Polar Coordinate Based Modulation: Concept, Performance Analysis and System Design

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Abstract—In this paper, a nonlinear base-band modulation scheme, namely POLar COordinate based Modulation (POLCOM), is proposed for intensity modulation with direct detection (IM/DD) based optical wireless communication (OWC) systems. Two detectors including the optimal maximum likelihood detector (MLD) operated in polar coordinate and the joint detector (JD) operated in rectangular coordinate are investigated. Over additive white Gaussian noise (AWGN) channels, the symbol error rate (SER) performances of POLCOM using these two detectors are derived in high signal-to-noise ratio (SNR) regime. To achieve good performance in terms of SER, a heuristic greedy walk algorithm (GWA) is proposed to design the signal constellations for POLCOM. Comprehensive comparisons are made with existing schemes under the same signal bandwidth and spectral efficiency. Through fair comparisons, it is shown that the proposed POLCOM outperforms most of existing schemes.

Index Terms—Polar coordinate based modulation, optical wireless communication, intensity modulation with direct detection (IM/DD), power allocation, constellation shaping, symbol error rate (SER), additive Gaussian white noise (AWGN) channel

I. INTRODUCTION

Optical wireless communication (OWC) is one of potential technologies to meet the increasing data transmission demand. It delivers information utilizing license-free optical spectrum with high data rate. To facilitate low-cost low-complexity implementation, intensity modulation with direct detection (IM/DD) is the most commonly used. There are mainly four categories of modulation techniques for IM/DD based OWC systems, base-band pulse amplitude modulation (PAM), digital pulse position modulation (PPM), sub-carrier intensity modulation (SIM) as well as their combinations and variants. Comprehensive overviews of these modulation schemes are available in [1] and [2].

In all IM/DD systems, the signals modulated on light intensity must be unipolar real. The unipolar characteristic usually requires the addition of a power-inefficient direct current (DC) bias, while the real characteristic inherently limits the signal space in one domain. Their impacts on system performance have been well investigated in optical orthogonal frequency multiplexing (OFDM) systems belonging to the SIM category in [3]–[5]. And in our recent work [6], we proposed a polar coordinate based optical (PCO-) OFDM systems that requires no DC bias and extends the signal space from one domain to two domains. It converts the signal after inverse Fourier transform (IFFT) onto the polar plane, and transmits the magnitude and angle signals in use of two degrees of freedom

(DoFs) with different powers. PCO-OFDM has been shown to obtain the best performance compared with other existing optical OFDM schemes. As is well known, the time domain signals after IFFT are approximately Gaussian distributed in OFDM systems if the number of carriers is sufficiently large. With an equiprobable signal constellation rather than Gaussian input, will the base-band polar coordinate based modulation (POLCOM) still achieve better performance?

This paper intends to explore the answer of the question. We build the system and signal model of POLCOM. Specially, in the built model, the transmitter sends the magnitude signal r and the angle signal θ of a complex symbol equiprobably drawn from a complex signal constellation, while the receiver either decodes the information in polar coordinate with maximum likelihood detector (MLD) or combines the direct detected signals in a specific way and jointly demodulates the data symbol in rectangular coordinate as [6]. Over additive white Gaussian noise (AWGN) channels, the symbol error rate (SER) performances of POLCOM using two detectors are derived in high signal-to-noise ratio (SNR) regime. To achieve good performance in terms of SER, the optimal power allocation and constellation design are also investigated in this paper. In particular, a greedy walk (GW) algorithm is proposed to design the signal constellations for POLCOM. The output constellations exhibit irregular shapes and show great differences with conventional constellations, especially for POLCOM with the joint detector (JD). Simulations are further provided to demonstrate the comparisons with existing schemes.

