Wave-equation Rayleigh wave inversion using fundamental and higher modes

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SUMMARY

Recorded surface waves often provide reasonable estimates of the Shear wave velocity in the near surface. However, these estimates tend to be low in resolution considering that they depend on dispersion nature of the fundamental mode of surface waves. We present a surface-wave inversion method that inverts for the S-wave velocity from the fundamental- and higher-modes of Rayleigh waves. The proposed method aims to maximize the similarity of the phase velocity ($f-v$) spectrum of the surface waves with all-Rayleigh wave modes (if they exist) in the inversion. The $f-v$ spectrum is calculated using the linear Radon transform and by using a local similarity-based objective function, we do not need to pick velocities in the spectrum plots. Thus, the best match between the predicted and observed data $f-v$ spectrum provides the optimal estimation of S-wave velocity. We derive the gradient of the proposed objective function using the adjoint-state method and solve the optimization problem using the LBFGS method. Our method can invert for lateral velocity variations, include all-mode dispersions, and mitigate the local minimum problem in full waveform inversion with a reasonable computation cost. Results with synthetic and field data illustrate the benefits and limitations of this method.

INTRODUCTION

Conventional surface wave inversion methods fall into three categories: 1) 1D inversion for a layered medium using semi-analytical solutions to the elastic wave equation (Nazarian et al., 1983; Xia et al., 2004; Milana et al., 2014) or global optimization methods including genetic algorithms (Feng et al., 2005; Dong et al., 2014), 2) full waveform inversion (Groos et al., 2014; Solano et al., 2014), and 3) wave equation dispersion-curve based inversions (Zhang et al., 2015, 2016; Li et al., 2016; Lu et al., 2017). Semi-analytical solutions can be used to robustly and efficiently invert for a 1D S-wave velocity model, but they are less accurate as the lateral variation in velocity is large in the subsurface. Global optimization methods can be used in practice for a layered 1D model, but the computational cost is not acceptable for the 2D and 3D cases, and especially for strong lateral variations in S-wave velocity. In contrast, waveform inversion estimates the velocity model that minimizes the misfit between the predicted and recorded data. However, the data-misfit function can be very sensitive to the accurate prediction of amplitudes, which is difficult to achieve with modeling methods that do not fully take into account the viscoelastic and anisotropic nature of the Earth. Moreover, a poor starting model will promote cycle-skipping and cause convergence to a local minimum (Virieux and Operto, 2009).

The wave-equation dispersion inversion method (also known

as *skeletonized* inversion) aims to match the dispersion curves of Rayleigh waves instead of complex waveforms. It might enjoy a more quasi-linear relationship between the model and the data and has a less bumpy misfit function than that corresponding to waveform inversion. However, the previously proposed wave-equation inversion algorithms highly depend on the automatic picking of the dispersion curves from the $f-v$ spectrum. Although the automatic picking can only apply to predicted data, in which case the approach is stable, it inevitably ignores the higher-mode Rayleigh waves and therefore cannot handle models with low-velocity layers (velocity reversal). The generation of higher modes is often attributed to the presence of low S-wave velocity layers (STOKOE II, 1994), and thus, such low velocity (or velocity reversal) can not be recovered without inverting such modes. Besides, higher modes penetrate deeper than the fundamental mode and can increase the resolution of the estimated S-wave velocities (Xia et al., 2003).

In this abstract, we adapt the wave equation dispersion inversion to include the fundamental- and higher-modes Rayleigh waves. Instead of picking the dispersion curve, we use the $f-v$ spectrum as input data. A local-similarity based objective function is introduced to measure the similarity of the observed and predicted $f-v$ spectrum. The $f-v$ spectrum is calculated using a high-resolution linear Radon transform (Luo et al., 2008). This abstract is divided into four sections. After the introduction, we introduce a novel objective function and solve the optimization problem. In the third section, we first test the synthetic model with S-wave velocity reversal and lateral variation, then apply the method to field data to analyze the effectiveness and limitations of our method. The last section presents the summary of our work.

THEORY

Objective functions intend to measure the mismatch between the predicted and the observed data. One of the most intuitive measurements is the L_2 norm distance, which is given by

$$\phi(m) = \|d^p(m) - d^o\|^2, \quad (1)$$

where ϕ measures the differences; d^p and d^o are predicted and observed data, respectively.

