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Wave-equation Traveltime Inversion with Multi-Frequency Bands: Synthetic and Land Data Examples

Han Yu, Sherif M. Hanafy, and Gerard T. Schuster

ABSTRACT

A wave-equation traveltime inversion method with multi-frequency bands is proposed to invert for the shallow or intermediate subsurface velocity distribution. Similar to the classical wave equation traveltime inversion, this method searches for the velocity model that minimizes the squared sum of the traveltime residuals using source wavelets with progressively higher peak frequencies. Wave-equation traveltime inversion can partially avoid the cycle skipping problem by recovering the low-wavenumber parts of the velocity model. However, we also utilize the frequency information hidden in the traveltimes for obtaining a more highly resolved tomogram. Therefore, we employ different frequency bands when calculating the Fréchet derivatives so that tomograms with better resolution can be reconstructed. Results are validated by the zero offset gathers from the raw data associated with moderate geometrical irregularities. The improved wave-equation traveltime method is robust and merely needs a rough estimate of the starting model. Numerical tests on both the synthetic and field data sets validate the above claims.

Keywords – Frequency; Wave equation; Traveltime inversion; Two-way traveltime

INTRODUCTION

Traditional inversion (Sun et al., 2017; Zhang and Toksöz, 1998) based on ray tracing is stable and converges quickly, but it sometimes fails due to the high-frequency assumption that is contradicted by the low-frequency data. Moreover, it cannot provide velocity models with reasonable resolution (Liu and Zhang, 2017; Zhou* et al., 2014), because the waveform information in the data is not utilized. As an alternative, conventional full waveform inversion (FWI) finds the velocity model that minimizes the L_2 norm of the waveform differences between the predicted and observed traces (Tarantola, 2005; Virieux and Operto, 2009). However, the iterative solution is easily trapped in a local minimum due to the non-linearity between the misfit function and the model's velocity variations. FWI succeeds in inverting some data sets (Zhu and Fomel, 2016; Dickinson et al., 2017; Xue et al., 2017), but significant efforts are required to overcome the cycle-skipping problems. Prior to the implementation of FWI tests, the data must be processed to eliminate the effects of unmodeled physics and coherent noise. Even with sufficient processing techniques, the success of FWI largely relies on the quality of the starting velocity model.

To mitigate the non-linearity in FWI, wave-equation traveltime inversion (WT) (Luo and Schuster, 1991a,b) was proposed to robustly estimate the low-wavenumber components of the background velocity model. By inverting the traveltimes instead of the full waveforms, it largely mitigates the cycle-skipping problem (Ma and Hale, 2013) and provides a robust convergence property compared to traditional FWI (Zhang et al., 2015; Van Leeuwen and Mulder, 2010). With long offset in the data acquisition, WT can reliably invert for the deeper parts of the earth's velocity model, which provides a good initial model for FWI. In general, WT bridges the gap (Zhou et al., 1995, 1997; Feng and Schuster, 2017) between the traveltime inversion and FWI.

A problem with the current implementation of WT is that it does not fully utilize the entire frequency band of information in the arrivals. For example, classical WT simultaneously invert-

s for both the high- and low-wavenumber parts of the velocity from each arrival. In contrast the multiscale approach of wave equation inversion first inverts for the low-wavenumber portion of the velocity model and then inverts for the high-wavenumber portion. This multiscale approach preserves the low-wavenumber character of the velocity while incorporating the more detailed velocity variations.

In a similar way we now propose a multiscale WT method where the low-wavenumber parts of the velocity model are first inverted. This is accomplished by using a low-pass wavelet for the computation of the predicted traveltimes. As the iterations proceed the higher frequencies in the wavelet are gradually incorporated and used to estimate the finer details of the velocity model. Our results suggest that multiscale transmission WT is less susceptible to cycle-skipping problems associated with the transmission FWI. To validate the WT tomogram we compare it to the zero-offset reflection section. Remarkably, the lateral variation of the ZO reflections from a strongly reflecting interface is almost identical to the velocity variations seen in the WT tomogram.

Previously, Wang et al. (2014) adapted the multiscale frequency strategy of Bunks et al. (1995) to wave equation traveltime inversion (Luo and Schuster, 1991b). They divided the frequency spectrum into overlapping bands, and first inverted the picked traveltimes for the low-frequency data associated with the picked traveltimes. This allowed for the updating of the low-wavenumber parts of the model and tended to avoid getting stuck in local minima. After several iterations, the frequency band for the forward modeling and backward propagation was widened to include higher frequencies, and the iterative inversion was restarted using the final model from the previous iterations as the starting model. The Wang et al. (2014) study was restricted to synthetic examples of direct traveltimes in crosswell data. Thus, the validation of this method is still waiting for field examples as well as its application to surface seismic data. In our study we have three innovations that advance the field of wave equation traveltime tomography with frequency dependent updates

of the velocity model.

