Full-waveform inversion in acoustic orthorhombic media and application to a North Sea data set

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ABSTRACT

Full-waveform inversion (FWI) in anisotropic media is challenging, mainly because of the large computational cost, especially in 3D, and the potential trade-offs between the model parameters needed to describe such media. By analyzing the trade-offs and understanding the resolution limits of the inversion, we can constrain FWI to focus on the main parameters the data are sensitive to and push the inversion toward more reliable models of the subsurface. Orthorhombic anisotropy is one of the most practical approximations of the earth subsurface that takes into account the natural horizontal layering and the vertical fracture network. We investigate the feasibility of a multiparameter FWI for an acoustic orthorhombic model described by six parameters. We rely on a suitable parameterization based on the horizontal velocity and five dimensionless anisotropy

parameters. This particular parameterization allows a multi-stage model inversion strategy in which the isotropic, then, the vertical transverse isotropic, and finally the orthorhombic model can be successively updated. We applied our acoustic orthorhombic inversion on the SEG-EAGE overthrust synthetic model. The observed data used in the inversion are obtained from an elastic variable density version of the model. The quality of the inverted model suggests that we may recover only four parameters, with different resolution scales depending on the scattering potential of these parameters. Therefore, these results give useful insights on the expected resolution of the inverted parameters and the potential constraints that could be applied to an orthorhombic model inversion. We determine the efficiency of the inversion approach on real data from the North Sea. The inverted model is in agreement with the geologic structures and well-log information.

INTRODUCTION

The objective of full-waveform inversion (FWI) is to recover a high-resolution model that is capable of matching the observed seismic data, trace by trace, through repetitive modeling and a local optimization technique (Lailly, 1983; Tarantola, 1984; Virieux and Operto, 2009). Over the past three decades, most FWI techniques have been designed to recover only a P-wave velocity model because of the high computational cost (e.g., Gauthier et al., 1986; Pratt, 1999). The recent progress in high-performance computing and the improvement in data acquisition resulted in the extension of FWI to 2D and 3D acoustic and elastic transverse isotropic media with a vertical axis of symmetry (VTI) (e.g., Warner et al., 2013; Operto et al., 2014; Wu and Alkhalifah, 2016; Djebbi and Alkhalifah, 2017; Kamath et al., 2017). This extension has resulted in

challenges related to the trade-offs between the model parameters in our inversion process. The trade-offs between the model parameters and their resolution limits have motivated several studies investigating suitable parameterizations for our inversion schemes (Gholami et al., 2013; Operto et al., 2013; Alkhalifah and Plessix, 2014; Alkhalifah, 2016; Kamath and Tsvankin, 2016; Djebbi et al., 2017; Kazei and Alkhalifah, 2018). Most of these studies rely on the analysis of the radiation (scattering) patterns based on the Born approximation (Wu and Aki, 1985; Panning et al., 2009).

Orthorhombic anisotropy is usually regarded as the most practical realistic approximation of the earth subsurface because it combines the anisotropy admitted by the natural, mostly horizontal layering of the earth (due to gravity), as well as the vertical aligned fractures, usually found in fractured reservoirs or horizontal stresses caused by the tectonic regime or salt intrusions (Cheadle et al.,

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1991; Wild and Crampin, 1991; Schoenberg and Helbig, 1997; Tsvankin, 1997; Bakulin et al., 2000). Contrary to the VTI case, orthorhombic media incorporate velocity variations with azimuths (azimuthal anisotropy), therefore, 3D wide-azimuth acquisitions are essential to estimate these variations (Tsvankin and Grechka, 2011). Azimuthal anisotropy can be described using the fracture weaknesses (Schoenberg and Sayers, 1995; Bakulin et al., 2000) or the deviations anisotropy parameters (Masmoudi et al., 2016).

Practical parameterizations for acoustic and elastic orthorhombic media have been investigated. In the acoustic case, Masmoudi and Alkhalifah (2016a) analytically analyze the radiation patterns of the anisotropic parameter perturbation and end up recommending different parameterizations for different FWI strategies, for instance, those using an initial model obtained from migration velocity analysis (MVA) and those obtained from inverting the diving wave energy. For VTI media, Alkhalifah and Plessix (2014) and Alkhalifah (2016) show that a combination of the horizontal velocity v_h and the parameters ϵ and η is optimal for an inversion that starts with the diving wave energy. In this parameterization, diving waves uniquely constrain the long-wavelength variations of the velocity model, ϵ is mainly sensitive to the model reflectivity, and η influences intermediate incidence angles. Furthermore, Guitton and Alkhalifah (2016) show that ϵ could absorb the density effect because these two parameters have similar scattering properties. Using these insights, Masmoudi and Alkhalifah (2016a) propose an acoustic orthorhombic parameterization based on the three VTI parameters (v_{h1} , ϵ_1 , η_1) defined in the (x, z) vertical plane and three additional azimuthal anisotropy parameters: ϵ_d and η_d describing the deviation in anisotropy between the (x, z) and (y, z) vertical planes and the parameter δ_3 describing the anellipticity in the horizontal (x, y) plane. As shown in Masmoudi and Alkhalifah (2016a), such parameterization preserves the attractive features of the three VTI parameters and maintains their scattering potential stationary as a function of azimuth. In the elastic case, Oh and Alkhalifah (2016b) propose an extension of this acoustic orthorhombic parameterization by adding three additional parameters: the vertical S-wave velocity v_{s1} polarized in the (x, z) plane, the parameter γ_1 in the same plane, and a deviation parameter $\gamma_d = (\gamma_2 - \gamma_1)/(1 + 2\gamma_1)$, where γ_2 is defined in the (y, z) plane. Recently, some applications of waveform inversion in elastic and acoustic orthorhombic media have been proposed (Albertin et al., 2016; Alkhalifah et al., 2016; Oh and Alkhalifah, 2016a; Wang and Tsvankin, 2016; Zhang et al., 2017).

