Robust $\mathcal{H}_\infty$ Pointing Error Control for Free Space Optical Communications in a Controlled Weak Turbulence Condition

W. Cai, I. N'Doye, X. Sun, W. G. Alheadary, M. S. Alouini, B. Ooi and T. M. Laleg-Kirati

Abstract—This paper proposes a robust $\mathcal{H}_\infty$ approach for free space optical (FSO) beam pointing error systems. First, a deterministic discrete-time model for pointing error loss due to misalignment is presented. This model considers detector aperture size and optical beam width. Then, the control problem that guarantees the stability of the closed-loop pointing error and the attenuation of the disturbance is investigated through a robust $\mathcal{H}_\infty$ control framework. Finally, simulation results of the FSO link under the effects of a controlled weak turbulence condition are performed to demonstrate the feasibility of the proposed robust pointing error approach.

Index Terms—Free Space Optical (FSO) communications, weak turbulence, $\mathcal{H}_\infty$ pointing control, stability, Linear Matrix Inequality (LMI).

I. INTRODUCTION

Free space optical communication (FSO) has attracted considerable attention in recent decades. Compared with radio frequency (RF) communication systems, FSO can provide high security, low cost, low power, and high rates [1], [2]. FSO links can be used for satellite-to-satellite cross links, up-and-down links between space platforms and aircrafts, ships, and other ground platforms, and among mobiles and stationary terminals [2]. However, FSO links suffer from misalignments due to atmosphere turbulence and pointing errors, which limit the link performance and availability [3]. Furthermore, pointing error is usually caused by thermal expansion, dynamic wind loads, weak earthquakes and misalignment. These factors impose exaggerated fading and power loss on FSO links [4], [5]. A large pointing loss can lead to intolerable signal fades and can significantly degrade the system performance. Hence, a very tight and pointing subsystem is required to reduce the loss due to misalignment between transmitters and receivers under atmosphere turbulence. Two research directions have been proposed to solve this problem: FSO channel analysis and modeling under atmosphere turbulence and pointing errors conditions; building theoretical models or practical pointing subsystems to mitigate the misalignment effects on FSO links.

Performance analyses on FSO over atmospheric turbulence and pointing error have been studied extensively [6]–[8]. Various theoretical turbulence and channel models have been proposed to describe bit error rate (BER), signal to noise ratio (SNR), ergodic capacities, outage probability and etc. The lognormal and the gamma-gamma distributions have been

found to be suitable for modeling the irradiance of the optical channels for weak-to moderate and moderate-to-strong turbulence channels respectively [9], since they provide good agreement between theoretical and experimental data. In [5], a statistical model for the optical intensity fluctuation at the receiver considering the effect of beam width, detector size, and jitter variance has been explicitly derived. In [10], a novel statistical fading channel model for multihop FSO systems using channel-state-information-assisted and fixed-gain relays is developed incorporating the atmospheric turbulence, pointing errors, and path-loss effects. Many pulse-position modulation has been investigated with impairments from atmospheric turbulence and pointing error with zero boresight in [11]. A nonzero boresight pointing error model [12] has been investigated for a Gaussian laser beam propagating through atmospheric channels.

Furthermore, many studies try to mitigate the misalignment effects on FSO links by theoretical or practical methods. In [13], a preliminary pointing and tracking subsystem design for an international space station ground link has been presented. The baseline pointing and tracking subsystems use a single focal plane array and a fine steering mirror. In [14], a beam tracking with a hybrid micro/nano manipulator driven by a control signal, which is generated by processing the beam intensity sensed from a four-quadrant photodiode, has been proposed. The beam tracking technique uses Kalman filter algorithm to estimate the state of the linear system, which is necessary for implementing the track-following control approach. In [15], a detection technique which can help to mitigate the effects due to turbulence-induced log-amplitude fluctuations in FSO links has been proposed. In [16], the novel performance analysis of repetition code and orthogonal space time block code in an FSO system over correlated lognormal channels is investigated, both schemes are shown to be useful in mitigating the turbulence fading effect and improving the BER performance. Other effective methods used to mitigate pointing error effects can also be found in [17]–[19]. However, these methods are usually too complex and computational expensive to be applied in controller design.

