Article type: Full Paper

Efficient Activation of Nano-mechanical Resonators

*M. A. A. Hafiz, N. Jaber, S. N. R. Kazmi, M. H. Hasan, F. Alsaleem, and M. I. Younis**

Dr. M. A. A. Hafiz[#], N. Jaber[#], Dr. S. N. R. Kazmi, and Prof. M. I. Younis.

[#]Equal contribution.

Physical Sciences and Engineering Division, King Abdullah University of Science and Technology, Thuwal 23955-6900, Saudi Arabia

*E-mail: mohammad.younis@kaust.edu.sa.

M. H. Hasan, Prof. F. Alsaleem

²Durham School of Architectural Engineering and Construction, University of Nebraska Lincoln, Lincoln, Nebraska, 68182-0816, USA.

Keywords: LC tank, nonlinear dynamics, resonators, and passive amplification.

Abstract

Electrostatically transduced nano-electromechanical systems (NEMS) resonators operating in the very-high and ultra-high (V/UHF) frequency bands are promising for many practical applications. However, electrostatically transduced nanoscale devices commonly suffer from poor transduction efficiency due to the reduced capacitive area for actuation and detection. Also, the requirement of ultra-high actuation forces renders exploiting their higher-order vibration modes and the desirable nonlinear behaviors practically beyond reach. Hence, it is imperative to develop a methodology that efficiently actuates nano and sub-micron scale highly stiff resonators with low voltages available in a standard integrated circuit (IC). Here, we propose and demonstrate the utilization of the passive voltage amplification across the inductor of an inductor-capacitor LC tank resonant circuit to efficiently actuate nano-resonators with high forcing amplitude. The proposed technique is simple and flexible in its implementation, and does not require any active electronic components. We experimentally show a forcing amplification up to 19 times (~ 25 dB), which can be improved further by reducing the electrical damping in the tank circuit. In addition, we show two independent

ports on the same device for force amplification, which if simultaneously activated, can increase the overall forcing amplitude by an order of magnitude exceeding hundreds of amplification gain.

1. Introduction

NEMS devices have invariably attracted great attention in the research community for various applications, such as ultra-sensitive sensors,[1-4] communication,[5-7] quantum information processing,[8,9] and logic processors.[10-15] Among other transduction techniques, electrostatic transduction is considered most practical for monolithic integration with the existing complimentary-metal-oxide-semiconductor (CMOS) technology.[16-19] However, with the continuous push toward GHz frequency range, these devices suffer from poor electromechanical coupling efficiency due to the reduced capacitive area for actuation and detection, as well as the tremendous stiffness increase of their structural components. Alternative to miniaturization to achieve higher operational frequencies is to exploit higher-order vibration modes of the nano structures. This however requires ultra-high actuation forces, which practically are not available. Moreover, considerable interest has grown recently on exploiting nonlinear vibration phenomena for developing devices with superior performances,[20, 21] which also requires large forcing. Active electronic components, amplifiers, can be used to boost the drive signal, however, it increases device footprint and fabrication cost. Also, state-of-the-art amplifiers are hindered by limited gain at high frequencies. The requirement of low voltage to actuate sensors and actuators imposed constraints on the design and the choice of material for sensors and actuators[22-24] and often complicates the circuit design in order to amplify the voltage.[25] Hence, it is fundamentally important to develop a methodology that efficiently excites nanoscale resonators with available low voltages, AC and DC, in electronic circuits and detect the generated motional signals produced by sub-nanometer scale vibrations with high signal-to-noise ratio.

