Regularized full-waveform inversion for salt-bodies

Mahesh Kalita, Vladimir Kazei, Yunseok Choi and Tariq Alkhalifah

King Abdullah University of Science and Technology

SUMMARY

Full-waveform inversion (FWI) aims to resolve an ill-posed non-linear optimization problem in order to retrieve unknown subsurface model parameters with high resolution from seismic data. The non-linearity, in the absence of low frequencies from the recorded seismic signal, tends to increase, especially for large-velocity structures, like salt bodies and the sediments beneath them, and often prevents the inversion from obtaining an adequate model. To alleviate the ill-posedness of FWI for salt-bodies, we propose to utilize model regularization in order to promote a limited variation in the inverted model and a salt-flooding regularization from the top of the salt (without picking). On that account, we split the optimization problem into two parts: first, we minimize the data misfit and the total variation in the model, seeking to achieve an inverted model with sharp interfaces; and second, we penalize sharp velocity drops in the model by a computational flooding of the velocity field. Unlike conventional industrial salt flooding, our proposed technique requires minimal human intervention and no information whatsoever about the top of the salt. Those features are demonstrated on a dataset of the Sigsbee2A model in which the lowest available frequency is 3 Hz. We retrieve most of the salt region and also some of the fine sedimentary layering beneath the salt.

INTRODUCTION

Despite the continued improvements of full-waveform inversion (FWI) over the recent years (Tarantola, 1984; Virieux and Operto, 2009), its theoretical underpinnings which are based on a single scattering assumption, known as Born approximation, often break down in the absence of low frequencies from recorded seismic signals given the initial model is very far from the true subsurface model. This limitation is aggravated in presence of a salt body whose position and shape the initial model most likely has no a priori knowledge, and whose physical properties are completely distinct from those of its neighborhood. As of now, the best ad-hoc industrial practice for salt inversion comprises of erroneous ‘migrate-pick-flood’ strategies (Dellinger et al., 2017). This conventional practice which is dominated by human decisions (prone to errors), often exacerbates the already difficult process of velocity building, and, hence the quality of the final subsurface image. To alleviate the human intervention in FWI, Lewis et al. (2012); Kadu et al. (2017) used a level-set approach in order to retrieve the salt geometries. Ovcharenko et al. (2017) attempted to reconstruct the salt-body model using variance based post-processing steps to conventional FWI. However, these methods have the shortcomings of over dependency on adaptive parameters throughout the iterations, or over-simplistic underlying surmises, which might not be feasible in a realistic com-

plicated model. Esser et al. (2016) prescribed a hinge-based constraint to perform the flooding, and thus, they modified the FWI objective function to include such a constraint by utilizing primal dual optimization, which mitigates some of the limitations mentioned above.

Here we push the envelope as we propose a new non-linear flooding procedure. Unlike Esser et al. (2016), we, actually, formulate the minimization problem into two cascading parts: we first match the data, promoting limited variation regions in the model and then flood from the top of the salt (without picking), a post processing optimization procedure. The splitting approach makes the algorithm way easier to adapt into the existing FWI flow. Moreover, in the flooding part, multiple regularization weights can be efficiently tested. We show the versatility of our proposed method on the synthetic datasets of SigsBee2A model with frequencies below 3 Hz muted to make the dataset more practical.

THEORY

Figure 1: Update direction (negative of gradient) of flooding regularization : a dummy velocity profile (red); its update direction (green) is not free from undesired directions; the black line shows the controlled update.

Figure 2: Sigsbee2A model with reduced water column thickness.

In order to reinforce sharp geological features in the salt-bodies without over-smoothing, FWI requires an edge preserving technique, such as minimization of model total-variation (TV) (Rudin

FWI for salt-bodies

Figure 3: Available streamer length for each shot. The acquisition coverage towards the west part of the model is very poor.

et al., 1992) or minimum gradient support norm (Kazei et al., 2017). Also, to prevent the update of velocity in wrong directions, FWI summons a regularization term that penalizes a velocity drop, especially in the vicinity of a large salt body (Esser et al., 2016). On that account, we propose to solve the following cascading minimization scheme incorporating the above mentioned a-priori information:

Figure 4: Shot gather from the source placed at $x = 23.0$ km and its average frequency spectrum.

