Accepted Manuscript

Thermal shale fracturing simulation using the Cohesive Zone Method (CZM)

Saeid Enayatpour, Eric van Oort, Tad Patzek

PII: S1875-5100(18)30214-2

DOI: [10.1016/j.jngse.2018.05.014](https://doi.org/10.1016/j.jngse.2018.05.014)

Reference: JNGSE 2572

To appear in: *Journal of Natural Gas Science and Engineering*

Received Date: 2 March 2018

Revised Date: 27 April 2018

Accepted Date: 8 May 2018

Please cite this article as: Enayatpour, S., van Oort, E., Patzek, T., Thermal shale fracturing simulation using the Cohesive Zone Method (CZM), *Journal of Natural Gas Science & Engineering* (2018), doi: 10.1016/j.jngse.2018.05.014.

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

Thermal Shale Fracturing Simulation using the Cohesive Zone Method (CZM)

Saeid Enayatpour^{a,*}, Eric van Oort^a, Tad Patzek^b

^a*Petroleum and Geosystems Engineering, The University of Texas at Austin, TX, USA*

^b*Earth Science and Engineering Department, King Abdullah University of Science and Technology*

Abstract

Extensive research has been conducted over the past two decades to improve hydraulic fracturing methods used for hydrocarbon recovery from tight reservoir rocks such as shales. Our focus in this paper is on thermal fracturing of such tight rocks to enhance hydraulic fracturing efficiency. Thermal fracturing is effective in generating small fractures in the near-wellbore zone - or in the vicinity of natural or induced fractures - that may act as initiation points for larger fractures. Previous analytical and numerical results indicate that thermal fracturing in tight rock significantly enhances rock permeability, thereby enhancing hydrocarbon recovery. Here, we present a more powerful way of simulating the initiation and propagation of thermally induced fractures in tight formations using the Cohesive Zone Method (CZM). The advantages of CZM are: 1) CZM simulation is fast compared to similar models which are based on the spring-mass particle method or Discrete Element Method (DEM); 2) unlike DEM, rock material complexities such as scale-dependent failure behavior can be incorporated in a CZM simulation; 3) CZM is capable of predicting the extent of fracture propagation in rock, which is more difficult to determine in a classic finite element approach. We demonstrate that CZM delivers results for the challenging fracture propagation problem of similar accuracy to the eXtended Finite Element Method (XFEM) while reducing complexity and computational

*Corresponding author

Email address: s.e@utexas.edu (Saeid Enayatpour)

effort. Simulation results for thermal fracturing in the near-wellbore zone show the effect of stress anisotropy in fracture propagation in the direction of the maximum horizontal stress. It is shown that CZM can be used to readily obtain the extent and the pattern of induced thermal fractures.

Keywords: Thermal Stimulation, Thermal Fracturing, Cohesive zone method (CZM), Hydraulic Fracturing

1. Introduction

Extracting hydrocarbons from 100-300 ft thick shale layers at a vertical depth of 6,000-10,000 ft can only be accomplished in an economically viable way through the combined technologies of horizontal drilling, typically in the direction of minimum horizontal in-situ stress in normal faulting environments [1, 2, 3], and well stimulation by hydraulic fracturing (HF) to increase rock permeability and improve connectivity to natural fractures [4, 5, 6]. Although our understanding of the factors governing HF stimulation is rapidly improving, it is fair to state that the general approach to HF optimization, e.g. determining optimum HF fluid formulations, pumping rates, proppant size, strength and density, fracture stage spacing etc., is predominantly through trial and error. It is therefore highly unlikely that the HF process, as currently practiced in field operations, and associated hydrocarbon recovery from shales is already at optimum. This is also because some factors and variables that influence HF have not yet been substantially addressed.

One such factor is temperature. Temperature has a profound effect on the magnitude of in-situ stresses, which in turn govern the extent and ease of HF. Few attempts have been made to date to deliberately manipulate temperature in horizontal wellbores and inside hydraulic fractures to benefit HF efficiency. Previously, the authors have presented a novel method for such intentional manipulation of temperature to create a more extensive fracturing pattern around the wellbore [7, 8]. The method exploits thermal gradients induced by cooling/freezing either the wellbore or the growing hydraulic fractures themselves.

Cooling/freezing lowers fracture initiation and propagation pressures and can induce thermal cracks at multiple sites, particularly in such heterogeneous media as shales. It was previously shown that deliberately inducing thermal cracks may increase hydrocarbon recovery by 15%-20% by conservative estimate. In this study, we are concerned with efficiently modeling the thermal fracturing process to optimize HF, and propose a physics-based continuum approach combined with fracture mechanics to simulate it. In our approach, cohesive zone elements acting in accordance with fracture mechanics laws can be used to simulate de-bonding between continuum elements. By combining both fracture cohesive and continuum elements, we show that the physics of both transport in a continuum framework as well as crack propagation can be effectively captured using an implicit integration scheme in a finite element model. This approach also enables us to investigate the uncertainties and variations associated with actual in-situ properties of inhomogeneous shales, with physical properties such as elastic moduli and heat conductivity, strength anisotropy resulting in preferred failure planes, etc. The quantitative approach presented here can be used to perform statistical sensitivity studies to investigate the sensitivity of the thermal fracturing process with respect to input parameters such as cooling/freezing temperatures, rock exposure time to cooling/freezing, etc.