LC tank resonant circuits have been widely proposed for efficient coupling between different physical domains, such as, electrical, mechanical, and optical.^[26-34] An LC tank circuit based impedance matching technique was shown to be effective in probing a single and arrayed nanoscale resonators.^[26] The LC tank resonance frequency, f_{LC} , was matched with the mechanical resonance frequency, f_m , enabling efficient RF power coupling at resonance with the detection circuit. This facilitated the detection of individual nano-resonator response in an array by measuring the reflected signal, which was otherwise overwhelmed by the parasitic signal. Bagci *et al.*^[27] used similar LC tank resonant circuit ($f_{LC} = f_m$) to facilitate strong coupling between the radio waves and a nano-optoelectromechanical transducer at room-temperature, and demonstrated the detection of weak radio wave signals with quantum-limited sensitivity. Sillanpää *et al.*^[28] used a GHz frequency LC tank circuit ($f_{LC} \gg f_m$) to block the external wiring capacitances in the electrical readout circuit, and efficiently detected the nano-resonator response by monitoring the sideband voltage created by the nanoscale motion. A similar technique was also shown to be effective for metallic beam resonators working at tens of megahertz.^[29] To facilitate radiation detection in the terahertz band, in a recent study,^[30] a coupled system comprised of an LC tank and a nanomechanical resonator was shown to be working as a microwave amplifier, capable of amplifying weak electrical signals. In a recent work, we have demonstrated the utilization of simultaneous electrical and mechanical resonances to achieve amplification on single-port MEMS devices.^[34] Note that the utilization of similar LC tank resonant circuits to enhance the detection bandwidth is well known in the electron transport community.^[32, 33] LC tank circuit coupled to Josephson junction has been utilized to model the electrical activities in the neurons^[35] and to understand the complex dynamics of particles moving in nonlinear time dependent potential field.^[36]

Despite the numerous studies, the potential for LC tank circuits to boost actuation powers for highly stiff nanoscale resonators has not been investigated. A clear understanding

on utilizing this technology to practically actuate and power nano-resonators is of paramount importance. Such an understanding will pave the way to enable the activation of their desirable higher-order vibration modes and non-linear resonances, which are otherwise impossible to activate due to the lack of forcing within the available voltage levels in a standard IC. We note also that most studies were conducted on 1-port configuration devices. Hence, further investigations are needed to examine the effectiveness of LC tank for actuation and detection techniques on devices working in the 2-port configuration, which is common for micro/nano-resonators.

In this work, we utilize the passive voltage amplification across the inductor of an LC tank resonant circuit and demonstrate experimentally ~ 25 dB increase in the forcing amplitude to actuate nanoresonators with high amplitudes. We also show two independent ports on a single device to induce force amplification, which if activated simultaneously, can lead to at least one order of magnitude increase in the overall forcing amplitude. This technique does not require any active electronic component and is limited only by the quality factor of the LC tank, which can be improved by reducing the resistive losses in the tank circuit.

2. Results

Figure 1(a) shows a scanning electron microscope (SEM) image of the fabricated clamped-clamped nanobeam resonator of length (l) $14.7\ \mu\text{m}$, width (w) $680\ \text{nm}$, thickness (t) $1.5\ \mu\text{m}$, and a $360\ \text{nm}$ transduction gap (g) between the nano-beam and the drive/sense electrodes. The doubly clamped nanomechanical beams of this study are fabricated out of a low-resistive ($0.1\text{--}0.5\ \text{m}\Omega\text{-cm}$) boron doped Si device layer of silicon-on-insulator (SOI) wafer. Using 950 PMMA A4 positive tone e-beam resist followed by reactive ion etching step, we define the dimension of the naomechanical beam, anchoring pads, and the sensing/driving electrodes. Finally, We employ HF vapor to etch the sacrificial oxide and release the

nanobeam. The details of the entire fabrication process are outlined in our previous work.^[37] Figure 1b and 1c show the experimental setup and a simplified equivalent electrical circuit model, respectively. Here, we use the mixing measurement technique to characterize the devices.^[6] A network analyzer (Agilent E5071C) is used as the first voltage source, $v_{AC1} = V_{AC1} \cos 2\pi f_1 t$, which is connected to an inductor, L , which in turn is connected to the drive electrode in series. An external capacitor of 10 pF is connected between the drive and the sense electrode, which can be varied to change the tank circuit resonance frequency. This external capacitor is in parallel with other parasitic capacitors (≈ 1.3 pF) present between the drive and sense electrodes forming the overall capacitor, C , for the LC tank circuit, highlighted with red shade in Figure 1c. A bias-tee is used to combine the second voltage source, $v_{AC2} = V_{AC2} \cos 2\pi f_2 t$, and the DC bias voltage, V_{DC} , which is connected to the nanobeam. The characterizations of the devices are performed in a ST-500 JANIS probe station under controlled vacuum pressure and temperature conditions. For the initial characterization of the nanoresonator, we bias the nanobeam with a DC voltage and drive the device with an AC signal from the network analyzer provided to the drive electrode. The sense electrode is connected to a low noise amplifier (LNA) with a gain of 25, which is coupled to the input port of the network analyzer. Figure 1d shows the measured S_{21} transmission parameters under a 50 V DC bias and -10 dBm of RF power at 4.8×10^{-4} mbar vacuum pressure, and at a chuck temperature of 25 °C. The resonator exhibits a resonance peak at $f_m = 20.048$ MHz with a quality factor $Q_m \approx 2500$.