1. We apply multiscale wave-equation traveltimes inversion (MWT) to surface seismic data and test its effectiveness for inverting refraction traveltimes in both synthetic data and field data. In comparison the Wang et al. (2014) study only tested MWT on synthetic data consisting of direct arrivals in a crosswell simulation.
2. We discover a new way to check field data tomograms against the ground truth associated with subsurface velocity variations. That is, we have discovered that comparing the zero-offset gathers (ZOGs) of reflection traces provides a raw estimate of the lateral velocity and reflectivity variations of the near surface. In two separate field examples, we show that the lateral velocity and reflectivity variations in the ZOGs largely agree with the velocity variations in the MWT tomograms.
3. We use the rationale frequency selection strategy of Boonyasiriwat et al. (2010) to select the frequency bands for an efficient inversion procedure.

This paper is organized into five sections. After the introduction, the second section briefly reviews wave equation traveltimes inversion where solutions to the visco-acoustic wave equation are used to compute the WT gradients. The third section presents the zero-offset gather that validate the accuracy of the WT tomogram with the synthetic data. In the fourth section, numerical tests of WT with different frequency bands for both synthetic and field data are presented and verified by converting the zero-offset gathers to depth. Conclusions are drawn in the last section.

THEORY AND METHODOLOGY

Let J denote the misfit function:

$$J = \frac{1}{2} \sum_{\mathbf{s}, \mathbf{g}} (\Delta\tau_{trans}(\mathbf{s}, \mathbf{g}))^2, \tag{1}$$

where $\Delta\tau_{trans}(\mathbf{s}, \mathbf{g}) = \tau_{obs}(\mathbf{s}, \mathbf{g}) - \tau_{pred}(\mathbf{s}, \mathbf{g})$ represents the traveltime difference between the observed and the calculated traveltimes recorded at $\mathbf{g}$ from the source at $\mathbf{s}$. For convenience we assume transmission traveltimes but this method can be used for reflection traveltimes as well. The conjugate gradient optimization method is used to iteratively update the velocity $c(\mathbf{x})$ by

$$c_{n+1}(\mathbf{x}) = c_n(\mathbf{x}) + \alpha_n \beta_n(\mathbf{x}), \tag{2}$$

where α, β and n are respectively the step length, the search direction and the iteration index. The Fréchet derivative $\frac{\partial \Delta\tau_{trans}}{\partial c}$ can be derived using the implicit function theorem (Luo and Schuster, 1991b) as

$$\frac{\partial \Delta\tau_{trans}}{\partial c} = - \frac{\partial \dot{F} / \partial c}{\partial \dot{F} / \partial \Delta\tau_{trans}}, \tag{3}$$

where $\dot{F}$ represents the time derivative of the cross-correlation of the predicted data $p_{pred}(\mathbf{g}, t + \Delta\tau_{trans}|\mathbf{s}, 0)$ and the observed data $p_{obs}(\mathbf{g}, t|\mathbf{s}, 0)$:

$$\dot{F}(\mathbf{s}, \mathbf{g}, \Delta\tau_{trans}) = \int_0^T \frac{p_{obs}(\mathbf{g}, t|\mathbf{s}, 0)}{A_{obs}(\mathbf{g}|\mathbf{s})} \dot{p}_{pred}(\mathbf{g}, t + \Delta\tau_{trans}|\mathbf{s}, 0) dt = 0. \tag{4}$$

Here, $A_{obs}(\mathbf{g}|\mathbf{s})$ represents the maximum amplitude of $p_{obs}(\mathbf{g}, t|\mathbf{s}, 0)$. The visco-acoustic wave equation characterized by the standard linear solid (SLS) mechanism (Blanch et al., 1995) for a

given velocity model c and a constant Q model in the spatial-temporal domain, is used to compute the pressure seismograms by numerically solving

$$\frac{\partial P}{\partial t} + \kappa(\tau + 1)(\nabla \cdot \mathbf{v}) + r_P = S(\mathbf{x}_s, t), \quad (5)$$