There are several methods available to investigate the process of thermal fracturing in rock, each method with its advantages and disadvantages. In a previous approach (Enayatpour *et al.* [9]), the discrete element method (DEM) has been used for thermal fracture simulation. DEM can overcome some of the limitations associated with continuum approaches, capturing HF patterns by incorporating local failure between particles. A potential problem, however, is that the simulation time could be hours or days for a sample of 100,000 mass particles (Enayatpour *et al.* [9]). Moreover, DEM may not properly represent the heat transport phenomenon and the energy balance, which are essential to proper simulation of thermal fracturing. Here, we use the Finite Element Method (FEM) to simulate the thermal fracturing in shale reservoir rock using embedded multiple-site cohesive zone elements. The intent of this paper is (1)

55 to further introduce thermal fracturing as a novel method of rock stimulation
to amplify HF, and (2) to present the idea and physical details of the CZM
used in the context of Finite Element Method (FEM) to study the pattern and
efficiency of thermal fractures, including delineating the sensitivity of thermal
fracturing to CZM input parameters. To assess the effect of thermal fracturing
60 on improved hydrocarbon production, advanced geomechanical simulations such
as thermal poro-elastic analysis, *i.e.*, stress deformation coupled with thermal
analysis and flow in porous media, are required. Moreover, the time-dependent
creep behavior of shale and its nonlinear rock properties must be included in the
simulation. The methodology presented here is capable of incorporating all of
65 these advanced simulation features; however, our focus remains on the perfor-
mance of the CZM method for thermal rock fracturing. Note that some fracture
topics, such as dynamic fracture propagation and fracture instability, are not
covered but ultimately will need to be addressed in a full dynamic description
of the thermal fracturing phenomenon.

70

1.1. Historical Work

The efforts by Inglis [10], Irwin [11] and Griffith [12] were fundamental to the
development of fracture mechanics in all engineering areas, including petroleum
engineering. In 1913, Inglis [10] obtained the stress around an elliptic crack,
75 which was used by Griffith in 1920 [12] to derive a criterion for unstable fracture
propagation. Griffith used the first law of thermodynamics to write an energy
balance equation, showing that a crack becomes unstable when the variation of
strain energy due to crack growth exceeds the surface energy of the material.
Griffith's theory clearly shows the relation between strength and fracture size,
80 for instance in glass specimens which are internally notched by a crack. Efforts to
extend Griffith's idea to metals were not successful, because his model assumes
that the fracturing energy is composed of surface energy only, which is true only
for brittle materials.

In 1957, Irwin [11] presented the idea of energy release rate. Based on the work

85 by Westergaard[13], Irwin proposed the stress intensity factor (SIF) whereby the
 stress and displacement fields around the crack tip can be related to the energy
 release rate. By 1960, the fundamentals of linear elastic fracture mechanics
 (LEFM) were fully understood, and studies of fracture mechanics started to
 focus on plasticity at the crack tip. Once plastic deformations at the crack tip
 90 become large, the assumptions of LEFM are no longer valid, and the results from
 LEFM analysis cannot be relied upon. Many researchers, including Irwin [11],
 attempted to better understand the effects of plasticity at the crack tip. Wells
 [14] introduced the displacement of the two crack surfaces as a replacement for
 the plastic failure criterion when plasticity effects are not negligible. His work led
 95 to the development of the Crack Tip Opening Displacement method (CTOD).
 In 1968, Rice [15] defined another parameter for description of the nonlinear
 material behavior at the crack tip. He introduced the J-integral, based on
 the Eshelby's work on development of path-independent integrals, showing that
 this integral is equal to the energy release rate. In the same year, Hutchinson
 100 put forward the so-called HRR (Hutchinson-Rice-Rosengren) theory of elastic-
 plastic stress fields in power hardening materials, which paved the road to the
 modern Non-Linear or Elasto-Plastic Fracture Mechanics (NLFM, Hutchinson,
 1968, and Rice and Rosengren, 1968). In HRR theory, the J-integral is related
 to the stress distribution in the crack tip of nonlinear materials. One of the
 105 applications of the J-integral method is to characterize the crack growth energy
 release rate in a rate-independent quasi-static fracture analysis. The J-integral
 thereby can provide a method for analyzing fractures in nonlinear materials.
 There are, however, some limitations associated with nonlinearity which can be
 overcome by using CZM. Based on the crack geometry shown in Figure 1, the
 110 J integral is defined as follows:

Figure 1: J-integral calculation

$$J = \int_{\Gamma} (W n_1 - \frac{\partial u_i}{\partial x_1} \sigma_{ij} n_j) ds \quad (1)$$

In this equation, W is the strain energy density, Γ is a contour beginning on the bottom crack surface and ending on the top surface, n is the outward normal to Γ , n_1 is the vector in the direction of q or along crack propagation direction, u_i is the crack tip displacement, q is a unit vector in the virtual crack extension direction, and σ_{ij} is the stress component on the path where integration is performed. With the contours taken around the crack tip in Figure 1, the singularity point will be eliminated and the calculation of J will be path-independent. There are inherent assumptions in defining the J integral method. These are: 1) the material is homogeneous in the crack direction, 2) the material is elastic. Assumption 2 simplifies calculations, but presents limitations. One of the benefits is that for elastic materials, the value of J is equal to the energy release rate associated with crack propagation; *i.e.*, $J = G$. Therefore G , which is a material property and is obtained from laboratory experiments, can be used instead of the Direct calculation of J for a mode I fracture in a linear elastic material results in the following relationships between J and K_I which is the mode I stress intensity factor:

$$J = G = \frac{1 - \nu^2}{E} K_I^2 \quad (\text{plane strain}) \quad (2)$$

and

$$J = G = \frac{1}{E} K_I^2 \quad (\text{plane stress}) \quad (3)$$

There are two cases to consider for mesh changes in numerical fracture simulations. When using the classic finite element method, the user has to conform the mesh to the cracked geometry. However, as a crack tip is approached, stress and strain gradients become large due to stress concentrations at crack tip. Mesh refinement is incapable of eliminating the stress singularity. In fact, in linear elastic simulation, strain exhibits a square root singularity. In other words,

$$\epsilon \propto r^{-1/2} \quad (4)$$

This means that mesh refinement, *i.e.*, reduction of dimension r , leads to larger values of strain and stress. Note that the J-integral is an energy measure. For Linear Elastic Fracture Mechanics (LEFM), accurate J values can generally be obtained with surprisingly coarse meshes, regardless of the inaccuracy associated with the high values of local strain and stress. However, the stress singularity problem is pronounced for nonlinear problems involving material plasticity and/or large deformation. For CZM, however, meshing is less dependent on the cracked geometry. With this method, assigning the cohesive zone properties to the element interface ahead of fracture tip provides freedom to assign the fracture propagation path. Hence, re-meshing of the domain is not required. The mesh requirement limitations of the J-integral clearly favor the use of CZM, which is therefore used in the simulations reported here.

The concept of a Cohesive Zone Law (CZL) was first put forward by Elliott [16] who introduced an inter-atomic attraction-repulsion force per unit area to investigate the amount of energy required to fracture a crystalline substance. Later, Barenblatt [17, 18] and Dugdale [19] investigated the CZL concept from an energy standpoint in brittle materials. Dugdale worked on the size of the plastic zone. Later, Smith [20] developed a general theory for cohesive behavior of rocks and provided a series of traction-separation plots based on the displacement along the fracture tip. An extended CZL model, the “fictitious crack model” has been employed for quasi-brittle materials such as fiber-reinforced concrete and rocks in cases where the size of fracture process zone is not negligible [21, 22, 23, 24, 25]. Hillerborg *et al.* [21] introduced a softening regime using

fracture energy and the tensile strength of rock. CZM has been widely used for
 135 several applications in rock fracturing studies, including prior work on thermal
 fracturing [26]. From a modeling perspective, recent literature typically shows
 a trade-off between the number of interacting fractures in the model and the
 accuracy with which the model solves the stress equilibrium equations. Mod-
 els which avoid the computational cost of explicit numerical solution typically
 140 rely on analytical approximations which describe fractures of a simple geom-
 etry (PKN, KGD, etc). In return for these simplifications, such models offer
 computational efficiency in capturing the interaction between a large number
 of hydraulic and natural fractures [27]. In contrast, full numerical models of-
 fer more realistic fracture simulations which require much more computational
 145 power, and as a result typically include a smaller number of fractures [28].
 CZM simulations benefit from relatively short simulation time and the ability
 to capture the thermo-mechanical behavior of rock.

1.2. Scale Dependent Rock Failure

Fracture creation occurs when connectivity is lost between constituents of a
 150 solid material. Cracks in solid bodies can be pre-existing, such as voids, notches
 and pore spaces, or exist as a set of randomly created and distributed openings
 in material during the rock deformation process. Cracks exist in solid materials
 at all scales. The existence of cracks plays a significant role in the strength of
 a solid material. Experiments performed as early as 1913 by Inglis[10], showed
 155 how defects and flaws control the strength of a material. Rock failure is such
 a complicated phenomenon because it encompasses a wide range of temporal
 and spatial scales (Figure 2), the latter including continuum scale and discrete
 atomic/molecular scale. At different observation scales, materials can behave
 differently as the load transfer mechanism and strength are both scale depen-
 160 dent. At the macro-scale, the effect of micro-structural crack development is not
 clearly observed in the strength formulation; rather, the contribution manifests
 through strength reduction. For example, the Mohr-Coulomb failure criterion
 compares the shear stress and shear strength at the most critical surface inside

a material and determines whether or not the body fails under shear stress;
 165 micro-scale material defects are not explicitly included. The final failure state
 of a material is the result of a series of small scale ruptures. During the small
 scale failure, the localized shear events of atomic lattice causes the micro-cracks
 to coalesce and this eventually leads to inter-granular failure. Therefore, the
 macro-scale shear displacement is the result of the coalescence of the micro-
 170 scale cracks.

Figure 2: Idealization of rock failure at different scales. Cohesive zone model and the lattice model are adapted from [29].

The structure of shale is composed of aluminium-silica plates and adsorbed water layers (Figure 3). The molecular forces of clay particles make the pore fluid pressure dissipation very slow [8].

Figure 3: (A) Shale complexity exhibited by the field emission electron photomicrograph of nano-scale pore architecture in the Barnett Formation [30], (B) Complex molecular structure of shale, Al+SiO₂ plates and water layers created in VMD, a LAMMPS post processor.