The circuit components under the red shaded area in Figure 1c show the electrical resonant circuit (RLC). Applying Kirtchoff's voltage law we get:

$$L \frac{d^2 Q}{dt^2} + R \frac{dQ}{dt} + \frac{1}{C} Q = v_{AC1}$$

(1)

where R is the total resistance in the circuit comprising of load resistance R_L and internal resistances of connectors and components used, C is the total capacitance, L is the total inductance, and Q is the charge in the circuit. By solving Equation 1, the steady state voltage at node D in Figure 1c, v_D , can be obtained using Equation 2:

$$v_D \approx \frac{v_{AC1}}{\sqrt{(2\pi f_1 RC)^2 + ((2\pi f_1)^2 LC - 1)^2}} \quad (2)$$

Similarly, the voltage at node B in Figure 1c is $V_B = V_{DC} + v_{AC2}$. The output voltage at node S (sense electrode) in Figure 1c is small, hence, can be neglected in the subsequent analysis. Now, the electrostatic force on the nanobeam acting in the transverse direction x (in-plane) is as follows:

$$F = \frac{\partial E}{\partial x} = \frac{1}{2} \left((v_D - V_B)^2 \frac{\partial C_{DB}}{\partial x} + (V_B)^2 \frac{\partial C_{SB}}{\partial x} \right) = \frac{1}{2} \left((v_D - V_{DC} - v_{AC2})^2 \frac{\partial C_{DB}}{\partial x} + (V_{DC} + v_{AC2})^2 \frac{\partial C_{SB}}{\partial x} \right) \quad (3)$$

where C_{DB} (C_{SB}) is the variable capacitor formed by the drive (sense) electrode and the nanobeam, as shown in Figure 1c, and $\partial_n C_{DB} / \partial x_n = (-1)_n (\partial_n C_{SB} / \partial x_n)$. The first term is the electrostatic attraction force between the nanobeam and the drive electrode while the second term represents the force between the nanobeam and the sense electrode. Now, expanding different terms in Equation 3, the applied force becomes:

$$F = \frac{1}{2} V_{DC}^2 \left(\frac{\partial C_{DB}}{\partial x} + \frac{\partial C_{SB}}{\partial x} \right) + \frac{1}{2} v_D^2 \left(\frac{\partial C_{DB}}{\partial x} \right) + \frac{1}{2} v_{AC2}^2 \left(\frac{\partial C_{DB}}{\partial x} + \frac{\partial C_{SB}}{\partial x} \right) + V_{DC} v_{AC2} \left(\frac{\partial C_{DB}}{\partial x} + \frac{\partial C_{SB}}{\partial x} \right) - V_{DC} v_D \frac{\partial C_{DB}}{\partial x} - v_D v_{AC2} \frac{\partial C_{DB}}{\partial x} \quad (4)$$

The last term in Equation 4 represents the mixing term of interest, which contains the mixing forces provided in Equation 5 below

$$F_{mix} = -\frac{1}{2}V_D V_{AC2} \frac{\partial C_{DB}}{\partial x} \cos[2\pi(f_1 + f_2)]t - \frac{1}{2}V_D V_{AC2} \frac{\partial C_{DB}}{\partial x} \cos[2\pi(f_1 - f_2)]t$$

(5)

By the proper choice of f_1 and f_2 , a time varying force around the mechanical resonance frequency of the nanobeam ($f_m = f_1 \pm f_2$) can be generated, which will eventually excite the device at resonance. In all of our experiments, we utilized the $f_m = (f_1 + f_2)$ term to excite the system at resonance. Finally, the output voltage signal across the load resistor, R_L , will have a frequency around $f_m = (f_1 + f_2)$ with an amplitude given by [16]

$$V_o \propto V_D \cdot V_{AC2} \cdot V_{DC}$$

(6)

