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Highlights

- We present a pointwise wave-mode separation method, which is independent of modeling stencils.
- We link the group-velocity angle with the polarization angle via the phase-velocity angle.
- The wavefields get separated through polarization angles.
- The numerical angle table is ready before wave extrapolation.

Fast Poynting-Vector based wave-mode separation and RTM in 2D elastic TI media

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Abstract

The wave-modes in isotropic elastic media are easy to get separated by using Helmholtz decomposition. This method, however, fails in TI (transverse isotropic) media due to the anisotropy, and more sophisticated operators are required. Most of these existing operators are implemented and limited in FD (finite-difference) stencil. We propose a Poynting-vector based method, which separates wave-modes pointwisely, independent of the modeling stencils. In TI media, the Poynting-vector indicates the group-velocity direction while the wave-modes get separated in the polarization-vector direction. We write the

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relationship between these two directions into a small numerical table by exploiting the phase-velocity direction as a bridge prior to wavefield propagation. During the modeling process, it is easy to estimate the group-velocity direction from the Poynting vector, and then we can get the polarization-vector direction to separate wave-modes by checking the numerical table. We test our method on several TI models. We furthermore apply our method to elastic reverse-time migration (RTM) in TI media.

Keywords: Wave propagation, Elasticity, Anisotropy, Wave separation,

1. Introduction

The anisotropic wave extrapolation plays an important role in seismic imaging [1, 2]. The first crucial step in elastic reverse-time migration (RTM) is wave-mode separation [3]. In isotropic elastic media, we can easily separate wave-modes by using Helmholtz decomposition [4]. However, this method does not work well in anisotropic media. To attack this issue, geophysicists have proposed several approaches. [5] separate wave-modes in vertical-transversely isotropic (VTI) media by projecting wave vectors to polarization vectors in the wavenumber domain. This method is extended by [6] to separate wave-modes in complex VTI and TTI media by convolving the wavefields with spatially polarized filters. [7] improve the efficiency of their methods by taking reference parameter values for interpolation. [8] furthermore improve the efficiency of wave-mode separation by using a low-rank approximation for anisotropic media. [9] develop an efficient pseudo-spectral approach to directly extrapolate the decoupled elastic waves with low-rank approximated mixed-domain integral operators. [10] extend the low-rank operator to wave-mode separation in orthorhombic media and encompass the case of triclinic media.

In recent years, unstructured-mesh based modeling schemes [11, 12, 13, 14, 15] get more attention due to their flexibility and accuracy in representing complex geometries [1]. Notably, the spectral element method [11, 12, 15] has got broad applications in geophysical problems at different scales. However, most of

the wave-mode separation methods aforementioned are limitedly implemented in the FD stencil, due to their implementations in the Fourier domain, which is inapplicable for the unstructured-mesh based modelings. In this paper, we introduce a Poynting-vector based wave-mode separation method, which is independent of modeling schemes. According to [16], the wave-modes get separated in the polarization directions. We thus propose to make a numerical table which links the polarization vector with the group-velocity vector via the phase-velocity vector. The group-velocity vector (i.e., Poynting vector) is easy to get during elastic wave propagation [17]. We can then implement wave-mode separation along the directions of polarization vectors.

The methods in [5, 6, 7, 8, 9, 10] work well in anisotropic wave-mode separation by computing the polarization vectors via numerically solving the Christoffel equation given the wave vector, $\mathbf{k}$, which points in the phase direction (perpendicular to wavefront). [5, 6, 7] achieve separation in the space domain via the use of non-stationary spatial filters. [8, 9, 10] achieve this in the Fourier domain with the help of low-rank decomposition to increase computational efficiency. However, all these methods are carried out in the Fourier domain, which is easy for the structured-mesh based modelings but not so applicable for the unstructured-mesh based ones. Poynting vector, a good indicator of group-velocity vector, is easy to calculate during wavefield propagation, even for the unstructured-mesh based modelings. If bridging the gap between the polarization and group angles, we can get the wave-modes separated even with the unstructured-mesh modeling schemes. Note that our method is also applicable to the structured-mesh modeling schemes.