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\rho} \nabla P = 0, \quad (6)$$

$$\frac{\partial r_P}{\partial t} + \frac{1}{\tau_\sigma} (r_P + \tau \kappa (\nabla \cdot \mathbf{v})) = 0, \quad (7)$$

where $\mathbf{x} = \{v_x, v_y, v_z\}$ is the particle velocity vector, P represents pressure, r_P indicates the memory variable, $\kappa = \rho c^2$, a product of the density ρ and the square of the velocity term c , represents the bulk modulus of the medium, and $S(\mathbf{x}_s, t)$ represents a bandlimited source wavelet for a point source at $\mathbf{x} = \mathbf{x}_s$ with the listening time t . The parameter τ is the ratio between the stress- and strain-relaxation parameters τ_σ and τ_ϵ , which can be calculated from the quality factor Q by

$$\tau_\sigma = \frac{\sqrt{1 + Q^2} - 1/Q}{\omega}, \quad (8)$$

$$\tau_\epsilon = \frac{\sqrt{1 + Q^2} + 1/Q}{\omega}, \quad (9)$$

$$\tau = \frac{\tau_\epsilon}{\tau_\sigma}. \quad (10)$$

Here, ω is the selected reference angular frequency and is usually chosen to be the centroid frequency of the source wavelet (Robertson and Walker, 1994). In this paper, the Ricker wavelet in equation 5 is used as the source wavelet with progressively higher frequencies as the iterations proceed. The centroid frequencies of these source wavelets coincide with their peak frequencies. The spatial resolution of the WT tomogram can be improved by increasing the peak frequencies of the source signals.

Combining equations 2 and 3 yields the gradient of the misfit function in equation 1:

$$\beta = -\frac{\partial I}{\partial c} \quad (11)$$

$$\begin{aligned} &= \sum_{\mathbf{s}, \mathbf{g}} \frac{\partial \dot{F} / \partial c}{\partial \dot{F} / \partial \Delta \tau_{trans}} \Delta \tau_{trans}(\mathbf{g}, t | \mathbf{s}, 0) \\ &= \sum_{\mathbf{s}, \mathbf{g}} \frac{1}{E} \int_0^T dt \dot{p}_{obs}(\mathbf{g}, t | \mathbf{s}, 0) \frac{\partial p_{pred}(\mathbf{g}, t + \Delta \tau_{trans} | \mathbf{s}, 0)}{\partial c(\mathbf{x})} \Delta \tau_{trans}(\mathbf{g}, t | \mathbf{s}, 0) \\ &= \frac{1}{c^3(\mathbf{x})} \sum_{\mathbf{s}, \mathbf{g}} \int_0^T dt [\dot{g}_{bk}(\mathbf{x}, t | \mathbf{g}, 0) * \dot{p}_{pred}(\mathbf{x}, t | \mathbf{s}, 0)] \delta \tau(\mathbf{g}, t | \mathbf{s}, 0) \\ &= \frac{1}{c^3(\mathbf{x})} \sum_{\mathbf{s}} \int_0^T dt \dot{p}_{pred}(\mathbf{x}, t | \mathbf{s}, 0) [\sum_{\mathbf{g}} \dot{g}_{bk}(\mathbf{x}, -t | \mathbf{g}, 0) * \delta \tau(\mathbf{g}, t | \mathbf{s}, 0)], \end{aligned} \quad (12)$$

where the asterisk * represents temporal convolution and $\delta \tau$ is the recorded data shifted in time and weighted by the cross-correlation of the time derivatives of the calculated traces E :

$$\delta \tau(\mathbf{g}, t | \mathbf{s}, 0) = -\frac{2}{E} \dot{p}_{obs}(\mathbf{g}, t - \Delta \tau_{trans} | \mathbf{s}, 0) \Delta \tau_{trans}(\mathbf{g}, t | \mathbf{s}, 0). \quad (13)$$

Here the back propagated residual wavefield g_{bk} in equation 12 is calculated by solving the adjoint visco-acoustic wave equations (Blanch et al., 1995):

$$\frac{\partial q}{\partial t} + \nabla \cdot \left(\frac{1}{\rho} \mathbf{u} \right) = -\Delta d(\mathbf{x}_g, t; \mathbf{x}_s), \quad (14)$$

$$\frac{\partial \mathbf{u}}{\partial t} + [\nabla \kappa (1 + \tau) q + \nabla \frac{\kappa \tau r_q}{\tau_\sigma}] = 0, \quad (15)$$

$$\frac{\partial r_q}{\partial t} - \frac{r_q}{\tau_\sigma} - q = 0, \quad (16)$$

where q , $\mathbf{u}$ and r_q are, respectively, the adjoint state variables of the pressure wavefield P , the particle velocity vector $\mathbf{v}$, and the memory variable r_p in the seismogram modeled by equations 5-7. Assuming only the pressure wavefields are recorded, the weighted pseudo traveltime residual vector $\Delta \mathbf{d}$ will have only one component as $\Delta \mathbf{d} = [\Delta d \ 0 \ 0]'$, which is also known as the adjoint source term

(Pratt et al., 1998). The gradient is then preconditioned to compensate for geometrical expansion in the forward modeling and back-propagation simulation, as suggested in Luo and Schuster (1991b). After computing the gradient in equation 11, the preconditioned conjugate gradient method is used to update the velocity tomogram (Luo and Schuster, 1991b; Nocedal and Wright, 2006).

