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Variance-based model interpolation for improved
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Running head: **Variance-based model interpolation**

ABSTRACT

When present in the subsurface salt bodies impact the complexity of wave-equation-based seismic imaging techniques, such as least-squares reverse-time migration, and full-waveform inversion (FWI). Typically, the Born approximation used in every iteration of least-squares-based inversions is incapable of handling the sharp, high-contrast boundaries of salt bodies. We develop a variance-based method for reconstruction of velocity models to resolve the imaging and inversion issues caused by salt bodies. Our main idea lies in retrieving useful information from independent updates corresponding to FWI at different frequencies. After several FWI iterations we compare the model updates by considering the variance distribution between them to identify locations most prone to cycle skipping. We interpolate velocities from the surrounding environment into these high-variance areas. This approach allows the model to gradually improve from identifying easily resolvable areas and extrapolating the model updates from those to the areas that are difficult to resolve at early FWI iterations. In numerical tests, our method demonstrates the ability to obtain convergent FWI results at higher frequencies.

INTRODUCTION

Despite rapid development of alternative energy technologies, the worldwide consumption of crude oil continues to increase (EIA, 2017). This increased demand together with the depletion of conventional geologically simple reservoirs have prompted energy companies to explore and develop new oilfields in challenging environments. Promising hydrocarbon reservoirs have, for instance, been found in traps beneath massive salt bodies (Lerche, 2013). Challenges in seismic imaging created by the presence of salt are primarily from the high acoustic impedance contrast and the sharp and steep boundaries between a salt body and its sedimentary environment. Salt is a mechanically weak and light rock, it thus “flows” along the weakest conduit when stress is applied (Leveille et al., 2011). Usually the direction of this flow is toward the surface where the salt later forms large salt canopies surrounded by relatively young sediments. Complex salt structures with sharp and steep flanks with high-velocity areas overriding the slower ones cause illumination issues when imaging salt-affected targets. Most of the propagating energy is reflected back from the surface of the salt, which results in insufficient illumination of the areas beneath the salt body. Successful salt and sub-salt imaging usually requires the integration of geologic knowledge with geophysical data (Jones, 2012) as well as broadband, long-offset and multiazimuth acquisition (Esser et al., 2016).

The problem of seismic inversion in salt-affected regions has been studied by several researchers (see, e.g., Leveille et al., 2011; Jones, 2012, and references therein). The conventional and most robust way to overcome the difficulties faced by full-waveform inversion in the presence of a salt body is to follow the so-called “top-to-bottom” salt-building technique, which incorporates multiple reverse-time migration iterations and extensive manual

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picking of the salt top on migrated images with the consequent flooding steps (Zhang et al., 2009). Despite the robustness of this commonly used approach in industry, it is time consuming and prone to errors often caused by the inherent bias of the interpreter (Dellinger et al., 2017).

Full-waveform inversion (FWI) is a frequently used industrial technique for high-fidelity subsurface imaging (Tarantola, 1984; Virieux and Operto, 2009). When properly implemented FWI allows inversion of complex, high-contrast models with minimal manual interventions. Low frequencies are crucial to conventional acoustic FWI convergence (Bunks et al., 1995; Kazei et al., 2013; Baeten et al., 2013; Kazei et al., 2016) and necessary for decoupling parameters in multiparameter FWI (Podgornova et al., 2015; Kazei and Alkhalifah, 2018). Unfortunately, in most cases, low frequencies are not present in seismic exploration data sets. The lack of low frequencies can trap FWI in a local minimum of the objective functional. The other challenge when inverting for the salt is lack of internal reflectivity which causes the ambiguity in inversion results within salt bodies. In the latter case, data misfit functional is not sensitive to changes in velocity. Prior assumptions about possible inversion results are therefore necessary.

There are two general approaches to address these challenges in FWI. The first approach is to assume that data misfit functionals should be modified. Smoother objective functions e.g., envelope or cross-correlation based functions lead to fewer local minima and therefore FWI is more likely to converge to the global minimum of the misfit functional (Van Leeuwen and Mulder, 2010; Bozdağ et al., 2011; Chi et al., 2014; Choi and Alkhalifah, 2015). Furthermore, adding regularizations (Kalita et al., 2018) and/or constraints (Baumstein, 2013) to the inversion can lead to better convergence toward the global minimum. Examples of constrained inversions include works by Esser et al. (2015, 2016) and Guo and

de Hoop (2013), who used a total variation norm to invert a salt-affected subsurface using FWI. Regularized inversion methods could be computationally expensive when multiple runs on a fine mesh are needed and these methods often require prior assumptions about the subsurface structure (Kadu et al., 2016).

The second approach involves image or gradient manipulations with single iteration updates of FWI. Processed gradients can lead to shorter paths toward the global minimum without being trapped in the local minima (Alkhalifah, 2015b, 2016). Image-guided inversion (Ma et al., 2012), gradient optimization (Wu and Alkhalifah, 2016) or gradient conditioning through scattering angle-based filters (Alkhalifah, 2015a; Kazei et al., 2016) can serve the same purpose.

The objective of FWI is to minimize both amplitude and phase differences between observed and modeled seismic data. For successful inversion the Born approximation requires the initial velocity model to deliver mismatches in travel times less than half the period (Beydoun and Tarantola, 1988). Larger errors cause cycle-skipping artifacts on the wavepaths specific to each source-receiver pair. These artifacts in the image domain can be identified as repeated contrast velocity anomalies that, in turn, lead the inversion to convergence toward a local minimum on an objective function (Virieux and Operto, 2009). Inversion of mono-frequency data allows features to be resolved at a specific scale. The inversion results from different mono-frequencies therefore do not match exactly.

When mono-frequency data are modeled, the phase of a harmonic plane wave at a given point on a wavepath depends on the frequency and travel time. Hence, at different frequencies, the phases of the waves propagating through the same wavepaths differ. While minimizing the misfit between observed and modeled data, FWI amends the velocity model

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to match the phase shift with its nearest zero crossing at a multiple of 2π . This phase shift varies at each frequency, causing the data to be fitted differently. These shifts cause cycle-skipping artifacts to vary from frequency to frequency for a particular wave.