The inverse problem is constrained by the first-order elastic wave equation, which is given by

$$\begin{pmatrix} \rho \mathbf{I}_3 & 0 \\ 0 & \mathbf{C}^{-1} \end{pmatrix} \frac{\partial \Psi}{\partial t} - \begin{pmatrix} 0 & E^T \\ E & 0 \end{pmatrix} \Psi - \mathbf{f} = 0, \quad (2)$$

where $\Psi = (v_1, v_2, v_3, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6)$ is a vector containing three particle velocities and six stresses, $\mathbf{C}$ is the stiffness matrix, E denotes space differentiation, and $\mathbf{f}$ is the source.

Due to the oscillatory nature of seismic waves, the L_2 norm objective function suffers from cycle skipping when the mismatches between the predicted and observed data exceed a half cycle. A natural remedy to this problem is to compare two events within a predefined extension. Theoretically, the critical value for cycle skipping in this approach is enlarged by the extension used. We propose a local-similarity based objective function replacing the L_2 norm based objective function, which is given by

$$\phi(m) = \frac{1}{2} \int_s \int_r \int_f \int_{f'} (W|C^p(f, v)| \cdot |C^o(f + f', v)|)^2 df' df dr ds, \quad (3)$$

where $|C^p|$ and $|C^o(f + f', v)|$ are normalized $f - v$ spectrum of the predicted and observed data, respectively. f' denotes frequency extensions. W is a polynomial-type weighting function, which satisfies the following boundary conditions: $W|_{\pm f'} = 0$; $W|_0 = 1$; $W'|_{\pm f'} = 0$; $W'|_0 = 0$.

The $f - v$ spectrum is calculated using a high-resolution linear Radon transform (Luo et al., 2008). After a temporal Fourier transform of the shot gather, the linear Radon transform can be calculated for each temporal frequency component f as:

$$C(f, v) = \int_{x_{min}}^{x_{max}} D(f, x) e^{-\frac{i2\pi fx}{v}} dx, \quad (4)$$

and its adjoint form is given by

$$D(f, x) = \int_{v_{min}}^{v_{max}} C(f, v) e^{\frac{i2\pi fx}{v}} dv. \quad (5)$$

The adjoint form of the optimization problem can be calculated using the adjoint-state method (Plessix, 2006). The model is updated iteratively using the L-BFGS method, which is written as

$$m = m_0 - \lambda H^{-1} g, \quad (6)$$

where λ is the step length calculated by the standard line-search method. H is the approximated Hessian matrix.

In summary, our proposed inversion approach includes the following steps:

1. Temporal Fourier transform of the shot gathers.
2. Calculate the corresponding $f - v$ spectrum (equation 4).
3. Solve the optimization problem using equations 3 and 6.

The implementation of the dispersion curve inversion is straightforward. The linear-radon transform enhances the signal-to-noise ratio of the surface waves, and thus, only a few preprocessing steps are needed. The benefits of the approach are: 1) No need to pick dispersion curves; 2) More wave modes (if exist) are included in the inversion, which can handle velocity reversals and provide a better estimation of the subsurface.

NUMERICAL EXAMPLES

The proposed inversion method is first tested on synthetic data and then applied to field data. The P-wave velocity used for

Figure 1: The actual v_p (a) and ρ (b) models used for the synthetic examples. They're also used as initial models but the initial v_p is equal to 90% of the true model.

Figure 2: The actual v_s (a) and the estimated v_s (b). Notice that there are low S-wave velocity zones which cannot be captured by previous automatic picking approaches (usually only the fundamental mode can be picked automatically).

the inversion is 10% lower than its actual value in the synthetic case. The P-wave velocity used for field data is a linearly increasing one. Initial S-wave velocities are both linearly increasing in the synthetic and field examples. The surface waves are simulated by solving the elastic wave equation (equation 2) with a free-surface boundary condition.

Synthetic S-wave velocity with reversals

We first test our method on a model with velocity reversals (low-velocity layers). The actual P-wave velocity and density are shown in Figure 1. There are 40 vertical sources and 200 receivers evenly distributed on the surface. The maximum frequency used in the inversion is 30 Hz and the spatial sampling is 4 m. The actual S-wave velocity as shown in Figure 2a is a layered one with velocity reversals, in which case the fundamental-mode based Rayleigh wave inversion fails. The estimated S-wave velocity using the proposed method (2b) has low-velocity zones as expected. For a better comparison, a vertical profile is provided in Figure 3. It's clear that the inverted result is close to the actual one although the initial velocity is far from perfect.