In this paper, we design a robust optimal approach control to reduce the turbulence effect and minimize the optical beam pointing errors. We derive a deterministic discrete-time model for pointing error loss due to misalignment which considers detector aperture size and optical beam width. Then, we investigate the $\mathcal{H}_\infty$ -norm optimization problem that guarantees the stability of the closed-loop pointing error

The authors are with Computer, Electrical and Mathematical Sciences and Engineering Division (CEMSE), King Abdullah University of Science and Technology (KAUST), Thuwal 23955-6900, Saudi Arabia (e-mail: taousmeriem.laleg@kaust.edu.sa)

and the attenuation of the disturbance. The proposed ideal control algorithm is capable of maintaining efficiently the center of the optical beam at the center of the receiving aperture.

Notations. M^T is the transpose of M . In symmetric block matrices, the symbol $(\star)$ in any matrix represents for any element that is induced by transposition. The space of square summable functions over the interval $[0, N - 1]$ is denoted by $\mathbf{L}_2[0, N - 1]$ and $\|u_k\|_2 = \left(\sum_{k=0}^{N-1} u_k^T u_k \right)^{\frac{1}{2}}$. $\mathbf{0}$ and $\mathbf{I}$ stand for the null matrix and the identity matrix of appropriate dimensions while I_n and $0_{n,n}$ stand for the identity matrix of dimensions $n \times n$ and the null matrix of dimensions $n \times n$, respectively. $\|x\| = \sqrt{x^T x}$ is the Euclidean vector norm.

II. EXPERIMENTAL SETUP FOR WEAK TURBULENCE FSO CHANNEL

The FSO link consists of a transmitter and a receiver separated by the atmospheric channel. In this paper, an experimental system has been set up whose schematic diagram is illustrated in Fig. 1. The optical signal amplitude and phase propagation through the FSO channel are affected by atmospheric turbulence. Many statistical models of the intensity fluctuation through FSO channels have been proposed in the literature for distinct turbulence regimes. For weak turbulence conditions, the most widely used model is the log-normal distribution, which has been validated through several studies [5], [20], [21]. In this paper we will focus on the weak turbulence, therefore the log-normal model will be considered. The laboratory atmospheric channel is a closed

Fig. 1. Block diagram of experiment setup.

glass chamber with a dimension of $100 \times 35 \times 42\text{cm}^3$ as depicted in Fig. 2. This chamber has been designed with the objective of observing the effect of atmospheric turbulence on the laser beam propagating through the channel. The main parameters of the FSO link are given in Table I.

The probability density function (PDF) of the received irradiance I due to the turbulence is derived by [22], [23]

$$P(I, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \frac{1}{I} \exp \left\{ -\frac{(\ln(I) + \sigma^2/2)^2}{2\sigma^2} \right\} \quad (1)$$

where σ^2 is the log-amplitude variance or scintillation index. The measured eye-diagrams for the received signal are depicted in Fig. 3 without turbulence and in Figs. 4, 5 and 6 with weak turbulence. We observe that the eye opening

Fig. 2. Experimental laboratory turbulence chamber.

TABLE I
PARAMETERS OF THE FSO LINK.

Description	Parameter	Value
Data	Format	OOK NRZ
	PRBS length	$2^{10} - 1$
	Signal intensity $V_{\text{peak-to-peak}}$	1.78V
	Data rate	622.082Mb/s
Laser diode	Type	LP642-SF20
	Peak wavelength	642nm
	Optical Output Power	20mW
	Operating current/voltage	0.089A/2.371V
Photodetector (PD)	Type	APD210
	Spectral range	400 – 1000nm
	Maximum gain	$2.5 \times 10^5 \text{ V/W}$
	Detector diameter	0.5mm
	Rise time	0.5ns
Lens	Type	LA1417-A
	Diameter	50.8mm
	Focal length	150mm
Receiver	Type	86100C-DCA-J
	Sampling time	0.2ns
Chamber	Dimension	$100 \times 35 \times 42\text{cm}^3$

is smaller in the presence of turbulence which results in a considerable level of signal intensity fluctuation and also reduces the FSO performance link.

Fig. 7 shows the histogram and the curve fitting plots of the received intensity signal without turbulence. As we can see, the PDF distribution is nearly Gaussian for lower values of σ^2 . Figs. 8, 9 and 10 illustrate the histograms and the curve fitting plots of the received intensity signal with turbulence. It is clear that the PDF has a good fitting with the log-normal distribution. The estimated scintillation index σ^2 fall within the range of $[0, 0.2]$ which is characterized by weak turbulence regime [23]. As σ^2 increases the distribution is more skewed with a long tail toward the infinity, and reduced peak probability density intensity as result of signal fading.