Here, we extend the second group of methods to address the challenges in FWI in the presence of salt bodies by manipulating updates acquired in parallel from several FWI iterations at different mono-frequencies rather than just at gradients. In particular, we construct problematic regions based on the variance distribution between these updates. Following Ovcharenko et al. (2017), we iteratively introduce corrections into these problematic regions in the model, leading FWI to better convergence. As a result, the proposed technique provides a more robust initial velocity model for conventional FWI.

In the following, we first discuss the general features of cost functions. We then present a method for variance-based salt body reconstruction using FWI. Finally we demonstrate the applicability of our method using crops from the left and central parts of the BP 2004 velocity benchmark model (Billette and Brandsberg-Dahl, 2005) and using a cross section of the SEAM Phase I model (Fehler and Keliher, 2011).

THEORY

Full-waveform inversion is a technique that performs high-resolution subsurface imaging by minimizing the difference between modeled and observed seismic data. Theoretically, FWI performs a search in the model space to find a subsurface velocity model that provides a minimum value of the so-called misfit or objective function, J (Tarantola, 1984; Virieux and Operto, 2009). The most familiar and simple cost function is defined by the squared L_2 norm of the differences between observed, $\mathbf{d}_0$, and generated, $\mathbf{d}$, seismic data for model

m:

$$J(\mathbf{m}) = \|\mathbf{d}(\mathbf{m}) - \mathbf{d}_0\|_2^2. \quad (1)$$

The function, J , is generally nonlinear (Sirgue and Pratt, 2004) and non-convex with multiple local minima (Mulder and Plessix, 2008). Gradient-based optimizations therefore pose a major challenge due to the fact that a search could prematurely stop in one of minima (Figure 1) and cause selection of an improper velocity model.

A popular technique to improve data fit delivered by FWI is the multiscale approach proposed by Bunks et al. (1995). The multiscale approach implies successive inversions of low-pass-filtered data, $\mathbf{d}_{0,\omega}$, from lower to higher frequencies, ω :

$$J(\mathbf{m}) \approx \sum_{\omega} \tilde{J}(\mathbf{m}, \omega) = \sum_{\omega} \|\mathbf{d}(\mathbf{m}, \omega) - \mathbf{d}_0(\omega)\|_2^2 \quad (2)$$

Cost functions, $\tilde{J}$, in inversions of low-frequency data tend to have local minima that are farther apart than cost functions at higher frequencies, as schematically illustrated in Figure 2. This allows an optimization technique to converge to a lower minimum of J and, thus, to recover an improved low-wavenumber model. Later, this low-wavenumber subsurface image is used as a starting model for FWI at higher frequencies to recover high-resolution features of the model. However, in the absence of low frequencies this approach is less reliable for complex velocity models. Even in the presence of low frequencies, the complexity of the model may require an impractical number of FWI iterations.

Within the multiscale approach, it is common practice to simultaneously invert for several neighboring frequencies instead of a single one (Brossier et al., 2009; Sourbier et al., 2009). According to this practice, the whole range of frequencies is split into groups (bands) and then gradients from different frequencies in the same band are summed at each itera-

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tion. This provides an enhanced signal-to-noise ratio in the gradient, thus improving the convergence when dealing with models that generate complex wave phenomena.

A lack of low-frequency data or of a decent initial model might result in synthetic data that are greater than a half-cycle away from observed data (Alkhalifah, 2016). This leads inversions to converge to a local minimum, which in the model domain could be associated with the presence of cycle-skipping artifacts (Virieux and Operto, 2009). Long-offset data that illuminates deep parts of the model are generally the most vulnerable to cycle-skipping issues (Mulder and Plessix, 2008). This happens due to larger phase shifts accumulated when seismic energy follows longer wavepaths. Deep targets are therefore more likely to be inverted erroneously.

In salt-body models, a significant part of the wave energy is reflected back from the top of the salt without propagating in the salt body. This reflection contributes to the finding that the top of the salt is well resolved in most FWI cases whereas the sub-salt areas become corrupted. Another scenario occurs when the margins of the salt body are properly identified from reflections but the inner content of the salt cannot be uniquely resolved. Corresponding objective functions of inversions of salt-body models at different single frequencies differ significantly in the same model. The inversion could prematurely stall in a local minimum of the objective function rather than searching for the best model. An important observation is that, in the synthetic case, the global minimum remains the same for misfit functions at all frequencies, whereas the local minima are different. Whenever the data are corrupted by noise the global minimum can shift when the frequency is changed, but variations in the local minima will, most likely, be larger. When the L_2 norm of the noise is higher than half of the difference between the misfit functional values at global and local minima, several global minima can potentially exist for a single misfit functional. Stacking several

neighboring frequencies helps in such a case (Brossier et al., 2009).

In our approach, we exploit the invariance of the global minimum for a selection of carefully determined frequencies. Consider a set of successive mono-frequency data, starting from 3 Hz which is feasible to be collected in the field with good signal-to-noise ratio (Maxwell and Lansley, 2011). Without very low-frequency data, such as below 2 Hz, independent inversions at each monofrequency will stop at their own local minimum in the vicinity of a point on the objective function corresponding to the initial guess, as is schematically shown in Figure 2. These minima are different due to the lack of global convexity of standard FWI functionals. The mismatch in the stopping points in the objective function of different frequencies indicates the convergence of FWI to different subsurface models. The key concept of our proposed approach is the measurement of mismatches between corresponding model updates. To this end, we use the variance between velocities at every point in the models.

We assume that the presence of cycle-skipping artifacts in the image domain indicates a local minimum of the objective function. We compare model updates computed in parallel for different single frequencies, using a variance criterion to identify regions in the image where the optimization has stalled with different model results. For salt-body reconstructions, we correct selected variable areas using the proposed “flooding” technique in the next section and repeat the process to obtain an improved initial model for FWI.

METHOD

In our method we use the variance between the resulting models at different frequencies to guide the flooding process. This includes identifying a sufficiently accurate starting model.

Variance-based salt flooding

Model updates from different frequencies deviate from each other. By comparing these updates, we can localize regions associated with global and local minima in the model domain. For this purpose, we compute in parallel updated subsurface models, $\mathbf{m}_k$, for a number, K , of distinct frequencies using FWI. Figure 3b-c shows such models, $\mathbf{m}_k$, for one of the chosen test cases. The number of distinct frequencies is arbitrary, but our tests revealed that four distinct frequencies are usually sufficient.