NUMERICAL TESTS

Source wavelets with gradually increasing peak frequencies are used to invert for velocity tomograms which are then compared with the zero-offset gathers from the raw data. In this section, all the quality factors Q are assigned constant values for ignoring strong variations of Q . The near-offset 10 traces in all the following examples are muted to avoid the effects of these traces on the inverted tomograms as they can easily dominate the energy.

A Synthetic Test

In the synthetic test, the data set is computed using the velocity model shown in Figure 1(a). This model is used to show the effects of the proposed method although its size is different from the actual one for exploration geophysics. The model size is 26 m in the vertical Z direction and 120 m in the horizontal direction with a grid point spacing of 1 m . The data are recorded by 60 receivers spaced at an interval of 2 m , and are triggered by 60 sources at every receiver's location. The source wavelet is a Ricker wavelet with a peak frequency of 30 Hz and with a constant quality factor model $Q = 200$. Each common shot gather (CSG) is recorded for 0.4 *second* with a sampling interval of 0.2 *millisecond*. A typical CSG is presented in Figure 1(b) and the ZOG (Zero Offset Gathers) extracted from each CSG is presented in Figure 1(c). To remove the strong source wavelet effects, another data set with the same acquisition geometry is also generated based on the homogeneous

velocity model such that $v = 1000\text{ m/s}$. The traces from the new are subtracted from the original traces and the resulting ZOG is shown in Figure 1(d) with red crosses marking the first arriving events. A frequency spectrum of the first trace of the CSG #1 is shown in Figure 2(a) and the first arrival traveltimes matrix is presented in Figure 2(b) after automatically picking the first troughs.

For inversion, the first arrivals are used to roughly estimate a very low resolved ray-based traveltimes velocity model, whose horizontal velocity distributions at each depth are then averaged to form an 1D gradient model as the initial model (Figure 3). The velocity model is reconstructed by a ray-based traveltimes inversion method and the proposed WT method using Ricker wavelets with peak frequencies at 30 Hz, 50 Hz, 70 Hz, and a source wavelet with the full frequency band (0-150 Hz) in Figure 4, respectively. The tomograms after 15 iterations are shown in Figures 5(a1-e1). For comparison, the wavenumber components of the true-velocity distribution and the reconstructed tomograms from 0 to 20 m in depth are also shown in Figures 5(a2-e2). Figures 5(b2-e2) are marked by the velocity contour at $v = 1500\text{ m/s}$ in depth converted from the red crosses labeling the first arriving events in Figure 1(d), and the results are shown in Figures 6(a-d). Figure 7 presents the first arrival matrices with tomograms inverted by ray-based method and WT using source wavelets with different peak frequencies and the full frequency band, and their residuals with the true traveltimes matrix in Figure 2b.

In this test, a series of the reconstructed tomograms demonstrate more accurate structures by using multi-scale frequency bands for inversion, and they also approach the real velocity model in the wavenumber domain. WT is less sensitive to the initial velocity model compared to FWI, and the starting gradient model can be estimated from the picked first-arrival traveltimes. The picked troughs or peaks of the first-arrival events are more accurate compared to the easily distorted waveforms. The ZOG obtained from the original data set validates the WT tomograms without the common image gathers. Besides the subsurface geometry, the highly resolved tomograms can also be obtained

using source wavelets with peak frequencies within the frequency band of the raw data set.

To further verify the effectiveness of the method, two field data sets recorded over exposed outcrops are tested in the next two subsections.

An Outcrop Example

A 2D seismic survey is conducted above an outcrop located to the south of KAUST. The survey consists of a line of 60 vertical component geophones (Figure 9), spaced every 1.0 *m*, and the shots are located at every receiver position so that there are $60 \times 60 = 3600$ traces. In this field experiment, a 50 *lb* weight drop is used to generate the seismic data with 4 stacks at each shot location. Each CSG was recorded for 0.5 *seconds* with a sampling interval of 1.0 *ms*. A small drone is used to take a picture (Figure 9(b)) of the field site, and a panorama view (Figure 9(c)) is taken from the front of the outcrop. The height of the outcrop above the ground is about 2.5 *meters*, and there are two layers consisting of semi-consolidated silt and clay for the upper part and gravel, silt and clay for the lower part (Figure 9(d)).

