Waveform inversion based target-oriented redatuming

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SUMMARY

Finding a velocity model that produces simulated data that fits the observed one is the main objective of full waveform inversion (FWI). To meet such an objective we often need to solve for a high-resolution delineation of the subsurface medium. The current algorithms are usually implemented over the entire model space with a consistent discretization and physical assumptions, which can be both complex and costly in practice. Alternatively, we develop an FWI framework that utilizes a split model to an overburden, like the medium above a reservoir, and the underlying represented by data at a datum at the bottom of the overburden. We simultaneously invert for the velocity model above the datum level, which effects the redatuming process but often owns to more simple physics, and the corresponding data at that datum, which may represent a complex reservoir region. We formulate the redatuming operator using a modified expression of the extended Born representation, which is a multi-dimensional crosscorrelation. The resulting modeling needed in such an inversion includes wavefields from a source and those ignited at the datum level. We estimate the overburden velocity using low-wavenumber updates along the modeled reflection wavepaths. The dimensionality of the model extension and the retrieved data helps us match data on the surface, which results in a robust implementation. Tests on a simple model and the Marmousi show that our method can build a good velocity model and also obtain redatumed data with reasonable amplitude accuracy.

INTRODUCTION

Current developed implementations of FWI focus on using gradient-based methods to achieve the model updates (Taratola, 1984; Pratt, 1999), which require the computation of the forward and adjoint wavefields. To fulfill the goal of FWI in providing high-resolution delineation of the reservoir, we often need to solve a multi-parameter optimization on a fine grid. The fine grid required by a high resolution reservoir makes the FWI process expensive and in many cases impractical. FWI becomes even more involved when we try to monitor time lapse changes in the reservoir. Some studies have been developed recently to reduce the computation cost of FWI by producing updates with more balanced illumination. So improving the convergence speed can help FWI be more practical (Métivier et al., 2013), but to a limit as such approaches requires costly Hessians, which is an issue for 3D models. Another approach, not categorized often as FWI, is to implement a target-oriented inversion by focusing on a localized wavefield (Yang et al., 2012; Vasconcelos et al., 2017). In order to highlight certain regions of interest, they propose deploying an optimal (virtual) survey or a modified objective function, which all rely on the data retrieval at shifted sources and/or receivers (van der Neut et al., 2015), referred to as seismic redatuming.

Redatuming has long been used in the seismic community to improve data processing and imaging of geologically complex surfaces. Its historic purposes are to remove elevation and near surface statics from the data collected from irregular surfaces and to bring the observed data to a more ideal datum below the complex near surface often associated with land data (Berryhill, 1979; Yilmaz and Lucas, 1986; Rothman, 1986; Zhu et al., 1992; Taner et al., 1998). Redatuming methods can be separated into model-based and correlation-based schemes that solve various imaging problems (Fink, 1992; Snieder, 2004; Wapenaar, 2004). More recently, Wapenaar et al. (2014) combined interferometric redatuming with Marchenko iterative methods to retrieve two-sided virtual data. With the importance of anisotropic parameterization of the medium and high resolution delineation of the reservoir, the redatuming process is becoming increasingly important as it provides with a venue to reduce the model space for the redatumed data, and thus, allow for more complex physics to be considered.

Based on the target-oriented spirit we split our classic inversion problem into the model above a datum level and the data at the datum, which represent the medium below the datum. The resulting modeling in such an inversion includes wavefields from a source and those ignited at the datum. Suppose we have a baseline model to calculate the Green's function between the original survey and the virtual survey, we can retrieve the virtual data by a waveform redatuming operator defined by some extensions to the interaction between down and upgoing wavefields. However, in practice the estimation of the overburden structure could be a complex task of its own. In this abstract, we redefine an optimization problem based on waveform inversion that inverts for the virtual data and the overburden model in a simultaneous fashion. To invert for the velocity above, we take advantage of the update along the reflection wavepath, which provides considerably low wavenumbers (Xu et al., 2012; Alkhalifah and Wu, 2016). To reproduce the reflections in the overburden model, we utilize the migration/demigration process with model extensions (Hou and Symes, 2015; Guo et al., 2017). The redatumed data will produce primary reflections originally from below the datum level. As we will demonstrate, the extended wavefields guarantee a good data match, which allows us to avoid converging to local minima, and produce effective low-wavenumber updates from the farthest offsets. We will describe our waveform redatuming and velocity inversion algorithm, and then use the Marmousi model to demonstrate the capabilities of our method in building a plausible velocity, even with our assumed recorded data are free of frequencies below 2 Hz.

