Subspace methods for Time-lapse elastic full waveform inversion

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SUMMARY

The application of elastic full waveform inversion on time-lapse seismic data is arising from the boom in conventional full waveform inversion. In the past few years, many different inversion strategies are introduced for the time-lapse case, taking advantage of the power of FWI in capturing small changes. However, all methods tend to suffer from the imperfect repeatability of the data acquisition and the weakness in FWI in focussing on the affected areas (i.e. the reservoir). Thus, we modify the subspace method and apply it to time-lapse elastic full waveform inversion. A soft mask calculated using the gradients of the baseline and monitoring data, which acts as a preconditioner, is introduced to localize the update area to the affected regions. Specifically, we suppress the similarities of the two gradients and at the same time highlight their differences when calculating the soft mask. The calculated soft mask can reduce the dimensionality of the inverse problem with a delicately selected threshold, which provides a feasible way to calculate the reduced Hessian matrix. Besides, it is a data-driven approach free of human intervention or *a priori* knowledge. For comparison, we also use a hard mask surrounding the injection area. The numerical example shows that the proposed soft mask performs better than the hard mask.

INTRODUCTION

Time-lapse (4D) seismic experiments have been adopted as an established technology for reservoir monitoring and developing. It attempts to capture the variations in the subsurface over time using several repeated seismic surveys in the same area. Conventional approaches to analyze the changes focus on the differences in the migration images, and thus, lack quantitative measurements of the changes in the rock and fluid properties. Elastic full waveform inversion (FWI) as a high-resolution quantitative imaging tool has been applied to time-lapse data sets to highlight the minor changes in the reservoir features.

Time-lapse full waveform inversion aside from its high-resolution potential suffers from inconsistency in seismic surveys, which can generate differences beyond what we are looking for such as illumination differences, ambient noise variations, and instrument-based changes. Many existing methods aim to minimize the influence of these differences. Zheng et al. (2011) proposed using the residual data between different time-lapse vintage to update the velocity drop caused by the injection. Raknes and Arntsen (2014) utilized migration images in a regularization term to isolate the injection area. Asnaashari et al. (2015) studied the robustness of three inversion strategies for time-lapse applications with regularized FWI. Yuan et al. (2017) introduced a localized inversion in the subsurface with reduced computational costs. In summary, all these inversion strate-

gies share the common goal of localizing the differences to highlight the weak signatures of the changes caused by the injection.

In many time-lapse applications related to injections, the velocity drops slightly in the reservoir region, and thus, the time-dependent inverse problem can be solved as a generalized linear problem. In this paper, we use the subspace method to split the subsurface model into a full-sized background model and a reservoir local-sized perturbation region (Oldenburg et al., 1993). The full-sized background model is retrieved from the baseline using a delicately-designed FWI approach (Zhang et al., 2018). The local-sized perturbation region is estimated from the monitoring line, and allows us to use a localized Hessian matrix to solve the inverse problem better (Ayeni and Biondi, 2012). We use a statistical approach to generate the soft mask (highlight the possible injection areas) and use the LBFGS method to approximate the localized Hessian matrix. We calculate them in the image domain (the gradients) for two main reasons: 1) the prestack data are non-repeatable in many cases and 2) the calculation of gradients is a focussing process, which increases the signal/noise ratio.

This abstract is divided into four sections. After the introduction, the theory of the proposed inversion strategy is described. Then we use a synthetic elastic model to verify the effectiveness of the proposed method and draw the conclusions from the numerical test.

THEORY

As wave-propagation velocity slightly drops in the reservoir due to fluid or steam injections and localized around the injection point, a localized Hessian matrix is needed to better estimate the update in this area since the full Hessian covering the whole domain is not feasible nor necessary. We use the subspace method to split the whole model into a whole background model and a much smaller perturbation model (Oldenburg et al., 1993). The time-lapse full-waveform inverse problem can be given by two sub-problems,

$$\phi(m_0) = \|d^p - d^b\|^2 \quad (1)$$

and

$$\phi(m_0 + Wm) = \|d^p - d^m\|^2, \quad (2)$$

where m_0 is the background model estimated from the baseline data, d^b . W is a selection operator highlighting the possible injection areas in m , d^m is the data collected using the monitoring line. With a linear assumption, $\phi(m_0 + Wm) = \phi(m_0) + \phi(Wm)$, the second problem can actually be solved by minimizing $\min \phi(Wm)$ after we solve the first problem.