The efficiency of wave simulators is a vital ingredient in FWI. Several techniques have been proposed to model P-wave propagation in orthorhombic media, among them solving coupled systems of second-order partial differential equations (Fowler and King, 2011; Cheng and Kang, 2014) and the mixed-domain wavefield extrapolators (Fowler and Lapilli, 2012; Fomel et al., 2013). Alkhalifah (2000) proposes a pseudoacoustic wave equation, which provides accurate simulation of the kinematics of P-wave propagation. Xu and Zhou (2014) suggest solving this pseudoacoustic VTI wave equation by decomposing the pseudodifferential operator into a scalar operator and a differential operator. The scalar operator involves an asymptotic approximation on calculating the wavefield propagation direction, which resulted in aliasing of the wavefield and dispersion artifacts in finite-difference implementations. To enhance the accuracy of this approach, Xu et al. (2015) propose an elliptic decomposition of the differential operator, which alleviates the magnitude of the scalar operator and provides accurate wave-

field solutions. Contrary to solving coupled-system equations, Xu and Zhou (2014) approach yields a wavefield solution free of S-wave artifacts. Furthermore, this method has been applied in a VTI migration (Mu et al., 2015) and inversion (Le et al., 2015).

In this paper, we focus on the feasibility of inverting the six acoustic orthorhombic model parameters proposed in Masmoudi and Alkhalifah (2016a). Through an inversion of the SEG-EAGE overthrust model, we assess the performance of this parameterization and discuss the resolution limits and the crosstalk between the parameters. The observed data in this inversion correspond to an elastic variable density version of the model. Therefore, we discuss the potential leakage of the elastic and density effects into the acoustic parameters. Furthermore, we apply our inversion to a 3D ocean bottom cable data set from the North Sea. We use total variation (TV) regularization and structural model constraints to resolve sharp interfaces and to push the inversion toward geologically consistent structures. To solve the pseudoacoustic orthorhombic wave equation, we use a similar approach to Xu and Zhou (2014) method. Here, the pseudodifferential operator is decomposed into two operators. The ellipsoid decomposition is used as well to enhance the accuracy. Furthermore, we formulate the adjoint problem and propose the gradients with respect to the model parameters.

We will start by reviewing the parameterization used throughout the paper and discuss its corresponding radiation patterns. Then, we will describe the forward and adjoint problems and discuss our FWI implementation. Finally, we will show applications of our acoustic orthorhombic inversion on a modified SEG-EAGE overthrust model and on 3D real data from the North Sea.

A SUITABLE CHOICE OF PARAMETERS

By analyzing the radiation patterns of the scattering of different orthorhombic parameterizations, Masmoudi and Alkhalifah (2016a) suggest a new parameterization for acoustic orthorhombic media with potentially interesting features for FWI and MVA. Here, we focus on the parameterization, which includes three VTI parameters: the horizontal velocity v_{h1} , ϵ_1 , η_1 , which are defined in the (x, z) vertical plane, and three azimuthal anisotropy parameters: two deviation parameters ϵ_d , η_d , and the parameter δ_3 defined from the x -axis TI plane (Tsvankin, 1997). The horizontal velocity is related to the vertical P-wave velocity v_v and to the NMO velocity v_{n1} in the (x, z) plane through the equations given by

$$v_{h1} = v_v \sqrt{1 + 2\epsilon_1}, \quad (1)$$

$$v_{h1} = v_{n1} \sqrt{1 + 2\eta_1}. \quad (2)$$

The two deviation anisotropy parameters used in this parameterization are

$$\epsilon_d = \frac{1}{2} \left(\frac{v_{h2}^2}{v_{h1}^2} - 1 \right), \quad (3)$$

$$\eta_d = \frac{\epsilon_d - \delta_d}{1 + 2\delta_d}, \quad (4)$$

where v_{h2} is the horizontal velocity in the (y, z) vertical plane and δ_d is another deviation anisotropy parameter defined as

$$\delta_d = \frac{1}{2} \left(\frac{v_{n2}^2}{v_{n1}^2} - 1 \right), \quad (5)$$

where v_{n2} is the NMO velocity in the (y, z) vertical plane. The parameters ϵ_d and η_d , as well as δ_3 , measure the deviation from a VTI medium; therefore, if they are set to zero, the orthorhombic model reduces to VTI.

Figure 1 shows the radiation patterns of the parameterization described above, as a function of the scattering angle θ , for different azimuths ϕ . Here, θ is the angle between the source and receiver measured at the scattering point and ϕ is the source-to-receiver azimuth of the scattering point measured from the (x, z) vertical plane. These radiation patterns correspond to perturbations from a horizontal reflector. They are also derived by assuming a small scatterer in a smoothly varying isotropic background medium (Masmoudi and Alkhalifah, 2016a). The effect of the background medium on the perturbation is expected to be small (He and Plessix, 2017). The resolution of the scattering point is governed by the diffraction tomography principles formula, which relates the model wavenumber k_m to the frequency ω and the scattering angle θ (Devaney, 1984; Miller et al., 1987; Jin et al., 1992), and is given by

$$|k_m| = 2 \frac{\omega}{v_0} \cos \frac{\theta}{2}, \quad (6)$$

where v_0 is the velocity at the scattering point of the background isotropic model. From this relation, we can predict that data from

large offsets (large scattering angles) should produce low-wavenumber updates, which are essential at the early stages of the model building process. On the other hand, small angles are associated with high-wavenumber updates. Keeping these facts in mind, we will infer some conclusions about the resolution of the parameters from the behavior of their radiation patterns.