THEORY

Least-squares waveform redatuming

We start with the procedure of modeling from datum. We as-

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sume the model above the datum level is the only priori information in redatuming, which controls all wave propagation in our optimization. Suppose we know the Green's function of the underlying medium, we can define the datuming wave equation as

$$\mathbf{F}\tilde{u}(s, \mathbf{x}, t) = \int d\mathbf{x}_s \int d\mathbf{h} u(s, \mathbf{x}_s - \mathbf{h}, t) \times g_{\mathbf{d}}(\mathbf{x}_s - \mathbf{h}, \mathbf{h}, t), \quad (1)$$

where $\tilde{u}$ is the particle displacement wavefield, u is the source wavefield, providing the pilot trace at the virtual source location $\mathbf{x}_s$; $g_{\mathbf{d}}$ is the Green's function in the underlying semi-infinite space, from $\mathbf{x}_s$ at datum level to subsurface offset $\mathbf{x}_s + \mathbf{h}$, and $\times$ states the convolution operation in time. Here, $\mathbf{F}$ is wave equation operator with velocity c :

$$\mathbf{F} = \left(\frac{1}{c^2} \frac{\partial}{\partial t^2} - \nabla^2 \right), \quad (2)$$

which satisfies $\mathbf{F}u(s, \mathbf{x}, t) = f$, where f is the true source function. This modeling formulation represents the adjoint of the extended imaging condition, and thus, provides a relation between data at the Earth surface and the virtual data at the datum, which can be derived from reciprocity theory (Wapenaar et al., 2004). An Integration over the virtual sources actually satisfies Huygen's principle, which reproduces the scattering pattern of the upgoing wavefield into the overburden medium. Our optimization problem with respect to $g_{\mathbf{d}}$ can, thus, be formed as

$$\min E = \frac{1}{2} \sum_{s,r} \int_t |d(c) + \tilde{d}(c, g_{\mathbf{d}}) - d_o|^2, \quad (3)$$

where d_o is the data recorded on the surface, d is the data modeled from the source and $\tilde{d}$ is the data modeled from the datum. Alternatively, it can be expressed as

$$\min E = \frac{1}{2} \sum_{s,r} \int_t |\mathbf{A}[u(s, \mathbf{x}, t) + \tilde{u}(s, \mathbf{x}, t)] - d_o|^2, \quad (4)$$

where the matrix $\mathbf{A}$ maps the wavefield to the receiver positions. Using the adjoint-state method (Plessix, 2006), we calculate the adjoint operator, referred to as waveform redatuming, and apply it to obtain the gradient with respect to $g_{\mathbf{d}}$:

$$\frac{\partial E}{\partial g_{\mathbf{d}}(\mathbf{x}_s, \mathbf{h}, t)} = - \sum_s \langle u(s, \mathbf{x}_s, t), \mu(s, \mathbf{x}_s + \mathbf{h}, t) \rangle_t, \quad (5)$$

with the adjoint wavefield μ satisfying

Figure 1: The velocity and survey design (green dots: acquisition level; pink stars: datum level).

$$\mathbf{F}^* \mu(s, \mathbf{x}, T - t) = \sum_r (\mathbf{A}[u + \tilde{u}] - d_o)(s, \mathbf{x}_r, T - t), \quad (6)$$

where $\langle \cdot, \cdot \rangle_t$ is the cross-correlation operator in time. The waveform redatuming operator corresponds to a modification to the extended imaging condition (Symes, 2008) that includes additional dimensions in space and time.

Figure 1 shows a simple two-layer model with an anomaly. The original survey is located at the surface and the datum survey is at 1.49 km depth, denoted by the pink stars. There are 32 shots evenly distributed and 320 receivers at every grid point on the surface. Three shots of the redatumed data, with $\mathbf{x}_s$ marked by yellow dots in Figure 1, are shown in Figure 2.

Figure 2: The inverted virtual data d_s at (a) $x=1.8$ km (b) $x=2.4$ km and (c) $x=3.0$ km.

Figure 3: The inverted redatuming d_s using an incorrect velocity at (a) $x=1.8$ km (b) $x=2.4$ km and (c) $x=3.0$ km.

Figure 4a compares a datum modeling (right) to an actual reflection data (left), in a mirror image plot. The residual, plotted at the same scale, shown on the right of Figure 4b demonstrates that the difference is small. If we use a velocity model with 10% higher velocity than the true one, we get the optimized virtual data shown in Figures 3. An inaccurate results a change in the stationary point as it admits different kinematics.

Velocity model inversion

In our optimization, we can take advantage of the update along reflection wavepath to improve the velocity estimate. Specifically, we include the migration/demigration, like in RWI, but with extension, into our modeling process to absorb the reflections from the overburden model.

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Figure 4: (a) The modeled data, on the right, and the observed reflection, on the left (yellow box) and (b) the misfit before (left) and after inversion (right).

The resulting modeling formula has the following form:

$$\begin{aligned} \mathbf{F}\tilde{u} &= \int d\mathbf{x}_s d\mathbf{h} u(s, \mathbf{x}_s - \mathbf{h}, t) \times g_d(\mathbf{x}_s - \mathbf{h}, \mathbf{h}, t) |_{\mathbf{x}_s \in \partial\Omega_{model}} \\ &+ \int d\mathbf{x}_s d\mathbf{h} u(s, \mathbf{x}_s - \mathbf{h}, t) \cdot \mathbf{I}(\mathbf{x}_s - \mathbf{h}, \mathbf{h}) |_{\mathbf{x}_s \in \Omega_{model}}, \end{aligned} \quad (7)$$

where the virtual source $\mathbf{x}_s$ for the migration/demigration includes the whole overburden space Ω_{model} and for the datum modeling includes data along the bottom boundary of overburden model $\partial\Omega_{model}$, which is the datum level (see Figure 5).