This complexity of shale structure shown in Figure 3 is largely responsible for the complexity in the process of fluid flow through shale and effective stress development in the shale matrix. Another level of unpredictability associated with shale fracturing simulation is that during hydraulic fracturing, even at the best controlled condition, multiple fractures grow at different lengths. The source of this unpredictable behavior is the rock material anisotropy and stress effect from propagating fractures in the vicinity of each fracture, known as the stress shadowing effect. Fracture propagation in tight formation exhibits an unpredictable propagation pattern for these reasons. To optimize recovery one should rely on enhancing and establishing the interconnectivity among existing and induced fractures. Here we propose the CZM method as a technique to quantify fracture interconnectivity in the context of thermal fracturing analysis. Later on, this method can be used for recovery enhancement studies and predictions.

1.3. Thermal Fracturing

New methods and techniques are continuously being explored to make hydraulic fracturing more effective and to increase ultimate hydrocarbon recovery. A novel method, which relies on the beneficial manipulation of thermal stresses around the wellbore to increase the crack density in hydrocarbon-bearing shale formations, was proposed recently by Enayatpour, *et al.* [8, 7]. Thermal cracks

can be formed by injection of cold / cooling fluids into hot reservoir shale rock.

195 The cooling effect will lower fracture initiation and propagation pressures of the
main hydraulic fracture(s). Moreover, cooling may initiate branching fractures
near the wellbore and the hydraulic fracture surfaces, leading to increased crack
density. Shales lend themselves well to such thermal fracturing. They have
well-known heat conduction which varies within a limited range for all shales.
200 Unlike pore pressure, heat can diffuse easily in shales. Moreover, shale fabric is
generally very weak in tension. This, in combination with the fact that cool-
ing temperature gradients in the near-wellbore region can cause large tensile
stresses in the shale, rock leads to effective thermally induced fracturing. This,
in turn, may significantly enhance the permeability and subsequent hydrocar-
205 bon recovery [8, 7].

2. Theoretical

2.1. Cohesive Zone Methodology

Material under mechanical loading fails in either tensile or shear failure
210 modes. There are a number of methods to study the failure mechanism of
solid materials including damage mechanics and fracture mechanics. Rock fail-
ure described by the CZM method occurs when the displacement or energy
imposed on the material body exceeds a threshold value. Therefore, failure in
CZM is based either on a traction-separation criterion or an energy dissipation
215 criteria. Here, we study stable fracture growth in which both fracture energy
and maximum displacement for de-bonding are constant. We explain the CZM
formulation and methodology in greater detail in the following section. The
first introductions to CZM were made by Dugdale [31] and Barenblatt [32] with
extensive research on fracture propagation in brittle materials. CZM provides
220 useful force-displacement relationships for fracture mechanics studies when the
shear and tensile stress of the interface zones exceed the strength of rock [33].
For example, the shear and tensile strength of cement and carbonates filling

natural fractures are generally smaller than the surrounding reservoir rock [34]. Such an interface weakness in highly jointed rocks such as shales can be successfully exploited for induced failure during matrix stimulation by hydraulic fracturing. The CZM finite element approach allows us to simulate the disparate nature of joints and interfaces in shale where thermal fracturing is performed. CZM essentially simulates the rupture process of originally bonded bodies using a traction-separation law, which can be regarded as a force-displacement relationship with nonlinear stiffness characteristics. When a load is applied to a fracture the material will initially behave elastically at the tip of the fracture. As the load is increased to the point where the shear/tensile stress of the bonding material between the fracture walls exceeds the shear/tensile strength of that material, rock matrix failure starts. Beyond this point, which is reflected by the declining portion of the traction-separation relationship, rock damage is accumulated until the maximum damage is attained and a fracture is initiated. One of the main challenges in fracture propagation is the fracture tip stress singularity mentioned earlier, which poses numerical issues in stress calculation at fracture tip as mentioned earlier. One of the advantages of CZM is eliminating this issue [35].

2.2. CZM Formulation

Fundamental to CZM application is the definition of the traction-separation relationship across fracture surfaces. CZM was developed to both eliminate stress singularity in Linear Elastic Fracture Mechanics (LEFM) and approximate the nonlinearity of material separation during fracturing [36, 37, 38, 39, 40]. Here we explain the formulation in detail to exhibit the parameters of the CZM formulation that we use in our finite element simulations. The main parameters of the traction-separation plot and the energy required for fracturing are defined first. Then, the formulation is extended to the case of mixed mode fractures, and the general form of a three-dimensional CZM model is presented. Figure 4 shows several possible forms of the traction-separation plot. For our simulations, we used plot (e) in Figure 4, the most widely used plot in literature [41].

Figure 4: Effective traction-separation relationships: (a) cubic polynomial, (b) trapezoidal, (c) smoothed trapezoidal, (d) exponential, (e) linear softening, and (f) bilinear softening [41].

Figure 5 shows this most common form of the CZM plot with the key parameters of the traction-separation behavior of a mode-I fracture.

Figure 5: The most common form of the CZM plot with linear elastic and linear softening segments

²⁵⁵ d_n in Figure 5 is the damage parameter, which is a scalar number representing the ratio of the number or density of broken bonds to that of intact bonds within the volume of material. Table 1 lists the key traction-separation plot parameters.

Parameter	Definition
T	normal contact (tensile) force
Δ_n	contact gap or normal separation
Δ_n^c	contact gap at the completion of de-bonding
$T_{\max}$	maximum normal contact (tensile) force
$\sigma_{\max}$	maximum normal contact (tensile) stress
δ_n	contact gap at the maximum normal contact (tensile) stress
d_n	damage or de-bonding parameter, 0 at A and 1 at point C
k	normal contact stiffness
k_{Reduced}	reduced normal contact stiffness
$\bar{\Delta}$	dimensionless displacement, $\frac{\Delta_n}{\delta_n}$

Table 1: Key parameters for the definition of a traction-separation plot.