The minimization problem is reformulated as

$$\phi(Wm_i + W\delta m_i) \approx \phi(Wm_i) + \delta m_i^T W^T g_i + \frac{1}{2} \delta m_i^T W^T H_i W \delta m_i, \quad (3)$$

where $W^T H_i W$ is referred as a localized Hessian matrix and $W^T g_i$ is the preconditioned gradient. Here W^T denotes the adjoint of the precondition operator W .

The inverse problem is constrained by the first-order elastic wave equation, which is given by (Virieux, 1986; Zhang et al., 2017)

$$\begin{pmatrix} \rho \mathbf{I}_3 & 0 \\ 0 & \mathbf{C}^{-1} \end{pmatrix} \frac{\partial \Psi}{\partial t} - \begin{pmatrix} 0 & E^T \\ E & 0 \end{pmatrix} \Psi - \mathbf{f} = 0, \quad (4)$$

where $\Psi = (v_1, v_2, v_3, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6)$ is a vector containing three particle velocities and six stresses, $\mathbf{C}$ is the stiffness matrix, E denotes space differentiation, and $\mathbf{f}$ is the source function.

We can impose a hard selection criterion (i.e. 0 or 1) to reduce the dimension of the Hessian matrix. However, restricting the activated region of the model may result in the loss of important model parameters necessary to converge to the global minimum, and thus, we obtain an inferior solution (Oldenburg et al., 1993). One possible remedy to this problem is to apply a soft-selection operator at the cost of an increased dimension of Hessian matrix. The soft-selection operator is build up from two criteria corresponding to variances in the shapes and updating directions of the gradients from the baseline and the monitoring line.

We use the local similarity (Fomel and Jin, 2009) to measure the similarities of two gradients from the baseline and monitoring line, which is given by

$$W_{sim} = e^{\alpha(1.0 - LocalSimilarity(\nabla_z g_b, \nabla_z g_m))}, \quad (5)$$

where W_{sim} is a Gaussian distribution, which aims to remove the similar parts of the gradients. ∇_z is the spatial derivative in z -direction, which acts like the migration image in Raknes and Arntsen (2014). The gradients g_b and g_s correspond to the baseline and the monitoring line, respectively, using the estimated background model m_0 as input.

Gradient calculation has a focussing component. Like migration, gradients attempt to focus energy to zero offset, which naturally enhances the signal/noise ratio and suppresses differences caused by non-repeatable acquisition realities. Thus, we utilize the difference in the gradients from the two data sets to highlight the possible injection zones, as follows

$$W_{diff} = e^{\beta(|g_b - g_m|)}. \quad (6)$$

The final selection operator is still a Gaussian distribution, which is given by,

$$W = W_{sim} * W_{diff}. \quad (7)$$

With more monitoring data sets included, the selection operator can be improved as there is a higher probability in this

case of finding the injection zones in a statistical perspective. The selection operator here is a soft mask of the possible injection zones; it balances the risks of removing the exact injection zones and the dimensionality of Hessian matrix.

Incorporating the proposed soft mask in FWI can be done by utilizing an iterative approach to build the Hessian matrix, which is similar to the classic LBFGS method (Ma and Hale, 2012). It can be written as

$$H'_{i+1} = H'_i + \frac{y'_i y'^T_i}{y'^T_i \delta m_i} - \frac{H'_i \delta m_i (H'_i \delta m_i)^T}{\delta m_i^T H'_i \delta m_i}, \quad (8)$$

where $H'_i = W^T H_i W$, $g'_i = W^T g_i$ and $y'_i = g'_{i+1} - g'_i$.