As we can see in Fig. 11, the experimental intensity sensed by the PD can be fitted with a nearly Gaussian distribution. We use an XYZ translation stage for mounting the photodetector, which has micrometers mounted on the rods. Thus, we adjust all the three axes to find the maximum intensity

Fig. 3. Measured screen shot eye-diagram of received intensity signal without turbulence

Fig. 5. Measured screen shot eye-diagram of received intensity signal under weak turbulence: $\sigma^2 = 0.0729$

Fig. 4. Measured screen shot eye-diagram of received intensity signal under weak turbulence: $\sigma^2 = 0.0529$.

Fig. 6. Measured screen shot eye-diagram of received intensity signal under weak turbulence: $\sigma^2 = 0.1156$.

Fig. 7. Gaussian PDF received distribution without turbulence (the curve fitting is shown by solid lines)

Fig. 8. Log-normal PDF received distribution under weak turbulence: $\sigma^2 = 0.0529$ (the curve fitting is shown by solid lines).

Fig. 9. Log-normal PDF received distribution under weak turbulence: $\sigma^2 = 0.0729$ (the curve fitting is shown by solid lines).

Fig. 10. Log-normal PDF received distribution under weak turbulence: $\sigma^2 = 0.1156$ (the curve fitting is shown by solid lines).

which means the optical beam is almost at the center of the photodetector in no turbulence case. Then, we move the X , Y positions to get the corresponding intensity. As it shows in Fig. 11, the signal intensity is significantly decreasing with optical beam moving away from the center of the photodetector.

III. SYSTEM DESCRIPTION AND DYNAMICAL MODELS

We consider a one-way optical link that consists of an optical transmitter and an optical receiver. Both are subject to relative motions. The transmitting laser source's azimuth

Fig. 11. Experimental intensity of the laser beam obeying to Gaussian distribution.

and elevation angles can be controller by a servo-driven pointing assembly. The emitted optical beam has a nonuniform intensity profile which is assumed to be Gaussian [24]. Normally, the aperture of the receiver is smaller than the received optical beam, so that the receiver can collect only a fraction of the optical beam [25]. This captured fraction can be enlarged by active pointing whose objective is to maintain the center of the optical beam at the center of the receiving aperture. A photodetector is used at the receiver to measure the intensity profile of the optical beam that strikes its aperture [25]. The output is then sent as feedback through an optical link or low-bandwidth RF channel and used to adjust the orientation of the transmitter. Fig. 12 illustrates the block diagram of this active pointing scheme under controlled weak turbulence.

Fig. 12. Pointing scheme for a short range free-space optical channel.

The discrete-time model considered in this study has been derived from the model structure that was introduced for spatial tracking systems [26] and stochastic state space model [25]. We denote the azimuth and elevation angles of the optical beam transmitter with respect to a fixed coordinate system by the vector θ_k and the azimuth and elevation angles of the receiving aperture of the stations with respect to the same coordinate system by α_k . We assume that the receiving aperture is held perpendicular to the line-of-sight optical beam. Under the assumption of a small pointing error $\epsilon_k = \theta_k - \alpha_k$ the relative displacement of the center of the optical beam with respect to the center of the receiving aperture is given by $y_k = d(\theta_k - \alpha_k)$, where d is the distance between the transmitter and receiver. Fig. 13 illustrates the optical beam in the plane of the receiving aperture and the displacement vector y_k .

The following linear discrete-time state-space equations de-

Fig. 13. Optical beam, receiving aperture and the displacement vector y_k .

scribes the optical beam transmitter system:

$$\begin{cases} x_{k+1}^p = A^p x_k^p + B^p u_k^p + R^p w_k^p, \\ \theta_k = C^p x_k^p, \end{cases} \quad (2)$$

where $k \in \mathbb{Z}^+$ is the set of all nonnegative integers, $x_k^p \in \mathbb{R}^{n^p}$ is the state vector, $\theta_k \in \mathbb{R}^2$ is the measurable output and $w_k^p \in \mathbb{R}^{q^p}$ is the external distributed process noise, $u_k^p : \mathbb{Z}^+ \rightarrow \mathbb{R}^2$ is the bounded control input. $A^p \in \mathbb{R}^{n^p \times n^p}$, $B^p \in \mathbb{R}^{n^p \times 2}$, $C^p \in \mathbb{R}^{2 \times n^p}$ and $R^p \in \mathbb{R}^{n^p \times q^p}$ are constant matrices. During the active pointing regime, the system operates over small angles, which justifies the use of a linear model for the pointing assembly.

