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Key Point:

• WDG method is more robust than full waveform inversion (FWI) and provides higher resolution than wave-equation traveltime tomography (WT)

Correspondence to:

J. Li,
inter.lijing@gmail.com

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Wave-Equation Dispersion Inversion of Guided *P* Waves in a Waveguide of Arbitrary Geometry

Jing Li^{1,2} , Sherif Hanafy^{2,3}, and Gerard Schuster²

¹College of Geo-Exploration Science and Technology, Jilin University, China, ²Department of Earth Science and Engineering, King Abdullah University of Science and Technology, Thuwal, Saudi Arabia, ³Geophysics Department, Faculty of Science, Cairo University, Giza, Egypt

Abstract We present a dispersion-inversion method which inverts for the *P*-velocity model from guided waves propagating in wave guides of arbitrary geometry. Its misfit function is the squared summation of differences between the predicted and observed dispersion curves of guided *P* waves, and the inverted result is a high-resolution estimate of the near-surface *P*-velocity model. We denote this procedure as wave-equation dispersion inversion of guided *P* waves (WDG), which is valid for near-surface waveguides with irregular layers and does not require a high-frequency approximation. It is more robust than full waveform inversion and can sometimes provide velocity models with higher resolution than wave-equation traveltime tomography. Both the synthetic-data and field data results demonstrate that WDG for guided *P* waves can accurately invert for complex *P*-velocity models at the near surface of the Earth.

Plain Language Summary In this paper, we present the theory for wave-equation inversion of guided *P* waves, where the misfit function is the sum of the squared differences between the wave numbers along the predicted and observed dispersion curves. This procedure, denoted as wave-equation dispersion inversion of guided *P* waves (WDG), is valid for near-surface waveguides with irregular layers. The importance of this work is that the WDG tomograms have higher resolution than the traveltime tomograms (WT) and mitigates the local minima problem of full waveform inversion (FWI). Numerical simulations and field data examples suggest that WDG is effective for selected 2-D *P*-velocity models where the dispersion curves can be readily identified.

1. Introduction

A waveguide is a structure that guides waves, such as electromagnetic waves (Strobbia & Cassiani, 2007), elastic waves (Y. G. Li & Leary, 1990; Roth et al., 1998), or ultrasonic waves (Foiret et al., 2014; Hosten et al., 2001), with minimal loss of energy by restricting the wave propagation to one or two dimensions (Robertsson et al., 1996; Sheriff & Geldart, 1982). For a source on the free surface of a quasi-layered Earth, guided waves mainly propagate along the interfaces. The guided waves consist of postcritical reflections and refractions where the deepest layer has a faster velocity than the one above it. The general characteristics of guided *P* waves are similar to surface waves where the propagating waves are dispersive and their geometrical spreading behaves as $O(r^{-1/2})$. There are many applications of guided waves in seismology such as imaging low-velocity faulted-zone layers (Y. G. Li et al., 1999, 2000), nondestructive evaluation and medical imaging with ultrasonic waves (Bochud et al., 2018; Moreau et al., 2017).

As an example of guided waves, Figure 1a shows a typical common shot gather (CSG) recorded over the Qademah fault system where there is a hidden low-velocity zone (LVZ) near the King Abdullah Economic City (KAEC) in Saudi Arabia. This LVZ is interpreted to be related to the Qademah fault (Hanafy, 2015; Roobol & Kadi, 2008). We can see the guided wave events that trail just behind the head-wave arrivals indicate resonance in the topmost layers. We denote a train of such events as a *railroad track*, because their traveltime curves are almost parallel to those of the refraction arrivals. The associated arrivals can also include *P*-wave reflections and refractions in the topmost system of layers, which can be isolated by a straightforward windowing of the early arrivals in Figure 1b. Guided waves disperse as they are trapped in the low-velocity layer of the near-surface, and the corresponding dispersion curves are shown in Figure 1d.

Figure 1. (a) Common shot gather recorded over the Qademah fault system near KAUST. The *railroad* tracks of guided-wave arrivals (GW) that trail just behind the head-wave arrivals indicate resonance in the topmost layers. The surface-wave arrivals are labeled as SW. Panels (b) and (c) are windowed guided waves and surface waves, respectively. (d) The spectrum of the shot gather reveals the dispersion curves for the guided waves and the surface waves.

The much slower arrivals in Figure 1a are the Rayleigh surface waves denoted as SW and propagate according to the S velocity of the associated layers. These surface waves are also known as guided waves because they are mainly postcritical P -SV reflections trapped in the near-surface layers (Figure 1c). Their corresponding dispersion curves, denoted as SW in Figure 1d, show that they are much slower than the guided P waves denoted as GW.

The ocean floor with soft sediments can be considered as a quasi-acoustic waveguide where the unconsolidated sediments underlain by rock only support P -wave propagation. Waveguides can also exist in foothill regions of the Earth in the form of a 5–20 m thick low-velocity layer underlain by a very fast rock (Buker et al., 1998; Chen et al., 2017; Kennett, 1984). In this case, seismic energy with frequencies higher than the cutoff frequency is mostly trapped in the waveguide. The consequence is that only a small percentage of the down-going source energy over a narrow downward pointing cone is transmitted into the deeper areas of interest. Therefore, it is important to invert for the velocity distribution in the waveguide so the seismic experiment and data processing can be designed to better illuminate the deeper strata. In addition, knowledge of the P -velocity distribution in the near-surface waveguides allows for the estimation of accurate static corrections (Boiero et al., 2013).

