Time-lapse waveform inversion regularized by spectral constraints and Sobolev space norm

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SUMMARY

Imperfect illumination from surface seismic data due to lack of aperture and frequency content leads to ambiguity and resolution loss in seismic images and in full-waveform inversion (FWI) results. The resolution of time-lapse velocity updates can, however, be improved enforcing the sparsity of the parameter changes. Edge-preserving regularizations and constraints are typically used to promote sparsity. However, different choice of regularization parameters leads to different inversion results and optimal parameters are hard to identify, especially for real data. In particular it is not straightforward to balance the inversion between sparsity constraint and data fit. Fortunately, it is possible to estimate local spatial wavenumbers in a velocity model that are best illuminated by the data. We propose to constrain part of the model difference spectrum that is well illuminated by the data and optimize the rest of the spectrum to enhance sparsity. We approximate correctly retrieved model wavenumbers by simply picking large enough values in the inverted spectrum and constrain that part of model spectrum. Then we adjust the rest of the wavenumbers to reduce the value of a Sobolev space norm (SSN). SSN reduction promotes sparsity of time-lapse updates, while spectral constraints ensure that the part of the modeled spectrum retrieved from the data is completely retained. Application to synthetic noisy data for a perturbation of the Marmousi II model shows that the model resolution can be improved by using our method to extrapolate the model spectrum.

INTRODUCTION

FWI suffers from the ambiguity in the results due to finite aperture and frequency bandwidth. Prior information needs to be incorporated through regularizations and constraints to steer inversion towards geologically realistic models. Edge-preserving regularizations or constraints such as minimum support (Portniaguine and Zhdanov, 1999; Kazei and Alkhalifah, 2017) or total variation (Rudin et al., 1992; Oghenekohwo et al., 2015) can provide sharper inversion results. Unfortunately, they also require parameter tuning, which is not straightforward for field data applications. Seismic reflection data are not sensitive to intermediate vertical wavenumbers in the model spatial spectrum (Claerbout, 1985), yet they are sensitive to low and high wavenumbers. Devaney (1984) estimated the missing spectrum in velocity models in the geophysical diffraction tomography and Mora (1989) expanded the technique to wavenumber coverage of FWI. Peter Mora's diagram shows (Fig. 1) particular set of the wavenumbers that are not illuminated by finite frequency and finite aperture data. Ten Kroode et al. (2013) used low frequencies and large offsets in the field data to successfully apply full-waveform inversion to a land seismic exploration dataset. Broadband and wide aperture data

Figure 1: 2D model wavenumber coverage by Mora (1989). v_0 here is the background velocity, θ_{max} is the maximum reflection angle defined by aperture and the target perturbation depth, $\mathbf{K} = (K_x, K_z)$ is the wavenumber vector and $\omega_{min}, \omega_{max}$ are the minimum and maximum angular frequencies used in inversion. We propose a technique to fill in the low accuracy part in a) and white gaps in b).

acquisition, however, present a relatively expensive solution to a reservoir monitoring problem.

We assume that the background velocity can be reconstructed from a data set with wider offset and frequency range and focus on the monitoring problem. One of the major challenges is the low resolution of time-lapse model updates, which commonly is treated by various regularizations (Raknes and Arntsen, 2014; Oghenekohwo et al., 2015; Maharramov et al., 2016; Kazei and Alkhalifah, 2017). Selection of the regularization parameters is a major challenge and several waveform inversions may be necessary to find optimal parameters, which is computationally costly. We address this problem by splitting the inversion problem into two stages. The first stage is a standard double-difference waveform inversion without regularization. The second stage is essentially denoising of the results of the first stage by spectrum extrapolation.

At the second stage we constrain part of the spatial spectrum that is well covered by the data (black region in Fig. 1) and update the rest of the wavenumbers so that the regularizing functional is minimized. Namely, we use Sobolev-Space Norms W_p^1 (SSN) as regularization functionals including as particular cases total-variation based penalty and classic Tikhonov regularization. We apply the proposed method to a synthetic data using Marmousi II model as baseline and add three anomalies to create a monitor model.