As shown in Figure 1, the scattering potential of the horizontal velocity is independent from the scattering and azimuth angles. On the other hand, the scattering potential of ϵ_1 and η_1 is stationary over azimuth angles. Moreover, ϵ_1 is associated with small scattering angles, while, η_1 is associated with intermediate scattering angles. We may infer that velocity could be well-resolved, at least its low-wavenumber model information because of the trade-off with ϵ_1 at small scattering angles. Because ϵ_1 affects vertically traveling waves, we may only update its high-wavenumber information. This is assuming density and other amplitude effects are ignored. However, in real cases, there is a trade-off between ϵ_1 and density. It has been suggested that ϵ_1 could absorb density effects (e.g., Alkhalifah and Plessix, 2014; Guitton and Alkhalifah, 2016). Since η_1 has weak scattering potential, it could probably be fixed during the inversion. Moreover, the scattering potential of the azimuthal anisotropy parameters changes with scattering and azimuth angles: The main scattering of δ_3 resides at $\phi = 45^\circ$, whereas the other two parameters have their maximum scattering energy at $\phi = 90^\circ$. Similarly to η_1 , η_d has weak scattering potential and might not be resolved. On the other hand, we have the chance to access the low-wavenumber

Figure 1. Radiation patterns for the parametrization proposed in Masmoudi and Alkhalifah (2016a) as a function of the scattering angle for different azimuth angles ϕ : 0° – 30° – 45° – 60° – 90° –All azimuths.

model update of ϵ_d and δ_3 if we have large enough crossline offsets. However, we still have to mitigate the trade-offs between these two parameters and v_{h1} .

THEORY

In FWI, we generally start with an initial model m_0 assumed to be close enough to the true model. The objective is to find a new model $m = m_0 + \delta m$, which reduces the size of a misfit function in the framework of an optimization problem. The key ingredients in this process are an efficient modeling operator and a local differentiation approach, in which the gradient and potentially the Hessian operators are estimated. In this section, we first present our modeling approach and its adjoint formulation with the aim of deriving the gradients of the orthorhombic model parameters. Then, we describe a specific objective function we use in this study, which includes model regularization and structural constraints that we use in the real-data example.

Forward modeling

The acoustic wave equation for anisotropic media satisfies a linear pseudodifferential equation of the form

$$[\omega^2 - \Phi(\mathbf{x}, \mathbf{k})]u(\mathbf{k}, \omega) = 0, \quad (7)$$

where $u(\mathbf{k}, \omega)$ is the pressure wavefield, ω is the frequency, $\mathbf{k}$ is the wave vector, and Φ is the pseudodifferential phase operator. The phase operator can be obtained from the dispersion relation using the Christoffel equation (Alkhalifah, 2003). In the orthorhombic case, this yields the cubic polynomial (Song and Alkhalifah, 2013):

$$-\Phi^3 + a\Phi^2 + b\Phi + c = 0. \quad (8)$$

The polynomial coefficients a , b , and c are given in Appendix A.

To solve equation 7 in the time-space domain, we propose similarly to Xu and Zhou (2014) and Xu et al. (2015) to decompose the pseudodifferential operator into an elliptic Laplacian operator ∇_E^2 and a scalar operator $S(\mathbf{x})$ as follows:

$$\partial_{tt}u(\mathbf{x}, t) - S(\mathbf{x})\nabla_E^2u(\mathbf{x}, t) = 0, \quad (9)$$

where

$$\begin{cases} \nabla_E^2u(\mathbf{x}, t) = \partial_{xx}u + (1 + 2\epsilon_d)\partial_{yy}u + \frac{1}{1+2\epsilon_1}\partial_{zz}u, \\ S(\mathbf{x}) = \Phi(\mathbf{x}, \mathbf{n}) / \left(n_x^2 + (1 + 2\epsilon_d)n_y^2 + \frac{1}{1+2\epsilon_1}n_z^2 \right), \end{cases} \quad (10)$$

where ∂_{tt} is the second-order time derivative, ∂_{xx} , ∂_{yy} , and ∂_{zz} are the second-order space derivatives, n_x , n_y , and n_z are the components of the unit vector $\mathbf{n}$ of phase direction $\mathbf{n} = \mathbf{k}/|\mathbf{k}|$. In this elliptic decomposition, the ellipticity of the wavefield is handled by a slightly modified Laplacian operator, whereas the anellipticity is corrected via the scalar operator. Therefore, $S = 1$ for an elliptic anisotropic media ($\eta_1 = \eta_d = \delta_3 = 0$). To compute the scalar operator in the space domain, we approximate the phase direction $\mathbf{n}$ from the components of $\nabla u/|\nabla u|$ (Xu and Zhou, 2014).

To test the accuracy of our modeling approach, we compare the wavefield solution from equation 9 with the low-rank approximation (Song and Alkhalifah, 2013), as well as with the traveltimes from the eikonal solution (Waheed et al., 2014). Figure 2a and 2b shows wavefield snapshots in an orthorhombic medium with the homogeneous model parameters $v_v = 2000$ m/s, $\epsilon_1 = 0.2$, $\epsilon_2 = 0.4$, $\delta_1 = 0.1$, $\delta_2 = 0.15$, and $\delta_3 = 0.05$. Here, δ_1 and δ_2 are the Thomsen's (1986) parameters in the (x, z) and (y, z) vertical planes, respectively. These values correspond to $v_{h1} \approx 2366$, $\epsilon_1 = 0.2$, $\epsilon_d \approx 0.143$, $\eta_1 \approx 0.0833$, $\eta_d \approx 0.093$, and $\delta_3 = 0.05$ in the parameterization considered in this paper. The new wavefield solution matches the kinematics of the low-rank solution, as confirmed by comparison with the traveltimes from the eikonal solution. Obviously, our proposed modeling is free of S-wave artifacts. Figure 2c shows the corresponding S operator. It is clear that S corrects for the anellipticity of the wavefield because its maximum influence resides at 45° propagation direction and is equal to one at the principal axes of symmetry.