We arrange the paper as follows. Firstly, we briefly review the theories about Poynting vector, the relationship between group-velocity and phase-velocity angles, the relationship between polarization-vector angle and phase-velocity angles in elastic anisotropic media. Then, we make in advance a numerical table, which links the group-velocity angle and polarization-vector angle via the phase-velocity angle. Next, we separate wave-modes during wave propagation by obtaining the polarization-vector angle via checking the numerical table and

interpolation. Finally, we examine and apply our method to elastic RTM in TI media.

2. Methods

2.1. Anisotropic elastic wave equation and Poynting vector

The anisotropic elastic wave equation reads

$$\begin{aligned} \rho \frac{\partial v_i}{\partial t} &= \frac{\partial \tau_{ij}}{\partial x_j}, \\ \frac{\partial \tau_{ij}}{\partial t} &= \frac{1}{2} c_{ijkl} \left(\frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k} \right). \end{aligned} \quad (1)$$

where ρ denotes the density, $\mathbf{v} = (v_x, v_y)$ is the particle-velocity vector and $\boldsymbol{\tau} = (\tau_{xx}, \tau_{xy}, \tau_{xz}; \tau_{yx}, \tau_{yy}, \tau_{yz}; \tau_{zx}, \tau_{zy}, \tau_{zz})$ the stress tensor, with $\tau_{xz} = \tau_{zx}$, $\mathbf{x}$ the Cartesian coordinates and $\mathbf{c}$ the fourth-order elastic stiffness tensor [16, 4]. We solve Eq. (1) with a so-called unstructured mesh based "grid method" [13]. The Poynting vector $\mathbf{p}$, which indicates the direction of group velocity (energy propagation) [17, 18], can be expressed as

$$\mathbf{p} = -\boldsymbol{\tau} \cdot \mathbf{v}, \quad (2)$$

where $\cdot$ is the inner-product operator. It is easy to get $\boldsymbol{\tau}$ and $\mathbf{v}$ during the wavefield extrapolation by Eq. 1.

2.2. Relationship between the phase-velocity and group-velocity vectors

In 2D case, according to [19], the relationship between phase velocity and group velocity in Fig. 1 reads

$$\mathbf{V}_G = \frac{\partial(kV)}{\partial k_x} \vec{i} + \frac{\partial(kV)}{\partial k_z} \vec{j}, \quad (3)$$

where $\mathbf{k} = (k_x, k_z)$ is the wave vector, k the magnitude of $\mathbf{k}$, $\vec{i}$ and $\vec{j}$ the unit coordinate vectors. Because we here just work on the elastic media, the group velocity in non-dispersive media is frequency-independent [16]. Elaboration of Eq. 3 gives

$$\begin{aligned} V_{Gx} &= \frac{\partial(kV)}{\partial k_x} = V(\theta) \cos \theta - \frac{\partial V(\theta)}{\partial \theta} \sin \theta, \\ V_{Gz} &= \frac{\partial(kV)}{\partial k_z} = V(\theta) \sin \theta + \frac{\partial V(\theta)}{\partial \theta} \cos \theta, \end{aligned} \quad (4)$$

where phase velocity $V(\theta)$ is the function of phase angle θ , with θ being measured with respect to the x axis, and z being the vertical, symmetric axis in VTI media. Eq. 4 is incomplete because the definitions of $V(\theta)$ and $\partial V(\theta)/\partial\theta$ remain unknown, which we will provide in the next subsection. However, we can still determine the group-velocity direction of $\mathbf{V}_G$ and its corresponding angle ψ by Eq. 2. For simplicity, we express the relationship between phase-velocity and group-velocity angles ψ and θ in function forms for qP- and qSV- waves, respectively, as

$$\psi_P = F_1(\theta_P), \quad (5)$$

and

$$\psi_{SV} = F_2(\theta_{SV}). \quad (6)$$

Regarding the unstructured-mesh based modeling, it is challenging to efficiently calculate the phase-velocity angles but not so challenging for the group-velocity angles.