The selection of frequencies is motivated by the need to have variation in the cycle-skipping artifacts between the updates, while the updates for the easily resolvable areas should be similar. The frequencies therefore should not be too close to each other to detect the variation and not too far apart to have similar wavenumber coverage. Because a constant ratio between subsequent frequencies is natural for frequency-domain inversions (Sirgue and Pratt, 2004; Mulder and Plessix, 2008), we scaled the frequencies with a small coefficient inside our bands.

We constrained the scaling ratio such that the maximum frequency used for flooding, f_{max} , would not exceed double the minimum frequency f_{min} . This allows some similarity of updates from multiples of f_{min} to be mitigated, which we observed in our experiments. This similarity can be roughly explained in the following way. When the frequency is doubled the phase shifts between the observed and modeled data are doubled. As a consequence, if the phase shift was very close to 2π for frequency f , it would be close to 4π for the frequency $2f$ and the updates would be very similar in approximately half of the cases. This suggests that the updates from these frequencies are very likely to be similar even for cycle-skipped areas.

This requirement suggests that the frequencies, f_k , required for flooding should be distributed in the range $f_k \in [f_{min} \dots 2f_{min})$. The coefficient that allows $K = 4$ frequencies, f_k , in this band is then defined

$$\frac{f_{max}}{f_{min}} < 2, \quad f_{k+1}/f_k < \sqrt[3]{2} \approx 1.259. \quad (3)$$

To satisfy equation 3, we take

$$f_{k+1}/f_k = 0.9^{-1} \dots \quad (4)$$

An example of this approach is visualized in Figure 4.

Updated models, $\mathbf{m}_k$, are together passed through the following three steps of the flooding procedure:

1. **Weighted model average.** Due to the local nature of cycle-skipping artifacts, they should vary in updates from different frequencies. To enhance regions that match among most models and to diminish those that are different, we sum up all models and find the weighted average of the FWI updates. As a result, we create a background model, $\mathbf{m}_b$, in which cycle-skipping artifacts are reduced and well-defined features are enhanced:

$$\mathbf{m}_b = \frac{\sum_{k=1}^K \mathbf{m}_k w_k}{\sum_{i=1}^K w_i}, \quad w_k = \frac{1}{f_k}, \quad (5)$$

where $\mathbf{m}_k$ is the model update from FWI at frequency f_k . The weighting term, w_k , obtains its largest value at the lowest frequency. Such averaging permits data from the lower frequencies to contribute more to the averaging because low-frequency content is less corrupted by numerical artifacts and provides, in general, smoother models. Figure 3d presents a weighted average model for the BP test case.

2. Weighted model variance. Because variance is a measure that indicates how much a variable alternates from its weighted average value, high values indicate large differences in model updates. High-variance areas mostly appear near or within salt bodies where cycle-skipping is the most prominent (Figure 5a). These uncertain areas are prone to be problematic and thus require additional processing to weaken the artifacts and to help to avoid local minima. We find the variance distribution, $\mathbf{V}$, point-wise among the model updates:

$$\mathbf{V} = \frac{\sum_{k=1}^K w_k (\mathbf{m}_k - \mathbf{m}_b)^2}{\sum_{i=1}^K w_i}, \quad (6)$$

where w_k has the same weighting term that was used to find the background model, $\mathbf{m}_b$, and $\mathbf{m}_k$ are the model updates for frequencies f_k .

Variable variance threshold. Let $\epsilon \in [0, 1]$ be a coefficient that separates a normalized variance distribution into normal and anomalous parts (Figure 5b). Anomalous parts are those where the variance among the updated models is relatively high, implying that these areas could be associated with local minima. Anomalous regions shape a “variance mask”, $\mathbf{V}_m$, which overlaps “suspicious” regions:

$$\mathbf{V}_m = \begin{cases} 1, & \text{if } \frac{\mathbf{V}}{V^{max}} > \epsilon \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

We change the threshold, ϵ , on the fly depending on the ratio between the mean V^{avg} and the maximum V^{max} values of the original variance distribution, $\mathbf{V}$. When the optimization technique finds the global minimum, the minimum is often stationary for all frequencies, f_k . This implies that the difference between updates is small and therefore the average and maximum values of the variance distribution are close to each other. For all flooding iterations, subscripted as i , we store the mean variance distribution, V_i^{avg} , normalized by

its maximum, V_i^{max} . Then, we modify the threshold, ϵ_n , depending on this ratio and its history. We define anomalous areas in the model as those that meet the relation given by equation 7, where:

$$\epsilon_n = \epsilon_0 \frac{\max_{i=1,n} \hat{V}_i^{avg}}{\hat{V}_n^{avg}}, \quad \hat{V}^{avg} = \frac{V^{avg}}{V^{max}} \quad (8)$$

Threshold, ϵ_n , is computed at each flooding iteration, n , and increases when the mean and maximum values of the variance distribution are close. Generally, ϵ_n increases with iterations as updates at given different frequencies become increasingly similar as model converges and $\hat{V}_n^{avg}$ decreases. The initial threshold, ϵ_0 , is set to the empirical value 0.2 found by the examination of the initial variance distribution, $\mathbf{V}$.

3. Flooding of high-variance areas. All regions covered by the variance mask, $\mathbf{V}_m$, are point-wise filled with the power mean (Bullen, 2003) of the values found within a circle with a radius of half a local wavelength. We assume that such a radius corresponds to the average size of a local cycle-skipping artifact. The power of the mean, p , defines how biased the result will be toward higher values. The empirical rule is to set p equal to the signal-to-noise ratio (SNR) in the observed data. SNR is defined as the ratio between the energies of the signal and the noise. The maximum local wavelength at each point of the model is determined by

$$\Lambda_{max} = \frac{\mathbf{m}_b}{f_{min}}, \quad (9)$$

where $\mathbf{m}_b$ is the weighted average background velocity model and f_{min} is the minimal frequency at which inversions are performed. The point-wise expression for flooding with a power mean is

$$m_b^{(i,j)} = \sqrt[p]{\frac{\sum_k \sum_l (m_b^{(k,l)})^p}{\sum_k \sum_l 1}}, \quad k, l \in S \quad (10)$$

$$S : \sqrt{(i-k)^2 + (j-l)^2} \leq \frac{\lambda_{ij}^{max}}{2}.$$

Depending on SNR, there are three natural outcomes from the equation 10. When the data are noise free, the power of the mean $p \rightarrow \infty$ delivers a maximum value from circle S to model update $m_b^{(i,j)}$. Whereas when the signal and noise are indistinguishable, $p = 1$, the output is an arithmetic average within the circle. The higher the SNR, the closer the value of the update to the maximum within the circle. The high-variance areas in Figure 5a are processed according equation 10. This allows the algorithm to average the cycle-skipping artifacts with relevant content from inside the circle region. A sample of a resulting flooded velocity update after the first flooding iteration is shown in Figure 6.