Note that the amplitude of the mixing force, F_{mix} in Equation 5, can be increased by increasing V_D and V_{AC2} , which increases the amplitude of motion of the nanobeam around resonance. In fact, when f_1 equals the electrical resonance frequency of the LC circuit f_{LC} , the voltage at node D is increased by Q_{LC} times, that is, $v_D \approx Q_{LC} \times v_{AC1}$. This produces amplified forcing on the nanobeam that results into amplified output voltage across R_L according to Equation 6. It is evident from Equation 5 and 6 that the mixing force around resonance and thus the output voltage can be amplified by introducing appropriate LC tank circuits independently from the drive side and from the nanobeam side, which will be shown experimentally in the following sections. If two LC tank circuits are formed from the drive and nanobeam sides with proper resonance frequencies, an overall effective force amplification of $Q_{LC1} \times Q_{LC2}$ can be expected if the force mixing technique is used to activate the device. However, if we only rely on the 5th term in Equation 4 for actuating the device, direct excitation using $V_{DC} \times V_{AC}$, at least force amplification of Q_{LC} times can be achieved by carefully tuning the electrical resonance frequency to match the beam natural frequency, $f_{LC} = f_m$. The significance of this technique can be well understood from the following example. For $V_{AC} = 1$ V, and $V_{DC} = 20$ V, a forcing amplitude of $V_{AC} \times V_{DC} = 20$ V² can be achieved. If an LC tank circuit is used with f_{LC}

$= f_m$, and $Q_{LC} = 20$, the effective forcing then becomes $Q_{LC} \times V_{AC} \times V_{DC} = 400 \text{ V}_2$. Without any amplification of the AC amplitude, a DC source voltage of 400 V will be necessary to achieve similar (400 V_2) forcing amplitude. This is not only impractical from the available resources point of view but also this large DC voltage can potentially destroy the device by static pull-in. The availability of this large forces can open the door for thorough investigation of higher order modes and nonlinear phenomena, such as, internal, parametric, and secondary resonances in highly stiff nanoscale devices, which are otherwise not accessible due to the voltage limitations in standard electronic characterization tools. [6] Next, we show several case studies to demonstrate the effectiveness and versatility of the proposed passive force amplification technique.

Figure 1: (a) An SEM image of the fabricated device. (b) Schematic of the experimental setup. (c) Simplified equivalent electrical circuit representation of the system, and (d) Linear response of the nanoresonator.

2.1 Case study 1

Figure 2a shows the frequency response of the electrical circuit, red shaded area in Figure 1c, with and without connecting the inductor L . Based on the experimental results, the circuit parameters are estimated as follows: $f_{LC} \approx 367$ kHz, $Q_{LC} \approx 19$, $R \approx 2.02$ k Ω , $L \approx 16.65$ mH, and $C \approx 11.3$ pF. Here, R represents the overall resistance in the resonant tank (LC) circuit comprising of the network analyzer source resistance ($R_s = 50\Omega$), load resistance ($R_L = 50\Omega$), and the internal resistance of the inductor (~ 1.92 k Ω). For the experimental setup in Figure 1b, we fix the frequency of the first voltage source, V_{AC1} , at $f_1 = f_{LC} = 367$ kHz with an input power of -5 dBm. However, according to Equation 2, the effective RF power at node D is ~ 20.6 dBm due to the electrical resonance at 367 kHz. The second voltage source, V_{AC2} , is swept, $f_2 = 19.634 - 19.759$ MHz, with an RF power of 23 dBm. This produces the mixing force around frequencies, $f_{mix} = (f_1 + f_2) = 20.001 - 20.126$ MHz, which is around the

mechanical resonance frequency ($f_m=20.048$ MHz) of the device. The DC voltage is fixed at 50 V. The output signal is monitored at the intermediate frequency (IF) range between (20.001 – 20.126 MHz) by the spectrum analyzer in MAX-HOLD mode that was connected to the sense electrode without any LNA. We use the resolution bandwidth (RBW) of 30 Hz and sweep time of 3.5 ks for each sweep to obtain sufficiently smooth spectra. Figure 2b shows the measured output signal in dBm for the case of activating electrical resonance (ER) and for the case where the inductor L is bypassed to deactivate the ER. From the measured spectra, it is clearly visible that at ER the forcing is strong enough (due to the amplified voltage at node D) to actuate the nanobeam nonlinearly, evident by a sudden jump at 20.081 MHz in the blue color curve. However, when the ER is deactivated the forcing is not strong enough to actuate the nanobeam, hence, the output signal shows no visible peak, as shown by the red curve. We even increase the amplitude of the input signal to 10 dBm (maximum power of the NA), however, the measured response shown in black does not reveal any noticeable peak.