2.3. Relation between the phase-velocity vector and anisotropic polarization vector in VTI media

Let us start with the 2D elastic VTI case, in which the Christoffel equation [16] reads

$$\begin{bmatrix} c_{11}\cos^2\theta + c_{55}\sin^2\theta - \rho V^2(\theta) & (c_{13} + c_{55})\cos\theta\sin\theta \\ (c_{13} + c_{55})\cos\theta\sin\theta & c_{55}\cos^2\theta + c_{33}\sin^2\theta - \rho V^2(\theta) \end{bmatrix} \begin{bmatrix} U_1 \\ U_3 \end{bmatrix} = 0, \quad (7)$$

where θ is the phase angle, $V(\theta)$ is the angle-dependent phase velocity. We rewrite Eq. 7 for short as

$$[\mathbf{G} - \rho V^2 \mathbf{I}] \mathbf{U} = 0, \quad (8)$$

in which the solution to V corresponds to the eigenvalues found from $\det[\mathbf{G} - \rho V^2 \mathbf{I}] = 0$, and $\mathbf{G}$ is a positive definite matrix representing the angle-dependent parameters of different wave modes. In the 2D VTI case, Eq. 8 has two eigenvalues about the phase velocities, corresponding to the qP-wave (fast mode) and qSV-wave (slow mode), respectively. The qP- and qSV-modes are uncoupled from

the SH-mode and are polarized in the vertical (x,z) plane. Under the parameterization of Thomsen notation [20, 16], we have

$$\begin{aligned} c_{11} &= \rho(1 + 2\varepsilon)V_{P0}^2, \\ c_{13} &= \rho V_{P0}^2 \sqrt{f(f + 2\delta)} - \rho V_{SV0}^2, \\ c_{33} &= \rho V_{P0}^2, \\ c_{55} &= \rho V_{SV0}^2, \\ f &= 1 - \frac{V_{SV0}^2}{V_{P0}^2}, \end{aligned} \quad (9)$$

where V_{P0} denotes the P-wave velocity and V_{SV0} the SV-wave velocity. The two solutions of Eq. 7 can be written as

$$\begin{aligned} V_P(\theta) &= V_{P0} \sqrt{1 + \varepsilon \cos^2 \theta + \frac{f}{2} \left(\sqrt{1 + \frac{4\cos^2 \theta}{f} (2\delta \sin^2 \theta - \varepsilon \sin 2\theta) + \frac{4\varepsilon^2 \cos^4 \theta}{f^2}} - 1 \right)}, \\ V_{SV}(\theta) &= V_{SV0} \sqrt{1 + \varepsilon \cos^2 \theta + \frac{f}{2} \left(-\sqrt{1 + \frac{4\cos^2 \theta}{f} (2\delta \sin^2 \theta - \varepsilon \sin 2\theta) + \frac{4\varepsilon^2 \cos^4 \theta}{f^2}} - 1 \right)}. \end{aligned} \quad (10)$$

Substituting Eq. 10 back to Eq. 7 gives the polarization vector of qP- and qSV-waves: $\mathbf{U}_P$ and $\mathbf{U}_{SV}$, respectively.

Similar to the relationships between the group-velocity angle ψ and phase-velocity angle θ in Eqs. 5 and 6, we express the polarization angles denoted by $\mathbf{U}_P$ and $\mathbf{U}_{SV}$ as $\tilde{\theta}_P$ and $\tilde{\theta}_{SV}$, respectively. For simplicity, we also express the relationships between phase-velocity angle θ and polarization angle $\tilde{\theta}$ in function forms as

$$\tilde{\theta}_P = F_3(\theta_P), \quad (11)$$

and

$$\tilde{\theta}_{SV} = F_4(\theta_{SV}). \quad (12)$$

We substitute Eq. 10 back to Eq. 4 as well to make it intact. Note that VTI media is just a special case of TI media. We will elaborate the applications to TI media in the next part.

2.4. Bridging group-velocity and polarization-vector angles via phase-velocity angle in TI media

By now we already have in function forms the relationships between group-velocity and phase-velocity angles in Eqs. 5 and 6, and relationships between

polarization-vector and phase-velocity angles in Eqs. 11 and 12. That means, we can bridge the gap between group-velocity angle and polarization-vector angle via phase-velocity angle. By combining Eqs. 5, 6, 11 and 12, we can have a new function relationship for qP- and qSV- waves, respectively, in a complete form:

$$\tilde{\theta}_P = F_P(\psi_P, \varepsilon, \delta, f), \quad (13)$$

and

$$\tilde{\theta}_{SV} = F_{SV}(\psi_{SV}, \varepsilon, \delta, f), \quad (14)$$

in which $\psi, \varepsilon, \delta, f$ are the independent variables and $\tilde{\theta}$ the dependent variable. Fig. 2 demonstrates how we connect the polarization-vector and group-velocity angles via the phase-velocity angle.

