Burning characteristics of candle flames in sub-atmospheric pressures: an experimental study and scaling analysis

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Nomenclature			
A	Pre-exponential factor	$T_{fuel,vap}$	Fuel vaporization temperature [K]
B	Spalding number	T_0	Initial fuel temperature at liquid phase [K]
c_l	Specific heat of condensed phase	u_f	Axial velocity at flame tip [m/s]
c_{pg}	Specific heat from [8, 25, 26]	u_z	Axial velocity at height z [m/s]
D	Wick diameter [m]	Y_{FT}	Fuel concentration in fuel supply stream
E	Activation energy [kJ/mol]	z	Vertical height [m]
g	Gravitational acceleration [m/s ²]	Greek symbols	
g'	Acceleration due to buoyancy [m/s ²]	$\delta(z)$	Boundary layer thickness
Gr_z	Grashof number ($Gr_z = g\Delta T z^3 / (T_\infty \nu^2)$)	γ	Exponential factor that accounts for temperature influence on soot formation
h_{fg}	Energy required to vaporize a unit mass of liquid at temperature $T_{fuel,vap}$	μ_g	Dynamic viscosity [kg/(m·s)]
k_g	Gas phase thermal conductivity	ρ	Density [kg/m ³]
L	Length of wick [m]	ρ_g	Gas phase density
L_D	Flame dark zone height [m]	τ_s	Residence time for soot formation [s]
$L_{w,e}$	Effective wick length inside flame [m]	ξ	Mixture fraction
L_f	Yellow flame height [m]	ξ_{st}	Stoichiometric mixture fraction
$\dot{m}$	Mass burning rate [g/s]	ψ_z	Total entrainment at height z
$\dot{m}_F''(z)$	Mass burning flux at given axial distance z	ψ_{L_f}	Total entrainment at flame tip L_f
P_∞	Ambient pressure [kPa]	Subscripts	
R	Universal gas constant	∞	Ambient
S	Stoichiometric mass ratio of air to fuel	f	Flame
T_∞	Ambient air temperature [K]		

Abstract

Burning characteristics (mass burning rate, natural convection boundary layer thickness, flame height and dark zone height) of laminar diffusion flames produced by a candle at sub-atmospheric pressures in the range of $P_{\infty}= 50\text{-}100$ kPa were experimentally studied in a reduced-pressure chamber; such data are not reported to date. Scaling analysis was performed to interpret the pressure dependence. The new experimental findings for candle flames in the sub-atmospheric pressures were well interpreted by the proposed scaling laws: (1) the mass burning rate was higher for a candle with larger wick length, and it increased with increasing ambient pressure, a stagnant layer B -number model based on natural convection boundary (flame boundary layer thickness) was developed to scale the mass burning rate of candle flames at various pressures; (2) the flame boundary layer thickness was wider in lower pressure and can be well represented by a natural convection boundary layer solution; (3) flame height was higher for a candle with larger wick length, meanwhile the ratio of flame height to burning rate was independent of pressure; (4) the flame dark zone height representing a soot formation length scale changes little with pressure, meanwhile its ratio to the total flame height is scaled with pressure by $P_{\infty}^{-1/2} / L_{w,e}^{3/4}$ ($L_{w,e}$ is effective wick length inside flame). This work provided new experimental data and scaling laws of candle flame behaviors in sub-atmospheric pressures, which provided information for future characterization and soot modeling for diffusion flames associated with melting and evaporation processes of solid fuels.

Keywords

Candle flame; Sub-atmospheric pressure; Stagnant layer B -number model; Mass burning rate; Flame height and dark zone height.