We applied this three-step flooding algorithm to results from FWIs computed in parallel at single frequencies. We iteratively repeated the three steps until the introduced V^{avg}/V^{max} ratio stagnated.

Selection of the starting model

The initial model for FWI should provide a fair approximation of the general background of the area under study. Here, we used a one-dimensional velocity model with a water layer on top as the initial model. The suitable slope, β , of the linear model was found by brute-force search. This minimizes the misfit between observed and modeled data. An analytical function describing the set of initial models can be written as

$$v(z) = v_w + \beta \max(0, z - w), \quad (11)$$

where v_w is the velocity in the water layer, w is the known water depth, and β is the variable slope of the one-dimensional velocity model. The gradient could start either from a flat sea floor, or from real bathymetry or be a combination of two linear models.

RESULTS

We demonstrate the applicability of our variance-based salt flooding technique in a two-dimensional, isotropic acoustic medium mimicking geological structures of the deepwater area of the Gulf of Mexico. Our first and second numerical examples demonstrate implementation of the method with synthetic noise-free and noisy data (SNR=10) generated from sections from the central and left parts of the BP 2004 model (Billette and Brandsberg-Dahl, 2005), respectively. In the last example, we operate with noise-contaminated data (SNR=5) generated for the east-west cross section of the SEAM Phase I velocity model (Fehler and Keliher, 2011). Moreover, we assume the water depth and the velocity to be known.

Central part of BP 2004

The central part of the BP 2004 model is challenging for FWI due to a massive salt body with steep flanks and a hidden low-velocity intrusion (Figure 3a). This intrusion is a potential trap for hydrocarbons. The primary goal is therefore to gain a satisfactory image of the area below the salt body. The acquisition involved 241 receivers and 121 sources uniformly distributed on the surface with respective spacing of 40 and 80 meters. The size of the submodel was $3.5 \text{ km} \times 12 \text{ km}$; the model was discretized by a Cartesian uniform

grid 175×600 with 20 m spacing in both directions. The frequency range for multiscale FWI was from 3 to 7.46 Hz with a multiplicative increment of 1.2.

The initial model for mono-frequency multiscale FWI was built using the proposed salt flooding technique. As input, the algorithm was given velocity updates from four single frequencies that were expected to provide optimal variance content. These frequencies were sampled according to equation 4 from the minimum available frequency of 3 Hz with the step size equivalent to the change in local wavelengths of 10%. The resulting set of frequencies was 3.0, 3.33, 3.7 and 4.12 Hz. The initial variance threshold ϵ_0 was 0.2. Any variation between the updates above this threshold was considered to be anomalous. This initial value was roughly selected by examination of the variance distribution (Figure 5b) such that it would separate the dominant peak.

Several input models to the flooding routine are shown in Figure 3b-c. These are the updates from FWI at the lowest and highest single frequencies after 15 iterations using a limited-memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS) algorithm (Liu and Nocedal, 1989). Cycle-skipping artifacts (repeated velocity anomalies) are clearly visible on each updated model.

At the lowest of available frequencies used for the flooding, we performed forward modeling on a coarser mesh to ensure that there were at least ten grid points per shortest wavelength in the model. This is equivalent to 88×300 grid points with 40 m spacing. This problem size reduction leads to faster computation for FWI only within the three-step flooding procedure. We start our flooding approach considering the background model shown in Figure 7a.

The variance distribution between updates is the key ingredient of the proposed approach. This distribution pinpoints regions in the model that were reconstructed in the most ambiguous way. Thus, we compute the variance between the updates from a set of single frequencies according to equation 6. Following the iterative flooding workflow described in the previous section, we reconstruct the shape of the salt body with its principal features, such as its internal cavity. The resulting starting model for multiscale FWI is shown on Figure 7b. It took 20 runs of the three-step flooding procedure to build this model. The proposed technique requires more iterations to converge when applied to noise-free data. With such data, a variance mask likely covers accurately a high-variance region of the velocity model where most of the cycle-skipping artifacts occurred. The flooding procedure performs weighted averaging inside a circle area with a radius of half a local wavelength. Thus, the more homogeneous area is covered by the circle, the less changes are introduced by weighted averaging. As a result, it either takes longer to populate the anomalous areas according to equation 10 or the process stagnates when the artifacts from distinct frequencies align completely.

Non-regularized conventional multiscale FWI including a series of successive mono-frequency inversions fails when started from a one-dimensional initial model (Figure 7a). As expected, the top of the salt is correctly constructed whereas other parts are corrupted by cycle-skipping artifacts (Figure 8a). In contrast, our proposed iterative flooding approach allows for the preparation of an adequate starting model (Figure 7b) that leads FWI to a better final result (Figure 8b). As shown in Figure 8b, the low-velocity anomaly beneath the salt body surrounded by thin salt “legs” is well resolved. Usually, such thin salt intrusions beneath a massive salt body can hardly be identified by conventional FWI. The mismatch between the inverted model and the true model in the right bottom corner is caused by

poor illumination in that area.

Left part of BP 2004

Potential hydrocarbon reservoirs in the left part of the BP 2004 velocity model (Figure 9a) can be found immediately beneath an elongated salt body (Billette and Brandsberg-Dahl, 2005). The impedance contrast at these areas is higher than elsewhere in the model, which suggests multiple difficulties when imaging these targets. The crop is $2.5\text{ km} \times 13\text{ km}$ in size and it is also discretized by a uniform Cartesian grid with 20 m spacing (175×650 grid points). The acquisition involved 314 receivers and 158 sources uniformly distributed along the surface with respective spacing of 40 and 80 m. SNR was set to 10, which is relatively high for surface seismic.