2.2.1. Mode I & II Traction-Separation Behavior

The normal critical fracture energy for mode I de-bonding is the energy required to fracture a material, and is equal to the area under strain-displacement curve as:

$$G_c I = \frac{1}{2} \sigma_{\max} \Delta_n^c \quad (5)$$

The equation for the AC segment of the plot is:

$$T = k \Delta_n (1 - d_n) \quad (6)$$

where $0 < d_n < 1$. For mode I de-bonding, d_n can be obtained as:

$$d_n = \left(\frac{\Delta_n - \delta_n}{\Delta_n} \right) \left(\frac{\Delta_n^c}{\Delta_n^c - \delta_n} \right) \quad (7)$$

with dimensionless displacement $\bar{\Delta}$ defined as:

$$\bar{\Delta} = \frac{\Delta_n}{\delta_n} \quad (8)$$

The de-bonding parameter is specified as follows;

$$\bar{\Delta} \leq 1 \Rightarrow d_n = 0$$

and

$$\bar{\Delta} > 1 \Rightarrow 0 < d_n \leq 1$$

Similar equations are valid for mode II de-bonding, except that the subscript n is changed to t for tangential mode and the normal stress σ is changed to the shear stress τ for mode II fracturing. Mode II de-bonding is referred to as the interface separation mode where tangential slip dominates the interface separation in the normal direction. The equation for the tangential contact stress and tangential slip distance is written as:

$$\tau_t = k_t \Delta_t (1 - d_t) \quad (9)$$

where d_t is the shear mode de-bonding parameter. The tangential critical fracture energy of mode II de-bonding is the energy required to fracture a material in shear mode, and is equal to the area under strain-displacement curve for mode II as:

$$G_{cII} = \frac{1}{2} \tau_{\max} \Delta_t^c \quad (10)$$

where τ is the shear stress and Δ_t^c is tangential slip distance at the completion of de-bonding.

2.2.2. Mixed Mode Traction-Separation Behavior

To obtain the above equations for the case of mixed mode fractures, the effective quantities for the force $\bar{T}(\bar{\Delta})$ and the displacement $\bar{\Delta}$ are defined as:

$$\bar{T}(\bar{\Delta}) = \sqrt{(\sigma)^2 + (\tau)^2} \quad (11)$$

and

$$\bar{\Delta} = \sqrt{(\Delta_n/\delta_n)^2 + (\Delta_t/\delta_t)^2} \quad (12)$$

The parameters of Equation (12) were defined in Table 1 with the exception that subscript t is used for shear mode and subscript n is used for normal mode.

Tvergaard [42] related the above effective quantities ($\bar{T}, \bar{\Delta}$) to the normal and tangential tractions (T_n, T_t) as follows:

$$T_n = \frac{\bar{T}(\bar{\Delta})}{\bar{\Delta}} \frac{\Delta_n}{\delta_n} \quad (13)$$

and

$$T_t = \frac{\bar{T}(\bar{\Delta})}{\bar{\Delta}} \alpha_e \frac{\Delta_t}{\delta_t} \quad (14)$$

where α_e is a dimensionless constant associated with mode mixing. For mode I, where $\Delta_t = 0$, the normal cohesive traction (T_n) is the same as $\bar{T}(\bar{\Delta})$. For mode II, where $\Delta_n = 0$, the tangential cohesive traction (T_t) is equal to $\alpha_e \bar{T}(\bar{\Delta})$, see Equation (14). Therefore, the dimensionless constant (α_e) is a scaling factor between tangential and normal cohesive tractions. The simulations presented in this paper are 2D analyses at this stage, but can be extended to 3D CZM simulation. The 3D formulation can be obtained by extending 2D traction-separation models (*i.e.*, the effective displacement model) to 3D cohesive zone models [43, 44] as explained in the following. The effective displacement in this case is defined as:

$$\bar{\Delta} = \sqrt{(\Delta_1/\delta_1)^2 + (\Delta_2/\delta_2)^2 + (\Delta_3/\delta_3)^2} \quad (15)$$

where Δ_1 , Δ_2 , and Δ_3 are separations and δ_1 , δ_2 , and δ_3 are characteristic lengths along the local coordinates. Figure 6 shows the cohesive fracture gap (Δ_1) and sliding separations (Δ_2, Δ_3) along the local coordinate system with axes normal and tangential to the fracture surface.

Figure 6: Cohesive fracture gap (Δ_1) and sliding separations (Δ_2, Δ_3) along the local coordinate system (a) two-dimensions and (b) three-dimensions.

2.2.3. Rock Failure Stages in CZM

The structure of rock can be ideally regarded as an assembly of pieces or blocks, also known as fragments, attached along their boundaries using cohesion and friction. The cohesive zone along the boundaries is called the contact zone. Depending on the rock material properties and the applied load, failure can occur within the contact zone when the material is subjected to external loads. In Figure 2, the fragments are idealized as mass particles or mass blocks, and the contact zones or the potential failure zones are idealized as springs. The rupture of the contacts is described by CZM following a traction-separation law, as shown in the right column of Figure 2. Cohesive zone failure follows four distinct stages:

1. Pre-failure; this stage is mainly linear deformation of the fragments and contacts. At this stage, the contacts are not broken yet, in other words, there is no separation normal to-or along-the contact boundaries (no gap or sliding).
2. Failure; when the normal and shear strength of contacts are exceeded by the external load (also known as traction), one or a number of fragments

undergo excessive displacements and local fractures are initiated. This
 stage is initiation of damage; *i.e.*, local rupture of contacts or bonds.
 Failure is the peak point in the CZM plot in Figure 5.

