographic component, k_{tomo_max} , for 6 Hz and 12 Hz, respectively. The dash blue, red and cyan circles indicate the maximum wavenumber of the migration component for 6 Hz, 12 Hz and 24 Hz, respectively. As we invert pressure data for velocity, the unit of the gradient is $Pa^2 \cdot s/m$, where Pa stands for Pascal.

109x93mm (300 x 300 DPI)

Figure 4. (a) The tomographic component of the gradient extracted with non-stationary smoothing frequency by frequency. (b) The wavenumber spectrum of (a).

109x46mm (300 x 300 DPI)

Figure 5. Extraction of the tomographic component for the one-source-one-receiver geometry. (a) Four frequency groups used for frequency selection. The reference frequencies labelled on the horizontal axis are 1.5 Hz, 3 Hz, 6 Hz, and 12 Hz. The four color curves indicate the filters used for frequency selection. The dashed curve represents the spectrum of the wavelet shown in Figure 3b. (b) The same as (a) but with two frequency groups. (c) The tomographic component extracted with the four frequency groups shown in (a). (d) The wavenumber spectrum of the gradient shown in (c). (e) and (f) show the raw gradients selected by the two frequency groups. (e) corresponds to the lower frequency group while (f) is for the higher frequency group. (g) The tomographic component extracted with the two frequency groups shown in (b). The triangle indicates weak artifacts. (h-j) show the wavenumber spectra of the gradients in (e-g), respectively. The blue and red circles represent the maximum wavenumber of the tomographic component, k_{tomo_max} , for the reference frequencies, 6 Hz and 12 Hz, respectively.

164x145mm (300 x 300 DPI)

Figure 6. Gaussian smoothing on the raw gradient shown in Figure 3a with the reference frequencies of (a) 6 Hz and (b) 12 Hz. (c) and (d) show the wavenumber spectra of (a) and (b), respectively. The blue and red circles represent the maximum wavenumber of the tomographic component, k_{tomo_max} , for the reference frequencies, 6 Hz and 12 Hz, respectively.

108x91mm (300 x 300 DPI)

Figure 7. (a) The true Marmousi model. (b) The 1D initial velocity model.

141x60mm (300 x 300 DPI)

Figure 8. (a) The source wavelet and (b) its frequency spectrum.

74x78mm (300 x 300 DPI)

Figure 9. One shot gather of (a) the raw and (b) muted record.

136x73mm (300 x 300 DPI)

Figure 10. The gradients of the first iteration for the background update. A multi-scale strategy is applied in the inversion. The data for the first iteration are extracted by a band-passed filter centered at 4 Hz. The two reference frequencies used for extracting the tomographic component is 2.67 Hz and 5.33 Hz, the raw gradients of which are shown in (a) and (c), respectively. (e) The stack of (a) and (c). (b), (d) and (f) are the counterpart of (a), (c) and (e) after the non-stationary smoothing. (g) The tomographic component extracted using stationary smoothing with a reference frequency of 4 Hz, a reference velocity of 2700 m/s and a minimum opening angle, a_{min} , of 160°. (h) The same as (g) but for a reference frequency of 12 Hz.

157x193mm (300 x 300 DPI)

Figure 11. The wavenumber spectra of the gradients shown in Figure 10. (a), (b), (e) and (f) correspond to panel (a), (c), (b) and (d) of Figure 10, respectively. (c) and (d) are an enlarged view of (a) and (b), respectively. The blue and red curves in (a), (c) and (e) represent the maximum wavenumber of the tomographic component, k_{tomo_max} , of the reference frequency of 2.67 Hz for the velocity of 1600 m/s and 4000 m/s, respectively. The blue and red curves in (b), (d) and (f) are the same as those in (a), (c) and (e) but for the reference frequency of 5.33 Hz.

161x147mm (300 x 300 DPI)

Figure 12. Recovered Marmousi models from the 1D initial model using RWI with (a) non-stationary smoothing, stationary smoothing at the reference frequency of (b) 4 Hz and (c) 12 Hz, the reference velocity of 2700 m/s and the minimum opening angle, a_{min} , of 160°.

104x206mm (300 x 300 DPI)

Figure 13. Recovered Marmousi models of FWI starting from (a) the initial 1D model, and starting from (b-d) the RWI inverted models depicted in Figures 12 (a-c), respectively.

140x107mm (300 x 300 DPI)

Figure 14. Alternating display of one shot gather, comparing the records with the corresponding predicted gathers from (a) the 1d initial model, (b) the recovered model of RWI with non-stationary smoothing shown in Figure 12a, and (c) the recovered model of further FWI shown in Figure 13b. From left to right in each panel, the recorded data are shown first followed by the predicted data, and then alternating them. The dashed lines indicate the mute position of the direct arrivals and refraction events.

70x203mm (300 x 300 DPI)

Figure 15. RTM images with (a) the initial model, (b) the recovered model from RWI with non-stationary smoothing shown in Figure 12a, (c) the recovered model from further FWI shown in Figure 13b, and (d) the true model shown in Figure 7a.

140x98mm (300 x 300 DPI)

Figure 16. Two common-image-gathers (CIGs) at a distance of (a-d) 2.5 km and (e-h) 8.75 km. (a) and (e) are from the initial model. (b) and (f) are from the RWI model with non-stationary smoothing shown in Figure 12a. (c) and (g) are from the further FWI model shown in Figure 13b. (d) and (h) are from the true model shown in Figure 7a.

131x143mm (300 x 300 DPI)

DATA AND MATERIALS AVAILABILITY

Data associated with this research are available and can be obtained by contacting the corresponding author.