Figure 5: Initial model. The velocity increases linearly with the depth from the water bottom.

PROBLEM 1 : FWI+TV

$$J_{FWI+TV}(\mathbf{m}) = \min_{\mathbf{m}} \{ \|\mathbf{d}_o - \mathbf{d}_m\|_2^2 + \lambda_{TV} \|\mathbf{m}\|_{TV} \} \quad (1)$$

with $\mathbf{d}_o$: observed data; $\mathbf{d}_m$: synthetic data that follows the wave equation, $\|\mathbf{m}\|_{TV} = \sqrt{\mathbf{m}_x^2 + \mathbf{m}_z^2}$ with $\nabla \mathbf{m} = \{\mathbf{m}_x, \mathbf{m}_z\} = \{\frac{\partial \mathbf{m}}{\partial x}, \frac{\partial \mathbf{m}}{\partial z}\}$ is TV regularization.

PROBLEM 2 : FLOODING

$$J_F(\mathbf{q}) = \min_{\mathbf{q}} \{ \|\mathbf{q} - \mathbf{u}\|_2^2 + \lambda_F \|\mathbf{q}\|_F \} \quad (2)$$

with $\mathbf{u} : \argmin_{\mathbf{m}} J_{FWI+TV}$; $\|\mathbf{q}\|_F$: flooding regularization defined as follows

$$\|\mathbf{q}\|_F = \frac{1}{2p} \|(|\mathbf{q}_z| - \mathbf{q}_z)^p\|_2^2 \quad \text{with } p > 1; \quad (3)$$

λ_F is the regularization coefficient of $\|\mathbf{q}\|_F$.

The first minimization problem referred to as ‘FWI+TV’ recovers the velocity field by promoting limited variation regions in the model (Anagaw and Sacchi, 2012). Here, λ_{TV} is a positive regularization parameter that controls the trade-off between the data misfit term and the amount of interface-preservation in the inversion process. Also, a positive smoothing parameter ϵ_{TV} needs to be added in the TV functional to make it differentiable at the origin $\nabla \mathbf{m} = 0$.

The second optimization problem, ‘flooding’, starts with the final output of ‘FWI+TV’ and penalizes velocity drops in the depth direction. We refer to this minimization problem as flooding because it smears the velocity field along the vertical direction. In addition to the advantage of minimal human intervention, flooding is applied anywhere and everywhere since it is governed by the model under consideration rather than the top of the salt. Thus, the high velocity is smeared across any region where there is a drop in the velocity field, rather than only the salt-body region. However, this might have a detrimental effect in the presence of low-velocity zones, especially over the top of the salt. In that case, a minimal human intervention might be required to prevent over-flooding from the low-velocity zone. However, the p value in equation (3) helps to maneuver the flooding procedure in the appropriate region of the model, i.e., from the top of the salt. The large value of p is immune to small-scale drops in the velocity field (Kalita et al., 2018). Therefore, we steer the flooding towards a region of steeper velocity drops by setting p high enough. A steeper drop, fortunately, is pronounced just below the top of the salt.