CDWI INVERSION STRATEGY

Time-lapse waveform inversion can be considered as two independent problems: the first is to invert the baseline model and the second is to invert the monitor model. Independent inversion for two models, however, leads to artifacts in the difference between them. Several strategies have been suggested to reduce these artifacts. In differential or double-difference waveform inversion the baseline model is inverted first and then the monitor model is constructed based on double-difference misfit functional (Denli et al., 2009), which allows us to reduce artifacts due to imperfect baseline model inversion. Oghenekohwo et al. (2015) and Hicks et al. (2016) proposed to exploit common information between the baseline and monitor data and models in joint inversion and common model inversion respectively. Common information makes inversion for reference model more robust. We follow our previous work (Kazei and Alkhalifah, 2017), where we combined the advantages of double-difference and common model approaches, and invert all data sets (baseline and monitor) simultaneously to build the background model, which is a straightforward way to use common information for all available surveys. The difference between the data flows is highlighted in the figs. 2(a) and 2(b).

We illustrate the CDWI advantages in an application to the acoustic Marmousi II model modified with time-lapse anomalies to investigate FWI resolution possibilities Fig. 3(a). The data 2-9Hz are assumed to be available for background inversion and data with max offset 4km 5-9Hz are assumed to be available for monitoring. All data are contaminated by random noise with power SNR = 20dB. 839 sources and the same number of receivers are spread equidistantly at the surface of the model.

We first reparametrize the observed data:

$$\mathbf{d}_{avg} = \frac{\mathbf{d}_{mon} + \mathbf{d}_{bas}}{2}, \quad \mathbf{d}_{diff} = \frac{\mathbf{d}_{mon} - \mathbf{d}_{bas}}{2}, \quad (1)$$

$\mathbf{d}_{mon}$ and $\mathbf{d}_{bas}$ are the monitor and the baseline data.

Figure 2: Time-lapse waveform inversion strategies (Kazei and Alkhalifah, 2017). Double-difference (differential) waveform inversion – a), centered differential waveform inversion – b).

We invert for the average model $\mathbf{m}_{avg-inv}$ using standard FWI to fit the average data $\mathbf{d}_{avg}$. The model shows some resolution enhancement comparatively to the baseline model as theoretically the SNR is about 3dB (twice) higher when two data sets are used for the same model inversion. This is pronounced into the baseline model quality in Fig. 3(c) comparatively to

Fig. 3(b). Then we perform differential waveform inversion to estimate the time-lapse changes. The misfit functional for the CDWI is the following (Kazei and Alkhalifah, 2017):

$$J_{CDWI}(\mathbf{m}) = ||(\mathbf{d}(\mathbf{m}) - \mathbf{d}(\mathbf{m}_{avg-inv})) - \mathbf{d}_{diff}||^2, \quad (2)$$

where $\mathbf{d}(\mathbf{m})$ is data for the velocity model $\mathbf{m}$ perturbed with velocity $\mathbf{m}$. The CDWI misfit functional (equation 2) is essentially that of DDWI where average model is used as a baseline. Here we utilize regularization when inverting for baseline and average models, but not for the inversion of the difference between baseline and monitor surveys. Due to better background model Fig. 3(c), inversion with CDWI scheme Fig. 3(e) has less artifacts than inversion with DDWI Fig. 3(d).

SPECTRUM EXTRAPOLATION

Even though the background medium is inverted relatively well Fig. 3(c), we observe artifacts in the inversion results Fig. 3(e). The spectrum retrieved in the CDWI Fig. 4(b) resembles that of the true perturbation Fig. 4(a). There are some wavenumbers don't match at all. In particular, high wavenumbers are not very well retrieved.

We propose the following algorithm to retrieve the missing part of the spectrum:

Firstly, we pick wavenumbers in the spectrum that have higher amplitudes and construct a mask

$$mask(\mathbf{K}) = \begin{cases} 1 & \text{if } |\mathbf{dV}_{inv}(\mathbf{K})| > \epsilon \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

that covers absolute values of the spectrum above certain threshold ϵ .

Secondly, we introduce SSN based functional

$$J_{W_p^1} = ||\nabla \mathbf{dV}||_p, \quad (4)$$

which provides natural generalization of the TV norm ($p=1$) and Tikhonov regularizing terms ($p=2$) and already proved to be useful in waveform inversion (Kazei et al., 2017; Zuberi and Pratt, 2018).

Finally, we solve an optimization problem to minimize $J_{W_p^1}$, while keeping well inverted wavenumbers constant:

$$\begin{cases} \mathbf{dV}_{reg} = \operatorname{argmin}_{\mathbf{dV}} J_{W_p^1}(\mathbf{dV}) \\ s.t. \mathbf{dV}(\mathbf{K}) = \mathbf{dV}_{inv}(\mathbf{K}) \quad \forall \mathbf{K} : mask(\mathbf{K}) = 1 \end{cases} \quad (5)$$

Technically, we implement the spectral constraint by projecting gradients of the regularizing functional onto the complement of the mask Fig. 4(c) found from inverted perturbation spectrum. The main feature of our algorithm is that the penalty function is set in standard spatial domain and the constraints are in the wavenumber domain. Fig. 4(d) shows the spectrum of the perturbation reconstructed from the one inverted by CDWI by solving the system (5) for $p=1.1$.