CSG #30 is presented in Figure 10, and the ZOG of this data set is extracted and aligned in Figure 10(b). The 30th trace of CSG #30 is shown in Figure 10(c), implying the possible frequency bands to be used for inversion. Picking the first-arrival traveltimes associated with the trough gives the travelttime matrix shown in Figure 10(d). The starting velocity model is also presented in Figure 11. We set the model size as 15 *m* in vertical direction and 60 *m* in the horizontal direction with a grid point of spacing of 0.25 *m*. For the computation, the recording time length is 0.01 *s* with a temporal interval of 0.05 *ms*, which ensures that the first arrivals can be recorded.

The WT tomograms are presented in Figures 12(a-c), whose zoom-view versions above 6.25 *m* in depth are shown in Figures 13(a-c). The velocity contours at $v = 750$ *m/s* in these models are marked by blue, green and black dots, which are converted using the root mean square velocity

above them to traveltimes. They are shown in Figure 15 along with the red crosses marked in the original ZOGs. The zoom view in Figure 13(c) indicates the shape of the ZOG in Figure 10(b). It is clear that the black dashed line from the 100-Hz WT tomogram most closely resembles the red dashed curve picked from the ZOG. Figure 14 also validates this conclusion by presenting the first arrival matrices with tomograms inverted by ray-based method and WT using source wavelets with 50 Hz and 100 Hz peak frequencies, and their residuals with the true traveltime matrix in Figure 10d.

The Qademah Fault Example

A 2D seismic survey is conducted 20 km north of KAUST and very close to the Red Sea (Figure 16). The 2D acquisition geometry consists of one line of vertical-component geophones (Figure 17) crossing the possible fault. Along this line, there are 240 receivers with a 2.5 m spacing, and the shots are located at every other receiver position. In this field experiment, we used a 200 lb weight drop to generate the seismic data with 5 stacks at each shot location. A common shot gather with the spectrum of its first trace is shown in Figures 18 (a) and (c), respectively. A total of 8 shot gathers out of the 120 CSGs are too noisy to be picked, so they are excluded from consideration. A typical ZOG is shown in Figure 18(b), and Figure 18(d) depicts the 21419 traveltimes picked from the traces. For this data set we estimate the dominant wavelength and the dominant frequency of the first-arrival head waves to be 20 m and 50 Hz, respectively, where the near-surface P-velocity at the geophones is estimated to be 1000 m/s.

We choose a starting velocity model (Figure 19) where the velocity increases from 0.3 km/s to 2.4 km/s in depth. The grid spacing is set to 1.25 m and the model dimensions are 600 m in the horizontal direction and 100 m in the vertical direction, which is consistent with our field experiment. Here, the recorded length of the calculated CSG is 0.35 s with a time step of 0.1 ms.

The tomograms constructed by ray-based tomography inversion, and WT with source wavelets of 20, 40 and 60 Hz are presented in Figures 20(a-d). The anomaly at $Z = 25$ m and $X = 150$ m is not quite discernible in the ray-based tomogram (Figure 20(a)); however it can be identified in Figure 20(b-d), consistent with the fault indicated in Figure 17(b). Two velocity contours at $v = 1.6$ km/s and $v = 2.3$ km/s are identified in these Figures and respectively marked by blue and cyan dots, as presented in Figures 20(a-d). Figure 21 demonstrates the first arrival matrices with tomograms inverted by ray-based method and WT using source wavelets with 20-Hz, 40-Hz, and 60-Hz peak frequencies, and their residuals with the true traveltimes matrix in Figure 18d. The zero-offset traveltimes associated with these contours are computed and overlaid on the ZOG in Figures 22(a-d). The cyan colored contours best resembles the zero-offset reflections in the ZOG, indicating that the higher frequency bands accurately recover the structures. All the tomograms indicate the fault position around $X = 150$ m in the photos.

In summary, the above two field data cases, together with their ground truths, have verified the effectiveness of our proposed method, and further advanced the study of frequency dependent WT with the crosswell case in Wang et al. (2014).