Modern Methods for Inverting Guided Waves.

Modern methods for P -wave velocity inversion of the velocity model can be grouped into four different classes (Huthwaite, 2014):

1. Ray-based tomography methods that assume a high-frequency approximation. In this case, the length scale of the velocity variations is assumed to be more than 3 times greater than the characteristic wavelength

Figure 2. Common shot gather $d(x, t)$ in (a) and fundamental ($n = 0$) dispersion curves in (b) $k - \omega$ and (c) $C(\omega) - \omega$ domains. Here the phase velocity is $C(\omega) = \omega/k(\omega)$ and $\kappa(\omega)$ is the skeletonized data (Schuster et al., 2017).

of the waves. Thus, ray-based tomography is typically limited to the resolution of no better than several wavelengths (Belanger & Cawley, 2009; Schuster, 2017; Volker et al., 2011; Williamson, 1991).

2. One-dimensional dispersion-inversion methods are limited by a layered-medium assumption. For earthquake seismology, a local 1-D layer assumption has proven effective for estimating lateral velocity variations by inverting the dispersion curves of guided waves (Boiero et al., 2013; Campbell et al., 2017; Roth & Holliger, 1999; Roth et al., 1998). This is an accurate approximation if the scale of the lateral velocity variations is much greater than the wavelengths under consideration. However, this assumption is inappropriate for strong lateral gradients in the subsurface velocity distribution.
3. Born approximation methods that assume weak scattering. For small P -velocity perturbations from the background model, the iterative Born approximation can provide a useful P -velocity model (Belanger et al., 2010; Huthwaite et al., 2012; Malyarenko & Hinders, 2001).
4. FWI methods (Mora, 1987; Virieux & Operto, 2009). The waveform inversion methods are potentially the most general in that they do not assume a high-frequency approximation or require small perturbations from the background velocity model. However, FWI suffers from a high computational cost and can lack robust convergence for complicated models. Iterative FWI can easily get stuck in a local minimum because the misfit function is highly nonlinear.

We now present the WDG method which eliminates the high-frequency approximation of ray-based tomography, removes the layered medium approximation of standard inversion of dispersion curves (Boiero et al., 2013; Campbell et al., 2017), and mitigates FWI's tendency of getting stuck in a local minimum. WDG is similar to the surface wave WD method of J. Li and Schuster (2016) and J. Li, Feng, and Schuster (2017), which is typically used to obtain the shear-wave velocity associated with Rayleigh-mode dispersion curves and a layered medium with irregular interfaces. Now we invert for the P -wave velocity model from guided P waves that propagate exclusively in the shallow layers. No Born, high-frequency or 1-D approximations are required, and it is more robust with complex field data compared to FWI. As noted by J. Li, Feng, and Schuster (2017), WDG is more robust than FWI because it uses the simpler dispersion curves in

the wave number domain to replace complicated guided wave arrivals. These dispersion curves are considered to be skeletonized data (Luo & Schuster, 1991a) that can be inverted by calculating solutions to the elastic wave equation.

The WDG theory is first presented for inverting for the P -velocity model from P waves propagating in a near-surface wave guide. We assume a few layers at the near surface where the guided waves associated with reverberating P reflections and refractions can be easily separated from the Rayleigh-like waves. We then present numerical results that validate the effectiveness of WDG applied to synthetic data and real data recorded over the Qadimah fault. The real data example provides several sanity tests that are used to test the validity of the final tomogram. The last section presents the conclusions and summary.

Figure 3. Dispersion curves associated with an acoustic slab of thickness h with a free surface on the top and a rigid boundary condition on the bottom. Illustration adapted from Grant and West (1965).

Figure 4. The workflow for wave-equation dispersion inversion of guided P waves.

2. Theory of Wave-Equation Dispersion Inversion of Guided Waves

The WDG formulas are now derived for inverting guided P waves in an elastic medium. These equations can also be used for the acoustic case where there are a few near-surface layers with strong velocity contrasts. For a certain range of Poisson's ratio and a two-layer near-surface model, Roth and Holliger (1999) show that the early-arrival guided waves mainly consist of reflected-refracted P waves, while the Rayleigh waves arrive much later in the records. The general formulas for updating the S -velocity models are presented in J. Li, Feng, and Schuster (2017).

We assume a 2-D elastic medium where there are P -velocity variations $v(\mathbf{x})$ along the x and z axes; for convenience, we assume that the P - and S -velocity ratio is constant. The source located at $\mathbf{s} = (x_s, 0)$ is a pressure line source perpendicular to the x - z plane with a bandlimited source wavelet. The vertical-component particle-velocity geophones at $\mathbf{g} = (x_g, 0)$ are evenly distributed along the free surface with the observed vertical-component spectrum denoted by $D(\mathbf{g}, \omega)_{obs}$, where ω denotes the harmonic frequency. This means that $D(\mathbf{g}, \omega)_{obs}$ satisfies the 2-D elastic equation with the P -velocity distribution denoted by $v(x_g, z)$. We also assume a line source so that efficient numerical solutions to the 2-D, not 3-D, wave equation can be used. This

Figure 5. Normalized phase-velocity derivatives with respect to the P -wave velocity variations $\partial C(\omega)/\partial c$ in the (a) first, (b) second, (c) third, and (d) fourth layers.