Adjoint formulation

The adjoint state method provides an efficient way to derive the gradient of the FWI misfit function through the solution of an adjoint equation (Liu and Tromp, 2006; Plessix, 2006). For simplicity, we consider here the least-squares misfit function defined as

Figure 2. (a) Wavefield snapshots computed with the low-rank approximation, (b) the new modeling approach (equations 9 and 10), and (c) its corresponding S operator. The dashed red contours represent the eikonal solution.

$$J_d(m) = \frac{1}{2} \sum_r \int_0^T \|u(\mathbf{x}_r, t) - d(\mathbf{x}_r, t)\|^2 dt, \quad (11)$$

where m denotes the model parameters, $[0 \ T]$ denotes the time series of interest, $\mathbf{x}_r$ are the receiver locations, $d(\mathbf{x}_r, t)$ is the observed data, and $u(\mathbf{x}_r, t)$ is the modeled data satisfying the wave equation 9, rewritten with a source term f :

$$\partial_{tt}u(\mathbf{x}, t) - S(\mathbf{x})\nabla_E^2 u(\mathbf{x}, t) = f, \quad (12)$$

with initial conditions given by

$$u(\mathbf{x}, 0) = 0, \quad \partial_t u(\mathbf{x}, 0) = 0. \quad (13)$$

Using the adjoint state method, we obtain the adjoint wavefield and the gradients of the misfit function 11. In this case, the adjoint wavefield λ satisfies the following equation:

$$\begin{aligned} & (1/v_{h1}^2)\partial_{tt}\lambda(\mathbf{x}, t) - S(\mathbf{x})\nabla_E^2\lambda(\mathbf{x}, t) \\ & - \nabla_E^2 u(\mathbf{x}, t)\nabla\lambda(\mathbf{x}, t)(\partial S/\partial\nabla u)^T \\ & = \sum_r [u(\mathbf{x}_r, T-t) - d(\mathbf{x}_r, T-t)]\delta(\mathbf{x} - \mathbf{x}_r), \end{aligned} \quad (14)$$

Figure 3. An elastic variable density orthorhombic model parameters obtained from the SEG-EAGE overthrust model.

with initial conditions given by

$$\lambda(\mathbf{x}, T) = 0, \quad \partial_t \lambda(\mathbf{x}, T) = 0, \quad (15)$$

where $\delta(\mathbf{x} - \mathbf{x}_r)$ is the Dirac delta function. The adjoint source in equation 14 is the data residual obtained by subtracting the observed and modeled data set at the receiver locations. The gradients with respect to the parameterization considered in this paper are given in Appendix B. The partial derivatives of S in equation 14 and in the gradients (equations B-1–B-6) are not easy to obtain from the solution of the cubic polynomial 8. Therefore, we approximate its P-wave solution using a first-order Taylor expansion. Specifically, by assuming the anisotropy parameters to be small, we expand the solution of Φ in terms of ϵ_1 , ϵ_d , η_1 , η_d , and δ_3 as follows:

$$\Phi \approx \Phi_0 + \Phi_{\epsilon_d} \epsilon_d + \Phi_{\epsilon_1} \epsilon_1 + \Phi_{\eta_1} \eta_1 + \Phi_{\eta_d} \eta_d + \Phi_{\delta_3} \delta_3, \quad (16)$$

where Φ_0 , Φ_{ϵ_1} , Φ_{ϵ_d} , Φ_{η_1} , Φ_{η_d} , and Φ_{δ_3} are the coefficients in this expansion. By inserting equation 16 into the cubic polynomial 8, one can solve for the coefficients and obtain an approximation of Φ , given by

$$\Phi \approx 1 + 2k_y^2(k_y^2 + k_z^2)\epsilon_d - 2k_z^2\epsilon_1 - 2k_z^2(k_x^2 + k_y^2)\eta_1 + 2k_y^2k_z^2\eta_d + 2k_x^2k_y^2\delta_3, \quad (17)$$

where k_x , k_y , and k_z are the components of the wave vector $\mathbf{k}$. Using the new approximation 17, we obtain expressions for the partial derivatives of S with respect to the model parameters (see Appendix B).

The objective function

In practice, FWI may require some kind of regularization and constraints to reduce the model null-space and to steer the inversion toward geologically meaningful models. In the scope of this paper, we rely on an objective function containing a data-misfit term, a damped TV term, and a structural constraint term:

$$J(m) = J_d(m) + \alpha J_{TV}(m) + \beta J_{\text{cons}}(m), \quad (18)$$

where α and β control the contribution of the penalty terms. To relax the requirement for amplitude matching, we use the normalized crosscorrelation objective function instead of the least-square misfit function. The normalized crosscorrelation evaluates the similarity between the observed and modeled data (Choi and Alkhalifah, 2012):

$$J_d(m) = -\hat{u} \cdot \hat{d}, \quad (19)$$

Figure 4. Example of one shot gather generated from the overthrust model shown in Figure 3.

Figure 5. Initial model parameters used in the overthrust model inversion.

where $\hat{d} = d/\|d\|$ is the normalized observed data and $\hat{u} = u/\|u\|$ is the normalized modeled data. Therefore, we replace the adjoint source in the adjoint wave equation 14 by the following adjoint source (Choi and Alkhalifah, 2012):

$$1/\|u\|((\hat{d} \cdot \hat{u})\hat{u} - \hat{d}). \quad (20)$$

Moreover, we use a smooth TV as a penalty in the objective function (Guitten, 2012):

$$J_{TV}(m) = \int \sqrt{\|\nabla m\|^2 + e^2} dx, \quad (21)$$

where e mitigates the singularity in the gradient. Finally, we use a cross-gradient constraint (Gallardo and Meju, 2003; Hu et al., 2009) to enforce the structural similarity between any two model parameter combinations m_i and m_j :

$$J_{\text{cons}}(m) = \int |\nabla m_i \times \nabla m_j| dx. \quad (22)$$

In the real-data application, as we will explain later, the structural similarity constraint is applied to ϵ_1 and ϵ_d .