II. SYSTEM MODEL

The system model of POLCOM is illustrated in Fig. 1. As shown in Fig. 1, a group of $\log_2 M$ bits is firstly mapped to a complex symbol drawn in M -ary signal constellation $\mathcal{X} = \{x_1, x_2, \dots, x_M\}$, whose magnitude set and angle set are denoted by $\mathcal{R} = \{r_1, r_2, \dots, r_M\}$ and $\Theta = \{\theta_1, \theta_2, \dots, \theta_M\}$, respectively. Provided x is the symbol chosen from $\mathcal{X}$, it can be expressed by

$$x = a + jb \quad (1)$$

where j is an imaginary unit. As it is complex if $b \neq 0$ and can not directly modulated on the light intensity. To facilitate low-complexity intensity modulation, the signal is converted onto the polar plane either by computing in digital domain or by

Fig. 1. Block diagram of a POLCOM system

going through a specific circuit [6]. After that, the magnitude signal and the angle signal are generated as

$$r = \sqrt{a^2 + b^2}, \quad r \geq 0, \quad (2)$$

and

$$\theta = \tan^{-1} \left(\frac{b}{a} \right), \quad 0 \leq \theta < 2\pi. \quad (3)$$

respectively. From (2) and (3), it is observed that both r and θ are non-negative and can be directly modulated onto the light intensity of a light-emitting diode (LED) or a laser source. r and θ are separately and independently modulated in use of two DoFs, e.g., two slots or two light sources. Here, for simplicity, it is assumed two slots are used and parameters ρ_1, ρ_2 are introduced to indicate the power allocation for two transmissions. The modulated optical signals then propagate over the optical wireless channel, arrive at the surface of the photodetector and are transformed to electrical signals for further information detection. Without loss of generality, it is assumed that two transmissions experience the AWGN channels, and the transform efficiency of light source and the responsibility of the photodetector are unit. Therefore, the received signals write as

$$y_1 = \rho_1 r + n_1, \quad (4)$$

and

$$y_2 = \rho_2 \theta + n_2, \quad (5)$$

respectively, where n_1 and n_2 are real-valued Gaussian noise with zero mean and variance σ_n^2 . Under the constraint of average optical power, the relationship between the power allocation factors ρ_1 and ρ_2 can be expressed as

$$\mathbb{E}\{\rho_1 r\} + \mathbb{E}\{\rho_2 \theta\} = 2P_o, \quad (6)$$

where P_o represents the average optical power per slot. As two slots are used for transmitting $\log_2 M$ bits, thus the overall transmission rate of POLCOM is

$$R = \frac{1}{2} \log_2 M \text{ (bits/channel use)}. \quad (7)$$

At the receiver, if MLD in polar coordinate is employed, $\hat{r}_k$ and $\hat{\theta}_k$ are detected by

$$\hat{r}_k, \hat{\theta}_k = \arg \min_{r_k, \theta_k} \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} \rho_1 r_k \\ \rho_2 \theta_k \end{bmatrix} \right\|_2, \quad (8)$$

where $\|\cdot\|_2$ is the Frobenius norm.

If the JD [6] in rectangular coordinate is employed, $\hat{x}_k$ is detected by

$$\hat{x}_k = \arg \min_{x_k} \left\| \frac{y_1}{\rho_1} \exp \left(j \frac{y_2}{\rho_2} \right) - x_k \right\|_2. \quad (9)$$

III. OPTIMAL POWER ALLOCATION AND SER PERFORMANCE ANALYSIS

For MLD in polar coordinate, r and θ play the same role in carrying information and the equal power should be allocated for transmitting r and θ . The SER of POLCOM using ML detector at high SNR regime can be approximately expressed as

$$P_s^{\text{ML}} \approx \frac{\lambda^{\text{P}}}{2M} \text{erfc} \left(\frac{d_{\min}^{\text{P}}}{2\sqrt{2}\sigma_n} \right), \quad (10)$$

where λ^{P} represents the number of closest-pairs and $d_{\min}^{\text{P}}$ is the minimum pairwise Euclidean distance in polar coordinate and defined by

$$d_{\min}^{\text{P}} = \min_{k, l=1,2,\dots,M, k \neq l} \left\| \begin{bmatrix} \rho_1 r_k \\ \rho_2 \theta_k \end{bmatrix} - \begin{bmatrix} \rho_1 r_l \\ \rho_2 \theta_l \end{bmatrix} \right\|_2. \quad (11)$$

For JD in rectangular coordinate, the two transmissions of r and θ can be expressed by one transmission as

$$\hat{x} = x + n_e, \quad (12)$$

where n_e denotes the effective noise. And we have the following proposition.