Figure 6. Normalized phase-velocity derivatives with respect to the thickness variations $\partial C(\omega)/\partial h$ in the (a) first, (b) second, (c) third, and (d) fourth layers.

is justified because WDG only inverts for the P -velocity model that minimizes the residual of the dispersion curves and does not attempt to explain amplitudes. A spatial-Fourier transform is applied to the data $D(\mathbf{g}, \omega)$ recorded along the x coordinate at $z = 0$ to give the spectrum $\tilde{D}(k, \omega)$. Here $k \rightarrow k_x$ and the notation for $z = 0$ is silent. The skeletonized data consist of the points picked from the fundamental dispersion curve $\kappa(\omega)_{\text{obs}}$, which is the red dashed line shown in Figure 2b. For a simplified exposition, we assume a single-shot gather and only invert the fundamental curve $\kappa(\omega)_{\text{obs}}$ associated with the guided P waves. This method is easily generalized to an arbitrary number of shot gathers and more detailed velocity models can be reconstructed from the higher-order dispersion curves.

The WDG gradient formulas are computed by numerical solutions to the elastic wave equation, but in most of our examples we only need to use solutions to the acoustic wave equation because guided P -wave reflections and refractions at the near surface are mostly sensitive to P -velocity variations, not to those of the S -velocity model.

1. *Form the misfit function.* Similar to the misfit function of surface-wave dispersion inversion (J. Li, Feng, & Schuster, 2017), the dispersion misfit function ϵ is defined as the sum of the squared dispersion-curve residuals:

$$\epsilon = \frac{1}{2} \sum_{\omega} \overbrace{(\kappa(\omega) - \kappa(\omega)_{\text{obs}})^2}^{\text{residual}=\Delta\kappa(\omega)}, \quad (1)$$

The P -velocity model $v(\mathbf{x})$ is computed by the iterative solution:

$$v(\mathbf{x})^{(k+1)} = v(\mathbf{x})^{(k)} - \alpha \sum_{\omega} \Delta\kappa(\omega) \frac{\partial \kappa(\omega)}{\partial v(\mathbf{x})}, \quad (2)$$

where v is the P -wave velocity model, α is the step length, and the superscript k indicates the iteration number. In practice, we use a preconditioned conjugate-gradient optimization method with the multiscale strategy described in Lu et al. (2017). A penalty function can be incorporated into the misfit function to penalize variations of the velocity model that are faster than the smallest wavelength in the data. For a simplified exposition, we now derive the Fréchet derivative for an acoustic model. The acoustic formula is useful

Figure 7. (a) Two-layer P -velocity model. (b) Common shot gather of data recorded over the synthetic model. The railroad tracks of guided wave arrivals (GW) that trail just behind the head-wave arrivals indicate resonance in the topmost layers. (c) Windowed shot gather of guided P waves. (d) Actual dispersion curves picked from the guided P -wave phase-velocity profile which is obtained by the FFT and Radon transform methods. (e) Two-dimensional dispersion-curve image of all shots, (f) WDG P -velocity tomograms for (g) acoustic and (f) elastic data. For (f) and (g), the input data were generated by finite-difference solutions to the elastic and acoustic wave equations, respectively. (h) WT P -velocity tomogram. FFT = fast Fourier transform.

because our numerical tests show that it gives almost the same P -velocity tomogram as that obtained from the Fréchet derivative for the elastic model (J. Li, Feng, & Schuster, 2017; Schuster, 2017). In Appendix A, we derive the Fréchet derivative in equation (2) and the final form of the Fréchet derivative is

$$\frac{\partial \kappa}{\partial v(\mathbf{x})} = \frac{2v(\mathbf{x})}{A} \text{Real} \left\{ \overbrace{W(\omega)G(\mathbf{x}, \omega | \mathbf{s}, 0)}^{\text{source field}=f(\mathbf{x}, \mathbf{s})_{\omega}} \int \overbrace{[\hat{D}(\mathbf{g}, \omega)_{\text{obs}} G(\mathbf{g}, \omega | \mathbf{x}, 0)^*]^*}_{\text{backprojected data}=b(\mathbf{x}, \mathbf{s})_{\omega}^*} d\mathbf{x}_g \right\}, \quad (3)$$

where the overbrace associated with $b(\mathbf{x}, \mathbf{s})_{\omega}$ indicates the weighted shot gather:

$$\hat{D}(x_g, \omega)_{\text{obs}}^* = \frac{-ix_g e^{-i\Delta \kappa x_g}}{2\pi} D(x_g, \omega)_{\text{obs}}^*. \quad (4)$$

Here $D(x_g, \omega)$ is the 2-D inverse Fourier transform of $\hat{D}(k, \omega)$, which is backprojected into the medium by an extrapolation operation. Multiplying the a conjugated Green's function $G(\mathbf{g}, \omega | \mathbf{x}, \omega)^*$ by the source time-history term $\hat{D}(\mathbf{g}, \omega)_{\text{obs}}$ and integrating over the geophone coordinate x_g computes the back-propagation of the recorded field into the interior of the velocity model (Schuster, 2017). Also, $f(\mathbf{x}, \mathbf{s})_{\omega}$ is the forward modeled wavefield for the source at $\mathbf{s}$ and recorded at $\mathbf{x}$.