3. Post-failure; this stage starts from the first bond rupture and may continue
 if the external load keeps increasing. This phase is the evolution of local
 failure or the damage evolution stage of rock fracturing, during which the
 rock loses its strength and exhibits a softening behavior. The softening
 behavior is due to the stiffness reduction, which is reflected by the declin-
 ing slope of stress-strain plot. The damage evolution is reflected by the
 declining section of the stress-deformation plot (also known as traction-
 separation behavior of CZM). The declining section of the CZM can be
 either linear or nonlinear (Figure 4). The evolution of damage is a complex
 phenomenon, and has been studied previously [45].
4. Fracture; if the external load keeps increasing, the post-failure stage termi-
 nates by creation of a populated crack density zone which leads to distinct
 failure surfaces.

Next we will discuss CZM for thermal fracturing and show CZM model valida-
 tion for wellbore rock fracturing in the oil and gas industry. Problems such as
 uniaxial tensile loading and wellbore stability will be used in the context of best
 numerical methods to validate and introduce the most efficient method of using
 CZM for rock fracturing purposes.

2.3. Near Well bore Thermal Stresses

At reservoir depth, all the stresses are compressive, and rock fails due to
 either shear stress or reduced compressive stress, as both reduce the margin
 of safety based on Mohr-Coulomb shear failure envelope. In other words, the
 stress in rock gets closer to shear failure envelope if the effective compressive
 load decreases or shear stress increases. The failures around a well bore are in
 the form of breakouts, breakdown, fractures, or coalescence of small fractures
 leading to failure. Understanding the stress changes in the near well bore zone

is essential to well design and stability management. Another important information is the well orientation which is necessary to obtain hoop stress around a well bore while overburden and in-situ stresses are available. In the case of vertical well, the two stresses acting on the well cross-section are maximum and minimum horizontal stresses, $S_{H\max}$ and $S_{h\min}$. For a horizontal well however, the direction of well determines the combination of stresses acting on well cross-section. For example if the well bore is aligned with the direction of maximum horizontal stresses, the two stresses acting on well cross-section are $S_{h\min}$ and S_v . S_v is the vertical stress from overburden pressure of rock layers above that point. The effective tangential stress for a vertical well in a homogeneous and isotropic elastic rock in which the total vertical stress S_v is the principal stress, at the wall of the well bore is given by Equation (16).

$$\begin{aligned} \sigma_{\theta\theta} = & S_{h\min} + S_{H\max} - 2(S_{H\max} - S_{h\min})\cos(2\theta) \\ & - 2P_p - \Delta P - \Delta\sigma_{\theta\theta}^{\Delta T} \end{aligned} \quad (16)$$

Figure 7 shows how the breakdown and breakout pressures in the near wellbore zone in terms of in situ stresses and wellbore pressure can be obtained from Equation (16).

Figure 7: Breakdown and breakout pressures in terms of the in-situ stress and wellbore pressure

In Equation (16), θ is measured from the azimuth of the maximum horizontal stress, S_{HMax} and S_{hmin} are the maximum and minimum horizontal stresses; P_p is the pore fluid pressure; ΔP is the difference between the wellbore pressure (mud weight) P_w , and the pore pressure, and $\Delta\sigma_{\theta\theta}^{\Delta T}$ is the variation of the tangential stress around the well bore, induced by cooling of the well bore by ΔT [46]. With the sign convention of compression being positive and tension being negative, the last term in Equation (16) takes the negative sign. The negative sign indicates that thermal cooling of ΔT degrees reduces the tangential stress $\sigma_{\theta\theta}$ by $\Delta\sigma_{\theta\theta}^{\Delta T}$. Reduction of temperature in a confined solid rock body changes the in-situ stresses of rock. The lower the temperature, the lower the in-situ stress and the easier it becomes to fracture the rock. In other words, reduction of temperature reduces the fracture initiation pressure or breakdown pressure. To obtain breakdown pressure, $\theta = 0$ is plugged into Equation (16).

$$\sigma_{\theta\theta} = 3S_{hmin} - S_{HMax} - P_w - P_p - \Delta\sigma_{\theta\theta}^{\Delta T} \quad (17)$$

For a rock sample of tensile strength T , the breakdown occurs under the condition of $\sigma_{\theta\theta} + T < 0$, in which case the right hand side of Equation (17) is defined as the breakdown pressure. This leads to the following relation for breakdown initiation around the wellbore.

$$P_w > 3S_{hmin} - S_{HMax} - P_p - \Delta\sigma_{\theta\theta}^{\Delta T} + T \quad (18)$$

For shale with very small tensile strength, T is taken as zero and the wellbore pressure at breakdown will be

$$P_w > 3S_{hmin} - S_{HMax} - P_p - \Delta\sigma_{\theta\theta}^{\Delta T} \quad (19)$$