In a different experiment, we fix the frequency of the second voltage source, V_{AC2} , at $f_2=19.682$ MHz with an RF power of 23 dBm, and sweep the first voltage source, V_{AC1} , between $f_1 = 292 - 442$ kHz with an input power of -5 dBm. The DC bias voltage is fixed at 50V. As opposed to the previous experiment where the frequency $f_1=367$ kHz, hence, the gain at node D is fixed at ~ 25 dB as shown in blue in Figure 2c, the gain at node D is varying according to Equation 2, plotted in red in Figure 2c. This produces a variable force between $f_{\text{mix}} = (f_1 + f_2) = 19.974 - 20.124$ MHz, with a maximum around 20.049 MHz. The recorded response using the spectrum analyzer is plotted in blue in Figure 2d, which also includes the result obtained with the fixed gain case, plotted in red for comparison. Expectedly, the response shows nonlinear jump around 20.068 MHz (blue) due to the reduced forcing at higher frequencies as opposed to a nonlinear response up to 20.081 MHz (red) for a fixed amplified force configuration. The reported results show a lower bandwidth in the case of a variable gain

drive, and for a linearly excited system, this will lead to increase in the quality factor, which can increase the sensitivity of a nanoresonator based sensor system. The variable gain drive system, as demonstrated here, can be used to minimize the hysteric (nonlinear) frequency band while maintaining maximum output signal similar to a fixed gain drive system.

One note worth mentioning here regarding the placement of the inductor L in the tank circuit arrangement and its implications on the overall performance of the system. The force amplification mechanism will work similarly if L is connected in series between the sense port and the load resistance instead of connecting it at the drive electrode without changing the position for C in the circuit. For this case the amplified force will be acting on the capacitor C_{SB} instead of C_{DB} , which will actuate the nanobeam with high forcing amplitude and is expected to produce similar output signal as the case for L connected at the drive port.

Figure 2: (a) Response of the *RLC* circuit and the corresponding voltage amplification. (b) Response of the device under activated and deactivated ER conditions. (c) Variation in the voltage gain at node *D* for the reported experimental case of (d), and (d). Response of the device under variable and fixed voltage gain at node *D*.

2.2. Case Study 2

Next, we introduce an extra independent port on the same structure to demonstrate the passive force amplification. Combination of force amplification from these two ports can lead to an order of magnitude increase in the overall forcing compared to the single port amplification case. For example, if an amplification factor of 20 (26 dB) is achieved from each port, the total force amplification then become $20 \times 20 = 400$ (52 dB). To experimentally demonstrate the activation of the second port, we place the inductor, *L*, in between the bias-tee and the nanobeam port *B* (Figure 1c) and connect the external capacitor of 10 pF between the nanobeam and the sense electrode, as shown in Figure 3a. For this experiment, we use the signal generator and the network analyzer as the first (V_{AC1}) and the second (V_{AC2}) voltage

sources, respectively. Note that the sinusoidal voltage at node B will be amplified due to ER activation effect and amplify the mixing force similarly according to the last term in Equation 5. Figure 3b shows the measured frequency responses of the LC tank circuit with and without connecting the inductor, L . Subsequently, the circuit parameters are estimated as follows: $f_{LC} \approx 306$ kHz, $Q_{LC} \approx 17$, $R \approx 2.99$ k Ω , $L \approx 26.5$ mH, and $C \approx 10.2$ pF. Here, we fix the frequency of the second voltage source, V_{AC2} , at $f_2 = 306$ kHz with an input power of -5 dBm, while the frequency of the first voltage source, V_{AC1} , is swept, $f_1 = 19.695$ – 19.820 MHz, with an RF power of 16 dBm. This produces the mixing force around frequencies, $f_{\text{mix}} = (f_1 + f_2) = 20.001$ – 20.126 MHz, which is around the resonance frequency of the device (20.048 MHz). Similar to the case study 1, the output is monitored in the intermediate frequency (IF) range between (20.001–20.126 MHz) by the spectrum analyzer. The DC voltage is fixed at 50 V. Figure 3c shows the measured motional signal in dBm for the case of activating ER and for the case where the inductor L is bypassed. From the measured spectra, it is clearly visible that at ER the forcing is strong enough due to the amplified voltage at node B , which drives the nanoresonator into the nonlinear regime, clearly visible with a sudden jump around 20.068 MHz, shown in blue curve. However, under the deactivated ER, no visible peak is observed either for a -5 dBm input power (in red) or even for an increased input power of 10 dBm (in black).