2. *Gradient update.* Plugging equation (A10) into equation (2) gives the iterative formula to update the P -wave velocity model from one shot gather:

$$v(\mathbf{x})^{(k+1)} = v(\mathbf{x})^{(k)} - \alpha \sum_{\omega} \overbrace{\frac{2s(\mathbf{x})\Delta \kappa(\omega)}{A} f(\mathbf{x}, \mathbf{s})_{\omega} b(\mathbf{x}, \mathbf{s})_{\omega}^*}^{\text{misfit gradient } f \text{ or a single CSG}}. \quad (5)$$

Figure 8. Plots of (a) fundamental dispersion curves for some of the predicted and observed shot gathers and (b) normalized misfit function versus iteration number.

Equation (5) says that the back-projected weighted data $b(\mathbf{x}, \mathbf{s})_\omega$ are conjugated and multiplied by the weighted source field $\frac{2s(\mathbf{x})\Delta\kappa(\omega)}{A}f(\mathbf{x}, \mathbf{s})_\omega$ to give the P -velocity update at $\mathbf{x}$. The modified source wavelet is defined as $\Delta\kappa(\omega)W(\omega)$ so that the P -velocity update is proportional to the zero-lag correlation between the back-projected data and the weighted source field.

In summary, the P -velocity model is updated by calculating the spectra of the guided wave dispersion curves for both the observed and predicted data. The observed data $D(\mathbf{g}, \omega)_{\text{obs}}$ of all sources are backprojected, where the forward-propagated source has the weighted source spectrum $W(\omega)\Delta\kappa(\omega)$. The gradients of all shot gathers are summed together to get the P -velocity model update (J. Li & Schuster, 2016). In practice, the velocity update at each iteration is computed by reverse time migration of the weighted traces in the space-time domain.

2.1. Resolution and Higher-Order Modes

Higher-order guided waves can be more sensitive to changes in the medium parameters than the lower-order modes. This claim can be demonstrated in the case of an acoustic slab with a free surface on the top and a rigid boundary at the bottom, and the slab thickness is h . In this case, Grant and West (1965) shows that the horizontal phase slowness $1/C(\omega)$ of a specified mode is given by

$$S(\omega) = \frac{1}{C(\omega)} = \left[\frac{1}{c^2} - \frac{(n + 1/2)^2 \pi^2}{\omega^2 h^2} \right]^{1/2}, \quad (6)$$

where n is the mode number, $C(\omega)$ is the phase velocity of a specified mode, c is the P wave velocity, and ω is the angular frequency. Differentiating the phase slowness in equation (6) with respect to the slab thickness h gives:

$$\frac{\partial S(\omega)}{\partial h} = \frac{(n + 1/2)^2 \pi^2}{S(\omega) \omega^2 h^3}. \quad (7)$$

This equation says that the apparent slowness becomes more sensitive to changes in the thickness h for increasing mode numbers. As an example, Figure 3 displays the dispersion curves for an acoustic slab, where

Figure 9. (a) True P -velocity model and (b) P -velocity tomogram after 25 iterations of the WDG method.

Figure 10. Plots of (a) dispersion curves for all of the predicted and observed shot gathers and (b) normalized misfit function versus iteration number.

the slope of the $n = 1$ curve at the frequency f_0 is steeper than the slope for the $n = 0$ curve. The large value of the slope at f_0 for $n = 1$ indicates a strong sensitivity of the data, that is, changes in $S(\omega)$, to changes in the slab thickness. Thus, the WDG method has the capability for high-resolution imaging of the slab thickness. This is not too surprising because the steep part of the dispersion curves corresponds to nearly vertically incident waves, which generate resonant reflections that can potentially provide superresolution imaging (Schuster & Huang, 2014).

The formula that relates depth resolution δh to mode number n can be obtained by defining the phase $\phi(x, t, \omega)$ of the traveling guided waves as

$$\phi(x, t, \omega) = \omega[xS(\omega) - t]. \quad (8)$$

Perturbing ϕ in equation (8) by δh , inserting equation (7), and after some rearrangement we get

A Base Map of the Study Area

Figure 11. Surface geology map showing the location of the Qademah fault. The zoom view of the orange recording line at Site 2 is shown on the right.

Figure 12. (a) A windowed CSG, and (b) its phase-velocity image. Hotter colors correspond to larger amplitude values in the spectrum.

$$\delta h = \frac{\delta \phi \omega h^3}{x} \frac{\left[\frac{1}{v_p^2} - \frac{(n+1/2)^2 \pi^2}{\omega^2 h^2} \right]^{1/2}}{(n+1/2)^2 \pi^2}, \quad (9)$$

where $\delta \phi$ is the phase perturbation due to a change in layer thickness δh . If $\delta \phi$ is set to a detectable level in the data, then we conclude that δh gets smaller as the mode order n becomes larger. This suggests that small changes in δh can be detected with a detectable change in the phase $\delta \phi$. Hence, superresolution estimation of the slab thickness might be possible by inverting higher-order modes in the dispersion curves.

2.2. Workflow for Wave-Equation Dispersion Inversion of Guided Waves

Figure 4 depicts the workflow for the WDG method.