We start our flooding approach based on the background model shown in Figure 9c. The highest variance between the updates occurred in the areas below the salt (see Figure 9b). This variance indicates that there is uncertainty in the inversion results in these areas. After 12 runs of the flooding procedure, the mean and maximum values of the variance distribution grew close and the stopping criterion for the flooding loop was reached. The flooded velocity model is shown in Figure 9d. In the noise-free case, the number of FWI iterations at each flooding step varies because the exit criterion is the stagnation of the functional decrease. In the presence of noise (SNR=10), this approach does not work well. We therefore set a constant number of FWI iterations preceding each flooding iteration. In this example, each flooding run required 10 L-BFGS iterations as this many iterations delivered stagnation of the gradient. The flooding was performed on a sparse, resampled computational grid with 88×325 grid points due to the involvement of only long-wavelength

data.

Conventional multiscale FWI with total variation (TV) regularization (Rudin et al., 1992) starting from 1D velocity model (Figure 9c) succeeded in imaging the top of the salt, but failed to reveal the sub-salt structures (Figure 10a). Cycle-skipping artifacts were easy to discern within the salt body where the initial velocity model differs significantly from the true one. Given that we used the flooded velocity model (Figure 9d) as the initial model for multiscale FWI, we could recover both the salt body and the area beneath the salt. Despite the relatively low frequencies used for FWI, most short-wavelength features could be distinguished in the final inversion result (Figure 10b). One of the targeted low-velocity areas below the salt, likely a reservoir, is inverted in its correct location and can clearly be identified. The small reservoir to the left in the area below the salt could not be recovered probably due to the presence of noise in the data.

Cross-section of SEAM Phase I

The subsurface model built on the SEAM Phase I dataset mimics a realistic geology of a salt-containing region in the Gulf of Mexico (Fehler and Keliher, 2011). There is a massive salt body with steep flanks embedded into a layered sediment environment (Figure 11a). We implement the variance-based technique on a resampled 2D subset of the model taken in the east-west direction from North 23.9 km with dimensions of $3 \text{ km} \times 8 \text{ km}$. We run FWI from 3 to 10 Hz. The target area was the sedimentary basin hidden beneath a hanging salt flank.

The model was discretized with 150×438 grid points with 20 m spacing. The acquisition involved 106 receivers and 53 sources uniformly distributed along the surface with respective

spacing of 40 and 80 m. The presence of noise is ubiquitous in real data applications, we therefore added random Gaussian noise with SNR of 5 to the data.

We initiated the flooding procedure on a sparse grid (75×298 grid points) starting from a linear velocity model with known sea floor bathymetry (Figure 11c). During seven flooding runs we performed 15 iterations of mono-frequency non-regularized FWI in parallel at 3.0, 3.33, 3.7 and 4.12 Hz. The algorithm computed variance between updated models from listed frequencies and applied corrections according to equation 10, which ultimately resulted in the low-wavenumber initial model shown in Figure 11d. The variance distribution before the first iteration of the flooding procedure was dominated by inconsistencies that were located primarily in the top part of the salt body (Figure 11b). This was because the most significant velocity contrasts occurred there. The variance threshold ϵ_0 was 0.2. The number of FWI iterations was selected after examination of the data misfit functions and remained constant for all inversions at all frequencies (Figure 12).

To provide full-scale FWI, we ran successive inversions of mono-frequency data at 3.00, 3.6, 4.32, 5.18, 6.22, 7.46 and 10.75 Hz in the framework of conventional multiscale FWI (Sirgue and Pratt, 2004). Conventional FWI regularized with TV failed due to the lack of low frequencies (Figure 13a). However, it could capture top of the salt although deeper parts, such as the complex basement and the salt body itself were not captured.

When started from the initial model produced by the flooding procedure, the final result from the TV-regularized inversion was improved (Figure 13b). While the resulting model is still mildly blurry, all boundaries of the main salt body, and the high-velocity arch in the basement are resolved in the final model.

DISCUSSION

The proposed variance-based reconstruction technique extends the applicability of conventional FWI to complex salt bodies. As input, it uses model updates at certain discrete frequencies. As output, the technique provides an alternative model update that can be used as an initial model for the next FWI iteration. The proposed procedure can be easily embedded into any conventional FWI routine, regardless of the solver or computational domain because only model updates for corresponding single frequencies are required as input.

When forward modeling is performed in the time domain, on-the-fly discrete Fourier transform (Goertzel, 1958) can be used to extract specific frequency content from the data. Apart from a set of standard parameters involved in frequency-domain FWI, we used the following additional tuning parameters in the proposed procedure:

Number of FWI iterations before flooding. We used L-BFGS with a stopping criterion given by the relative decrease in the functional that was less than 10^{-4} . This empirically resulted in 10 to 20 L-BFGS iterations per flooding cycle. This parameter is dependent on the FWI realization and problem. It should therefore be determined directly from examining a specific misfit function. The number of iterations depends on the choice of optimization algorithm. In a reasonably complex model, quasi-Newton methods are typically the best choice for FWI implementation. A truncated Newton method in some cases can be a better option to handle multiscattering (Métivier et al., 2013). Potentially, a single but computationally more expensive iteration of the truncated Newton could replace several L-BFGS iterations currently used for variance map construction. Another option would be to include multi-scattering in the inversion as suggested by Alkhalifah and Wu

(2016).

Step size between single frequencies. The step size has to be sufficient to cause a visible shift of the cycle-skipping artifacts. We scaled the frequencies such that the wavelength at the same location for the next frequency would be 0.9 of the previous one, which provides a bound on the frequency scaling factor that depends on the number of frequencies.

Initial variance threshold. The parameter $\epsilon_0 \in [0, 1]$ separates normal and anomalous values in the variance distribution. An arbitrary case can be found by algorithmic or visual examination of the variance distribution such that the ϵ plane cuts off the dominating high-amplitude values. From our tests, an empirical value of 0.2 is recommended. This threshold should be amended if the flooding process does not converge.

Power of the mean. According to equation 10, parameter p indicates how biased the flooding will be toward higher values. We set p equal to SNR in the observed data as noise adds ambiguity to the inversion, which then suggests that the averaging in unstable areas be milder to avoid unrealistic high values in the model.