Next, we further examine the two different configurations shown in the previous case studies. Figure 3d shows the responses for the inductor L connected at node D and node B , shown in red and blue, respectively. We use $V_{DC} = 50$ V, $V_{AC1} = -5$ dBm, $V_{AC2} = 23$ dBm, and $V_{AC1} = 23$ dBm, $V_{AC2} = -5$ dBm, for the results shown in red and blue, respectively. Due to $f_1 = f_{LC} = 367$ kHz and $f_2 = f_{LC} = 306$ kHz, respectively, for red and blue curves, the amplitude for the mixed forcing term is similar for both the cases. However, the response in blue shows an extended nonlinear response compared to the red, which is an indication that the nanobeam is driven strongly by placing the inductor L at node B compared to the case where it is placed at

node D . Note that the excitation force is composed of a direct driving force, $V_{AC1} \times V_{DC}$, acting on the nanobeam due to V_{AC1} at the driving electrode plus the combination force, $V_{AC1} \times V_{AC2}$, which is being swept around the natural frequency (20.048 MHz). The direct driving force, $V_{AC2} \times V_{DC}$ due to V_{AC2} is being cancelled due to the force symmetry. For the case of inductor L at node D , the direct driving force is at a fixed frequency of $f_1 = f_{LC} = 367$ kHz with an effective power of 20 dBm (due to the ER effect). Since this forcing frequency is far away from the resonance frequency of the nanobeam, it does not contribute to the nanobeam vibration. In contrast, for the case of inductor L at node B , the direct driving force acting on the nanobeam from the drive electrode is swept around 19.695 – 19.820 MHz with an RF power of 23 dBm, which is much closer to the resonance frequency of the nanobeam. With the combination of this direct driving force with the mixing force around resonance, an increase in the nanobeam response can be expected, which is evident from the blue color curve shown in Figure 3d at which the nonlinear response extended beyond the measurement window.

(a)

(b)

Figure 3: (a) Experimental setup used for the second set of experiments. (b) Response of the *RLC* circuit and corresponding voltage amplification. (c) Response of the device under activated and deactivated ER conditions, and (d) Response of the device under similar forcing conditions with different places for the inductor *L*.

4. Analytical model

The dynamic response of electrostatically actuated nanobeam can be modeled as a single degree of freedom system as follows:[38]

$$m_{eff} \ddot{x}(t) + c\dot{x}(t) + k_{eff}x(t) + \alpha_c x^3(t) = F(t)$$

(7)

where c is the damping coefficient, k_{eff} is the effective stiffness, and $F(t)$ is the total electrostatic force acting on the nanobeam defined in Equation 3. The cubic nonlinearity coefficient α_c is extracted by solving the beam equation for different values of α_c until we

match the simulation jump down frequency with the experimental one depicted in Figure

2b.[39] The effective mass is defined as $m_{eff} = \lambda m$, where m is the total beam mass calculated from the nanobeam dimensions and the tabulated material properties and $\lambda=0.3959$ for clamped-clamped nanobeam.[40] For convenience we divide Equation 7 by the effective mass. Hence, the following equation is obtained

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2x(t) + \frac{\alpha_c}{m_{eff}}x^3(t) = F(t)/m_{eff} \quad (8)$$

where $c/m_{eff} = 2\zeta\omega_n$ such that $\zeta = 1/2Q_m$ is the damping ratio calculated based on the experimentally extracted quality factor Q_m shown in Figure 2a. To find the natural frequency $\omega_n = \sqrt{k_{eff}/m_{eff}}$, we derived the eigenvalue problem of Equation 8 under static voltage V_{DC} as follows:[38]

$$f = \frac{1}{2\pi} \sqrt{\frac{\epsilon l t V_{DC}^2}{m_{eff}} \left(\frac{1}{(g-x)^3} - \frac{1}{(g+x)^3} \right) - \omega_n^2} \quad (9)$$

where l is the beam length, t is beam thickness, g is the gap, and ϵ is air permittivity. At $V_{DC} = 50$ V, we experimentally found the resonance frequency f to be 20.048 MHz, hence, the unforced natural frequency ω_n can be calculated by plugging the parameters values in Equation 9. Table 1 shows a summary of the parameters used in the simulation. The simulated dynamic response is based on the long time integration of Equation 8 until reaching the steady state.

Figure 4a shows the simulated response using the simple lumped parameter model for the experimental condition used to generate Figure 2d. The results show qualitative agreement

with experimental results showing that the fixed gain drive is stronger than the variable gain drive. Also, a qualitative agreement is seen between the simulated results shown in Figure 4b and Figure 3d.