Similarly, we model the receiving aperture α_k using a discrete-time model given by the following state-space equations

$$\begin{cases} x_{k+1}^l = A^l x_k^l + R^l w_k^l, \\ \alpha_k = C^l x_k^l, \end{cases} \quad (3)$$

where $x_k^l \in \mathbb{R}^{n^l}$ is the state vector and $w_k^l \in \mathbb{R}^{q^l}$ is a distributed process noise. A^l , R^l and C^l are constant matrices with appropriate dimensions.

The displacement vector $y_k = d(\theta_k - \alpha_k)$ is a linear function of x_k^p and x_k^l . It can be written as the following augmented system form

$$\begin{cases} x_{k+1} = \mathbb{A}x_k + \mathbb{B}u_k + \mathbb{R}w_k, \\ \epsilon_k = \mathbb{C}x_k, \end{cases} \quad (4)$$

where

$$\mathbb{A} = \begin{bmatrix} A^p & \mathbf{0} \\ \mathbf{0} & A^l \end{bmatrix}, \mathbb{B} = \begin{bmatrix} B^p \\ \mathbf{0} \end{bmatrix}, \mathbb{R} = \begin{bmatrix} R^p \\ R^l \end{bmatrix}, \mathbb{C} = \begin{bmatrix} C^p & -C^l \end{bmatrix},$$

and $x_k = \begin{bmatrix} x_k^p \\ x_k^l \end{bmatrix} \in \mathbb{R}^{n^p+n^l}$ is the augmented state vector,

$w_k = \begin{bmatrix} w_k^p \\ w_k^l \end{bmatrix}$ is the augmented disturbance vector and $y_k = d\epsilon_k$ is the pointing error.

The objective of an optimal pointing system is to maintain the centroid of the optical beam as close as possible to the center of the photodetector. This pointing problem can be interpreted as finding a set-point $u_k = -\mathbb{K}x_k$ depending on x_k such that the following $\mathcal{H}_\infty$ norm of the pointing error y_k with respect to disturbance w_k is satisfied i.e:

$$\sum_{k=0}^{N-1} \|y_k\|^2 \leq \varepsilon^2 \sum_{k=0}^{N-1} \|w_k\|^2, \quad (5)$$

over the time interval $[0, N-1]$, $N \in \mathbb{N}$ with ε being the smallest positive real to be minimized.

IV. ROBUST OPTIMAL POINTING ERROR CONTROL FOR FSO

In this section, we consider the $\mathcal{H}_\infty$ -norm optimization problem that guarantees the closed-loop pointing error y_k is stable and ensures the disturbance attenuation level $\|y_k\|_2 \leq \varepsilon \|w_k\|_2$ for a prescribed attenuation level $\varepsilon > 0$.

The following lemma will be frequently used to obtain the main results in this section.

Lemma 1: (The Schur complement lemma [27]). Given constant matrices Q , J , K of appropriate dimensions where Q and K are symmetric, then $K > 0$ and $Q + J^T K^{-1} J < 0$ if and only if

$$\begin{bmatrix} Q & J^T \\ J & -K \end{bmatrix} < 0, \quad \text{or equivalently} \quad \begin{bmatrix} -K & J \\ J^T & Q \end{bmatrix} < 0.$$

The following theorem ensures that the augmented system (4) is stable and the pointing error is achieved for the attenuation index ε .