Number of flooding iterations. The process should stop automatically when the high-variance regions are gone. Equation 8 introduces a variance-dependent function that stagnates with iterations when the maximum and mean values of the variance distribution converge. The relative decrease in the result of equation 8 serves as a stopping criterion for flooding iterations.

In total, there is only one purely user-defined parameter – the initial variance threshold, $\epsilon_0 \in [0, 1]$, whose recommended default value is 0.2. The number of FWI iterations before

1
2
3
4 flooding depends on the optimization technique used. The step between frequencies is
5
6 selected from the empirical wavelength ratio (equation 4). The iterative flooding procedure
7
8 terminates automatically when the variance distribution in the model appears to be close
9
10 to homogeneous.

11
12 The computational cost of our proposed variance-based salt-flooding procedure is de-
13
14 termined by the lowest frequency available in the data. The flooding routine itself only
15
16 computes the variance between the updated models and then performs correction on the
17
18 most unstable areas. The execution time of three-step flooding iteration in most cases is
19
20 therefore negligible. Even for large models this process takes only a few seconds. The major
21
22 computational cost is introduced by the preceding iterations of the optimization technique
23
24 that searches for minima of the objective functions in the implemented FWI routine. The
25
26 numerous FWI iterations required for the salt flooding procedure should be performed on a
27
28 sparse computational grid. This is because only data in the vicinity of the lowest available
29
30 frequency are required to prepare the starting velocity model for full-scale FWI. The pro-
31
32 posed approach is only expected to produce an adequate initial model for full-scale FWI,
33
34 therefore only lowest available frequencies need to be modeled on a coarse mesh. Model
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36 coarsening dramatically reduces the computational costs of FWI iterations preceding the
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38 flooding procedure, making the proposed workflow feasible. All results presented in this
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40 work were computed on a laptop with a quad-core CPU. The complete iterative salt flood-
41
42 ing procedure on a sparse grid for each of the examples took between five and ten minutes
43
44 whereas the subsequent multiscale FWI on a dense grid took about 40 minutes.

45
46 When there is noise in the observed data, the performance of the variance-based ap-
47
48 proach is improved. This is because more flooding iterations are likely necessary with the
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50 noise-free data for the algorithm to interpolate from the robustly defined regions into the
51
52

ones covered by the variance mask.

To verify if the model created by the flooding procedure is ready for FWI, some quality control (QC) procedures might be necessary. These may include examination of extended images or angle gathers (Sava and Fomel, 2003) to avoid misinterpretation of salt bodies in practice.

As a final remark, the starting one-dimensional velocity model for FWI has to be reasonable (as is the case for any other model used for FWI). A completely inconsistent initial guess of the velocity would inhibit FWI from recovering any part of the salt body that should be partly resolved in realistic cases, such as the top of the salt. In a worst case scenario, the variance-based approach will not work properly when an adequate variance distribution among the updates cannot be retrieved, which may happen when several iterations of FWI are not able to provide any information about the salt body. Nevertheless, the use of robust data at the lowest available frequency together with the salt flooding procedure on a corresponding coarse mesh allows us to obtain an adequate initial model for FWI in a cost-effective way.

CONCLUSIONS

Conventional FWI fails in the presence of the salt bodies without the aid of low-frequency data. Here, we proposed a new approach to frequency-domain FWI that allows us to perform inversions without low frequencies, without significant modifications to the optimization workflow and with only a moderate increase in computational complexity. The novelty of the approach lies in looking at nonlinear updates of FWI from several single frequencies rather than just at gradients or migration images. These updates show significant similarity

if the model is mature enough to be handled by Born approximation, unlike reverse-time migration images that are different for different frequencies in a complicated medium. The variance distribution between these updates is used as an indicator to reveal areas where FWI fails. We update these areas through a simple, automated flooding process with the power mean of velocities from nearby locations.

The only inputs for a single iteration of the proposed procedure are several model updates from conventional FWI. Incorporating our flooding procedure into any existing inversion routine is therefore simple. Designed to work with the lowest frequencies available, the procedure allows the use of sparser grids for modeling and inversion for the preparation of an improved initial model to be used at later FWI stages. Moreover, these additional computational costs are negligible within a complete multi-scale FWI scenario. The proposed technique could be easily extended to 3D and more realistic physics as it operates exclusively in the model space.

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REFERENCES

Alkhalifah, T., 2015a, Scattering-angle based filtering of the waveform inversion gradients: Geophysical Journal International, **200**, 363–373.

———, 2016, Full-model wavenumber inversion: An emphasis on the appropriate wavenumber continuation: Geophysics, **81**, no. 3, R89–R98.

Alkhalifah, T., and Z. Wu, 2016, Multiscattering inversion for low-model wavenumbers: GEOPHYSICS, **81**, no. 6, R417–R428.

Alkhalifah, T. A., 2015b, Full model wavenumber inversion: ASEG Extended Abstracts, **2015**, 1–1.

Baeten, G., J. W. de Maag, R.-E. Plessix, R. Klaassen, T. Qureshi, M. Kleemeyer, F. t. Kroode, and Z. Rujie, 2013, The use of low frequencies in a full-waveform inversion and impedance inversion land seismic case study: Geophysical Prospecting, **61**, 701–711.

Baumstein, A., 2013, POCS-based geophysical constraints in multi-parameter full wavefield inversion: Presented at the 75th EAGE Conference & Exhibition incorporating SPE EUROPEC 2013.

Beydoun, W. B., and A. Tarantola, 1988, First born and rytov approximations: Modeling and inversion conditions in a canonical example: The Journal of the Acoustical Society of America, **83**, 1045–1055.

Billette, F., and S. Brandsberg-Dahl, 2005, The 2004 BP velocity benchmark: Presented at the 67th EAGE Conference & Exhibition.

Bozdağ, E., J. Trampert, and J. Tromp, 2011, Misfit functions for full waveform inversion based on instantaneous phase and envelope measurements: Geophysical Journal International, **185**, 845–870.

Brossier, R., S. Operto, and J. Virieux, 2009, Seismic imaging of complex onshore structures

by 2D elastic frequency-domain full-waveform inversion: *Geophysics*, **74**, no. 6, WCC105–WCC118.