Although $(\psi, \varepsilon, \delta, f)$ in Eqs. 13 and 14 span a 4D parameter space, this is not a source of concern. However, note that the 4D table for VTI in the proposed method will grow to 9D for P waves in orthorhombic media, which is characterised by 3 angles and 6 anisotropic parameters [10]. This number will increase in the media of lower symmetry. For ψ , we just take the range $[0, \pi/2]$, although in the 2D case ψ has a full range of $[0, 2\pi]$. The reason is that the horizontal and vertical Poynting vectors have four sign combinations in the four quadrants: $+/+$, $+/-$, $-/+$, $-/-$. Given the angle range in one quadrant, we can infer the angles in the other three quadrants by axis-symmetry and rotation-symmetry. We take the symmetry property among different quadrants due to the symmetry of TI media considered. Note that when the media are of lower symmetry (e.g, orthorhombic or monoclinic), this property no longer holds [16, 21, 10]. For Thomsen parameters ε and δ in weak anisotropy approximation, generally, they change in small ranges. For the parameter f , it also has a small range [16]. In practical applications, we evaluate in advance the distributions of the parameters ε, δ, f , and then discretize them together with ψ to get the discrete numerical table. We need some interpolation work to get polarization angle $\tilde{\theta}$ in the parameter space spanned by $(\psi, \varepsilon, \delta, f)$. The interpolation in the parameter space is similar to that in the Cartesian space [7]. Note that although

we can perform the eigen-decomposition per imaging point, the resulting cost is expensive, especially for the large-scale problems.

The methods above are applicable to 2D VTI media. It is worth noting that we can easily extend them to 2D TI media. The reason is that VTI media is nothing but a special case of TI media with vertical symmetry. Via angle rotation regarding the tilt parameter, we can rotate the Poynting-vector angle in a TI media into another Poynting-vector angle in a pseudo-VTI media, in which the methods above work well. After the determination of the polarization angle, we rotate the angle map back to that in TI media. Another theoretical background accounting for the angle rotation is that the group-velocity, phase-velocity and polarization-vector angles are coincident at the direction of symmetry axis [16]. Note that because we can handle the tilt angle via a simple rotation, the tilt parameter in TI media is not included as the independent variables in Eqs. 13 and 14, which reduces the parameter dimension by one.

2.5. Pointwise wave-mode separation in elastic media

We start the use of Poynting vector for wave-mode separation from the elastic isotropic case. Since P-wave is longitudinal and SV-wave is transverse, their propagation directions are orthogonal in isotropic elastic media [4], we can separate the wave-modes only with the Poynting-vector based group-velocity angle ψ :

$$\begin{bmatrix} U_P \\ U_{SV} \end{bmatrix} = \begin{bmatrix} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \end{bmatrix} \begin{bmatrix} v_x \\ v_z \end{bmatrix}, \quad (15)$$

because we know from Eq. 4 that the group and phase velocities are the same, and no polarization vector exists in isotropic media [16]. We have one numerical example regarding the isotropic elastic case in Fig. 3, in which the wave-modes get separated by Eq. 15. For P waves, the polarization vectors are in the same direction as the group and phase direction. For S waves in the 3D case, from the physics point of view, they exist but are non-uniquely defined within the plane perpendicular to the P-wave polarization. For the S waves in the 2D case, luckily, their polarizations are perpendicular to the P-wave polarizations.