1. Isolate the near-surface guided waves from the other arrivals and apply a 1-D temporal Fourier transform to these events to get the frequency spectrum of each trace.
2. Apply a linear Radon transform (LRT) to the predicted and observed CSGs to get the dispersion curves $\kappa(\omega)$ and $\kappa(\omega)^{\text{obs}}$ (Luo et al., 2008). Compute the dispersion curve residuals in equation (1). The traces recorded within about one fourth of the wavelength from the source are excluded. For large data sets, an adaptive machine-learning algorithm can be used to efficiently and accurately pick dispersion curves (J. Li et al., 2018).
3. Use equation 4 to compute the weighted data $\hat{D}(\mathbf{g}, \omega)_{\text{obs}}$, which is then used for computing the backprojected data $b(\mathbf{x}, \mathbf{s})_{\omega}$ in equation (5). The source field $f(\mathbf{x}, \mathbf{s})_{\omega}$ in equation (5) computed by a finite-difference solution to the elastic wave equation.
4. Employ the zero-lag cross correlation of the forward- and backward-propagated wavefields to calculate the misfit gradient (see equation (5) for each shot gather).
5. Use equation (5) with a line search to get the update for the P velocity. The preconditioned conjugate-gradient method will lead to a fast convergence.
6. In addition, we use a multiscale strategy to invert the dispersion curves. We first invert the dispersion residuals at higher frequencies and shorter offsets to reconstruct velocities for the shallow part of the model. Then we increase the offsets and lower the frequencies to invert for the deeper areas of the model (J. Li, Feng, & Schuster, 2017).

3. Numerical Tests

3.1. Sensitivity Analysis

We use a four-layered model to test the sensitivity of the phase velocity $C(\omega)$ to changes in the P -wave velocity $\partial C(\omega)/\partial c(x)$ and the layer thickness $\partial C(\omega)/\partial h$, where $c(\mathbf{x})$ and h represent the P -wave velocity and layer thickness, respectively. The result plotted in Figure 5 shows that the sensitivity of the phase-velocity with respect to changes in the P velocity is frequency-dependent, where the phase velocity variations $\partial C/\partial c$ in the shallower (deeper) layers are more sensitive to higher (lower) frequencies. This indicates that lower frequencies are needed to image deeper structures in the near surface. This is also true for estimating layer thicknesses as shown in Figure 6, where the low-frequency phase-velocity derivatives $\partial C/\partial h$ are most sensitive to thickness variations in the deeper layers (Boiero et al., 2013; Campbell et al., 2017).

3.2. Two-Layer Model With an Irregular Interface

The WDG method is tested on elastic data computed for the two-layer model shown in Figure 7a. The layer interface is between the depths of 10 m and 12 m, where the P velocities are 1,000 m/s and 2,500 m/s,

Figure 13. (a) *P*-velocity tomogram by multiscale wave-equation traveltimes inversion of first arrivals, (b) *P*-velocity tomogram obtained by WDG applied to the Qademah data, and (c) recorded COGs at the source-receiver offset of 0 m. Dashed lines indicate abrupt discontinuities in the reflectivity interface, which might indicate the presence of cracks, faults, or LVZs in the subsurface. COGs = common offset gathers; LVZs = low-velocity zones.

respectively. A Ricker wavelet with the peak frequency of 30 Hz is used as the source wavelet for computing the data, and the associated dominant wavelength is 33 m in the first layer. For the observed data, we use 2-D finite difference solutions to the elastic wave equation to compute the 30 shot gathers (Figure 7b) with 60 receivers located every 4 m on the free surface. The initial model is a linear velocity-gradient model $v(z) = v_0 + \alpha z$, where v_0 is the *P* velocity at the surface, $\alpha = (v_{\max} - v_{\min})/h$ approximates the velocity gradient, z is the depth, h is the maximum depth of the model, and $v_{\max}$ and $v_{\min}$ are the maximum and minimum *P*-velocity values in the model.

We first window around the guided-wave arrivals shown in Figure 7c, use the 1-D FFT and the linear Radon transform (LRT) to transform each windowed shot gather into the $C - f$ domain, and then pick the dispersion curve of the fundamental mode for each shot gather (Figure 7d). Here $C(f)$ is the phase velocity obtained by computing the LRT of the windowed traces in the frequency domain (J. Li, Feng, & Schuster, 2017). In the example of Figure 7c, the window size of the LRT is approximately one-wavelength wide and includes about 15 traces. All 60 shot gathers can be windowed and transformed into 60 distinct dispersion curves, one for each of the 60 shots. These dispersion curves are concatenated together to get a pseudo-section of the

Figure 14. Comparison of (a) the recorded COG, (b) and (c) the synthetic COGs computed from the WDG and WT tomograms, respectively. In this example the source-receiver offset is 50 m and the dashed red line is picked from the maximum amplitudes in the COG shown in (a).

phase-velocity distribution as a function of frequency (Figure 7e). We use the multioffset WDG strategy to invert for the P -velocity model using the conjugate-gradient method (J. Li, Lin, et al., 2017). Figure 7f shows the WDG tomogram after 25 iterations, and the predicted dispersion curves shown in Figure 8a (black solid lines) resemble the observed values (black dashed lines). The normalized misfit function in Figure 8b shows rapid convergence, and the inverted P -velocity tomogram in Figure 7f is almost identical to the true model in Figure 7a. Figure 7g shows the inverted result by using acoustic WDG. We find that the inverted P -velocity tomogram is almost identical to the one where the observed data are generated by using the elastic finite-difference method in Figure 7f, which suggests that the WDG method is valid for both acoustic and elastic data.

We also compare the WDG tomogram to the tomogram computed by wave-equation traveltime inversion (WT). In the example, the multiscale WT tomogram is slightly less accurate than the WDG tomogram but has about the same resolution as the WDG tomogram. However, the travel-times of the first arrivals must be tediously picked for WT while the WDG method only requires a rough estimate of the muting window and an automatic picking of the dispersion curve with maximum amplitudes.