1. Introduction

Candle flames have long been associated with human history. A candle produces a laminar diffusion flame providing light, behind which there are fundamental nature of combustion involving chemical reactions, gas-phase diffusion processes, fluid mechanics, and phase changes. It has long been a scientific subject to understand burning behaviors, flame characteristics, and emissions. The first scientific study on candle flames can be dated to 150 years ago by Faraday [1]. Later in the 1990s, microgravity experiments were conducted in space and drop towers [2-4], revealed well-known images of nearly spherical candle flames instead of their typical elongated shape on Earth due to the buoyancy. Arai and Amagai [2], Oostra et al. [3], and Ross et al. [4] reported the influence of gravity on burning characteristics of candle flames, including soot production and flame shape. Subsequently, Hamins et al. [5] measured the mass burning rate, candle regression rate, flame height, and heat flux. Allan et al. [6] investigated laminar smoke points of candle flames by changing the wick diameters and lengths for various waxes. Sunderland et al. [7] investigated the height and width of candle flames with varying wick lengths and diameters, in which a stagnant layer model was developed based on the burning for a finite cylinder and the laminar flame model proposed by Roper et al. [8].

Although candle flames have drawn attention for a long time with extensive works conducted to provide insight into the fundamentals of their combustion behavior, its burning characteristics in sub-atmospheric pressures have not been explored yet. Recently, several experiments [9-11] were carried out for turbulent diffusion flames produced by hydrocarbon liquid pool fire and gaseous fuel jet flames in a sub-atmospheric atmosphere in Lhasa, Tibet (atmospheric pressure of 64 kPa). The results revealed that combustion behaviors, including flame shape and radiation [9, 10] and burning rate [11], differ appreciably for these turbulent flames from those at the standard atmospheric pressure. However, there

is still no study on combustion characteristics of candle flames in sub-atmospheric pressures. The relevance of such a problem is to provide candle flame characteristics in sub-atmospheric pressures at high altitude utilization. Note that reducing pressure mitigates the influence of buoyancy [12], such that a systematic experiment with decreasing pressure could provide a link between the flame behavior in the normal gravity and microgravity. Additionally, the scientific significance lies in that its data under various sub-atmospheric pressures as well as its scaling laws on pressure dependence could provide baseline data for flame and soot modeling, which involves both gas phase diffusion and complex phase change behavior for wax, as compared with those for liquid or gaseous fuels.

In the present study, experiments were carried out to study candle flame behaviors, including mass burning rate, boundary layer thickness as well as flame height and its dark zone height under various sub-atmospheric pressures. Scaling analysis was conducted to interpret the pressure dependence.

2. Experiment

Figure 1 shows a schematic of the experimental apparatus and measurement setup, consisting of a candle, an electronic balance, a CCD digital camera and a reduced-pressure chamber [13]. The inner dimensions of the chamber are 3 m (width) $\times$ 2 m (length) $\times$ 2 m (height). The ambient temperature and relative humidity inside the chamber were about 25°C and 65%, respectively. The ambient pressure inside the chamber (P_∞) can be regulated and maintained by a vacuum pump based on a real-time pressure sensor. The designed pressures were ranged from 50 to 100 kPa and at a specified pressure, the variation was less than $\pm 1\%$ for this small candle flame tests. The mass of candle was measured in real time by an electronic balance (0.01g resolution, 1 s sampling interval), which was connected to a computer. The candles used in this work are made of solid paraffin wax, which is similar to the ones

used previously [5, 7]. The chemical composition of the wax is a mixture of alkane fuels (C_nH_{2n+2} , $n=19\sim36$) [5], and the density is about 900 kg/m^3 . Two different wick lengths (7 mm and 12 mm) with the same diameter of 1 mm were used. The diameter of the candle body was 35.5 mm. Six ambient pressures conditions were tested from 50 to 100 kPa with 10 kPa interval. Each case was repeated 3 times showing good repeatability and averaged values were used for analysis and discussion.

Figure 1. Schematic of experimental setup.

For each test, the candle was ignited after the pressure maintained at a desired level for over 5 min to ensure the inner environment was stable. A heating wire installed on a stepping motor ignited the wick and moved out immediately after the ignition. All the measurements were conducted for 1000s. The chamber was flushed with fresh air before each experiment.