Table 1. The designed and experimentally extracted parameters of the nanobeam.

<i>Parameters</i>	<i>Values</i>
Length (l)	14.7 [μm]
Width (w)	680 [nm]
Thickness (t)	1.5 [μm]
Gap (g)	360 [nm]
Natural frequency f	20.97 [MHz]
Density (ρ)	2330 [kg/m ³]
Quality factor (Q_m)	2500
Damping ratio (ζ)	0.0002
Cubic nonlinearity α_c	655 $\times 10^{12}$ [N/m ³]

(a)

Figure 4. (a) Simulation results of the device response under variable and fixed voltage gain at node *D*. (b) Simulated response of the device under similar forcing condition with different places for the inductor *L*.

5. Conclusion

In conclusion, we have proposed and demonstrated an *LC* tank resonant circuit based technique to boost the actuation force up to ~ 25 dB to efficiently excite NEMS resonators with high forcing amplitude without using any active electronic components. Through the case studies, we have shown the effectiveness, simplicity, and flexibility of this technique to induce force amplification from different ports of a nanodevice. With the use of controllable on-chip inductors and capacitors to form the *LC* tank circuits, an unprecedented level of forcing amplitude can be achieved, which can give access to otherwise inaccessible higher order modes of highly stiff nanoresonators. This can also facilitate the exploration of nonlinear dynamics, such as, internal, secondary, and parametric resonances in NEMS devices that usually demand high forcing. Also, this can support the continuous miniaturization effort to increase the mechanical resonance frequency of the devices without stringent capacitive gap requirement for efficient transduction. Though our results were demonstrated using a silicon nanobeam, the same technique can be easily extended to actuate and probe other interesting resonators based on 2D materials and carbon nanotubes. This may reduce the

overall fabrication cost, for example, by lowering the required lithography resolution while defining the capacitive gap for nanoscale devices. Note that although we have shown this method on electrostatically transduced devices, we believe this technique can be tailored for other actuation and detection mechanisms, such as, piezoelectric, which warrants further research.

Acknowledgements

This research has been supported through KAUST research fund.

References

- [1] K. Ekinici, X. Huang, M. Roukes, *Applied Physics Letters*, **2004**, 84, 4469.
- [2] B. Ilic, Y. Yang, H. G. Craighead, *Applied Physics Letters*, **2004**, 85, 2604.
- [3] J. A. Sidles, J. L. Garbini, K. J. Bruland, D. Rugar, O. Züger, S. Hoen, C. S. Yannoni, *Reviews of Modern Physics* **1995**, 67, 249.
- [4] [1] D. Rugar, R. Budakian, H. J. Mamin, B. W. Chui, *Nature*, **2004**, 430, 329.
- [5] F. D. Bannon, J. R. Clark, C. T. C. Nguyen, *IEEE Journal of Solid-State Circuits*, **2000**, 35, 512.
- [6] W. Ark-Chew, C. T. C. Nguyen, *Journal of Microelectromechanical Systems*, **2004**, 13, 100-112.
- [7] C. T. C. Nguyen, *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, **2007**, 54, 251-270.
- [8] E. K. Irish, K. Schwab, *Physical Review B*, **2003**, 68, 155311.
- [9] A. Cleland, M. Roukes, *Journal of Applied Physics*, **2002**, 92, 2758.
- [10] R. L. Badzey, G. Zolfagharkhani, A. Gaidarzhy, P. Mohanty, *Applied Physics Letters*, **2004**, 85, 3587.
- [11] I. Mahboob, H. Yamaguchi, *Nature Nanotechnology*, **2008**, 3, 275.
- [12] D. N. Guerra, A. R. Bulsara, W. L. Ditto, S. Sinha, K. Murali, P. Mohanty, *Nano Letters*, **2010**, 10, 1168.
- [13] J.-S. Wenzler, T. Dunn, T. Toffoli, P. Mohanty, *Nano Letters*, **2014**, 14, 89.
- [14] I. Mahboob, E. Flurin, K. Nishiguchi, A. Fujiwara, H. Yamaguchi, *Nature communications*, **2011**, 2, 198.
- [15] M. A. A. Hafiz, L. Kosuru, M. I. Younis, *Nature Communications*, **2016**, 7, 11137.
- [16] A. Uranga, J. Verd, J. L. Lopez, J. Teva, G. Abadal, F. Torres, J. Esteve, F. Perez-Murano, N. Barniol, *Electronics Letters*, **2007**, 43, 452.
- [17] J. L. Lopez, J. Verd, J. Teva, G. Murillo, J. Giner, F. Torres, A. Uranga, G. Abadal, N. Barniol, *Journal of Micromechanics and Microengineering*, **2009**, 19, 015002.
- [18] C. Wen-Chien, F. Weileun, L. Sheng-Shian, *Journal of Micromechanics and Microengineering*, **2011**, 21, 065012.