The gradient direction of the data misfit term in FWI is very well known to our community (Lailly, 1983). We can obtain the gradient direction of TV and flooding regularization using variational calculus as follows:

$$\frac{\delta \|\mathbf{m}\|_{TV}}{\delta \mathbf{m}} = -\nabla \cdot \frac{\nabla \mathbf{m}}{\sqrt{\mathbf{m}_x^2 + \mathbf{m}_z^2 + \epsilon_{TV}}}; \quad (4)$$

$$\frac{\delta \|\mathbf{q}\|_F}{\delta \mathbf{q}} = -\frac{\partial}{\partial z} \{ (\mathbf{q}_z^\epsilon - \mathbf{q}_z)^{2p-1} \left(\frac{\mathbf{q}_z}{\mathbf{q}_z^\epsilon} - 1 \right) \} \quad \text{with } \mathbf{q}_z^\epsilon = \sqrt{\mathbf{q}_z^2 + \epsilon_F}; \quad p \geq 1. \quad (5)$$

Like TV, equation (3) requires a small positive smoothing parameter ϵ_F in order to alleviate the ill-conditioning of functional arising during the gradient evaluation process given by equation (5). Figure 1 shows the update direction of a dummy velocity profile, sinusoidal in nature to imitate the realistic update inside the vicinity of a large salt-body. As expected, there is no update while velocity increases with depth. However, when the velocity drops, the non-zero update is not always

FWI for salt-bodies

Figure 6: Left panel shows the outcomes of ‘FWI+TV’ scheme for different frequency bandwidths and they are used for subsequent ‘flooding’ procedure; middle displays their results; right panel compares an observed seismic trace (black) with counterpart predicted traces using the velocity models obtained from ‘FWI+TV’ (red) and ‘flooding’ approach (green). Middle panel is subjected to ‘FWI+TV’ scheme with higher frequency bandwidth to obtain the updated model displayed in the next row first column subplot. The rationale behind indicators (blue ink) is described inside the text section of Numerical example.

positive. To prevent the update in the reverse direction, we set it to zero whenever negative, as shown by the black profile in Figure 1.

The smoothing parameter ϵ and the regularization coefficient λ manipulates the success and convergence rate of the optimization problem of our consideration. A large ϵ leads to a smoother solution, which is undesirable in the prospect of preserving sharp edges in a model with high resolution. Here, we set ϵ to be very small, depending on the model parameters. For example, we set ϵ_{TV} to 0.1% of $\langle \nabla \mathbf{m}, \nabla \mathbf{m} \rangle$ and ϵ_F to 0.1% of $\langle \mathbf{q}_z, \mathbf{q}_z \rangle$ throughout the inversion process. $\langle \cdot, \cdot \rangle$ denotes the dot product of two vectors divided by its length. We observe that smaller values of ϵ reduces the convergence rate of inversion and does not impact the results in a significant way if the same decrease in the misfit is achieved.

The regularization coefficient λ controls the confidence level between the terms entailed in the functional. We assign a stationary number for each bandwidth of the data. For example, $\lambda_{TV} = 0.5$ for the first bandwidth, $\lambda_{TV} = 0.4$ for the second bandwidth of data, and so on. Therefore, the data misfit term in ‘FWI+TV’ gradually dominates the inversion, which yields results that are less biased by the initial values of the regularization parameters. In addition, the final stage of the inversion using the full-bandwidth of the data is purely data driven without any model regularization. In other words, we solve only ‘FWI+TV’ with $\lambda_{TV} = 0$ for the full bandwidth of the data. For the second optimization problem, ‘flooding’, we heuristically assign $p = 2$ and $\lambda_F = 10 * \lambda_{TV}$.

NUMERICAL EXAMPLE

We demonstrate an application of our method to the Sigsbee2A model (Paffenholz et al., 2002). However, we reduce the thickness of the water column of the original model by 1.5 km. Figure 2 displays the true model of our consideration. Besides the complex salt structure with its variable size and shape, this model contains a sedimentary sequence broken up by a number of normal and thrust faults, attempting to emulate the deep-water prospect areas in the Gulf of Mexico. Data are recorded for 10 seconds in a 16 km long streamer starting from 0.16 km at the near offset, placed at a depth of 0.016 km. Figure 3 confirms the poor acquisition coverage towards the west part of the model. A Ricker wavelet of 5.5 Hz dominant frequency and low-cut filter at 3 Hz is used in order to prepare the observed data with the free surface boundary condition. Figure 4 shows a shot gather from the source placed at $x = 23.0$ km and its average frequency content. We prepare the initial model (Figure 5) by increasing the velocity linearly from the water bottom. We follow a multiscale approach (Bunks et al., 1995), starting the inversion at a lower frequency bandwidth (< 3.5 Hz) and progressively including the higher frequencies (< 4.5 Hz, < 6 Hz, < 9 Hz and the full bandwidth).