Flame characteristics were recorded by the CCD camera (resolution 1920×1280 pixels) at 25 fps. To accurately determine the flame edge, which will be used to obtain flame length scales, the CCD "blooming" effect had been minimized by setting the camera's aperture at a suitable level to filter a blooming light and the background was set to be dark to improve distinguishability. The combined usage of these two methods helped to weaken the "blooming" in the recorded flame images. Further, a mean shift algorithm [14] was used to eliminate the "blooming" from the recorded images. A program based on OTSU algorithm [15], as applied in [9,13] to ensure local contrast maximum-likelihood

between a flame and the background, was used to demarcate flame edge that separated flame from the background, as well as to demarcate edges of dark and yellow regions.

3. Results and Discussion

3.1 Flame evolution

Figure 2 shows direct photographs, typically taken when the mass loss rate reached a steady state for the two wick lengths in the pressure range of 50-100 kPa. It was observed that, for all flames with the same wick size at a specified ambient pressure, the flame shape was relatively unchanged as the wick size was kept constant to control the fuel vaporization rate. Soot leakage along the flame tip was not observed as all the flame heights do not reach the smoke point [6]. With the variation of the pressure, the flame height was in general monotonically increased with the increase in ambient pressure for both wick lengths. The flame in higher pressure showed more intense yellow luminosity with larger yellow flame area, which was consistent with the previously observed sooting tendency with pressure [16, 17].

Figure 2. Photographs of candle flames with pressure variation for wick length of (a) 12 mm and (b) 7 mm; (c) specification of flame length scales.

The combustion process of a candle flame is complicated as reported in the previous study [18]. The heat transferred from a flame leads to the melting of solid wax and a pool of liquid wax is formed at the top of the candle. The melted wax moves upward along the wick driven by the capillary action and the Marangoni convection, and then vaporizes into gas phase as further heated by the flame. The mixing of gaseous fuel and air is driven by the buoyant convection. The base of the flame has blue color due to chemiluminescence, a dark zone relative to the yellow luminous zone above where soot is formed. At higher height, soot is formed mainly from polycyclic aromatic hydrocarbons (PAHs) in the high temperature region on the fuel side from the flame [19, 20]. These soot particles are transported by convection upward and finally oxidized by the reaction with oxidizing species (OH and O₂) supplied by diffusive-convective processes from the surrounding air. These complex processes are influenced considerably by the ambient pressure, which will be discussed based on the measured values of burning rate $\dot{m}$, flame natural convection boundary layer thickness $\delta(z)$, flame height L_f , effective wick height inside the flame $L_{w,e}$ and flame dark zone height L_D , as illustrated in Fig. 2c.

3.2 Burning rate and flame natural convection boundary layer thickness

Figure 3 presents a typical variation of candle mass with time (a) and the mass burning rate with ambient pressure (b). The mass variation with time shows that after the ignition transient, the candle mass is linearly decreasing with time for such a small laminar candle flame, which is an indication of stable steady burning. The mass burning rate $\dot{m}$ was obtained from the slope of the mass versus time plot during the steady burning period. The result shows that the mass burning rate for the wick length of 12 mm is larger than that of 7 mm and both increases with increasing ambient pressure.

Figure 3. Typical mass-time history and the determination of mass burning rate $\dot{m}$ (a) and its variation with ambient pressure (b)

For a steady burning of a condensed fuel dominated by convective heat feedback, the conventional stagnant layer model (also known as B -number theory) can be employed to predict the burning rate for a wide range of condensed fuel. In the present, the candle flame configuration can be assumed as a flame with the vertical wick providing fuel vapor from liquified wax with a vertical laminar flow boundary dominated by convection. Then, the mass flux at a given height (z) along the wick has [21]:

$$\dot{m}_F''(z) = \left[k_g / (c_{pg} \delta(z)) \right] \ln(1 + B) \quad (1)$$

where B is the fuel mass transfer number (Spalding number),

$$B = \frac{Y_{O_2, \infty} (\Delta H_c / r) - c_{pg} (T_{fuel, vap} - T_\infty)}{h_{fg} + c_l (T_{fuel, vap} - T_0)} \quad (2)$$