CONCLUSIONS

We have presented a multiscale WT method that accurately inverts for both the low- and high-wavenumbers parts of the velocity model. Synthetic tests show that multiscale WT can produce more accurate tomograms using higher frequency bands within the recorded data spectra. Moreover, the final multiscale WT tomogram is less sensitive to the starting model. Comparisons of field data results with the ZOGs suggest that the MWT is more accurate in estimating the high-wavenumber features of the velocity model than the WT or ray-based inversion methods. The liability of MWT is that it can be more costly than WT. For example, the synthetic tests showed that MWT is about 3

or 4 times more costly than WT depending on the frequency bands used. A similar increase in cost was incurred for the field data tests. The two land data cases showed the effectiveness of MWT by comparing the inverted tomograms with the known ground truths.

The multiscale WT method can be generalized to reflection WT to improve the resolution in the deeper parts as long as reliable reflection traveltimes can be identified and picked, because they are the only required precise information. The peak frequency of the Ricker source wavelet cannot be higher than the range of the frequency band. The main reason is the picked first arrivals in the predicted data may not match those in the observed data, and easily lead to the bulk depth errors in the depth of the reflector image.

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REFERENCES

Blanch, J. O., J. O. Robertsson, and W. W. Symes, 1995, Modeling of a constant q: Methodology and algorithm for an efficient and optimally inexpensive viscoelastic technique: *Geophysics*, **60**, 176–184.

Boonyasiriwat, C., G. T. Schuster, P. Valasek, and W. Cao, 2010, Applications of multiscale wave-

- form inversion to marine data using a flooding technique and dynamic early-arrival windows: *Geophysics*, **75**, R129–R136.
- Bunks, C., F. M. Saleck, S. Zaleski, and G. Chavent, 1995, Multiscale seismic waveform inversion: *Geophysics*, **60**, 1457–1473.
- Dickinson, D., F. Mancini, X. Li, K. Zhao, and S. Ji, 2017, A refraction/reflection full-waveform inversion case study from north west shelf offshore australia, *in* SEG Technical Program Expanded Abstracts 2017, 1368–1372, Society of Exploration Geophysicists.
- Feng, S. and J. T. Schuster, 2017, Skeletonized wave equation inversion in vti media without too much math: *Interpretation*, **5**, 1–33.
- Liu, Z. and J. Zhang, 2017, Joint traveltimes, waveform, and waveform envelope inversion for near-surface imaging: *Geophysics*, **82**, R235–R244.
- Luo, Y. and G. T. Schuster, 1991a, Wave equation inversion of skeletalized geophysical data: *Geophysical Journal International*, **105**, 289–294.
- , 1991b, Wave-equation traveltimes inversion: *Geophysics*, **56**, 645–653.
- Ma, Y. and D. Hale, 2013, Wave-equation reflection traveltimes inversion with dynamic warping and hybrid waveform inversion, *in* SEG Technical Program Expanded Abstracts, 871–876.
- Nocedal, J. and S. Wright, 2006, *Numerical optimization*: Springer Science & Business Media.
- Pratt, G., C. Shin, and Hicks, 1998, Gauss-newton and full newton methods in frequency space seismic waveform inversion: *Geophysical Journal International*, **133**, 341–362.
- Robertson, S. E. and S. Walker, 1994, Some simple effective approximations to the 2-poisson model for probabilistic weighted retrieval: *Proceedings of the 17th annual international ACM SIGIR conference on Research and development in information retrieval*, 232–241.
- Sun, M., J. Zhang, and W. Zhang, 2017, Alternating first-arrival traveltimes tomography and waveform inversion for near-surface imaging: *Geophysics*, **82**, R245–R257.

Tarantola, A., 2005, Inverse problem theory and methods for model parameter estimation: SIAM.

Van Leeuwen, T. and W. Mulder, 2010, A correlation-based misfit criterion for wave-equation traveltime tomography: *Geophysical Journal International*, **182**, 1383–1394.

Virieux, J. and S. Operto, 2009, An overview of full-waveform inversion in exploration geophysics: *Geophysics*, **74**, WCC1–WCC26.

Wang, Y., Y. Zheng, X. Chang, and Z. Yao, 2014, Frequency-dependent wave equation traveltime inversion: *Journal of Seismic Exploration*, **23**, 367–378.

Xue, Z., H. Zhu, and S. Fomel, 2017, Full waveform inversion using seislet regularization: *Geophysics*, **82**, 1–40.

Zhang, J. and M. N. Toksöz, 1998, Nonlinear refraction traveltime tomography: *Geophysics*, **63**, 1726–1737.

Zhang, S., Y. Luo, and G. Schuster, 2015, Shot-and angle-domain wave-equation traveltime inversion of reflection data: Synthetic and field data examples: *Geophysics*, **80**, S79–S92.