Proposition 1: The effective n_e can be approximately considered as a complex AWGN with zero mean and variance σ_e^2 , which can be given by

$$\sigma_e = (2 - 2e^{-\frac{\sigma_n^2}{2\rho_1^2}}) \mathbb{E}\{r^2\} + \frac{\sigma_n^2}{\rho_1^2}. \quad (13)$$

That is to say $n_e \sim \mathcal{CN}(0, \sigma_e^2)$.

The proof of this proposition can be found in Appendix A.

Based on the proposition, the two transmissions can be treated as one transmission over AWGN channel. From (13), it can be seen that the effective noise σ_e^2 is relative to the power allocation factors ρ_1 and ρ_2 . Next, we will derive the optimal ρ_1 and ρ_2 to minimize σ_e^2 . Specifically, if we reformulate the average optical power constraint in (6), we can get

$$\rho_2 = \frac{2P_o - \rho_1 \mathbb{E}\{r\}}{\mathbb{E}\{\theta\}}, \quad (14)$$

$$\sigma_e^2 = \frac{\left(\sqrt[3]{\mathbb{E}\{r^2\}\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2 + \mathbb{E}\{r\}\right)^2 \left[\mathbb{E}\{r^2\}\mathbb{E}\{\theta\}^2 + \left(\sqrt[3]{\mathbb{E}\{r^2\}\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2}\right)^2\right]}{4 \left(\sqrt[3]{\mathbb{E}\{r^2\}\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2}\right)^2 \gamma_{\text{elec}}} \quad (24)$$

By inserting this reformulation into (13), σ_e^2 is obtained as

$$\sigma_e^2 = 2\mathbb{E}\{r^2\} - 2\mathbb{E}\{r^2\} \exp\left(-\frac{\sigma_n^2 \mathbb{E}\{\theta\}^2}{2(2P_o - \rho_1 \mathbb{E}\{r\})^2}\right) + \frac{\sigma_n^2}{\rho_1^2}. \quad (15)$$

Taking the first derivative of (15), we have

$$\frac{d\sigma_e^2}{d\rho_1} = \frac{2\mathbb{E}\{r^2\}\mathbb{E}\{r\}\sigma_n^2 \mathbb{E}\{\theta\}^2}{(2P_o - \rho_1 \mathbb{E}\{r\})^3} \exp\left(-\frac{\sigma_n^2 \mathbb{E}\{\theta\}^2}{2(2P_o - \rho_1 \mathbb{E}\{r\})^2}\right) - \frac{2\sigma_n^2}{\rho_1^3}. \quad (16)$$

By solving the equation $d\sigma_e^2/d\rho_1 = 0$ using numerical method such as a bisection method [7], the optimal ρ_1^{opt} can be obtained.

For high SNR cases with fixed P_o , σ_n^2 approaches to 0, and $\lim_{x \rightarrow 0} \exp(x) = 1 + x$. Then,

$$\exp\left(-\frac{\sigma_n^2 \mathbb{E}\{\theta\}^2}{2(2P_o - \rho_1 \mathbb{E}\{r\})^2}\right) \approx 1 - \frac{\sigma_n^2 \mathbb{E}\{\theta\}^2}{2(2P_o - \rho_1 \mathbb{E}\{r\})^2}. \quad (17)$$