with a value of 1.89 for the wax [7], and those of the thermal conductivity k_g and c_{pg} are referred from [5, 7, 22]. The boundary layer thickness, $\delta(z)$, in which most of the chemical reaction is experienced, can be assumed to be the horizontal length scale from the wick surface to the flame sheet (Fig. 4). As can be visually confirmed in Fig. 2, the smaller $\delta(z)$ at higher ambient pressure can be attributed to the overall increase in mass burning rate with pressure. The relationship between boundary layer thickness $\delta(z)$ and Gr_z for a laminar flame dominated by convection can be expressed as [21]:

$$\delta(z)/z \sim Gr_z^{-1/4} \sim \left[\left(\rho_g^2 g \Delta T z^3 \right) / \left(T_\infty \mu_g^2 \right) \right]^{-1/4} \quad (3)$$

Here, the temperature difference between flame and ambient air ΔT is taken as constant (878 K) [7, 10]. As $\rho \sim P$, it becomes:

$$\delta(z)/z \sim \left[\left(P_\infty^2 g \Delta T z^3 \right) / \left(T_\infty \mu_g^2 \right) \right]^{-1/4} \quad (4)$$

The relation between the experimentally measured boundary layer thickness $\delta(z)$ (up to the effective wick height inside the flame near the leading edge of the flame sheet where the mass flux originated from) with $Gr_z^{-1/4}$ is shown in Fig. 4, showing a reasonable agreement with Eq. (4).

Figure 4. Correlation of flame natural convection boundary layer thickness ($\delta(z)$) normalized by vertical location (z) against modified Grashof number based on Eq. (4).

Substituting Eq. (4) into Eq. (1) and integrating along the effective wick length, $L_{w,e}$, which was shown to be $L_{w,e} = 0.8e^{-0.6D/L}L$ [7], the overall mass burning rate, can be expressed as,

$$\dot{m} \sim \int_0^{L_{w,e}} \dot{m}_F''(z) dx = \int_0^{L_{w,e}} \left[\frac{k_g}{c_{pg} \delta(z)} \right] \ln(1+B) dx = \frac{L_{w,e}^{3/4} k_g}{c_{pg}} \left(\frac{\rho_g^2 g \Delta T}{T_\infty \mu_g^2} \right)^{1/4} \ln(1+B) \quad (5)$$

As $\rho \sim P$, then it gives,

$$\dot{m} \sim \frac{L_{w,e}^{3/4} k_g}{c_{pg}} \left(\frac{P_\infty^2 g \Delta T}{T_\infty \mu_g^2} \right)^{1/4} \ln(1+B) \quad (6)$$

Figure 5. Scaling of mass burning rate based on stagnant layer model for various wick sizes and ambient pressures.

Figure 5 shows that the mass burning rate obtained in this work at various pressures, as well as those reported previously [5, 23] at normal pressure can be well correlated by the proposed theory (Eq. 6). Note that the wick and flame sizes in [5] and [23] are appreciably larger than those in our work. In [5], the flame height was 40 mm and the wick diameter was nearly 2 mm, both about two times to the present ones. According to ref. [23], the wick size was quite the same with [5]. So, their burning rate data are much larger than ours.