- [19] A. Uranga, J. Verd, N. Barniol, *Microelectronic Engineering*, **2015**, 132, 58.
- [20] D. Antonio, D. H. Zanette, D. López, *Nature Communications*, **2012**, 3, 806.
- [21] J. V. Warner, J. C. Matthew, R. E. Stephen, *Nanotechnology*, **2014**, 25, 425501.
- [22] P. A. Truitt, J. B. Hertzberg, C. C. Huang, K. L. Ekinici, K. C. Schwab, *Nano Letters*, **2007**, 7, 120.
- [23] Z. Wang, Z. Liu, L. Chen, Y. Yang, J. Ma, X. Zhang, Y. Guo, G. Zhang, D. Zhang, *Advanced Electronic Materials*, **2018**, 4, 1800025.
- [24] S. Lai, P. Cosseddu, L. Basiricò, A. Ciavatti, B. Fraboni, A. Bonfiglio, *Advanced Electronic Materials*, **2017**, 3, 1600409.
- [25] F. M. Weiss, T. Töpper, B. Osmani, S. Peters, G. Kovacs, B. Müller, *Advanced Electronic Materials*, **2016**, 2, 1500476.
- [26] K. Fukuda, T. Minamiki, T. Minami, M. Watanabe, T. Fukuda, D. Kumaki, S. Tokito, *Advanced Electronic Materials*, **2015**, 1, 1400052.
- [27] T. Bagci, A. Simonsen, S. Schmid, L. G. Villanueva, E. Zeuthen, J. Appel, J. M. Taylor, A. Sørensen, K. Usami, A. Schliesser, E. S. Polzik, *Nature*, **2014**, 507, 81.
- [28] M. A. Sillanpää, J. Sarkar, J. Sulkko, J. Muhonen, P. J. Hakonen, *Applied Physics Letters*, **2009**, 95, 011909.
- [29] J. Sulkko, M. A. Sillanpää, P. Häkkinen, L. Lechner, M. Helle, A. Fefferman, J. Parpia, P. J. Hakonen, *Nano Letters*, **2010**, 10, 4884.
- [30] F. Massel, T. T. Heikkilä, J. M. Pirkkalainen, S. U. Cho, H. Saloniemi, P. J. Hakonen, M. A. Sillanpää, *Nature*, **2011**, 480, 351.
- [31] R. J. Schoelkopf, P. Wahlgren, A. A. Kozhevnikov, P. Delsing, D. E. Prober, *Science*, **1998**, 280, 1238.
- [32] M. A. Sillanpää, L. Roschier, P. J. Hakonen, *Physical Review Letters*, **2004**, 93, 066805.
- [33] N. Ares, T. Pei, A. Mavalankar, M. Mergenthaler, J. H. Warner, G. A. D. Briggs, E. A. Laird, *Physical Review Letters*, **2016**, 117, 170801.
- [34] M. H. Hasan, F. M. Alsaleem, N. Jaber, M. A. A. Hafiz, M. I. Younis, *AIP Advances*, **2018**, 8, 015312.
- [35] S. Meister, M. Mecklenburg, V. Gramich, J. Stockburger, J. Ankerhold, B. Kubala, *Physical Review B*, **2015**, 92, 174532.
- [36] J. Ma, J. Tang, *Nonlinear Dynamics*, **2017**, 89, 1569.
- [37] S. N. R. Kazmi, M. A. A. Hafiz, K. N. Chappanda, S. Ilyas, J. Holguin, P. M. F. J. Costa, M. I. Younis, *Nanoscale*, **2017**, 9, 3449.
- [38] Younis, M. I. *MEMS Linear and Nonlinear Statics and Dynamics* (Springer US, 2011).
- [39] D. K. Agrawal, J. Woodhouse, A. A. Seshia, *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, **2013**, 60, 1646.
- [40] B. D. Hauer, C. Doolin, K. S. D. Beach, J. P. Davis, *Annals of Physics*, **2013**, 339, 181.