We report the evolution of the velocity model through our proposed method in Figure 6: the left panel shows the ‘FWI+TV’ results for different frequency bandwidths of the data and the middle panel shows the subsequent flooding results. In addition, we show the effect of flooding on the predicted traces in the right panel of Figure 6. We manage to retrieve a portion of the top of the salt from the first bandwidth of data under consideration, but fail to retrieve the rest of the salt-body as

FWI for salt-bodies

Figure 7: An intermediate inverted model.

Figure 8: Final inverted model using the Figure 7 as the initial model, with preconditioning the gradient equals to zero whenever the velocity is larger than 4.2 km/s.

displayed by the encircled region of Figure 6(a). Next, the velocity model is subjected to the flooding procedure to wipe out the sudden drop in velocity, especially below the top of the salt. However, a careful observation reveals that flooding is inevitable to any region of the model depending on the velocity drop, e.g., the areas pointed out by the blue circles in Figure 6(b). Nevertheless, the subsequent ‘FWI+TV’ scheme attempts to eradicate the undue over-flooding (Figure 6(d)). Although cycle-skipping at this stage of inversion is prevalent due to the lack of low frequencies and the appropriate initial model (no arrival in the predicted traces during the time window marked by blue circle in Figure 6(c)), the updated velocity model after flooding generates seismic records with less cycles skips, compared to that using the velocity before flooding. We observe a clear shift in arrivals, indicated by the blue line in Figure 6(c), before and after the ‘flooding’ procedure. As expected, the bottom of the salt is strikingly evident with the maturity of inversion process, as displayed in Figure 6(g). Also, the synthetic records start resembling better with the observed counterparts due to the improvements in the model and in particular, the salt province (Figure 6(i)).

The inverted model, displayed in Figure 7, shows that our proposed method attempted to reconstruct the salt province well, but a smooth version of the subsalt, which looks very promising for imaging. In order to retrieve the reflectors in high resolution, we run basic FWI on the model displayed in Figure 7 by preconditioning the gradient to zero whenever the velocity exceeds 4.2 km/s (Figure 8). Besides the improvements, an enlarged area in Figure 9 accentuates the region where FWI fails to render the velocity field accurately. By taking into account the innate limitations of FWI, such as poor subsurface illumination, and limited acquisition aperture, we are able to retrieve

most of the model.

Figure 9: Enlarged sections of the (a) true, (b) initial, (c) intermediate (Figure 7), and (d) final inverted (Figure 8) models.

CONCLUSIONS

We proposed an FWI with a two-stage model regularization in order to reconstruct salt-bodies and the sediments beneath them. We split the minimization problem into two parts, namely ‘FWI+TV’ and ‘flooding’. The ‘FWI+TV’ promotes the condensed variations in the model obtained from the data misfit function. The ‘flooding’ smears the high velocity vertically in a controlled manner wherever there is a drop of velocity with depth. Any undesired flooding will be eradicated by the data misfit term in the subsequent ‘FWI+TV’ stage of our cascading strategy. Our proposed method is mathematically more robust and less cumbersome than the standard erroneous ‘migrate-pick-flood’ approach. Regularization parameters need to be properly selected for the inversion set up, yet the computationally fast flooding stage allows us to find optimal parameter choice without extra FWI runs. We also observe that the choice of ϵ parameter does not impact the results if it is small enough. We have shown the application of the proposed method to the Sigsbee2A model, in which the lowest available frequency of the observed synthetic datasets was 3 Hz. We also plan to present the results of a field application in the meeting.