Fig. 5 SSN-based enhanced velocity changes for several p values. Fig. 5(a) and Fig. 5(b) present the reconstruction filtered

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Figure 3: FWI inputs – a). Marmousi II model is used as baseline, three anomalies are designed to investigate time-lapse waveform inversion. Inverted baseline – b) and average – c) models. Inverted time-lapse changes following DDWI and CDWI schemes – d) and e) respectively with offset limitation of 4km. CDWI with offsets up to 10km – f).

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Figure 4: Absolute values of the true perturbation spectrum – a), inverted perturbation spectrum without regularization – b), spectral mask designed based on the inverted spectrum – c), extrapolated spectrum of the perturbation – d).

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with spectral filter based on the spectral mask (Fig. 4(c)) and retrieved by optimization of Tikhonov stabilizing functional $J_{W_2^1}$ respectively. Fig. 5(c) and Fig. 5(d) show velocity changes regularized by $J_{W_{1.1}^1}$ and $J_{W_1^1} = J_{TV}$ respectively. We observe that the perturbations located in the middle of the model become sharper with the decrease of p value, while the shallow perturbation in the left part of the model vanishes when TV optimization is applied. Fig. 5(e) shows the case when $p=0.8$, which is similar to the usage of minimum gradient support regularizer, the contrast and sharpness of the anomalies in the middle is improved further. In our synthetic case the optimal p value appears to be 1.1, which presents a natural blend of TV and Tikhonov regularizations in a single regularizer.

Figure 5: Time-lapse changes in the model reconstructed by CDWI scheme without regularization and filtered with the spectral mask – a), regularized by spectral mask constrained optimization of first order Tikhonov – b), $W_{1.1}^1$ – c), $W_1^1 \equiv TV$ – d), $W_{0.55}^1$ penalty functionals.

DISCUSSION & CONCLUSIONS

Illumination of model wavenumbers by seismic data can easily be estimate from data aperture and frequency content (Mora, 1989,e.g.). We constrained the part of the model spectrum that is well illuminated by the data and updated the rest to optimize sparsity of the velocity changes. The algorithm can in principle be used with an arbitrary edge-preserving regularization. We implemented it by using Sobolev-Sapace W_p^1 norms as regularizing functionals.

We tested the algorithm on a noisy synthetic data set for perturbed Marmousi II model. To reduce the effects of the noise on inversion we utilized CDWI inversion strategy (Kazei and Alkhalifah, 2017), which combines advantages of DDWI (Denli et al., 2009) and common model (Hicks et al., 2016) approaches. Average model was retrieved by using standard FWI with standard Sobolev-Space (total variation like) regularization and had better resolution than baseline due to improved signal-to-noise ratio.

We first inverted the difference between baseline and monitor model without any regularization. We picked high absolute values in the spectrum of the inverted difference and constrained those wavenumbers. Then we optimized spectrum values at the rest of the model spectrum to improve the resolution of the model difference. Namely, we minimized several Sobolev space norms W_p^1 to promote sparsity of the model difference. We, predictably, observed that the smaller the p value is, the sharper is the result of optimization. Interesting observation is that if we use Tikhonov regularization ($p=2$) then then the whole procedure is very similar to simple spectral filtering and there is no resolution enhancement. At the same time any value of p less than 2 makes time-lapse updates sharper.

The computational costs of the proposed denoising algorithm are slightly larger than those in conventional Rudin-Osher-Fatemi (Rudin et al., 1992) denoising as forward and inverse Fourier transforms need to be computed at every iteration, therefore in-situ image processing for online applications is not, probably, the best place to apply the algorithm. The computational costs are still, however, negligible comparatively to all modelings necessary to perform full-waveform inversion. For example modeling of one shot in time-domain using pseudo-spectral method involves number of time steps Fourier transforms forward and backward and is more expensive than the whole post-processing routine. The latter is important when in a field data application one will need to test several options for wavenumber mask choice and SSN type. In future we plan to improve the wavenumber mask estimation algorithms and apply the proposed denoising of time-lapse updates to field data.

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