Zhou, C., W. Cai, Y. Luo, G. T. Schuster, and S. Hassanzadeh, 1995, Acoustic wave-equation traveltime and waveform inversion of crosshole seismic data: *Geophysics*, **60**, 765–773.

Zhou, C., G. T. Schuster, S. Hassanzadeh, and J. M. Harris, 1997, Elastic wave equation traveltime and waveform inversion of crosswell data: *Geophysics*, **62**, 853–868.

Zhou*, D., W. Wang, J. Zhang, and D. R. O’Connell, 2014, 3d joint inversion of seismic traveltime and gravity data: A case study, *in* SEG Technical Program Expanded Abstracts 2014, 3148–3152, Society of Exploration Geophysicists.

Zhu, H. and S. Fomel, 2016, Building good starting models for full-waveform inversion using adaptive matching filtering misfit: *Geophysics*, **81**, U61–U72.

Figure 1: (a) The true velocity model, (b) CSG #1, the zero offset gather (c) with and (d) without direct waves. The red crosses mark the first arrivals in the zero offset gathers (d).

Figure 2: (a) The frequency spectrum of the first trace of CSG #1, and (b) the matrix of picked first-arrival traveltimes.

Figure 3: The starting velocity model.

Figure 4: (a) A source wavelet with the full frequency band, and its (b) spectrum.

Figure 5: The (a1) true model, (b1) ray-based tomogram, WT inverted velocity models using Ricker wavelets peaked at (c1) 30 Hz, (d1) 50 Hz, (e1) 70 Hz, and (f1) the wavelet with full frequency band, and wavenumber Figures (a2-f2) associated with models (a1-f1).

Figure 6: The red crosses mark the converted zero-offset gathers in Figure 1(d) to depths using the (a) traveltime velocity model, and the WT velocity models (b-d) using source wavelets with different peak frequencies and (e) the full frequency band.

Figure 7: (a1-e1) The first arrival matrices with tomograms inverted by ray-based method and WT using wavelets with different peak frequencies and the full frequency band, and (a2-e2) their traveltime residuals.

Figure 8: The first gradients in inversion using source wavelets with (a-c) different peak frequencies and (d) the full frequency band.

Figure 9: (a) The location of the seismic survey, (b) the outcrop viewed from a drone, (c) panorama front view of the outcrop, and the (d) geology of the outcrop above the ground. The white arrows in (b) and (c) mark the cracks or vugs which are seen above the ground and they extend into the subsurface.

Figure 10: (a) CSG #30, (b) zero offset gathers, (c) frequency spectrum of a trace in CSG #30, (d) matrix of first-arrival traveltimes in the raw data. The red crosses in the (b) ZOG mark the strong reflection signals.

Figure 11: The starting gradient model.

Figure 12: The (a) ray-based, (b) 50-Hz WT, and (c) 100-Hz WT tomograms.

Figure 13: Three zoom views of the Figures 12(a-c). The blue, green and black dots mark the velocity curve at $v = 750 \text{ m/s}$ in each figure.

Figure 14: (a1-c1) The first arrival matrices with tomograms inverted by ray-based method and by WT using wavelets with 50 Hz and 100 Hz peak frequencies, and (a2-c2) their traveltime residuals.

Figure 15: A comparison of the zero-offset reflection times picked from the raw data and the converted times from the depths picked from Figures 13(a-c). It is obvious that the picks (dashed black curve) from the 100-Hz WT tomogram most closely agree with the traveltime picks (red curve) from the ZOG.

Figure 16: The acquisition location.

Figure 17: (a) The acquisition line, and (b) the line passing through the conjectured Qadema fault.

Figure 18: (a) CSG #1, (b) zero-offset gather, (c) frequency spectrum of the first trace of CSG #1, and (d) the traveltime matrix for first arrivals.

Figure 19: The starting velocity model for inverting the Qadema fault.

Figure 20: The (a) ray-based traveltime tomogram and the (b) 20-Hz, (c) 40-Hz, and (d) 60-Hz WT tomograms. The velocity curves at $v = 1600 \text{ m/s}$ and $v = 2300 \text{ m/s}$ are marked by the blue and the cyan dots in depths on these tomograms.

Figure 21: (a1-d1) The first arrival matrices with tomograms inverted by ray-based method and WT using wavelets with 20-Hz, 40-Hz and 60-Hz peak frequencies, and (a2-d2) their traveltime residuals.

Figure 22: The marked depths in Figures 20(a-d) converted to time (a-d) in the zero offset space

using the RMS velocity distribution.