The effect of temperature on the wellbore wall is time-dependent, such that the longer the rock is in contact with the wellbore fluid the further away from the hole the temperature perturbation will propagate. In a simplified approach, one can assume that the material is impermeable (a reasonable assumption for ultra-low permeability materials such as shales). In this case, a relatively simple integral equation can be written for the magnitude of $\Delta\sigma_{\theta\theta}^{\Delta T}$, the variation of the

tangential stress around the wellbore as a function of radial position r and time t due to temperature effects ([46]). Although the exact solution for the temperature distribution near a constant-temperature wellbore is a series expansion, solutions exist which approximate the temperature in the near wellbore region using only the first two terms of the expansion as shown in Equation (20), [47]:

$$\Delta\sigma_{\theta\theta}^{\Delta T} = \left[\frac{\alpha E \Delta T}{1 - \nu} \right] \left[\left(\frac{1}{2\rho} - \frac{1}{2} - \ln\rho \right) I_0^{-1} - \left(\frac{1}{2} + \frac{1}{2\rho} \right) \right] \quad (20)$$

where: $\rho = \frac{r}{a}$; $I_0^{-1} = \frac{1}{2\pi i} \int_{-\infty}^0 \frac{e^{[4\tau z / \sigma^2]z}}{z \ln z} dz$

Once steady state has been reached, the change of the tangential stress around the wellbore is given by Equation (21), [48, 49]:

$$\Delta\sigma_{\theta\theta}^{\Delta T} = \left[\frac{\alpha E \Delta T}{1 - \nu} \right] \quad (21)$$

Where α is the linear coefficient of thermal expansion, E and ν are the static Young's Modulus and the Poisson's Ratio of the rock respectively, and ΔT is the temperature difference between the borehole fluid and the in-situ formation (note that since we chose to use $-\Delta\sigma_{\theta\theta}^{\Delta T}$ in Equation (16), ΔT is positive if the fluid in the borehole is cooler than the formation temperature). Cooling of the wellbore (*i.e.*, positive values of ΔT) reduces the value of the tangential stress $\sigma_{\theta\theta}$ around the wellbore, thus lowering the threshold for placing the wellbore in a state of effective tension and lowering the threshold for hydraulic fracture initiation. It will also lower the value of S_{hmin} , the minimum horizontal stress. For a rock fracture that is effectively cooled, this will reduce the fracture propagation pressure. The fact that cooling leads to reduction of fracture initiation and propagation pressure is well-known from deepwater drilling operation, where circulating cold drilling fluid across downhole formations triggers induced tensile wellbore fracturing and associated mud losses (see, e.g., [50]) As shown in Figure 8, the upper figure shows the variation of tangential and radial stresses around a well bore for a well condition in which the vertical stress S_v

is the principal stress. It can be seen that the tangential stress is minimum at the azimuth of the maximum far-field principal stress direction. Therefore, creating tensile stress at this point is the easiest and can take place by reducing the temperature in the near well bore zone. The bottom Figure 8 shows the compressive and tensile failures due to in-situ stresses at the well cross-section, with our focus being on tensile failure.

Figure 8: (Up) Variation of tangential and radial stresses around a well bore for a wellbore condition in which the vertical stress S_v is the principal stress. (Bottom) Compressive and tensile failures due to in-situ stresses at the well cross section. (Adapted from GeoMechanics Intl. Inc.)

2.4. Mathematical Model

As described earlier, materials fail if the applied load either exceeds the material yield strength or the displacement exceeds an ultimate value. Figure 9-(a) shows the load displacement of a bonded pair of material which undergoes tensile or shear stress. When the load is less than T_{ult} there is no damage developed in material. Loading and unloading below T_{ult} does not create any residual strains

in material and displacement will be back to the original displacement. T_{ult} is the maximum tensile stress that a material can undergo before failure. Beyond failure, material experiences a progressive damage until it fails at maximum displacement δ_f . The relation between traction causing separation of fracture faces, can be written in the general form of Equation (22) in which m is the cohesive law behavior index.

$$T = T_{ult} \left(1 - \frac{\delta}{\delta_f} \right)^m \quad (22)$$

Figure 9-(b) exhibits the realistic behavior of stress at the tip and ahead of a fracture tip. For quasi-brittle to ductile materials, there is an inelastic zone also known as fracture process zone (FPZ) which is the zone of micro-structural damage ahead of fracture tip. In classic fracture mechanics, dealing with energy dissipation process in the FPZ is challenging. However, CZM formulation eliminates these challenges, hence easier to utilize. Figure 9-(c) shows the cohesive parameters of a typical traction-separation model [51]. In general, a propagating fracture can undergo three forms of external mechanical load; these are normal tensile (t_n), shearing mode (t_s), and tearing mode (t_t). The stiffness components associated with each of these loadings or corresponding deformations result in the following matrix form of constitutive law of Equation (23). The traction and separation vectors are reflected in Equation (23) by $\vec{t}$ and $\vec{\delta}$. For uncoupled failure modes 1 to 3, the off-diagonal members of the matrix become zero and the system of equations can be solved as a de-coupled system. In addition to Equation (23), the progressive damage has to be incorporated in the solution process. The damage parameter D in Equation (24) is reflected by the declining portion of the traction-separation plot (Figure 9-(a)). D is the damage parameter which is a scalar parameter that varies between 0 at the maximum stress T_{ult} and 1 at the maximum separation δ_f .