Based on this approximation, (15) and (16) are rewritten as

$$\sigma_e^2 \approx \frac{\mathbb{E}\{r^2\}\mathbb{E}\{\theta\}^2 \sigma_n^2}{(2P_o - \rho_1 \mathbb{E}\{r\})^2} + \frac{\sigma_n^2}{\rho_1^2}, \quad (18)$$

and

$$\frac{d\sigma_e^2}{d\rho_1} \approx \frac{2\mathbb{E}\{r^2\}\mathbb{E}\{r\}\sigma_n^2 \mathbb{E}\{\theta\}^2}{(2P_o - \rho_1 \mathbb{E}\{r\})^3} - \frac{2\sigma_n^2}{\rho_1^3}, \quad (19)$$

respectively. By solving $d\sigma_e^2/d\rho_1 = 0$, ρ_1^{opt} can be expressed as

$$\rho_1^{\text{opt}} = \frac{2P_o}{\sqrt[3]{\mathbb{E}\{r^2\}\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2 + \mathbb{E}\{r\}}}. \quad (20)$$

By substituting (20) into (14), the optimal ρ_2^{opt} is got by

$$\rho_2^{\text{opt}} = \frac{2P_o \sqrt[3]{\mathbb{E}\{r^2\}\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2}}{\mathbb{E}\{\theta\} \left(\sqrt[3]{\mathbb{E}\{r^2\}\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2} + \mathbb{E}\{r\}\right)}. \quad (21)$$

Utilizing ρ_1^{opt} and ρ_2^{opt} for two transmissions, the average optical power for the first slot is given by

$$P_{o1} = \mathbb{E}\{\rho_1^{\text{opt}} r\} = \frac{2P_o \mathbb{E}\{r\}}{\sqrt[3]{\mathbb{E}\{r^2\}\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2 + \mathbb{E}\{r\}}}, \quad (22)$$

and that for the second slot is derived as

$$P_{o2} = \frac{2P_o \sqrt[3]{\mathbb{E}\{r^2\}\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2}}{\sqrt[3]{\mathbb{E}\{r^2\}\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2} + \mathbb{E}\{r\}}. \quad (23)$$

Therefore, the power ratio distributed to the two slots should be $\left(\frac{\mathbb{E}\{r\}}{\sqrt[3]{\mathbb{E}\{r^2\}\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2 + \mathbb{E}\{r\}}}, \frac{\sqrt[3]{\mathbb{E}\{r^2\}\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2}}{\sqrt[3]{\mathbb{E}\{r^2\}\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2 + \mathbb{E}\{r\}}}\right)$. It is different from the case when optimal MLD in polar coordinate is employed.

And also, we can derive the minimum σ_e^2 as (24) at the top of this page, where γ_{elec} is the electrical SNR and defined by $\gamma_{\text{elec}} = P_o^2/\sigma_n^2$. Given the M -ary signal constellation $\mathcal{X} = \{x_1, x_2, \dots, x_M\}$ with normalized power $\mathbb{E}\{r^2\} = 1$, let $d_{\min}$ denote the minimum distance, i.e., $d_{\min}^R = \min_{k \neq l, k, l=1,2,\dots,M} \|x_k - x_l\|_2$ and the SER with the JD can be derived as

$$P_s^{\text{JD}} = \frac{\lambda^R}{2M} \text{erfc}\left(\frac{d_{\min}^R}{2\sqrt{2}\sigma_e}\right). \quad (25)$$

where λ^R is the total number of the closest-pairs in rectangular coordinate. As $\mathbb{E}\{r^2\}$ is normalized, the effective noise power σ_e in (25) can be written as

$$\sigma_e^2 = \frac{\left(\sqrt[3]{\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2} + \mathbb{E}\{r\}\right)^2 \left[\mathbb{E}\{\theta\}^2 + \left(\sqrt[3]{\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2}\right)^2\right]}{4 \left(\sqrt[3]{\mathbb{E}\{r\}\mathbb{E}\{\theta\}^2}\right)^2 \gamma_{\text{elec}}}, \quad (26)$$

and one can verify that $\partial\sigma_e^2/\partial\mathbb{E}\{r\} > 0$ and $\partial\sigma_e^2/\partial\mathbb{E}\{\theta\} > 0$. In other words, the effective noise power will increase as either $\mathbb{E}\{r\}$ or $\mathbb{E}\{\theta\}$ increases. And simultaneously,

$$d_{\min}^R = \min_{k \neq l, k, l=1,2,\dots,M} \|r_k \exp(j\theta_k) - r_l \exp(j\theta_l)\|_2, \quad (27)$$

will increase as either $\mathbb{E}\{r\}$ or $\mathbb{E}\{\theta\}$ increases. This indicates that there is a trade-off in determining $\mathbb{E}\{r\}$ or $\mathbb{E}\{\theta\}$.