3.3. Complex Three-Layered Model Test

Data are now generated for the complex three-layer velocity model in Figure 9a to assess the resolution capabilities of the WDG method. The first-layer interface is at the depth of 2 m with the P -wave velocities $c_1 = 900$ m/s and $c_2 = 1,100$ m/s, respectively. The guided waves have very weak amplitudes due to the subtle velocity contrasts.

The second interface is at the depth of 8 m and the P -wave velocity of the third layer is 1,800 m/s. Here the guided waves are trapped above the second interface, and have strong amplitudes. A Ricker wavelet with a 30-Hz peak frequency is used as the source wavelet for generating these data,

and has the dominant wavelength of 37 m in the second layer. We still use the velocity-gradient model as the initial model and the inverted model is shown in Figure 9b, where the white dashed lines depict the velocity interfaces. The predicted dispersion curves shown in Figure 10a (black solid lines) largely agree with the observed values shown in Figure 10a (black dashed lines), and the normalized misfit function plotted in Figure 10b shows a rapid convergence. However, the WDG method cannot reconstruct the topmost thin layer because the weak-velocity contrast is not able to generate guided waves with strong amplitudes.

3.4. Qadimah Field Data

The WDG method is applied to the field data recorded over a hidden LVZ near the King Abdullah Economic City (KAEC) in Saudi Arabia. This LVZ is denoted as the Qadimah fault and strikes in a north-south direction (Figure 11). The direct evidence for the fault is not seen at this location, but it is inferred from the geophysical data as a LVZ (Hanafy, 2015). The shot spacing is 5 m, the vertical-component receiver spacing is 2.5 m, and there are 240 traces per shot gather to give a total of 120 shot gathers. Inverting the Love waves gives a low- S -velocity tomogram in the middle of site 2 (J. Li, Lin, et al., 2017).

The guided P -wave dispersion curves are inverted to get the P -velocity tomogram. Figure 12a depicts a typical shot gather with guided P waves and its associated dispersion image. It is easy to identify the fundamental-mode dispersion curve (white dashed line) with the maximum amplitudes in the velocity-frequency domain (Figure 12b).

Figure 13a shows the P -wave tomogram computed by first-arrival wave-equation traveltime inversion with the steeply dipping dashed lines interpreted from the zero-offset common offset gathers (COGs) in Figure 13c. Figure 13b shows the P -wave velocity tomogram inverted from the dispersion curves. This interpretation is consistent with the red dashed lines seen in the COGs (Figure 13c). Here a LVZ is suggested by a rapid lateral change in the P -velocity values.

Figure 15. Comparison of the observed (black line) and inverted shot gathers computed from the (a) WT and (b) WDG tomograms, respectively.

WT traces. This suggests that the WDG tomogram might be more accurate than the WT tomogram. However, the blocky features in the WT tomogram seem more geologically reasonable than the smoothly varying features of the WDG tomogram. Resolving this apparent contradiction is a subject of research.

4. Discussion and Conclusions

We present the theory for wave-equation inversion of dispersion curves computed from guided P waves, where the misfit function is the difference between the wave numbers along the predicted and observed dispersion curves. The P -velocity model is updated by migrating the weighted data, where the weight is proportional to the dispersion residual. Our synthetic examples show that WDG avoids getting stuck in a local minimum, does not require a high-frequency approximation, and achieves high-resolution imaging of the velocity model. A field data example suggests that WDG is effective for inverting for 2-D P wave velocity models. The comparison between the WT and WDG tomograms suggests that the COGs simulated from the WDG tomogram more accurately reveal lateral variations in P velocity than provided by the WT tomogram. Practical uses of this high-resolution methodology include the estimation of the near-surface P velocity model for statics estimation, hazard assessment for building and dam construction, hydrological studies, and imaging of fluid contaminants. Future research should explore the possibility of getting even better resolution by inverting the higher-order modes in the dispersion curves. An interesting possibility is to use WDG to invert the strong-amplitude guided waves obtained by cross-correlation of passive seismic data. In this case, first-arrival refractions are too weak to be picked and inverted. An interesting possibility is to use WDG to invert the strong-amplitude guided waves obtained by cross-correlation of passive seismic data. In this case, first-arrival refractions are too weak to be picked and inverted.

There are several challenges with the WDG.

1. WDG cannot invert for the P -velocity model unless there are significant velocity contrasts in the LVZ. Strong velocity contrasts lead to sufficiently strong dispersion in the guided waves so they can be reliably identified and inverted.
2. WDG requires a wide-frequency band so that the thickness of the thinnest layer is larger than about one fourth of the shortest wavelength. Any frequency lower than the cutoff frequency will not lead to dispersive guided waves for that layer.

The WT tomogram in Figure 13a indicates a blocky high-velocity structure at the offset of about 100 m, and it also shows a mostly horizontal interface with offsets $150 \text{ m} < x < 600 \text{ m}$ and the depth of $z \approx 10 \text{ m}$. In contrast, the WDG tomogram in Figure 13b shows a more rounded high-velocity zone (HVZ). In addition, the WDG tomogram shows a low-velocity (blue colors at $0 < z < 10 \text{ m}$) zone that becomes deeper from the offset $x = 600 \text{ m}$ to the area around $x = 200 \text{ m}$. This dipping low-velocity feature appears to be more consistent with the zero-offset COG in Figure 13c where there is an increasing delay in the reflection events from right to left. The HVZ in both tomograms is in agreement with the zero-offset common offset gather (ZOG) in Figure 13c, which indicates faults, cracks, or LVZs adjacent to the HVZ. The ZOG indicates a sharply defined velocity contrast at the HVZ, which is more consistent with the WT tomogram. However, the ZOG does not appear to be consistent with the WT tomogram in that a sharp horizontal interface at $z = 10 \text{ m}$ and $150 \text{ m} < x < 600 \text{ m}$.