Bullen, P., 2003, *Handbook of means and their inequalities*, vol. 260 Kluwer Academic Publisher: Dordrecht, The Netherlands.

Bunks, C., F. M. Saleck, S. Zaleski, and G. Chavent, 1995, Multiscale seismic waveform inversion: *Geophysics*, **60**, no. 5, 1457–1473.

Chi, B., L. Dong, and Y. Liu, 2014, Full waveform inversion method using envelope objective function without low frequency data: *Journal of Applied Geophysics*, **109**, 36–46.

Choi, Y., and T. Alkhalifah, 2015, Unwrapped phase inversion with an exponential damping: *Geophysics*, **80**, no. 5, R251–R264.

Dellinger, J., A. Brenders, X. Shen, I. Ahmed, J. Sandschaper, and J. Etgen, 2017, What mistakes are we making while interpreting salt? Could FWI help?: Presented at the 79th EAGE Conference and Exhibition 2017-Workshops.

EIA, 2017, Annual Energy Outlook 2017 with projections to 2050: [https://www.eia.gov/outlooks/aeo/pdf/0383\(2017\).pdf](https://www.eia.gov/outlooks/aeo/pdf/0383(2017).pdf).

Esser, E., L. Guasch, F. J. Herrmann, and M. Warner, 2016, Constrained waveform inversion for automatic salt flooding: *The Leading Edge*, **35**, 235–239.

Esser, E., L. Guasch, T. v. Leeuwen, A. Y. Aravkin, and F. J. Herrmann, 2015, Automatic salt delineation - Wavefield Reconstruction Inversion with convex constraints, *in* SEG Technical Program Expanded Abstracts 2015: Society of Exploration Geophysicists, 1337–1343.

Fehler, M., and P. J. Keliher, 2011, SEAM Phase 1: Challenges of subsalt imaging in tertiary basins, with emphasis on deepwater Gulf of Mexico: Society of Exploration Geophysicists.

Goertzel, G., 1958, An algorithm for the evaluation of finite trigonometric series: *The*

American Mathematical Monthly, **65**, 34–35.

Guo, Z., and M. V. de Hoop, 2013, Shape optimization and level set method in full waveform inversion with 3D body reconstruction, *in* SEG Technical Program Expanded Abstracts 2013: Society of Exploration Geophysicists, 1079–1083.

Jones, I. F., 2012, Tutorial: Incorporating near-surface velocity anomalies in pre-stack depth migration models: First Break, **30**, 47–58.

Kadu, A., T. Van Leeuwen, and W. Mulder, 2016, A parametric level-set approach for seismic full-waveform inversion, *in* SEG Technical Program Expanded Abstracts 2016: Society of Exploration Geophysicists, 1146–1150.

Kalita, M., V. Kazei, Y. Choi, and T. Alkhalifah, 2018, Regularized full-waveform inversion with automated salt-flooding: Geophysics, submitted.

Kazei, V., and T. Alkhalifah, 2018, Waveform inversion for orthorhombic anisotropy with p waves: feasibility and resolution: Geophysical Journal International, **213**, 963–982.

Kazei, V., E. Tessmer, and T. Alkhalifah, 2016, Scattering angle-based filtering via extension in velocity, *in* SEG Technical Program Expanded Abstracts 2016: Society of Exploration Geophysicists, 1157–1162.

Kazei, V., V. Troyan, B. Kashtan, and W. Mulder, 2013, On the role of reflections, refractions and diving waves in full-waveform inversion: Geophysical Prospecting, **61**, 1252–1263.

Lerche, I., 2013, Dynamical geology of salt and related structures: Elsevier. (p. xi).

Leveille, J. P., I. F. Jones, Z.-Z. Zhou, B. Wang, and F. Liu, 2011, Subsalt imaging for exploration, production, and development: A review: Geophysics, **76**, no. 5, WB3–WB20.

Liu, D. C., and J. Nocedal, 1989, On the limited memory BFGS method for large scale optimization: Mathematical Programming, **45**, 503–528.

- Ma, Y., D. Hale, B. Gong, and Z. J. Meng, 2012, Image-guided sparse-model full waveform inversion: *Geophysics*, **77**, no. 4, R189–R198.
- Maxwell, P., and M. Lansley, 2011, What receivers will we use for low frequencies?, *in* SEG Technical Program Expanded Abstracts 2011: Society of Exploration Geophysicists, 72–76.
- Métivier, L., R. Brossier, J. Virieux, and S. Operto, 2013, Full waveform inversion and the truncated newton method: *SIAM Journal on Scientific Computing*, **35**, B401–B437.
- Mulder, W., and R.-E. Plessix, 2008, Exploring some issues in acoustic full waveform inversion: *Geophysical Prospecting*, **56**, 827–841.
- Ovcharenko, O., V. Kazei, D. Peter, and T. Alkhalifah, 2017, Variance-based salt body reconstruction: Presented at the 79th EAGE Conference and Exhibition 2017.
- Podgornova, O., S. Leaney, L. Liang, et al., 2015, Analysis of resolution limits of VTI anisotropy with full waveform inversion, *in* SEG Technical Program Expanded Abstracts 2015: Society of Exploration Geophysicists, 1188–1192.
- Rudin, L. I., S. Osher, and E. Fatemi, 1992, Nonlinear total variation based noise removal algorithms: *Physica D: Nonlinear Phenomena*, **60**, 259–268.
- Sava, P. C., and S. Fomel, 2003, Angle-domain common-image gathers by wavefield continuation methods: *Geophysics*, **68**, no. 3, 1065–1074.
- Sirgue, L., and R. G. Pratt, 2004, Efficient waveform inversion and imaging: A strategy for selecting temporal frequencies: *Geophysics*, **69**, no. 1, 231–248.
- Soubier, F., S. Operto, J. Virieux, P. Amestoy, and J.-Y. L'Excellent, 2009, Fwt2d: A massively parallel program for frequency-domain full-waveform tomography of wide-aperture seismic datapart 1: Algorithm: *Computers & Geosciences*, **35**, 487–495.
- Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation:

Geophysics, **49**, no. 8, 1259–1266.