IV. GREEDY WALK CONSTELLATION DESIGN ALGORITHM

To achieve good SER performance, a heuristic greedy walk (GW) constellation design algorithm will be developed in this part and suitable for systems either with the MLD in polar coordinate or with the JD in rectangular coordinate. Due to the limitation of length, we take the signal constellation design for systems with JD in rectangular coordinate as an example to introduce the algorithm.

The key idea of GW algorithm is to add the constellation point one by one into the constellation symbol set $\mathcal{X}$ in rectangular coordinate. The minimum Euclidean distances of the new added constellation point to $\mathcal{X}$ must be larger than or equal to that in $\mathcal{X}$. The new constellation point are selected from a few of candidates at the boundary of the cover area of $\mathcal{X}$. The detailed algorithm is listed in Algorithm 1.

This algorithm is equivalent to walking from the constellation cover area to one of the boundary points in a greedy manner to enlarge the cover area. This is why we name it GW algorithm. For clear viewing, we illustrate the design steps of GW algorithms in Fig. 2. In Fig. 2, the red pentagram marker represents the constellation point and the cyan asterisk marker stands for the candidate point. From the figure, it is directly observed that the constellation points are one by

Fig. 2. Design steps including the 1st, 2nd, 3rd and 13th steps of GW algorithm

Algorithm 1 Greedy Walk (GW) Algorithm

Input: M , γ_{elec}

Output: M -ary signal constellation for POLCOM

Initiation: $\mathcal{X} = \{0\}$ and

while $|\mathcal{X}| \leq M$ **do**

 Draw $|\mathcal{X}|$ circles with centers being the points in $\mathcal{X}$ and radius $d = 1$.

 Find the cross points of two circles or that between circle and the radial $\theta = 0$ as candidates. Let $\mathcal{D}$ denote the candidate point set.

for $p \in \mathcal{D}$ and $p \notin \mathcal{X}$ **do**

 Add one candidate into $\mathcal{X}$ to generate $\mathcal{X}_{\text{tem}}$, normalize $\mathcal{X}_{\text{tem}}$, calculate the value of P_e according to (25).

end for

 Add the p^{opt} with smallest P_e into $\mathcal{X}$ as a new $\mathcal{X}$.

end while

Normalize and output $\mathcal{X}$.

one selected from the candidates. The output constellations of $M = \{4, 8, 16, 32, 64, 128, 256, 512, 1024\}$ are demonstrated in Fig. 3. It is shown that for $M = \{4, 8\}$, POLCOM with JD has the same optimal constellations as conventional pulse amplitude modulation (PAM). When M is greater than 8, the constellations for POLCOM with JD show significant differences from existing signal constellations and asymptotically exhibit a beautiful “half heart” shape. Such constellations are irregular, the pseudo Gary mapping [8], [9] can be used for the bits-to-constellation mapping.

Utilizing GW algorithm to design the signal constellations for POLCOM with MLD in polar coordinate, the output constellations of $M = \{4, 8, 16, 32, 64, 128\}$ are illustrated in Fig. 5 on the last page. It is noted that Fig. 3 is depicted in rectangular coordinate and with normalized power $\mathbb{E}\{x^2\} = 1$, while Fig. 5 in polar coordinate and with normalized optical

power $(\mathbb{E}\{r\} + \mathbb{E}\{\theta\})/2 = 1$. The constellations in Fig. 5 exhibits the similar shape as the designed constellations in [10], but the exact constellation points are mostly different. For comparison, we list the minimum distances among constellation points of both designs in Table I. It is shown that the constellation design for spatial-collaborative coded (CC) systems in [10] is slightly superior to the proposed design in this paper when $M = 4$ and $M = 8$, while the proposed design outperforms the design in [10] a bit when $M > 8$ in terms of the minimum distance among constellation points.