IMPLEMENTATION

We developed a 3D finite-difference modeling algorithm and the associated FWI infrastructure valid for an acoustic constant density orthorhombic medium. Our forward modeling relies on solving equation 9 using an explicit time-domain finite-difference algorithm. The main extra step in our modeling approach compared with an isotropic wavefield propagator is the computation of the scalar operator $S(\mathbf{x})$. Furthermore, we use a second-order finite-difference scheme for time stepping and a fourth-order scheme for the space derivatives. The boundary condition for 3D modeling requires computationally efficient implementation, with as little extension to the computational domain as possible. In this study, we rely on the absorbing boundary conditions (Clayton and Engquist, 1977), which compute simplified one-way wave equations to predict the wavefield beyond the boundary. To compute the gradients, we use a strategy based on making the forward (source) wavefield available at the same time step as the adjoint wavefield. We implement here the boundary saving approach (Dussaud et al., 2008; Yang et al.,

Figure 6. Inverted model parameters and their vertical profiles at $(x = 4 \text{ km}, y = 3 \text{ km})$.

2014), in which the final two time steps of the forward wavefield as well as data recorded during the forward propagation step at the boundary of the computation domain are used to run the source wavefield backward in time. During the backward propagation, we use the regenerated source wavefield information to compute the diagonal of the approximate Hessian (Shin et al., 2001). The diagonal of the approximate Hessian (the pseudo-Hessian) is then used to precondition the gradients. Such preconditioning approximately compensates for illumination effects. In other words, it enhances the gradient where the amplitude of the source wavefield is weak and reduces it where the wavefield amplitude is large. Moreover, we improve the descent direction by using the nonlinear conjugate gradient algorithm. Finally, global step lengths that scale each parameter gradient can be estimated using a line-search method in which the second-order approximation of the objective function is evaluated (Pica et al., 1990; Sambridge et al., 1991). This estimation step requires solving an additional forward modeling problem for each parameter, as well as solving a linear system of equations (Masmoudi and Alkhalifah, 2017). Although this estimation can slightly mitigate some of the trade-off issues, in practice, we find it challenging to apply for our objective function (equation 18). Therefore, in this study, we rely on simple decreasing step lengths as a function of iterations.

Our FWI code is designed to run efficiently on large high-performance computing systems by using the Message Passing

Interface library across the shots and OpenMP to accelerate the computation within a compute node across the available cores of the processor. Typically, one shot is run on one compute node. When few nodes are available, we may run in parallel several shots on each node. When this is not possible due to limited memory or a small number of cores per node, we may sequentially run several shots on the same node. Proper synchronization between the compute nodes and input/output management are essential to maintain the code scalability. Retaining as much as possible data in local memory, rather than writing and reading from the disk, preserves the performance of the FWI code. In our implementation, a compute node holds in local memory the model parameters and the observed data for one shot and then it computes the forward wavefield and the adjoint source; it also stores the recorded source wavefield at the boundary of the computation domain and then it computes the adjoint wavefield and crosscorrelates it with the regenerated source wavefield to form the gradients and the pseudo-Hessians. Once all local gradients and pseudo-Hessians are computed, they are spread over all compute nodes and stacked to form a global gradient and a global pseudo-Hessian. Therefore, to reduce the communication through the interconnect and to maintain the scalability of the code, the main information that needs to travel across the network is the gradient and the pseudo-Hessian. Finally, one single node is writing to the disk the updated model at every iteration.

NUMERICAL EXAMPLES

An SEG-EAGE overthrust model inversion

We first show an application of the acoustic orthorhombic inversion on a modified 3D SEG-EAGE overthrust model. By scaling the original P-wave velocity, we generate the elastic variable density orthorhombic model given in Figure 3. The parameters in Figure 3 correspond to the elastic parameterization proposed in Oh and Alkhalifah (2016b). In addition, we add a water layer of 375 m and a transition bumping layer of 375 m on top of the original model. To reduce the computational time, we decrease the total size of the original model and end up using an inversion domain of 8×6 km laterally and 4.75 km in depth. We use this elastic orthorhombic model to generate streamer data. Specifically, 60 shots are distributed on the water surface (at 25 m depth) in three crossline (y-axis) directions at 1.5, 3, and 4.5 km, with 20 shots in each line spaced by 400 m. Data are collected from receivers on the water surface placed at a 25 m depth at all the grid points of the computational domain. One shot gather example is shown in Figure 4. A 12.5 m grid spacing in all three dimensions is used to generate the observed data; however, a 25 m grid spacing is used in our acoustic inversion. The initial model is obtained from smoothing the true acoustic model parameters except δ_3 , which is set to zero (see Figure 5). In practice, MVA or tomography combined with log information are necessary to obtain such an initial model; δ_3 is generally hard to estimate even with these conventional techniques.

In terms of inversion strategy, we rely on the normalized cross-correlation objective function (see equation 19). We also use a hierarchical selection of frequencies by successively inverting three frequency bands (2–6, 2–8, and 2–12 Hz). Moreover, we use a multistage model inversion strategy: We update first the isotropic parameter v_{h1} (20 iterations) at the first frequency band, then the VTI model by including η_1 and ϵ_1 (five iterations) on the second frequency band, and finally the orthorhombic model by including ϵ_d, η_d ,

Figure 7. (a) Example of shot gathers from six different cables and (b) the same shot gathers after a band-pass filter and time windowing.

and δ_3 (five iterations) on the last frequency band. One can also simultaneously update all parameters in one stage. This strategy requires proper gradient preconditioning using the second-order derivatives of the objective function, specifically the Hessian matrix (Pratt et al., 1998; Pan et al., 2016). However, the size of the Hessian matrix makes this computationally challenging (Métivier et al., 2014). On the other hand, if we update parameters that have strong effect on the data first then include additional parameters with less effect at later stages, we will have the potential to reduce the crosstalk between the parameters. In the selected parameterization, we have such an opportunity because there is a continuity in the scattering potential of the model parameters as we move from higher symmetry anisotropy to lower ones (Masmoudi and Alkhalifah, 2016b).