Theorem 1: If there exist matrices $\mathbb{Y} = \mathbb{Y}^T > 0$, $\mathbb{S}$ and scalar ε such that the following LMI condition

$$\min_{\mathbb{S}, \mathbb{Y} > 0} \varepsilon \quad \text{subject to} \quad \begin{bmatrix} -\mathbb{Y} & \mathbf{0} & \mathbb{Y}\mathbb{A}^T - \mathbb{S}^T\mathbb{B} & \mathbb{Y}\mathbb{C}^T \\ (\star) & -\varepsilon^2 I & \mathbb{R}^T & \mathbf{0} \\ (\star) & (\star) & -\mathbb{Y} & \mathbf{0} \\ (\star) & (\star) & (\star) & -I \end{bmatrix} < 0 \quad (6)$$

has a feasible solution with $\mathbb{K} = \mathbb{S}\mathbb{Y}^{-1}$, then

- i) the closed-loop error system (4) is stable.
- ii) and the pointing error y_k satisfies the disturbance attenuation condition for a specific attenuation factor $\varepsilon > 0$

$$\sum_{k=0}^{N-1} \|y_k\|^2 \leq \varepsilon^2 \sum_{k=0}^{N-1} \|w_k\|^2. \quad (7)$$

□

Proof: Let us define the Lyapunov function $V_k = x_k^T \mathbb{P} x_k$ with $\mathbb{P} = \mathbb{P}^T > 0$. If $V_0 = 0$, i.e., all the initial conditions are null then, inequality (7) holds if the following satisfies along the trajectories of system (4)

$$\begin{aligned} & \sum_{k=0}^{N-1} \left(\|\mathbb{C}x_k\|^2 - \varepsilon^2 \|w_k\|^2 \right) + V_N - V_0 \\ &= \sum_{k=0}^{N-1} \left(\|\mathbb{C}x_k\|^2 - \varepsilon^2 \|w_k\|^2 + V_{k+1} - V_k \right) < 0. \end{aligned} \quad (8)$$

A sufficient condition to fulfill the inequality (8) is to guarantee for all $k \in \mathbb{Z}_{\geq 0}$

$$\|\mathbb{C}x_k\|^2 - \varepsilon^2 \|w_k\|^2 + V_{k+1} - V_k < 0. \quad (9)$$

Inequality (9) can be written as follows

$$\begin{aligned} & x_k^T \left[(\mathbb{A} - \mathbb{B}\mathbb{K})^T \mathbb{P} (\mathbb{A} - \mathbb{B}\mathbb{K}) - \mathbb{P} \right] x_k + x_k^T (\mathbb{A} - \mathbb{B}\mathbb{K})^T \mathbb{P} \mathbb{R} w_k \\ &+ w_k^T \mathbb{R}^T \mathbb{P} (\mathbb{A} - \mathbb{B}\mathbb{K}) x_k + w_k^T \mathbb{R}^T \mathbb{P} \mathbb{R} w_k \\ &+ x_k^T \mathbb{C}^T \mathbb{C} x_k - \varepsilon^2 w_k^T w_k < 0. \end{aligned} \quad (10)$$

Inequality (10) can be expressed as

$$\begin{bmatrix} x_k \\ w_k \end{bmatrix}^T \begin{bmatrix} \Xi & (A - BK)^T P R \\ (\star) & -\varepsilon^2 I + R^T P R \end{bmatrix} \begin{bmatrix} x_k \\ w_k \end{bmatrix} < 0, \quad (11)$$

where $\Xi = -P + (A - BK)^T P (A - BK) + C^T C$.

Using Schur complement, we obtain the following inequality

$$\begin{bmatrix} -P + C^T C & 0 \\ (\star) & -\varepsilon^2 I \end{bmatrix} + \begin{bmatrix} \tilde{A}^T P \\ R^T P \end{bmatrix} P^{-1} \begin{bmatrix} P \tilde{A} & P R \end{bmatrix} < 0, \quad (12)$$

where $\tilde{A} = A - BK$.

Applying again the Schur complement to (12), then we obtain the following sufficient condition

$$\begin{bmatrix} -P & 0 & (A - BK)^T P & C^T \\ (\star) & -\varepsilon^2 I & R^T P & 0 \\ (\star) & (\star) & -P & 0 \\ (\star) & (\star) & (\star) & -I \end{bmatrix} < 0. \quad (13)$$

Pre- and post-multiplying inequality (13) by $\text{diag}[Y, I, Y, I]$ and using $Y = P^{-1}$ and $S = KY$ which yields to the LMI (6). The fulfillment of inequality (6) implies the fulfillment of the optimality condition:

$$\sum_{k=0}^{N-1} \|y_k\|^2 \leq \varepsilon^2 \sum_{k=0}^{N-1} \|w_k\|^2, \quad u_k \neq 0, \quad w_k \neq 0. \quad (14)$$