As a sanity test, Figure 14 compares the predicted COGs computed from the WDG and WT tomograms to the recorded COG. The COG predicted from the WT tomogram simulates the smoothly varying events in the recorded COG but is not able to capture the sudden jumps in the recorded events. However, the WDG COG profile (offset = 50 m) in Figure 14b is more consistent with such jumps in the recorded COG in Figure 14a. We also select one shot gather in Figure 15 to compare the predicted and recorded traces. It is observed that the WDG traces agree more with the recorded traces than the

3. If the near-surface model is too complex, then coherent dispersion curves might not be identifiable. Further tests are needed to assess the robustness of this method for more complex models.
4. The final dispersion tomogram is sensitive to the accuracy of the picked dispersion curves and the offset range of traces used for calculating the Radon transform.
5. In our work we assumed that attenuation is negligible. However, if the attenuation model is known, then its effects can be accounted for by solving the viscoelastic wave equation to compute the theoretical dispersion curves. Accounting for attenuation can be important for fault zones, where attenuation is typically quite significant near the surface and in damaged faulted-zone layers. In these cases, attenuation acts as a nonlinear low-pass filter and can modify considerably the dispersion curves (Peng et al., 2003). Ben-Zion and Aki (1990) provided an analytical solution for guided waves in a simple geometry consisting of parallel faulted-zone layers between two quarter-spaces, using the acoustic wave equation and including attenuation. Peng et al. (2003) demonstrated the effect of attenuation and other model parameters on the dispersion of acoustic guided waves using the analytical solution of Ben-Zion and Aki (1990) and analyzed dispersion curves of guided waves in the rupture zone of the 1992 Landers CA earthquake. Additional studies of fault zones using guided waves in the *P* and *S* waveforms can be found in Qin et al. (2018) and Qiu et al. (2017). In these works the waveguide model has a planar geometry.
6. Instead of inverting just for velocity, the WDG method can, in principle, be modified to invert for both the velocity and attenuation models. In this case the viscoelastic wave equation and its numerical solutions must be computed to estimate the gradients for the attenuation parameters (J. Li, Dutta, & Schuster, 2017; Schuster et al., 2017). However, there is an inherent nonuniqueness problem in inverting for both velocity and attenuation models, so it is likely that both dispersion curves and normalized amplitudes should be used as input data.

Appendix A : Fréchet Derivative

The Fréchet derivative $\frac{\partial \kappa(\omega)}{\partial v(\mathbf{x})}$ is obtained by forming a connective function (Luo & Schuster, 1991a) that relates the wave number residual $\Delta \kappa(\omega)$ to the velocity model. This connective function is the crosscorrelation between the observed $\tilde{D}(k, \omega)_{\text{obs}}^*$ and predicted $\tilde{D}(k, \omega)$ spectra in the wave number domain:

$$\Phi(\Delta \kappa, v(\mathbf{x})) = \text{Real} \int \dot{\tilde{D}}(k + \Delta \kappa, \omega)_{\text{obs}}^* \tilde{D}(k, \omega) dk = 0, \quad (\text{A1})$$

where $\dot{\tilde{D}}(k, \omega)_{\text{obs}} = \frac{\partial \tilde{D}(k, \omega)_{\text{obs}}}{\partial k}$ and $\Delta \kappa$ is the wave number lag that aligns the observed and predicted real parts of the spectra with one another for a specified ω , which is also equal to the dispersion residuals.

The derivation of the Fréchet derivatives for *P*-velocity and *S*-velocity variations is given in J. Li, Feng, and Schuster (2017) and Schuster (2017). For a simplified exposition, we now derive the Fréchet derivative for the *P* velocity in an acoustic mode. The acoustic formula is useful because our numerical tests show that it gives almost the same *P*-velocity tomogram as that obtained from the Fréchet derivative for the elastic *P* velocity. The implicit function theorem (Luo & Schuster, 1991b) implies from equation (A1) that κ is an implicit function of $v(\mathbf{x})$ so that

$$d\Phi = \frac{\partial \Phi}{\partial v} dv + \frac{\partial \Phi}{\partial \kappa} d\kappa = 0. \quad (\text{A2})$$

So, the Fréchet derivative

$$\frac{\partial \kappa}{\partial v(\mathbf{x})} = -\frac{\partial \Phi / \partial v}{\partial \Phi / \partial \kappa}, \quad (\text{A3})$$

where the denominator is the normalization term

$$A = \frac{\partial \Phi(\Delta \kappa, v(\mathbf{x}))}{\partial \kappa} = \text{Real} \left\{ \int \dot{\tilde{D}}(k + \Delta \kappa, \omega)_{\text{obs}}^* \tilde{D}(k, \omega) dk \right\}, \quad (\text{A4})$$

the numerator is

$$\frac{\partial \Phi(\Delta \kappa, v(\mathbf{x}))}{\partial v(\mathbf{x})} = \text{Real} \left\{ \int \dot{\tilde{D}}(k + \Delta \kappa, \omega)_{\text{obs}}^* \frac{\partial \tilde{D}(k, \omega)}{\partial v(\mathbf{x})} dk \right\}. \quad (\text{A5})$$