Figure 5: An illustration of the modeling process: the overburden migration/demigration (on the left) is extended with sub-surface offset (on the right); the medium below the dash line is represented by the virtual data g_d .

Note that despite the zero-lag extension over time, the extended migration $\mathbf{I}$ shares a similar approach to the redatuming, which is originally proposed to improve the data match Guo et al. (2017). These extensions in model in the overburden or data at datum offer additional opportunities to fit the data, and thus, mitigate the cycle-skipping and improve the robustness of velocity building. Using the same objective function in equation 3, we derive the gradient with respect to the overburden velocity as

$$\begin{aligned} \frac{\partial E}{\partial c(\mathbf{x})} &= - \sum \frac{1}{c} \int dt \nabla^2 \mu(s, \mathbf{x}, t) \tilde{u}(s, \mathbf{x}, t) \\ &+ \nabla^2 u(s, \mathbf{x}, t) \tilde{\mu}(s, \mathbf{x}, t) + \nabla^2 u(s, \mathbf{x}, t) \mu(s, \mathbf{x}, t) \end{aligned} \quad (8)$$

with $\tilde{\mu}$ satisfying

$$\begin{aligned} \mathbf{F}^* \tilde{\mu}(T-t) &= \int d\mathbf{h} \langle g_d(\mathbf{x}_s, \mathbf{h}, T-t), \mu(s, \mathbf{x}_s + \mathbf{h}, T-t) \rangle_t \\ &+ \int d\mathbf{h} \mu(s, \mathbf{x}_s + \mathbf{h}, T-t) \cdot \mathbf{I}(\mathbf{x}_s, \mathbf{h}). \end{aligned} \quad (9)$$

The first two terms in equation 8 are updates along reflection wavepath, which is utilized in RWI to provide low-wavenumber updates; the third term is the FWI term. We show the sensitivity kernel for a single shot in the middle, calculated using the first two terms in Figure 6. The source function is a Ricker's wavelet with peak frequency of 8 Hz. As we can see, reflection from the redatumed data generates low-wavenumber updates.

Figure 6: The sensitivity kernel calculated by a single shot: the first term $u(s, \mathbf{x}, t)\mu(s, \mathbf{x}, t)$ (a) and the second term $\mu(s, \mathbf{x}, t)u(s, \mathbf{x}, t)$ (b).

MARMOUSI EXAMPLE

We apply our proposed datum waveform inversion to the Marmousi model, shown in Figure 7. The virtual sources and receivers are indicated by the yellow and black dots at 2 km depth and the recording acquisition is near the surface, marked by green dots. We generate the surface seismic data using

Figure 7: The true model used to generate the data with datuming survey at 2 km depth.

a finite difference modeling on the entire model and in our implementation we start with a linearly increasing overburden model, which is shown in Figure 8. We use a Ricker's wavelet with 15 Hz peak frequency as our source function and filter out data below 2 Hz to imitate practical conditions. First, we apply velocity inversion using the first two terms in equation 8. With reflections generated from datum modeling and migration/demigration, the updates along the reflec-

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tion wavepath provide a reasonably low-wavenumber estimation of the background model, which is shown in Figure 9a. Throughout the velocity model building process, we simultaneously update both the redatumed data and overburden image; we specially remove the contribution of diving waves to emphasize the role of reflections.

Figure 8: The initial overburden model.

As the velocity improves, we turn on the FWI term to update the scattering part. The optimized high-resolution model, which is shown in Figure 9b, will reproduce reflections and multiples caused by overburden model, thus, yields Green's functions capable of datuming.

Figure 9: The inverted model using the background updates (a); the final FWI result (b).

The final redatumed data at $x = 2.74$ km and 3.35 km are shown in Figures 10a and 10d. For comparison, we show the redatuming using the true overburden model and the synthetic data modeled in the true underlying model in Figures 10b, 10e and Figures 10c, 10f, respectively. Figure 11 shows, from the middle to the outer sides, the recorded data at the surface, the modeled data using our optimization and the migration/demigration data, respectively. It can be noticed that our datum modeling provides a good match of the reflection events corresponding to the underlying medium.

Figure 10: The virtual data at $x = 2.74$ km (a) and 3.35 km (d) calculated using the estimated model; (b) and (e) calculated by the true model; (c) and (f) the synthetic data

CONCLUSIONS

We introduced an FWI framework to retrieve the Green's functions (virtual data) corresponding to a target medium, as well as the velocity in the overburden. With the Marmousi example, we demonstrate our proposed datum waveform inversion promises robust low-wavenumber velocity estimates, using reflections modeled by the limited model extension in the overburden and redatumed data. We ended up a high-resolution baseline model from a poor initial guess, and thus, succeeded in retrieving a virtual dataset with reasonable amplitude information considering acquisition limitations. The well reserved dynamic features could help us focus our sequential inversion in the target region considering more complicated physics.

Figure 11: The observed data at the surface (middle), modeled data from both the model and redatuming (two sides next to the observed) and only from the overburden model (outermost).

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