This completes the proof. ■

V. NUMERICAL SIMULATIONS

In this section, we investigate the pointing error performance of the proposed FSO link under the influence of the weak atmospheric turbulence through numerical simulation. Many statistical models of the intensity fluctuation through FSO channels have been proposed in the literature for distinct turbulence regimes. However, these models are usually too complex to be applied in controller design. Simpler dynamic models suitable for control design can be obtained using system identification method. The identification of the optical beam transmitter channel states under weak turbulence conditions was carried out using real-time field data obtained from the experimental FSO setup. The intensity was measured in open-loop control operation and the 2D axes x and y were obtained using the Gaussian autocorrelated distribution function which is shown in Fig. 11. The identification results of the intensity response from an On-Off Keying (OOK) input excitation signal along the optical beam have been depicted in Fig. 14. As one can see the identified linear discrete-time model approximates around 50% the useful portions of the original data. Fig. 15 shows the 2D optical beam motion guided by the reduced-order autocorrelated beam intensity along x - y axes. From the autocorrelated position models, we performed a reduced-order model using a convex optimization method [28]. It is shown that the disturbed system can be approximated by a reduced order system with similar disturbance properties to the original plant. The model reduction strategy preserves the input disturbance nature of the model. It guarantees a sufficiently small error between the outputs of the original

and the reduced-order systems, and also maintains stability properties [28]. As it can be seen, the trajectory position of the optical beam is moving around the center. Besides, the occurrence probability of different position is different which also justify the log-normal PDF and nearly Gaussian distribution.

Fig. 14. Intensity data obtained from the voltage position using an OOK.

Fig. 15. 2D optical experimental beam motion guided by the autocorrelated beam intensity along x - y axes.

The receiving aperture motion (3) is assumed similar to the Brownian motion of a particle subjected to excitation as showed in Fig. 16. The Brownian motion is given by the generalized differential equation [14], [29], [30]

$$m \frac{d^2 x(t)}{dt^2} = -\gamma_1 \frac{dx(t)}{dt} - k_1 \frac{dx(t)}{dt} + \sqrt{2k_B T \gamma_1} W(t), \quad (15)$$

where $x(t)$ is the trajectory of the particle with respect to the center, m is the particle mass, γ_1 is the friction exerted by the surrounding medium on the particle, k is the optical trap stiffness, $k_B T$ is the thermal energy unit, k_B is the Boltzmann constant, T is the absolute temperature and $W(t)$ is a Wiener process. The discrete-time state space of the receiving aperture model (3) is derived from the discretized particle Brownian motion (15). It is given as follows [14], [29], [30]

$$\begin{cases} x_{k+1}^l = (1 - \frac{k_1 \Delta T}{\gamma_1}) x_k^l + \sqrt{2k_B T \gamma_1} w_k^l, \\ \alpha_k = x_k^l, \end{cases} \quad (16)$$

x_k^l is the source position and ΔT is the discretization time step. The y -axis can be modeled in the same manner as the x -axis, though with different dynamics. Fig. 16 shows the 2D position of the receiver motion along x and y axes.

To evaluate the quality of the pointing error obtained by

Fig. 16. Position of 2D receiver motion.

the LMI condition given in (6). The YALMIP interface [31] to MATLAB 8.5 was used to provide solutions. Figs. 17 and 18 show the pointing error plots. The closed-loop pointing maintains good alignment between the optical beam transmitter and the receiver aperture system with a relatively absolute error amplitude less than $\pm 0.08\text{mm}$ and $\pm 0.06\text{mm}$ in x and y directions respectively.

Fig. 17. Closed-loop pointing error in x direction versus Time.

Fig. 18. Closed-loop pointing error in y direction versus Time.

VI. CONCLUSION

In this paper, the performance of a controlled FSO link under the influence of weak atmospheric turbulence has been investigated. An atmospheric FSO chamber has been characterized theoretically and experimentally. In addition, a deterministic discrete-time model for pointing error loss due to misalignment which considers detector aperture size and optical beam width has been described. A robust optimal approach that guarantees the stability of the closed-loop pointing error and the attenuation of the disturbance has been proposed. Simulation results of the FSO link under

the effects of controlled weak turbulence conditions have been performed to demonstrate the feasibility of the proposed pointing error approach. Future research work will include the design of control strategies based on more accurate models to improve the pointing error performance results.

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