Plugging equations (A4) and 7 into equation (A3) gives the Fréchet derivative:

$$\begin{aligned}\frac{\partial \kappa}{\partial v(\mathbf{x})} &= -\frac{\partial \Phi / \partial v}{\partial \Phi / \partial \kappa} \\ &= \frac{-1}{A} \text{Real} \left\{ \int \dot{D}(k + \Delta \kappa, \omega)_{\text{obs}}^* \frac{\partial \tilde{D}(k, \omega)}{\partial v(\mathbf{x})} dk \right\} \\ &= \frac{-1}{A} \text{Real} \left\{ \int \hat{D}(\mathbf{g}, \omega)_{\text{obs}}^* \frac{\partial D(\mathbf{g}, \omega)}{\partial v(\mathbf{x})} d\mathbf{g} \right\}.\end{aligned}\quad (\text{A6})$$

In equation (A5), we can use the Fourier transforms to replace the functions in the integrand:

$$\begin{aligned}\dot{D}(k + \Delta \kappa, \omega)_{\text{obs}} &= \frac{1}{2\pi} \int [i y' D(y', \omega)_{\text{obs}} e^{i(k + \Delta \kappa)y'}] dy', \\ \tilde{D}(k, \omega) &= \frac{1}{2\pi} \int D(y, \omega) e^{iky} dy,\end{aligned}\quad (\text{A7})$$

to obtain

$$\begin{aligned}\frac{\partial \Phi(\Delta \kappa, v(\mathbf{x}))}{\partial v(\mathbf{x})} &= \frac{1}{4\pi^2} \left\{ \int d\mathbf{x}_g \left\{ \int d\mathbf{x}'_g \left[\int e^{ik(x_g - x'_g)} dk \right] (-ix'_g) D(x'_g, \omega)_{\text{obs}}^* e^{-i\Delta \kappa x'_g} \right\} \frac{\partial D(x_g, \omega)}{\partial v(\mathbf{x})} \right\}, \\ &= \frac{1}{4\pi^2} \left\{ \int d\mathbf{x}_g \left\{ \int d\mathbf{x}'_g [2\pi \delta(x_g - x'_g)] (-ix'_g) D(x'_g, \omega)_{\text{obs}}^* e^{-i\Delta \kappa x'_g} \right\} \frac{\partial D(x_g, \omega)}{\partial v(\mathbf{x})} \right\}, \\ &= \frac{1}{2\pi} \left\{ \int d\mathbf{x}_g [-ix_g D(x_g, \omega)_{\text{obs}}^* e^{-i\Delta \kappa x_g}] \frac{\partial D(x_g, \omega)}{\partial v(\mathbf{x})} \right\}, \\ &= \left\{ \int d\mathbf{x}_g \left[\frac{-ix_g e^{-i\Delta \kappa x_g}}{2\pi} D(x_g, \omega)_{\text{obs}}^* \right] \frac{\partial D(x_g, \omega)}{\partial v(\mathbf{x})} \right\},\end{aligned}$$

here,

$$\hat{D}(y, \omega)_{\text{obs}}^* = \frac{-iye^{-i\Delta \kappa y}}{2\pi} D(y, \omega)_{\text{obs}}^*.\quad (\text{A8})$$

Here $D(y, \omega)$ is the 2-D inverse Fourier transform of $\tilde{D}(k, \omega)$. For computational efficiency, we will assume that the guided waves mostly consist of P -wave reflections and refractions with little influence from the S -wave reflections. This convenient assumption means that the computationally more expensive Fréchet derivative computed by the elastic wave-equation solutions can be replaced by that computed by acoustic wave-equation solutions. Therefore, we use the Born approximation for the Helmholtz equation (Luo & Schuster, 1991a)

$$\frac{\partial D(y, \omega)}{\partial v(\mathbf{x})} = -2v(\mathbf{x})W(\omega)G(\mathbf{g}, \omega|\mathbf{x}, 0)G(\mathbf{x}, \omega|\mathbf{v}, 0),\quad (\text{A9})$$

where $W(\omega)$ is the source wavelet spectrum and $G(\mathbf{g}, \omega|\mathbf{x}, 0)$ is the harmonic solution to the 2-D scalar wave equation for the z displacement at the point $\mathbf{x} = (x, z)$ and a z component particle-velocity recording at $\mathbf{g} = (x_g, z_g)$. Recognizing that $\mathbf{g} = (x_g, z_g)$ is associated with the geophone locations $\mathbf{g} = (x_g, 0)$ on the surface, and plugging equation (A9) into equation (A6) gives the final form of the Fréchet derivative:

$$\frac{\partial \kappa}{\partial v(\mathbf{x})} = \frac{2v(\mathbf{x})}{A} \text{Real} \left\{ \overbrace{W(\omega)G(\mathbf{x}, \omega|\mathbf{s}, 0)}^{\text{source field}=f(\mathbf{x}, \mathbf{s})_{\omega}} \int \overbrace{[\hat{D}(\mathbf{g}, \omega)_{\text{obs}} G(\mathbf{g}, \omega|\mathbf{x}, 0)^*]^* d\mathbf{g}}^{\text{backprojected data}=b(\mathbf{x}, \mathbf{v})_{\omega}^*} \right\},\quad (\text{A10})$$

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