ADAPTIVE METHOD FOR MRI ENHANCEMENT USING SQUARED EIGENFUNCTIONS OF THE SCHRÖDINGER OPERATOR

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ABSTRACT

Recently, a Magnetic Resonance image denoising method, based on squared eigenfunctions of the Schrödinger operator, has been presented. However, its performance depends on the choice of a filtering parameter called h . We propose an adaptive selection of the filtering parameter by a grid segmentation of the noisy input image. The latter will follow an appropriate distribution along the different sub-images allowing the adaptation of its value to the spatial variation of noise and responded efficiently to the denoising objectives. Numerical tests using a synthetic dataset from BrainWeb and real MR images show the effectiveness of the proposed approach compared to the standard case with one fixed parameter.

Index Terms— Magnetic Resonance Imaging (MRI), adaptive image denoising, Semi-Classical Signal Analysis (SCSA), eigenfunctions of the Schrödinger operator.

1. INTRODUCTION

Magnetic Resonance Imaging (MRI) signal enhancement is one of the most active medical imaging research fields. This is mainly due to the growing challenges related to the newest evolution of MR technologies seeking the reduction of the acquisition time and the enhancement of the resolution of the image for potential clinical use. However, high-speed MRI image acquisition techniques suffer from the high level of noise. Many research works have been conducted treating MR image denoising from diverse perspectives which have been classified in different denoising categories [1], for instance Non-Local Mean (NLM) based method [2] [3] [4], wavelet based method [5] [6] and the statistical approaches [7]. However, most of the state-of-art MRI denoising methods need insights about noise, by estimating noise distribution...etc., to be able to deal with the noise efficiently.

Recently, the so-called Semi-Classical Signal Analysis (SCSA) [8] presents a new concept of image denoising based on the Schrödinger operator. It shows a powerful ability to enhance image quality and specially preserve image details. In the SCSA method, the image is decomposed through image

dependent functions; these functions are the L_2 normalized squared eigenfunctions associated with the discrete spectrum of a Schrödinger operator, where its potential is considered to be the image. These functions exhibit interesting localization properties and provide new parameters that can be used to extract relevant features of image variations. They can be an efficient analysis tool as the information about the image is reflected on these localized image dependent functions. This method has been successfully applied for MR image denoising [9] where a soft threshold, indicated by an appropriate choice of a parameter denoted h , is used. A previous paper that applies the method to Magnetic Resonance Spectroscopy (MRS) denoising [10] through optimizing an appropriately weighted cost function that consists of a fidelity term and a smoothing term. The choice of this parameter is related to a compromise between accuracy and noise removal. Indeed, with larger h values, more noise is removed but with more image blurring due to over-smoothing the image details. However, the application of this optimization to image denoising will lead to a single value for this parameter which would give good results when the noise is uniform along the whole image. Nevertheless, it is known that the noise in MR image is not uniform, for most SENSE-based and GRAPPA-based reconstructions [11], and a single value for h may not be enough to reach desired denoising performance. In this paper, we propose an adaptive method based on grid segmentation of the image to improve the selection of the parameter h . The idea is that h follows an appropriate grid distribution that adapts its values to the noise level for each sub-image. The performance of the proposed strategy on synthetic and real MR image dataset is studied using standard performance criteria. The approach is compared to the SCSA Method with fixed h .

This paper is organized as follows: Section 2 gives a brief introduction to the SCSA method and the proposed method. Section 3 presents the results of the proposed method using synthetic and real MRI images. A general conclusion is given in the section 4.

2. MATERIAL AND METHODS

2.1. Standard SCSA-fixed h denoising method

The new image denoising, so-called SCSA, decomposes the image using an appropriate number of squared eigenfunctions of the Schrödinger operator whose potential is given by the image. The number of eigenfunctions depends on a design parameter h that has to be fixed to reach desired denoising performance while conserving important information. This method uses a fixed values for h , with the semi-classical Schrödinger operator $\mathcal{H}_{2,h}(I)$ whose potential is the noisy image I and which depends on the parameter h as follows:

$$\mathcal{H}_{2,h}(I) = -h^2 \Delta - I \quad \text{with} \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad (1)$$

The denoised image $I_{h,\gamma}$ is defined such as:

$$I_{h,\gamma}(x, y) = \left(\frac{h^2}{L_{2,\gamma}^{cl}} \sum_{k=1}^{K_{h,\gamma}} (-\mu_{k,h,\gamma})^\gamma \psi_{k,h,\gamma}^2(x, y) \right)^{\frac{1}{1+\gamma}} \quad (2)$$

where $L_{2,\gamma}^{cl} = \frac{1}{2^2\pi} \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+2)}$, $I_{h,\gamma}(x, y)$ is the pixel (x, y) in the denoised image $I_{h,\gamma}$. Γ refers to the standard Gamma function. Moreover, $\mu_{k,h,\gamma}$ and $\psi_{k,h,\gamma}$ denote the negative eigenvalues, $\mu_{1,h,\gamma} < \dots < \mu_{K_{h,\gamma},h} < 0$, and associated L^2 -normalized eigen functions of the operator $\mathcal{H}_{2,h}(I)$, $K_{h,\gamma}$ is a finite number of negative eigenvalues and $\gamma = 4$ is a positive constant such that:

$$\mathcal{H}_{2,h}(I) \psi_{k,h,\gamma} = \mu_{k,h,\gamma} \psi_{k,h,\gamma}, \quad k = 1, \dots, K_{h,\gamma} \quad (3)$$

An efficient numerical implementation of this algorithm has been proposed for images reconstruction in [8]. The idea consists in splitting the 2D Schrödinger operator into two one dimensional operators [12] and in solving the eigenvalues problem row by row and column by column. The squared eigenfunctions of these one dimensional operators are then combined using a tensor product approach. It is known from [8] that as k increase as the eigenfunctions presents more oscillation. Therefore represent the noise. So for denoising, our aim is to remove the highest order eigenfunctions but tuning the parameter h . Indeed, that as h decrease as the number of eigenfunctions increases hence the oscillations increase.