Complexity analysis: The complexity of system design is easily analyzed as the constellation candidates are at most $2n - 1$ for the n -th step, and the number of steps is M . Thus the complexity of signal constellation is $\mathcal{O}(M^2)$. The complexity of pseudo Gray mapping is also $\mathcal{O}(M^2)$ [9]. As a result, the total complexity of the system design is $\mathcal{O}(M^2)$.

V. SIMULATION

In this part, we compare the proposed POLCOM with systems including spatial multiplexing (SMX), spatial modulation (SM) [11], repetition code (RC) [12] and CC [10] that also require 2 DoFs under the same signal bandwidth, spectrum efficiency of 3 (bits/channel use) and unit average optical power, in terms of SER versus γ_{elec} . The results are illustrated in Fig. 4.

Fig. 4 demonstrates that POLCOM with MLD is capable of achieving the-state-of-art performance as similar as SMX and CC over two AWGN channels. POLCOM with JD [6] is less comparable in terms of SER. This can be explained as follows. The expectation power of the jointly detected $\hat{x}$ in rectangular coordinate is given by

$$\mathbb{E}\{|\hat{x}|^2\} = \mathbb{E}\{r^2\} + \frac{\sigma_n^2}{\rho_1^2}. \quad (28)$$

From (28), it is observed that the received signal power in rectangular coordinate is only related to the power of the

Fig. 3. Signal constellations for POLCOM with joint detection in rectangular coordinate.

Fig. 4. Comparisons among various schemes in terms of SER versus γ_{elec} .

magnitude signal and that of the noise. This indicates that the power allocated to the angle signal is wasted as it play no

roles in determining the receive SNR. This also explains why its optimal constellation shape exhibit a “half heart” shape, which is to make $\mathbb{E}\{\theta\}$ as small as possible. Compared with SM and CC, the proposed POLCOM with either MLD or JD is superior over them in saving optical power to obtain a same SER performance.

VI. CONCLUSION

In this paper, we proposed a polar coordinate based modulation (POLCOM) and investigated its power allocation, SER performance as well as system design using MLD and JD. The SER performances with these detectors were derived in a closed-form expression. An heuristic algorithm was proposed to optimize the constellation. Through comparisons with existing schemes, it was found that POLCOM with MLD achieves the state-of-the-art best performance, and outperforms that with JD.

Fig. 5. Signal constellations for POLCOM with ML detector in polar coordinate.

TABLE I
COMPARISONS WITH THE CONSTELLATION DESIGNS IN [10] IN TERMS OF THE MINIMUM DISTANCE AMONG CONSTELLATION POINTS

Minimum distance among constellation points						
-	$M = 4$	$M = 8$	$M = 16$	$M = 32$	$M = 64$	$M = 128$
$d_{\min}^P$ of the proposed design for POLCOM	1.9521	1.1271	0.7145	0.4690	0.3163	0.2166
$d_{\min}^R$ of the constellation design for CC in [10]	2.0000	1.1429	0.7111	0.4571	0.3048	0.2065

APPENDIX

Proof: The expectation of n_e and $|n_e|^2$ are calculated by

$$\mathbb{E}\{n_e\} = 0 \quad (29)$$

and

$$\begin{aligned} \mathbb{E}\{|n_e|^2\} &= \mathbb{E}\left\{\left|r \exp(j\theta) - \left(r + \frac{n_1}{\rho_1}\right) \exp\left[j\left(\theta + \frac{n_2}{\rho_2}\right)\right]\right|^2\right\} \\ &= (2 - 2e^{-\frac{\sigma_n^2}{2\rho_2^2}})\mathbb{E}\{r^2\} + \frac{\sigma_n^2}{\rho_1^2} \triangleq \sigma_e^2. \end{aligned} \quad (30)$$

respectively. By Kolmogorov-Smirnov test [13], n_e can be approximately considered as complex Gaussian, i.e., $n_e \sim \mathcal{CN}(0, \sigma_e^2)$. ■

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