3.3 Flame height

Figure 6 show the visible yellow flame height (averaged from 1000 images) against ambient pressure with its RMS variation marked as the error bar. The flame height was defined as the vertical height of visible flame from the base of the dark zone to the top of yellow luminous zone. Although there is a difference between a visible flame height and the stoichiometric height corresponding to a main reaction zone height, a visible flame height is often adopted as the stoichiometric height [8, 24] for these stable candle flames. The results show that the flame height with the wick length of 12 mm is larger than that of 7 mm wick, and both increase as the ambient pressure increases. The present flame height data for the 7 mm wick at the normal pressure (although with wick curvature) is in good agreement with that reported by Sunderland (wick stays upward) [7], indicating that the effect due to

the curvature of the wick is shown to be reasonably negligible for similar wick size. For a longer wick, liquid fuel on the wick can be vaporized from a larger heated area (length), which results in higher burning rate and hence the flame height. The pressure dependence of the flame heights found here is quite different from those of turbulent diffusion flames of hydrocarbon liquid fuel pool fire and gaseous jets, whose flame heights both showed to be increased as the ambient pressure is reduced [9, 10]. Note that the height of diffusion flame is determined by coupling effects of fuel supply rate and air (oxidizer) entrainment. The height of diffusion flame increases with increasing fuel supply rate, or the decrease in air entrainment mass flux. As the ambient pressure decreases, the fuel mass supply (evaporation) rate decreases (for liquid and solid fuels, which will lead to flame height decrease) or does not change (for gaseous jet fuel), meanwhile air entrainment also decreases (which will lead to flame height increase). Then, the overall outcome of the flame height dependence on pressure should be determined by which effect (changes in mass burning rate or air entrainment) is dominating. For turbulent diffusion flames of hydrocarbon liquid pool fire or gaseous jet, the effect due to change in air entrainment should overcome that due to the change in fuel mass supply rate. Meanwhile, for the laminar candle flames, the effect due to the change in mass burning rate should be dominating as implied by Fig. 6. So, it is of interest to investigate how the ratio between the flame length and burning rate scales with pressure.

Figure 6. Measured flame height at various sub-atmospheric pressures.

The total entrainment ψ_z at height z can be characterized as [25]:

$$\psi_z \propto \mu_g z \quad (7)$$

in which the μ_g is independent of pressure. The combustion will cease when the mixture fraction at the centerline becomes equal to the stoichiometric value $\xi_{st} = 1/(1+S)$. Then, the flame height and its ratio with the mass burning rate can be scaled as:

$$L_f \propto \psi_{L_f} / \mu_g = (\dot{m} / \xi_{st}) / \mu_g = [\dot{m}(1+S)] / \mu_g \quad (8a)$$

or

$$L_f / \dot{m} \propto (1+S) / \mu_g \propto P_\infty^0 \quad (8b)$$

This indicates that the ratio of flame height to mass burning rate is independent of ambient pressure for this laminar candle flame. The experimentally measured ratio is shown in Fig. 7, confirming the independence of the ratio with pressure with the average value about 19.6 m/(g·s⁻¹). Although there are large variations in the wick heights among the present and those reported Hamins [5], the present correlation implies that the ratio is minimally affected by the wick length, wick diameter and wax composition. Although the data by Yuan [26], is for methane jet with varying pressure, it is interesting to see the ratio is reasonably similar. This implies that the scaling law based on buoyancy-induced mixing and entrainment is still applicable for a candle flame once the dependence of its mass loss rate on pressure is quantified.

Figure 7. Correlation of flame height normalized by mass burning rate against ambient pressure based on Eq. (8b).

3.4 Flame dark zone height and its ratio to flame height

Figure 8 shows the measured flame dark zone height (along the centerline to soot zone) against ambient pressure, which shows that this height keeps almost unchanged for the candle flame as the ambient pressure decreases. Note that this dark zone height could represent a characteristic length scale for soot *formation* in the flame.

Figure 8: Measured flame dark zone height at various sub-atmospheric pressures.