Figure 1: (a) The true velocity model, (b) CSG #1, the zero offset gather (c) with and (d) without direct waves. The red crosses mark the first arrivals in the zero offset gathers (d).

107x56mm (300 x 300 DPI)

Figure 2: (a) The frequency spectrum of the first trace of CSG #1, and (b) the matrix of picked first-arrival traveltimes.

79x30mm (300 x 300 DPI)

Figure 3: The starting velocity model.

55x29mm (300 x 300 DPI)

Figure 4: (a) A source wavelet with the full frequency band, and its (b) spectrum.

69x18mm (300 x 300 DPI)

Figure 5: The (a1) true model , (b1) ray-based tomogram, WT inverted velocity models using Ricker wavelets peaked at (c1) 30 Hz, (d1) 50 Hz, (e1) 70 Hz, and (f1) the wavelet with full frequency band, and wavenumber Figures (a2-f2) associated with models (a1-f1).

173x118mm (300 x 300 DPI)

Figure 6: The red crosses mark the converted zero-offset gathers in Figure 1(d) to depths using the (a) traveltime velocity model, and the WT velocity models (b-d) using source wavelets with different peak frequencies and (e) the full frequency band.

169x136mm (300 x 300 DPI)

Figure 7: (a1-e1) The first arrival matrices with tomograms inverted by ray-based method and WT using wavelets with different peak frequencies and the full frequency band, and (a2-e2) their traveltime residuals.

210x124mm (300 x 300 DPI)

Figure 8: The first gradients in inversion using source wavelets with (a-c) different peak frequencies and (d) the full frequency band.

106x54mm (300 x 300 DPI)

Figure 9: (a) The location of the seismic survey, (b) the outcrop viewed from a drone, (c) panorama front view of the outcrop, and the (d) geology of the outcrop above the ground. The white arrows in (b) and (c) mark the cracks or vugs which are seen above the ground and they extend into the subsurface.

319x165mm (300 x 300 DPI)

Figure 10: (a) CSG #30, (b) zero offset gathers, (c) frequency spectrum of a trace in CSG #30, (d) matrix of first-arrival traveltimes in the raw data. The red crosses in the (b) ZOG mark the strong reflection signals.

128x79mm (300 x 300 DPI)

Figure 11: The starting gradient model.
65x44mm (300 x 300 DPI)

Figure 12: The (a) ray-based, (b) 50-Hz WT, and (c) 100-Hz WT tomograms.

61x12mm (300 x 300 DPI)

Figure 13: Three zoom views of the Figures 12(a-c). The blue, green and black dots mark the velocity curve at $v = 750$ m/s in each figure.

64x14mm (300 x 300 DPI)

Figure 14: (a1-c1) The first arrival matrices with tomograms inverted by ray-based method and by WT using wavelets with 50 Hz and 100 Hz peak frequencies, and (a2-c2) their traveltime residuals.

104x31mm (300 x 300 DPI)

Figure 15: A comparison of the zero-offset reflection times picked from the raw data and the converted times from the depths picked from Figures 13(a-c). It is obvious that the picks (dashed black curve) from the 100-Hz WT tomogram most closely agree with the traveltimes picks (red curve) from the ZOG.

84x43mm (300 x 300 DPI)

Figure 16: The acquisition location.

192x216mm (300 x 300 DPI)

Figure 17: (a) The acquisition line, and (b) the line passing through the conjectured Qadema fault.
102x33mm (300 x 300 DPI)

Figure 18: (a) CSG #1, (b) zero-offset gather, (c) frequency spectrum of the first trace of CSG #1, and (d) the traveltime matrix for first arrivals.

121x72mm (300 x 300 DPI)

Figure 19: The starting velocity model for inverting the Qadema fault.

103x60mm (300 x 300 DPI)

Figure 20: The (a) ray-based traveltime tomogram and the (b) 20-Hz, (c) 40-Hz, and (d) 60-Hz WT tomograms. The velocity curves at $v = 1600$ m/s and $v = 2300$ m/s are marked by the blue and the cyan dots in depths on these tomograms.

250x140mm (300 x 300 DPI)

Figure 21: (a1-d1) The first arrival matrices with tomograms inverted by ray-based method and WT using wavelets with 20-Hz, 40-Hz and 60-Hz peak frequencies, and (a2-d2) their traveltime residuals.

233x212mm (300 x 300 DPI)

Figure 22: The marked depths in Figures 20(a-d) converted to time (a-d) in the zero offset space using the RMS velocity distribution.

247x141mm (300 x 300 DPI)

DATA AND MATERIALS AVAILABILITY

Data associated with this research are available and can be obtained by contacting the corresponding author.