Figure 6 shows the inverted model as well as vertical profiles at the center of the model comparing initial, true, and inverted model parameters. Figure 6a shows that the horizontal velocity is generally well recovered. Indeed, most of the overthrust model structures are captured at their correct position. The parameter ϵ_1 , which affects the data mainly at narrow scattering angles, is not well-recovered here (Figure 6b). Some of the true ϵ_1 structures have been captured; however, additional events leaked to this parameter. Because ϵ_1 and density have a similar influence on the data, it is possible that ϵ_1 has absorbed the density effect. On the other hand, the shallow part of ϵ_d (up to 3 km depth) is relatively well-recovered as shown in Figure 6d. This is in agreement with the radiation pattern analysis, which predicts that large offsets are necessary to update this parameter. Similar conclusions hold for the inverted δ_3 . The main shallow variations (up to 1.5 km depth) at the center of the model are retrieved (Figure 6f). Finally, η_1 and η_d are not well-recovered (Figure 6c and 6e), as predicted, by the radiation patterns analysis and some previous studies pin-pointing the weak influence of η on surface seismic data as a scatterer in this parameterization (Alkhalifah, 2016; Alkhalifah and Guitton, 2016).

We further explain the low-resolution of the inverted η_1 and η_d by the potential trade-offs with the elastic parameters. Indeed, Oh and Alkhalifah (2016b) and Kazei and Alkhalifah (2018) analyze the scattering radiation patterns of the nine orthorhombic elastic parameters; six of them are the acoustic parameters that we are interested in here, and the three others are v_{s1} , γ_1 , and γ_d . Their studies reveal strong trade-offs between v_{s1} and η_1 , as well as between η_d and γ_d when using PP and PS surface seismic data. Therefore, in our acoustic model inversion using elastic data, η_1 and η_d might have absorbed some elastic effects due to these two additional parameters.

Application to real data from the North Sea

We test the 3D acoustic orthorhombic FWI on the Volve data from the North Sea. These data are

acquired on the seafloor (at a 90 m depth) using 12 cables, with 400 m separation. Each cable contains 240 receivers, separated by 25 m. The source vessel has provided shots over an area extending 3 km to the left and right of each cable. The maximum offset in the inline direction is 5 km, whereas the maximum offset in the crossline direction is 1.2 km. The short offset, mainly in the crossline direction, will limit our ability to invert for some of the long-wavelength information of parameters describing azimuthal information. Moreover, the data have been processed by the data owners for migration purposes. An example of a vertical-component shot gather is shown in Figure 7a. One can notice that parts of the diving waves are removed. On the other hand, a tomographic VTI model (see Figure 8) was provided.

We invert for the vertical-component data with frequencies ranging between 2 and 10 Hz. Figure 7b shows one shot gather example used in this inversion after a band-pass filter and a time windowing

Figure 8. The VTI tomographic model parameters: (a) v_{h1} , (b) ϵ_1 , and (c) η_1 .

up to 4 seconds of the original data. We smooth the tomographic model, as shown in Figure 9, and use it as an initial model for FWI. The size of the model is 12.3 (inline), 6.8, (crossline) and 4.5 km (depth). The initial velocity corresponds to a generally increasing velocity with depth with a mild low-velocity zone at approximately 3 km in the target reservoir area (Szydluk et al., 2007). Figure 9b and 9c shows the initial ϵ_1 and η_1 parameters. These two parameters reveal significant anisotropy, especially approximately 2 km depth. We set ϵ_d , η_d , and δ_3 to zero as azimuthal anisotropy information is not available. Given the short crossline offset, the influence of η_d

and δ_3 on the data is expected to be weak; therefore, we do not update these two parameters and we focus only on the inversion of the three VTI parameters (v_{h1} , ϵ_1 , and η_1) and the azimuthal anisotropy parameter ϵ_d .

At every FWI iteration, we randomly select 200 shots from 17123 available shots so as to limit the source and receiver imprints on the inverted model and to reduce the computational cost (Diaz and Guitton, 2011). We invert velocity first (20 iterations), and then we simultaneously update all parameters in a second stage (10 iterations). To resolve sharp interfaces, we apply TV regularization to

Figure 9. The initial model parameters: (a) v_{h1} , (b) ϵ_1 , (c) η_1 , and (d) ϵ_d .

Figure 10. The inverted model parameters: (a) v_{h1} , (b) ϵ_1 , (c) η_1 , and (d) ϵ_d .

the inversion (see equation 21). Moreover, we apply structural constraints (see equation 22) to ϵ_d by constraining its shallow structural layers to be similar to ϵ_1 . The consistency of the structural layering is generally expected. In addition, the deeper part of ϵ_d (less than 2 km depth) is not updated.

The inverted parameters are shown in Figure 10. From the inverted velocity (Figure 10a), one can clearly notice the low velocity (at 3 km depth), which corresponds to the target reservoir area. In

Figure 11. A vertical profile of the initial (red), tomographic (rose), inverted (blue), and the well-log (black) velocity extracted from the well location.