However, in the elastic TI media, we cannot do wave-mode separation using a generic polarization angle $\tilde{\theta}$ as that in Eq. 15, because the group-velocity directions of the qP- and qSV-waves are not orthogonal [16]. Therefore, we need to tell apart group-velocity angles and to prepare for the mappings of $\psi_P \rightarrow \tilde{\theta}_P$, $\psi_{SV} \rightarrow \tilde{\theta}_{SV}$. Based on the fact that [4]

$$\begin{bmatrix} v_{x(P)} & v_{z(P)} \\ v_{x(SV)} & v_{z(SV)} \end{bmatrix} = \begin{bmatrix} \cos \tilde{\theta}_P & -\sin \tilde{\theta}_{SV} \\ \sin \tilde{\theta}_P & \cos \tilde{\theta}_{SV} \end{bmatrix} \begin{bmatrix} U_{qP} & 0 \\ 0 & U_{qSV} \end{bmatrix}, \quad (16)$$

if the wave-modes get separated well, the following relationships should hold:

$$\begin{aligned} \|U_{qP}\|_2 &= \|v_{x(P)}^2 + v_{z(P)}^2\|_2, \\ \|U_{qSV}\|_2 &= \|v_{x(SV)}^2 + v_{z(SV)}^2\|_2, \end{aligned} \quad (17)$$

in which $\|\cdot\|_2$ means norm-2. Eq. 17 suggests that given some tolerance threshold, we can tell apart ψ_P and ψ_{SV} by measuring the equality between the left-hand and right-hand sides. That is, we first run the qP- wave-mode separation over the global space using $\psi_P \rightarrow \tilde{\theta}_P$; we then measure the level of equality in Eq. 17 within some tolerance threshold to pick out ψ_P ; we finally run the qS- wave-mode separation over the rest space using $\psi_{SV} \rightarrow \tilde{\theta}_{SV}$. The relations $\psi_P \rightarrow \tilde{\theta}_P$ and $\psi_{SV} \rightarrow \tilde{\theta}_{SV}$ have been well prepared in Eqs. 13 and 14. Subsequently, similar to Eq. 15, we separate wave-modes in elastic TI media as follows:

$$\begin{bmatrix} U_{qP} \\ U_{qSV} \end{bmatrix} = \begin{bmatrix} \cos \tilde{\theta}_P & \sin \tilde{\theta}_P \\ -\sin \tilde{\theta}_{SV} & \cos \tilde{\theta}_{SV} \end{bmatrix} \begin{bmatrix} v_x \\ v_z \end{bmatrix}. \quad (18)$$

To demonstrate our method on wave-mode separation in elastic TI media, we first handle an elastic VTI case, as demonstrated in Figs. 4 and 5. Fig. 4 shows individually the x- and z- wavefield components, x- and z- Poynting vectors, and separated P- and S- wavefields. Fig. 5 shows individually the different angles occurring in this numerical example: group-velocity angle by Poynting vector, polarization-vector angle obtaining by checking the numerical table regarding the group-velocity angle, and the difference between the polarization-vector and group-velocity angles. Note that the polarization-vector angle is the one used for wave-mode separation.

We then have two elastic TI cases in Figs. 6 and 7. The difference between the TI and VTI media is that the TI media is more likely a general case with some varying tilt parameters while the tilt in the VTI media are fixed vertical. VTI media is nothing but a special case of TI media. We can first rotate the Poynting-vector angle in a TI media via tilt rotation to get another Poynting-vector angle in a pseudo-VTI media; then implement the conversion from the group-velocity angle to polarization-vector angle; next rotate the updated angle back via tilt rotation; finally carry out the wave-mode separation on the x- and z- wavefield components in the elastic TI media. We can see in Figs. 6 and 7, the anisotropic wave modes get separated well.

2.6. Elastic RTM in TI media

With polarization angle $\tilde{\theta}$ ready and wave-modes separated, we attempt to image the elastic TI media with the cross-correlation imaging condition:

$$\begin{aligned} I_{PP} &= \int U_{qP}^{R*} U_{qP}^S dt, \\ I_{PS} &= \int U_{qSV}^{R*} U_{qP}^S dt, \end{aligned} \quad (19)$$

in which U_{qP}^{R*} and U_{qSV}^{R*} denotes the backward propagated receiver qP- and qSV-waves, and U_{qP}^S the forward propagated source qP- wave. Because the most commonly used exciter in practice is the explosive source, we do not consider I_{SP} and I_{SS} here. For the suppression of low-wavenumber noises, we follow [18], using an angle-dependent filter. For the polarity correction in PS imaging, we follow [22], doing the correction according to the relationship between the direction vectors of source and receiver wavefields. The difference between the direction vectors in isotropic imaging and TI imaging is that we take Poynting vector directly in the former case but take polarization vector instead in the latter case.