2.2. The proposed adaptive method

In MRI images, noise has a non-uniform distribution [11]. As a consequence, the standard SCSA denoising technique, will be unsufficient in reducing noise, since it uses a constant h value throughout the image. To address this problem, we propose a modified SCSA method. where the parameter h is spatially adapted to noise. Therefore, the parameter h takes the form of a set of distributed values $h(i, j)$. Where each

Fig. 1. Image square grid of size $N_s \times N_s$

value of $h(i, j)$, applied to a specific sub-image of coordinates (i, j) , depends on the local noise level. The proposed method, so-called SCSA with Distributed h (SCSA-Dh), is mainly based on three concepts:

- **Image grid segmentation** where the image is subdivided into equal sub-images or blocks in a grid form of size $N_s \times N_s$ blocks, where N_s is the number of blocks per row or column.
- **Square Padding:** where each block will be surrounded by additional pixels of size N_p to avoid the visual grid effect. For the border blocks, the extra pixels are the replicates.
- **Adaptive distribution** : where each block will be denoised using an optimal $h(i, j)$ value that ensures the best possible denoising in this location.

Hence the adopted strategy consists of two steps: First, the noisy image is divided into noisy sub-images I_{ij} . Secondly, the squared eigenfunctions of the Schrödinger operator $\mathcal{H}_{2,h(i,j)}(I_{ij})$ are used for denoising each sub-image $I_{h(i,j)}$ with the corresponding $h(i, j)$ value (see Fig. 1).

3. RESULTS AND DISCUSSION

3.1. Experiemental MRI data

The synthetic dataset simulated is noisy normal brain dataset from `Brainweb.bic.mni.mcgill.ca` where the following characteristics have been chosen: [modality=T2, slice thickness = 3mm, intensity non-uniformity= 20%, noise= 5%]. The noiseless image has the same characteristics but noise= 0%. For the real MRI data, the experiments

are performed on one healthy male subject, on a 3T scanner (MAGNETOM Tim-Trio, Siemens Healthcare) equipped with a 32-channel head coil for signal reception. Turbo Flash sequence is used with the following parameters: TR/TE = 250/2.46 ms; matrix size, 256x256 resolution and 33 contiguous slices; FOV: 220mm; voxel size: 0.9x0.9x3 mm³; flip angle:10; and receiver bandwidth set to 320 Hz/pixel.

3.2. Image evaluation metrics

For simulated data, the image denoising performance has been evaluated using three standard metrics: the Structural SIMilarity index (SSIM) [13] which evaluates the preservation of image details, the Global Phase Coherence (GPC) to measure the sharpness of the image [14] and the Peak Signal to Noise Ratio (PSNR) to estimate the amount of removed noise:

$$PSNR = 10 \log \left(\frac{Max(I_{ref})}{\frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N (I_{ref}(i,j) - I_{h,\gamma}(i,j))^2} \right) \quad (4)$$

For the synthetic dataset, a set of tests has been performed using various grid sizes to evaluate the performance of the proposed SCSA-D h approach. The optimal h distribution and grid size , that ensure the highest $PSNR$ such that $0.1 < h(i,j) < 2.5$, have been selected to compare the proposed method with the optimal standard SCSA using the optimal h as well. For the real data, the selection is driven by maximizing $SSIM$ and SI . Moreover, the results have been evaluated visually to ensures small detail preservation.

Figure 2 and figure 3 display the comparison between the denoised images with fixed and distributed h for synthetic and real MRI data respectively. The zoomed areas illustrate the preservation of the details for each method, where a noticeable improvement can be observed using the SCSA-D h method. The proposed method shows that the adaptive SCSA can acheive better results due to its ability to access locally the parts of the image with different noise levels. The proposed method will be compared to other methods to check its potential comparing the existing adaptive denoising methods.

4. CONCLUSION

A new adaptive method for MR image enhancement has been proposed. It uses the squared eigenfunctions of the Schrödinger operator which potential is the noisy image. The preliminary results provides very encouraging insight and show good potential in MRI noise removal while preserving the small details. The experimental results demonstrate that the proposed method with distributed h , improves significantly the performance of the SCSA-fixed h method ; it

Fig. 2. (a.1) Original simulated MRI image, (b.1) noisy simulated MRI noisy image, Denoised image using (c.1) Standard SCSA with fixed h , (d.1) SCSA-D h . The zoomed area of (a.2) Original image, (b.2) Noisy image, Denoised image using (c.2) Standard SCSA with fixed h , (d.2) SCSA-D h .

achieved higher sharpness (SI) while preserving structural similarity (SSIM) and enhanced (PSNR). As future work, further improvement will studying the optimal selection of the sub-images size and adapting the distribution of h to more general case for instance where the spatial variation of noise has a Rican distribution.

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Fig. 3. (a.1) Real MRI noisy image, Denoised image using (b.1) Standard SCSA with fixed h , (c.1) SCSA-Dh. The zoomed area of (a.2) Noisy image, Denoised image using (b.2) Standard SCSA with fixed h , (c.2) SCSA-Dh.

6. REFERENCES

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