Apart from the difficulties introduced by the time-lapse problem, conventional FWI requires a kinematically-accurate initial model as input. Due to the oscillatory nature of seismic waves, the L_2 norm objective function suffers from cycle skipping when the mismatch between the predicted and observed data exceed a half-cycle. Thus, we use a normalize non-zero lag crosscorrelation type objective function to build an initial model for the optimization problem (equation 1), which is written as (Zhang et al., 2018)

$$\phi = - \int \int \int \int \left(W(\tau) \frac{C[d^p, d^b, \tau]}{\sqrt{\int d^p(t)^2 dt} \sqrt{\int d^b(t+\tau)^2 dt}} \right)^2 d\tau dx_r dx_s dt, \quad (9)$$

where τ is the time lag used for crosscorrelation. $C[d^p, d^b, \tau] = \int d^p(t) d^b(t + \tau) dt$ denotes the crosscorrelation in time. $W(\tau)$ is the weighting function, which satisfies the following boundary conditions: $W|_{\pm\tau} = 0$; $W|_0 = 1$; $W'|_{\pm\tau} = 0$; $W'|_0 = 0$. The normalization terms can help compensate the energy loss at far offsets.

NUMERICAL EXAMPLE

We use a modified Marmousi model to verify the effectiveness of the proposed strategy. The actual S-wave velocity is generated by setting $v_s = v_p / \sqrt{3} + 0.1 * (v_p - 2.4)$ and it is not independently updated here. The P-wave velocity model before and after injections are shown in Figures 1a and 1b, respectively. The P-wave velocity drop zone is shown in Figure 1c and it is inhomogeneous. The source wavelet is a Ricker wavelet ($f_p = 5\text{Hz}$) with frequencies below 3 Hz muted, in which case a conventional L_2 norm based FWI cannot converge to the global minimum. A mono time-lag (for equation 8), $\tau = 0.06\text{s}$, is used to get a better initial model for conventional elastic FWI. Only one frequency band, 3-5 Hz, is used for the crosscorrelation-objective inversion. The monitoring data have a shifted source and receiver locations and also have moderate-level random noises. There are 110 explosive sources and 660 receivers deployed in a regular manner. The recorded data are two-component particle velocities. The staggered finite-difference method is implemented to solve the elastic wave equation. The initial models are 1D linearly increasing models, which are far from the exact ones. We conduct a conventional inversion for both the baseline and monitoring line as shown in Figures 2a and 2b, respectively. Their

differences are contaminated with those differences due to acquisition geometry changes between the baseline and the monitoring line and possibly the added noise in the monitoring line. The results of the proposed method as shown in Figures 3a, 3b and 3c highlight the difference caused by the injection. For comparison, we add a hard mask (0 or 1) surrounding the injection area and repeat the inversion process as shown in Figures 4a, 4b and 4c. Aside from the difficulties in recognizing the exact injection influenced area, the hard mask cast strong boundary effects on the inversion results.

CONCLUSIONS

We use a statistical approach to calculate the soft mask to highlight the possible injection areas in the subsurface. The soft mask allows us to focus the inversion on the key areas in which changes are expected but compared to hard mask it requires a larger dimension localized Hessian matrix. However, the localized Hessian matrix can be approximated using an LBFGS method. The proposed method is data-driven, and thus, helps us match the observed data better with predicted data. Our numerical example shows that the proposed soft mask performs much better than a hard mask to the injection area. Besides, the soft mask has a higher probability of finding the actual areas with velocity drops. The success of FWI in finding the base model is crucial to time-lapse applications. Thus, we used a normalized non-zero lag crosscorrelation based objective function to refine the initial model for conventional FWI. The proposed method is based on a linear assumption in changes, which might not be valid for complicated changes in the data, and thus, additional tests are planned to figure out the range of its validity.

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Figure 1: Actual P-wave velocities. a) Before injection, b) after injection and c) velocity drops.

Figure 2: Estimated P-wave velocities in a regular approach. a) baseline, b) monitoring line and c) their differences.

Figure 3: Estimated P-wave velocities using the proposed approach. a) baseline, b) monitoring line and c) their differences.

Figure 4: Estimated P-wave velocities using the proposed approach. a) baseline, b) monitoring line and c) their differences.

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