3. Numerical examples

3.1. 2D TI Hess model

In this section, we will test the applicability of our methods to the complex media in wave-mode separation and the following RTM imaging. We select out

part of the total VTI Hess model, as shown in Fig. 8, and set the tilt parameter as -60° , thus making it a TI model. The model is 6-km deep and 9-km long, containing several reverse faults. The steep dip of the fault is a useful touchstone to examine the capability of imaging methods.

We first investigate the ability of our wave-mode separation method in TI Hess model. Before wavefield simulation, we evaluate in advance the ranges of ε, δ, f , and then design the discrete numerical table. In this model, because the ratio takes an empirical constant 1.732, so f has only one value; although ε and δ have discrete values, we here still treat them as continuous models and discretize them with interval 0.01. We discretize the group angle with interval. The resulting numerical table relating group-velocity angle with polarization-vector angle is at size $1 \times 15 \times 30 \times 91 = 40950$. The full storage of the numerical table in float requires an affordable 160-KByte memory. Even after a $1000\times$ amplification, a 160-Mbyte memory cost is still not expensive in scientific computing. We have two such kinds of numerical tables for qP- and qSV- waves, respectively. Statistical evaluation of the parameter distributions can furthermore reduce the size of the numerical table. We can also remove the two ends of the angle dimension points, as suggested by Fig. 2 and [16].

After obtaining the numerical table, we set off the wavefield modeling. One P-wave source together with one S-wave source is excited at the center of the original model. We capture at 1.2s the snapshots of x- and z- wave components, x- and z- Poynting vectors, separated P- and S- waves, as shown in Fig. 9. The wave-modes basically get separated. However, we notice that the wave-mode separations are imperfect in Figs. 9e and 9f. The overlapping of the Poynting vectors causes this imperfectness. To partially reduce overlapping effects caused by the model complexity we slightly smooth the velocities and density models within one wavelength. Then, we notice that in Fig. 10, the x-, z- waves and x-, z- Poynting vectors become less complicated, and the P- and S- waves get better separated. Note that in the wave-mode separation, we just need to check the numerical table in the memory and do some linear interpolations. The arising computational costs are negligible compared with seismic modeling.

With the separated wave-modes, we attempt to carry out elastic RTM in TI media. In this geophysical survey, we deploy on the surface 80 explosive shots with a 100-m interval, and 800 receivers with a 10-m interval. The recording duration is 7 s. PMLs surround the surveyed area. The true model in Fig. 8 generates the observed data, and the corresponding wave features can be seen in Fig. 9. Then, in the imaging procedure, we slightly smooth the media to facilitate better wave-mode separations for both source- and receiver- wave extrapolations. The corresponding wave features can be seen in Fig. 10. With polarization-vector at hand, we just take the elastic TI RTM following the framework of isotropic elastic RTM. We suppress the low-wavenumber noises following [18], and we correct the polarity of PS imaging following [22]. Fig. 11 shows that the basic structures of the Hess model are depicted well with elastic RTM in TI media.

4. Discussions

Although the choice of unstructured mesh provides more flexibility and accuracy for seismic modeling, its anisotropic wave-mode separation is not straightforward due to the inapplicability of regular FFT for unstructured mesh. To solve this problem, we propose a Poynting-vector based point-wise wave-mode separation method, in which we make a numerical table to link the group and polarization angles. However, there are some prices paid for working with wave-mode separation in this method such as overlapping Poynting vectors. Also, the cost of making the numerical table depends on the range of model parameters and the number of parameter combinations. For example, a wide parameter range will make a massive numerical table, and the 4D table for elastic TI media in the proposed method will grow to 9D for P-waves in orthorhombic media with 3 angles and 6 anisotropic parameters. Besides, S-wave singularities will occur where the two S waves have similar velocity for lower-symmetry (e.g., orthorhombic and monoclinic) media. Therefore, the separated wavefields will not be true-amplitude and neither is the resulting RTM image. Note that the

notion of "true-amplitude" here is defined in the high-frequency sense due to the use of polarization vectors from Christoffel equation. We will attack these limitations in our future study.