Synthetic S-wave velocity with lateral variations

Considering practical applications, the proposed method should be able to invert lateral inhomogeneities. We use a checkerboard model to verify its effectiveness. The actual S-wave velocity in Figure 4a has both lateral variations and depth reversals. The same geometry is used as the first synthetic example and the initial S-wave velocity is a linearly increasing one. Figure 4b shows the inverted S-wave velocity. Although the resolution is not high, the anomalies in the velocity model can be detected. For a better comparison, three vertical profiles (indicated by yellow triangles in Figure 4b) are plotted in

Figure 3: The vertical profile across the middle of the model. The low S-wave zones are captured.

Figure 4: The actual v_s (a) and the estimated v_s (b). The high-velocity zones (red dashed squares) and low-velocity zones (yellow dashed squares) can be detected using the proposed method. The yellow triangles indicate the locations of the vertical profiles.

Figure 5.

Field data example

A seismic land survey was carried out near the Red Sea coast (across the Qademah fault) in Saudi Arabia, and we show a representative shot gather in Figure 6. It is slightly preprocessed by adding a selection window to the raw data set and applying a bandpass filter. The source wavelet is estimated in each iteration. The geophone spacing is 10 meters, the source is a hammer on a metal plate, and the dominant frequency in the traces is about 40 Hz, but we filtered the data to a maximum frequency of 30 Hz. 60 vertical sources and 60 receivers are used in this inversion and the grid spacing is 5 m. The $f-v$ spectrum calculated using equation 4 for the first shot is shown

Figure 5: The vertical profile across the middle of the model. The low S-wave velocity zones can be captured.

Figure 6: A single shot gather.

Figure 7: The calculated $f-v$ spectrum of the shot gather shown in Figure 6. The fundamental mode has strong energy, and we can detect higher modes with weak energy.

in Figure 7. The initial S-wave velocity (Figure 8a) is a linearly increasing model and the $f-v$ spectrum roughly decides its minimum and maximum values. The estimated S-wave velocity is plotted in Figure 8b. There are some low-velocity zones as expected since the line is across the fault area and these low-velocity zones cannot be recovered using conventional fundamental-mode methods. As a quality control, we also plot the predicted data from the inverted S-wave velocity and its $f-v$ spectrum in Figures 9 and 10, respectively. The predicted data from the estimated S-wave velocity have similar moveouts with the field data and its $f-v$ spectrum has higher-mode Rayleigh waves as the field data do. For reference, the reader can compare the results to those attained by Li et al. (2016) of the same line.

CONCLUSIONS

We presented a wave-equation method for inverting the dispersion spectrum associated with surface waves. The main benefits of this approach are that it mitigates cycle skipping problems associated FWI of surface waves, it includes the fundamental- and higher-modes (if they exist), and it is applicable to 2D and 3D velocity models. Higher modes help increase the penetra-

Figure 8: The initial S-wave velocity (a) and estimated S-wave velocity (b). The initial S-wave velocity can be roughly determined using the range of velocities in the $f - v$ spectrum.

Figure 9: The predicted data using the estimated S-wave velocity. The predicted data are not expected to fully match the observed one considering the objective function but they are close.

Figure 10: The calculated $f - v$ spectrum of the shot gather shown in Figure 9. It's clear that higher-order modes exist. Notice that the $f - v$ spectrum are not modified by normalization or other processing.

tion depth and resolution of the estimated model and they're necessary for inverting for S-wave low-velocity layers. The proposed method is insensitive to ambient noise and P-waves in the observed data since the linear Radon transform enhances the surface waves with linear moveout. The $f - v$ spectrum is inverted using the LBFGS waveform inversion method in conjunction with finite-difference solutions of the elastic wave equation. Results for both synthetic and field data verify the effectiveness of this method and reveal some of its limitations. The $f - v$ spectrum itself has lower resolution than a picked dispersion curve, and thus, the estimated results based on this method also have relatively lower resolution. As a surface-wave targeted inversion method, it requires a sufficient number of shot gathers with wide aperture recording, and a wide enough bandwidth in the source wavelet. The proposed method is also applicable to dispersion spectrum inversion of Love waves.

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