Figure 12. A shot gather comparing (a) the initial modeled data and (b) the final modeled data with the real data.

the inverted ϵ_1 and η_1 , a thin and low anisotropy layer (almost an isotropic layer) at approximately 3 km depth is recovered (see Figure 10b and 10c). This layer is consistent with the low-velocity layer in the reservoir area. Moreover, higher resolution details and sharper boundaries are obtained in the inverted three VTI parameters. On the other hand, we favor smooth updates for the azimuthal anisotropy parameter ϵ_d (Figure 10d) as predicted by the analytical analysis and the synthetic data inversion. A slightly deviating well of the vertical P-wave velocity (Figure 11) allows us to validate the overall inversion results. The well is located at approximately 8.2 km in the inline and 3 km in the crossline directions. Also included in Figure 11 are the initial, tomographic, and inverted vertical velocity. The inverted velocity is consistent with the well-log information and provides a better fit than the tomographic result, especially at depth. However, the high-resolution variations at approximately 2 km depth are not captured in our inverted model. Those details may require higher frequencies to be captured. Furthermore, we show in Figure 12 an initial and a final modeled shot gather using the starting and final models. Data generated with the inverted model contain many reflections that match the real shot gather. Finally, the common image angle gathers (CIGs) from different locations along an inline direction (at $y = 3$ km) and a crossline direction (at $x = 6$ km) are shown in Figures 13 and 14. We notice that the inverted model better flattens the CIGs compared with the initial model at several locations.

Figure 13. Angle gathers for (a) the initial and (b) the inverted model. The angle gathers are shown for multiple locations along the inline direction at $y = 3$ km.

DISCUSSION

The scattering potential of the selected parameterization reveals that the six orthorhombic parameters can be classified into three types depending on the strength of their influence on the data: one parameter that influences the data at all azimuths and all scattering angles (v_{h1}), two parameters that affect the data at all azimuths and certain scattering angles (ϵ_1 and η_1), and three parameters that affect the data at certain azimuths and certain scattering angles (ϵ_d , δ_3 , and η_d). These three classes correspond to three level of model symmetry, which are the isotropic, the VTI, and the orthorhombic model representation. Moreover, the scattering potential of these parameters is stationary (remains unchanged) as we move from a high symmetry model to a lower one, which gives us the opportunity to hierarchically update the three parameter classes.

We generally find that four parameters out of the six describing the orthorhombic medium may be recovered from wide-azimuth P-wave surface seismic data with different resolution scales. Specifically, these parameters are v_{h1} , ϵ_1 , ϵ_d , and δ_3 . The velocity v_{h1} should be well-resolved. It has an isotropic radiation pattern and has the most influence on the data, among all other parameters. The parameter ϵ_1 could be partially recovered. Indeed, this parameter affects vertically traveling waves; therefore, we may only recover its high-wavenumber information. It follows that a good initial ϵ_1 is a requirement; otherwise, ϵ_1 would just play the role of a “garbage” collector

variable in absorbing the density and other data mismatch effects. Finally, smooth ϵ_d and δ_3 could be recovered from wide-azimuth data. However, we find that ϵ_d could be better recovered than δ_3 because it has stronger influence on large-offset data. The two remaining parameters η_1 and η_d might be inverted as well; however, we may not trust their updates given their weak influence of the data and the potential leakage with other parameters. In a nutshell, we could use these initial conclusions to guide the real data applications, in terms of the parameters we should focus on, the resolution expected from each parameter, as well as the potential model regularization that could be applied during the inversion process.

The real data set used in this study is challenging for an orthorhombic model inversion due to the limited offsets especially in the crossline direction. This limitation affects the recovery of the three parameters describing the azimuthal anisotropy. Therefore, we excluded from the inversion δ_3 and η_d and we focused only on ϵ_d , which we constrained to have a similar structure to ϵ_1 . Moreover, diving waves are needed to update the low-wavenumber content of the model. This is essential at the early stages of the inversion process (Bunks et al., 1995). Unfortunately, some of the diving waves are missing from the real data. This issue is compensated thanks to the availability of a good background model.

The inversion examples discussed here are based on a specific objective function, which is mostly sensitive to the phase information. Therefore, the resolution of the inverted model might be limited because of this choice. However, we find it to be reasonably practical for the acoustic approximation based modeling in which the dynamics are not accurate. Finally, the usage of more sophisticated objective functions should mitigate the requirement of accurate initial models.

CONCLUSION

Guided by the insights from the scattering potential analysis and the synthetic data inversion, we successfully applied a 3D acoustic orthorhombic waveform inversion to a field data set. The use of a specific parameterization that preserves the continuity of the scattering potential as we move from higher symmetry anisotropy to lower ones allowed us to propose a multistage model inversion strategy in which the isotropic, then the VTI, and finally the orthorhombic parameters are hierarchically updated. We have shown that P-wave surface seismic data have different sensitivity to the orthorhombic model parameters. The recovery of those depends on data quality and prior information. The synthetic data inversion using a variable density orthorhombic model illustrated the advantages and the limitations of the acoustic approximation. It also helped us explain the potential leakage of the elastic parameters into the acoustic ones. The case study from the North Sea demonstrated the challenges of inverting for the azimuthal anisotropy parameters especially when wide-azimuth information is limited. In this case, we have demonstrated the utility of using model constraints to steer the inverted model toward a more plausible subsurface representation. We have shown that the inverted model matches well data, generates synthetic data that match field data, and better flattens CIGs compared with the initial model.

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Figure 14. Angle gathers for (a) the initial and (b) the inverted model. The angle gathers are shown for multiple locations along the crossline direction at $x = 6$ km.