5. Conclusions

For wave-mode separation in elastic TI media, we present a Poynting-vector based method, which is point-wise such that independent of modeling schemes. The main idea is to make a numerical table, via phase-velocity angle, relating the group-velocity angle with the polarization-vector angle. We evaluate in advance the numerical table. We get the group-velocity angle from Poynting-vector during wavefield simulation, and then access the polarization-vector angle by checking the existing numerical table. We perform the wave-mode separation using the polarization-vector angle. The arising computation and memory costs in wave-mode separation by our method are negligible compared with the seismic modeling. We also test the applicability of our method for elastic RTM in TI media. In future applications to lower-symmetry media (e.g., orthorhombic and monoclinic), there are two S waves and they intertwine to create complex wavefields with singularities points where the two S waves have similar velocity. Also, in TI media, such singularities can also exist on SV wave field depending on the model parameters. For these singularities, we may seek solutions from multi-direction group vectors or do interpolation from their adjacent points.

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Figure 1: Schematic diagram of the relationship between phase-velocity vector (wave vector) and group-velocity vector (ray) in non-attenuative anisotropic media. We can see they two differ in directions.

Figure 2: (a) numerical table between the phase-velocity and group-velocity angles (in red), and numerical table between the phase-velocity and polarization-vector angles (in blue). Then, via the phase-velocity angle as a bridge, we can get a new numerical table relating group-velocity and polarization-vector angles in (b).

Figure 3: Elastic wave-mode separation in isotropic media by using Poynting vector. (a) and (b): x- and z- component wavefields. (c) and (d): x- and z- component Poynting vectors. (e) and (f): Separated P- and S- wavefields. Note that the angle-based wave-mode separation method preserves the amplitude and phase features of the original wavefields well.

Figure 4: Elastic wave-mode separation in VTI media by our method. (a) and (b): x- and z- component wavefields. (c) and (d): x- and z- component Poynting vectors. (e) and (f): Separated P- and S- wavefields. $Epsilon = 0.3, Delta = 0.1, Tilt = 0^\circ$.

Figure 5: Angle information regarding the wave-mode separation in Fig. 4. (a) Group-velocity angle determined by the Poynting vectors in Figs. 4c and 4d. (b) Polarization-vector angle obtained via checking the numerical table. (c) Angle difference between (b) and (a). Note that (b) is responsible for wave-mode separation.

Figure 6: Elastic wave-mode separation in VTI media by our method. (a) and (b): x- and z- component wavefields. (c) and (d): x- and z- component Poynting vectors. (e) and (f): Separated P- and S- wavefields. $\epsilon = 0.3, \Delta = 0.1, \text{Tilt} = 45^\circ$.

Figure 7: Elastic wave-mode separation in VTI media by our method. (a) and (b): x- and z- component wavefields. (c) and (d): x- and z- component Poynting vectors. (e) and (f): Separated P- and S- wavefields. $Epsilon = 0.1, Delta = 0.3, Tilt = 45^\circ$.

Figure 8: The 2D TI Hess model with tilt parameter -60° in our numerical testing. (a) P-wave velocity model. (b) S-wave velocity model. (c) Density model. (d) Delta model. (e) Epsilon model.

Figure 9: Elastic wave-mode separation in TI Hess media by our method. (a) and (b): x- and z- component wavefields. (c) and (d): x- and z- component Poynting vectors. (e) and (f): Separated P- and S- wavefields. (a) and (b) are for the generation of the observed data. From (e) and (f) we observe the imperfectness of the wave-mode separation caused by the overlapping of Poynting vectors.

Figure 10: Elastic wave-mode separation in TI Hess media by our method. (a) and (b): x- and z- component wavefields. (c) and (d): x- and z- component Poynting vectors. (e) and (f): Separated P- and S- wavefields. To alleviate the overlapping of Poynting vectors, we slightly smooth the Hess media, ready for the imaging of elastic RTM in TI media.

Figure 11: (a) PP and (b) PS images in elastic TI media. In this testing, we have 80 shots evenly placed near the surface. PML layers surround the imaging area. The basic structures of the Hess model are depicted well, except for the reverse fault in the bottom part of the PS image. This phenomenon can be explained with the radiation pattern of the PS wave: the energy strength of PS wave increases with incident angle. To have a better PS image, we may need a survey with longer offsets in the future research.