Physically, the visual characteristic yellow flame corresponds to soot *formation and oxidation* processes in laminar diffusion flames. The dark zone height L_D is typically controlled by characteristic soot formation time τ_s , which can be determined from the integration of the traveling time of buoyant flow u_z in the dark zone as:

$$\tau_s = \int_0^{L_D} u_z^{-1} dz \propto \int_0^{L_D} (g'z)^{-1/2} dz \propto L_D^{1/2} / \sqrt{g'} \quad (9)$$

where u_z is the axial velocity at the vertical height z and g' is an acceleration constant due to buoyancy [21]. According to Beji et al. [27], the soot formation time can also be determined by the chemical reaction time based on mixture fraction:

$$\tau_s^{-1} = \int_{\xi_{st}}^1 \frac{A}{\xi} \left(\frac{\xi_{st}}{\xi} \right)^{1/2} \rho T^r \left(Y_{FT} \frac{\xi - \xi_{st}}{1 - \xi_{st}} \right) e^{-E/RT} d\xi \quad (10)$$

This one-step global reaction model based on mass fraction and mixture fraction successfully captured experimental result on soot concentration and related length scales [27, 28]. All the parameters are

independent of ambient pressure except the density, resulting in:

$$\tau_s \propto P_\infty^{-1} \quad (11)$$

At sub-atmospheric pressure, concerning the reduced buoyant condition due to the change of dynamic viscosity or Grashof number ($Gr_z = g\Delta Tz^3 / (T_\infty \nu^2)$), the acceleration varies proportional to the square of pressure, $g' \propto p^2$ [29], which is also the base that the reduced gravity conditions can be simulated by sub-atmospheric pressure conditions [29]. Combining Eqs. (9) and (11) yields:

$$L_D \propto (\tau_s \sqrt{g'})^2 \propto P_\infty^0 \quad (12)$$

This is consistent with the observation in Fig. 8, as well as that reported in [30] for ethylene flames at pressures from 50 kPa to 100 kPa. However, as the pressure is less than 50 kPa, it shows that there is a remarkable increase of flame dark zone height as the pressure is further reduced [30], which should be due to the fact that the diffusion is controlled by dispersion rather than buoyancy at very low pressure. As $L_f \propto \dot{m} \sim L_{w,e}^{3/4} P_\infty^{1/2}$ (indicated in Eqs. (6) and (8b)), the pressure dependence of the ratio of two flame length scales has:

$$L_D / L_f \propto P_\infty^{-1/2} / L_{w,e}^{3/4} \quad (13)$$

Figure 9 shows that the measured ratio of L_D/L_f correlate well to the proposed function (Eq. (13)).

Figure 9. Correlation of the ratio of flame dark zone height to flame height against ambient pressure based on Eq. (13).

4. Conclusions

This paper experimentally investigated the burning characteristics (mass burning rate, flame boundary layer thickness, flame height, and dark zone height) of laminar candle flames at sub-atmospheric pressures (50-100 kPa), which have not been investigated to date. Scaling laws were then derived to interpret the pressure dependence of these combustion characteristics. Major findings include:

(1) The mass burning rate is higher for a candle with longer wick and it increases with increasing ambient pressure. A stagnant layer B-number model based on flame natural convection boundary, whose length scale (flame boundary layer thickness) is wider in higher ambient pressure, is developed to scale the mass burning rate of candle flames at various pressures.

(2) The flame height is larger for a candle with longer wick, while the ratio of flame height to mass burning rate is independent of pressure.

(4) The flame dark zone height shows to change little with pressure for the candle flames. The ratio of the flame dark zone height to the total flame height scales with pressure by $P_{\infty}^{-1/2} / L_{w,e}^{3/4}$.

This work provides new data and basic scaling laws for candle flame behaviors in sub-atmospheric pressures, which can be further referred for flame characteristics and soot modeling development in the future as well as its pressure dependence in laminar flames produced by solid combustibles involving solid-liquid-vapor phase changes.

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Figure Captions

Figure 1: Schematic of experimental setup.

Figure 2: Photographs of candle flames with pressure variation for the wick length of (a) 12 mm and (b) 7 mm; (c) specification of flame length scales.

Figure 3: Typical mass-time history and the determination of mass burning rate $\dot{m}$ (a), as well as its variation with ambient pressure (b).

Figure 4: Correlation of boundary layer thickness normalized by vertical location (z) against modified Grashof number based on Eq. (4).

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