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APPENDIX A

THE PHASE OPERATOR

The acoustic wave equation for anisotropic media satisfies a linear pseudodifferential equation of the form

$$[\omega^2 - \Phi(\mathbf{x}, \mathbf{k})]u(\mathbf{k}, \omega) = 0, \quad (\text{A-1})$$

where $u(\mathbf{k}, \omega)$ is the pressure wavefield, ω is the frequency, $\mathbf{k}$ is the wave vector, and Φ is the pseudodifferential phase operator. In the orthorhombic case, this yields the cubic polynomial (Song and Alkhalifah, 2013)

$$-\Phi^3 + a\Phi^2 + b\Phi + c = 0, \quad (\text{A-2})$$

where the polynomial coefficients a , b , and c are given below in terms of the parameterization considered in this paper:

$$a = v_{h_1}^2 \left(k_x^2 + (1 + 2\epsilon_d)k_y^2 + \frac{1}{1 + 2\epsilon_1}k_z^2 \right), \quad (\text{A-3})$$

$$b = v_{h_1}^4 \left(((1 + 2\delta_3) - (1 + 2\epsilon_d))k_x^2k_y^2 - \frac{2\eta_1}{(1 + 2\eta_1)(1 + 2\epsilon_1)}k_x^2k_z^2 - \frac{(1 + 2\epsilon_d)((1 + 2\eta_d)(1 + 2\eta_1) - 1)}{(1 + 2\eta_d)(1 + 2\eta_1)(1 + 2\epsilon_1)}k_y^2k_z^2 \right), \quad (\text{A-4})$$

$$c = v_{h_1}^6 \left(-\frac{k_x^2k_y^2k_z^2}{(1 + 2\epsilon_1)(1 + 2\eta_1)} \left((1 + 2\eta_1)(1 + 2\delta_3) - 2\sqrt{\frac{(1 + 2\epsilon_d)(1 + 2\delta_3)}{1 + 2\eta_d}} + (1 + 2\epsilon_d) \left(-2\eta_1 + \frac{1}{1 + 2\eta_d} \right) \right) \right). \quad (\text{A-5})$$

APPENDIX B

THE GRADIENTS

Starting from the least-squares misfit function (equation 11) and using the adjoint state method, one can obtain the adjoint wavefield λ (equation 14) as well as the gradients with respect to the model parameters v_{h_1} , ϵ_1 , ϵ_d , η_1 , η_d , and δ_3 given below:

$$\frac{\partial J_d}{\partial v_{h_1}} = \frac{2}{v_{h_1}^3} \int_0^T \lambda(\mathbf{x}, T - t) \left[\frac{\partial^2 u}{\partial t^2} \right] dt, \quad (\text{B-1})$$

$$\frac{\partial J_d}{\partial \epsilon_1} = \int_0^T \lambda(\mathbf{x}, T - t) \left[-2S \frac{\partial^2 u}{\partial z^2} + \frac{\partial S}{\partial \epsilon_1} \nabla_E^2 u \right] dt, \quad (\text{B-2})$$

$$\frac{\partial J_d}{\partial \epsilon_d} = \int_0^T \lambda(\mathbf{x}, T - t) \left[2S \frac{\partial^2 u}{\partial y^2} + \frac{\partial S}{\partial \epsilon_d} \nabla_E^2 u \right] dt, \quad (\text{B-3})$$

$$\frac{\partial J_d}{\partial \eta_1} = \int_0^T \lambda(\mathbf{x}, T - t) \left[\frac{\partial S}{\partial \eta_1} \nabla_E^2 u \right] dt, \quad (\text{B-4})$$

$$\frac{\partial J_d}{\partial \eta_d} = \int_0^T \lambda(\mathbf{x}, T - t) \left[\frac{\partial S}{\partial \eta_d} \nabla_E^2 u \right] dt, \quad (\text{B-5})$$

$$\frac{\partial J_d}{\partial \delta_3} = \int_0^T \lambda(\mathbf{x}, T - t) \left[\frac{\partial S}{\partial \delta_3} \nabla_E^2 u \right] dt, \quad (\text{B-6})$$

where the operators S and ∇_E^2 are given in equation 10.

The partial derivatives of S with respect to the model parameters are obtained using the approximate phase expression (equation 17)

$$\frac{\partial S}{\partial \epsilon_d} = -\frac{2n_y^2(1 + 2n_y^2(n_y^2 + n_z^2)\epsilon_d - 2n_z^2\epsilon_1 - 2(n_x^2 + n_y^2)n_z^2\eta_1 - 2n_y^2n_z^2\eta_d + 2n_x^2n_y^2\delta_3)}{A^2} + \frac{2n_y^2(n_y^2 + n_z^2)}{A}, \quad (\text{B-7})$$

$$\frac{\partial S}{\partial \epsilon_1} = \frac{2n_z^2(1 + 2n_y^2(n_y^2 + n_z^2)\epsilon_d - 2n_z^2\epsilon_1 - 2(n_x^2 + n_y^2)n_z^2\eta_1 - 2n_y^2n_z^2\eta_d + 2n_x^2n_y^2\delta_3)}{(1 + 2\epsilon_1)^2 A^2} - \frac{2n_z^2}{A}, \quad (\text{B-8})$$

$$\frac{\partial S}{\partial \eta_1} = \frac{-2n_z^2(n_x^2 + n_y^2)}{A}, \quad (\text{B-9})$$

$$\frac{\partial S}{\partial \eta_d} = \frac{-2n_x^2 n_z^2}{A}, \quad (\text{B-10})$$

$$\frac{\partial S}{\partial \delta_3} = \frac{2n_x^2 n_y^2}{A}, \quad (\text{B-11})$$

where

$$A = (n_x^2 + n_y^2(1 + 2\epsilon_d + 1/(1 + 2\epsilon_1)n_z^2))^2, \quad (\text{B-12})$$

where n_x , n_y , and n_z are the components of the unit vector $\mathbf{n}$ of phase direction $\mathbf{n} = \mathbf{k}/|\mathbf{k}|$, approximated from the components of $\nabla u/|\nabla u|$ (Xu and Zhou, 2014).

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