ACKNOWLEDGMENTS

We would like to thank KAUST for its support, all the members of Seismic Wave Analysis Group for fruitful discussions, the Supercomputer laboratory and IT Research Computing at KAUST for computer time. We also thank Antoine Guitton for his suggestions.

REFERENCES

- Anagaw, A. Y., and M. D. Sacchi, 2012, Edge-preserving seismic imaging using the total variation method: *Journal of Geophysics and Engineering*, **9**, 138–146, <https://doi.org/10.1088/1742-2132/9/2/138>.
- Bunks, C., F. M. Saleck, S. Zaleski, and G. Chavent, 1995, Multiscale seismic waveform inversion: *Geophysics*, **60**, 1457–1473, <https://doi.org/10.1190/1.1443880>.
- Dellinger, J., A. J. Brenders, J. Sandschaper, C. Regone, J. Etgen, I. Ahmed, and K. Lee, 2017, The garden banks model experience: *The Leading Edge*, **36**, 151–158, <https://doi.org/10.1190/tle36020151.1>.
- Esser, E., L. Guasch, F. J. Herrmann, and M. Warner, 2016, Constrained waveform inversion for automatic salt flooding: *The Leading Edge*, **35**, 235–239, <https://doi.org/10.1190/tle35030235.1>.
- Kadu, A., T. van Leeuwen, and W. A. Mulder, 2017, Salt reconstruction in full-waveform inversion with a parametric level-set method: *IEEE Transactions on Computational Imaging*, **3**, 305–315, <https://doi.org/10.1109/TCI.2016.2640761>.
- Kalita, M., V. Kazei, Y. Choi, and T. Alkhalifah, 2018, Full-waveform inversion for automated salt flooding: Presented at the 80th Annual International Conference and Exhibition, EAGE, Extended Abstracts.
- Kazei, V., M. Kalita, and T. Alkhalifah, 2017, Salt-body inversion with minimum gradient support and Sobolev space norm regularizations: Presented at the 79th Annual International Conference and Exhibition, EAGE, Extended Abstracts, <https://doi.org/10.3997/2214-4609.201700600>.
- Lailly, P., 1983, The seismic inverse problem as a sequence of before stack migrations: *Conference on inverse scattering: Theory and application: Society for Industrial and Applied Mathematics*, 206–220.
- Lewis, W., B. Starr, and D. Vigh, 2012, A level set approach to salt geometry inversion in full-waveform inversion: 82nd Annual International Meeting, SEG, Expanded Abstracts, 1–5, <https://doi.org/10.1190/segam2012-0737.1>.
- Ovcharenko, O., V. Kazei, D. Peter, and T. Alkhalifah, 2017, Variance-based salt body reconstruction: Presented at the 79th Annual International Conference and Exhibition, EAGE, Extended Abstracts, <https://doi.org/10.3997/2214-4609.201700832>.
- Paffenholz, J., J. Stefani, B. McLain, and K. Bishop, 2002, SIGSBEE 2A synthetic subsalt dataset-image quality as function of migration algorithm and velocity model error: 64th Annual International Conference and Exhibition, EAGE, Extended Abstracts.
- Rudin, L. I., S. Osher, and E. Fatemi, 1992, Nonlinear total variation based noise removal algorithms: *Physica D: Non-linear Phenomena*, **60**, 259–268, [https://doi.org/10.1016/0167-2789\(92\)90242-F](https://doi.org/10.1016/0167-2789(92)90242-F).
- Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation: *Geophysics*, **49**, 1259–1266, <https://doi.org/10.1190/1.1441754>.
- Virieux, J., and S. Operto, 2009, An overview of full-waveform inversion in exploration geophysics: *Geophysics*, **74**, no. 6, WCC1–WCC26, <https://doi.org/10.1190/1.3238367>.