Van Leeuwen, T., and W. Mulder, 2010, A correlation-based misfit criterion for wave-equation travelttime tomography: Geophysical Journal International, **182**, 1383–1394.

Virieux, J., and S. Operto, 2009, An overview of full-waveform inversion in exploration geophysics: Geophysics, **74**, no. 6, WCC1–WCC26.

Wu, Z., and T. Alkhalifah, 2016, The optimized gradient method for full waveform inversion and its spectral implementation: Geophysical Journal International, **205**, 1823–1831.

Zhang, Q., I. Chang, and L. Li, 2009, Salt interpretation for depth imaging-where geology is working in the geophysical world, *in* SEG Technical Program Expanded Abstracts 2009: Society of Exploration Geophysicists, 3660–3664.

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8 The resulting velocity model for the central part of the BP 2004 model after 30 iterations of conventional FWI at a range of frequencies from 3 to 7.46 Hz, starting from the (a) linear and (b) flooded initial models.

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12 Misfit functions decay during flooding iterations. Each spike except the first one reflects a flooding run with the preceding 15 iterations of mono-frequency FWI on a coarse grid at 3.0, 3.33, 3.7 and 4.12 Hz for a cross section of the SEAM Phase I model.

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Figure 1. Data misfit of the linear velocity model approaching the true central part of the BP 2004 model as $\kappa v_0 + \lambda v$. Dark zones are the minima where inversion could stall prior to reaching the proper result.

106x94mm (300 x 300 DPI)

Figure 2. Schematic projection of local minima of the objective function onto the model domain: Given an initial model (left), objective functions at different frequencies (central column) will guide inversions from the initial model (indicated by the blue line) to the closest local minimum (red line), where the inversion stops. Corresponding final velocity models and variance distribution between them are plotted in the images in the right column.

168x107mm (300 x 300 DPI)

Figure 3(a). Central part of the BP 2004 benchmark velocity model: the true velocity model

57x19mm (300 x 300 DPI)

Figure 3(b). Central part of the BP 2004 benchmark velocity model: updated velocity models from FWI at minimum 3 Hz used for flooding

57x19mm (300 x 300 DPI)

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57x19mm (300 x 300 DPI)

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57x19mm (300 x 300 DPI)

Figure 4. The dependence of cycle-skipping artifacts on local wavelengths. Frequencies (left column) increase from top to bottom with corresponding wavelengths (central column) and associated cycle-skipping artifacts (right column) in velocity models

115x42mm (300 x 300 DPI)

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57x19mm (300 x 300 DPI)

Figure 5(b). Model variance distribution: the variance threshold, $\epsilon = 0.2$, where all areas with values above the threshold ϵ (red plane) are considered as anomalous

148x111mm (300 x 300 DPI)

Figure 6. Salt flooding: the velocity model after the first run of salt flooding procedure on the central part of the BP 2004 velocity model

57x19mm (300 x 300 DPI)

Figure 7(a). Starting models for multiscale FWI for the central part of the BP 2004 model: the initial, one-dimensional velocity model with a constant gradient

57x19mm (300 x 300 DPI)

Figure 7(b). Starting models for multiscale FWI for the central part of the BP 2004 model: the velocity model obtained after 20 flooding iterations using our variance-based salt flooding technique

57x19mm (300 x 300 DPI)

Figure 8(a). The resulting velocity model for the central part of the BP 2004 model after 30 iterations of conventional FWI at a range of frequencies from 3 to 7.46 Hz, starting from the linear initial model

57x19mm (300 x 300 DPI)

Figure 8(b). The resulting velocity model for the central part of the BP 2004 model after 30 iterations of conventional FWI at a range of frequencies from 3 to 7.46 Hz, starting from the flooded initial model

57x19mm (300 x 300 DPI)

Figure 9(a). Left part of the BP 2004 benchmark velocity model: the true velocity model

54x17mm (300 x 300 DPI)

Figure 9(b). Left part of the BP 2004 benchmark velocity model: the variance distribution among updates from 3.0 to 4.12 Hz used for salt flooding

54x17mm (300 x 300 DPI)

Figure 9(c). Left part of the BP 2004 benchmark velocity model: the initial, one-dimensional velocity model with a constant gradient

54x17mm (300 x 300 DPI)

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54x17mm (300 x 300 DPI)

Figure 10(a). The resulting velocity model for the left part of the BP 2004 model after 30 iterations of conventional FWI at 3–7.46 Hz, starting from the linear initial model

54x17mm (300 x 300 DPI)

Figure 10(b). The resulting velocity model for the left part of the BP 2004 model after 30 iterations of conventional FWI at 3–7.46 Hz, starting from the flooded initial model

54x17mm (300 x 300 DPI)

Figure 11(a): Cross section of the SEAM Phase I velocity model: the true velocity model

64x24mm (300 x 300 DPI)

Figure 11(b): Cross section of the SEAM Phase I velocity model: the variance distribution among updates from 3.0 to 4.12 Hz used for salt flooding

64x24mm (300 x 300 DPI)

Figure 11(c): Cross section of the SEAM Phase I velocity model: the initial, one-dimensional velocity model with a constant gradient

64x24mm (300 x 300 DPI)

Figure 11(d): Cross section of the SEAM Phase I velocity model: the velocity model obtained after 7 flooding iterations using our variance-based salt flooding technique

64x24mm (300 x 300 DPI)

Figure 12. Misfit functions decay during flooding iterations. Each spike except the first one reflects a flooding run with the preceding 15 iterations of mono-frequency FWI on a coarse grid at 3.0, 3.33, 3.7 and 4.12 Hz for a cross section of the SEAM Phase I model

105x90mm (300 x 300 DPI)

Figure 13(a). The resulting velocity model for the cross section of the SEAM Phase I model after a few iterations of FWI with total-variation regularization at each listed frequency ranging from 3 to 7.46 Hz and SNR of 5, starting from the linear model

64x24mm (300 x 300 DPI)

Figure 13(b). The resulting velocity model for the cross section of the SEAM Phase I model after a few iterations of FWI with total-variation regularization at each listed frequency ranging from 3 to 7.46 Hz and SNR of 5, starting from the flooded initial model

64x24mm (300 x 300 DPI)

DATA AND MATERIALS AVAILABILITY

Data associated with this research are available and can be obtained by contacting the corresponding author.