Figure 9: (a) Traction-Separation behavior of materials, (b) Inelastic zone at fracture tip, and (c) Cohesive parameters of a typical traction-separation model [51]

$$\vec{t} = \begin{Bmatrix} t_n \\ t_s \\ t_t \end{Bmatrix} = \begin{bmatrix} K_{nn} & K_{ns} & K_{nt} \\ K_{ns} & K_{ss} & K_{st} \\ K_{nt} & K_{st} & K_{tt} \end{bmatrix} \begin{Bmatrix} \delta_n \\ \delta_s \\ \delta_t \end{Bmatrix} = \vec{K} \cdot \vec{\delta}, \quad (23)$$

$$t = (1 - D)t_{\max} \quad (24)$$

2.5. Thermal Fracturing using Embedded Multiple-Site Cohesive Zone Elements

To create a branching fracture pattern around the wellbore to amplify hydrocarbon production, a novel method was proposed by authors [8]. This method introduces a thermal gradient around the wellbore by cooling/freezing the wellbore. It imposes thermal stress and causes multiple-site thermal fractures. Complexity and high cost of field testing for this process motivated us to explore different simulation approaches to design thermal fracturing process in virtual simulation environment, so we utilized CZM as an alternative to Discrete Element Method (DEM). As explained earlier, for simulation of thermal fracturing using continuum models we assume that a block of rock is made up of smaller cubic blocks, Figure 10(a). These blocks are in contact using contact elements possessing the fracture properties of rock, Figure 10(b). Thermal stresses and deformations occur in the blocks and load transfer between blocks is carried out by contact elements. There are several criteria for fracture initiation and propagation. Here we focus on strain and energy criteria. In these two criteria, whenever the strain or energy in contact elements exceeds a threshold value, the

contacts break in shear or tension. To obtain a critical value for deformation of blocks at contact points, we use an energy approach in which as the potential energy of contacts or springs reaches the fracture energy of rock, fracturing initiates.

(a) The Rubik's cube model of rock (b) Geometry and contact elements

Figure 10: Rock model for thermal fracture propagation simulation

The critical value of crack opening for fracture initiation can be obtained using Equations (25), (26), and (27) in which, K is the stiffness of springs, a is the size of rock blocks, Δ is the deformation of the opening between the two blocks, G_c is the fracture energy release rate, E is the Young's modulus of rock and A is the area of each block. The stiffness of spring can be related to the Young's modulus using elastic Equation (25).

$$K = \frac{F}{\Delta} = \frac{\sigma A}{\Delta} = \frac{E \varepsilon A}{\Delta} = \frac{E \frac{\Delta}{a} A}{\Delta} = \frac{EA}{a} \quad (25)$$

From fracture mechanics, we know that the energy release rate is the energy required to create a unit surface of fracture. Then, to obtain the deformation, the energy release rate is made equal to the potential energy of springs as they deform.

$$G_c = \frac{1}{2} K \Delta^2 \Rightarrow \Delta = \sqrt{\frac{2G_c}{K}} = \sqrt{\frac{2G_c a}{EA}} \quad (26)$$

Finally, the critical value of deformation is obtained from Equation (25). For the case of Shale using typical values, Δ is obtained as follows;

$$\Delta = \sqrt{\frac{2 \times 100(J/m^2) \times 1(m)}{50 \times 10^9 \times 1(m^2)}} = 63\mu m \quad (27)$$

This value is used as the maximum opening between the two contact surfaces at the onset of de-bonding.

2.5.1. Fracture Propagation Criterion

Fracture propagation criterion that we use in our simulations is based on the fracture energy. This requires two parameters to define the traction-separation plot. These two parameters are the maximum contact stress and the fracture energy for the two modes of failure. We could also use the maximum contact stress and the contact gaps at completion of de-bonding (Δ_n^c), as the two parameters required to define the damage and failure criteria; however, we use the fracture energy based criterion for the simulations performed in this paper, due to availability of the fracture energy values for shale in the literature.

3. Results and Discussion

3.1. Model Validation

Here we use the finite element code ANSYS 16.0 and compare the results with the finite element method used by Mergheim *et al.* [52], who use discontinuous elements with additional degrees of freedom at the crack tip to make the solution mesh independent.

3.1.1. Verification Model 1- Tensile Mode Failure

In the first example, we use a 1mm×1mm square plate and apply a uniform tensile stress at one end. Geometry, loading and boundary condition of the problem are shown in Figure 11. The top edge is subjected to uniform displacement and can freely move upward. The lateral movement of the two top corner nodes

are constrained by two rollers and the bottom edge is fully constrained against
displacement in x and y directions. The material parameters in our simulation
are taken from the reference paper [52] and are as follows; Young's modulus
 $E = 100 \text{ N/mm}^2$, Poisson's ratio $\nu = 0$, tensile strength $f_t = 1.0 \text{ N/mm}^2$, and
fracture energy $G_f = 0.1 \text{ N/mm}$ which is equal to 100 J/m^2 . In this example,
if the fracture zone is defined along the entire length of the domain, the propa-
gation of fracture takes place instantly and the solution does not converge. To
avoid this, a small seed of fracture is placed on the left side of the fracture shown
in Figure 11 and from there, fracture propagates through the domain. G_I , the
first mode fracture energy, indicates that we assume no shear mode of failure
occurs and all the energy imparted to the block is used to fracture the block
in tensile mode. We obtain the global load-displacement plot for pure mode I
failure and compare it against the plot obtained from the reference paper [52].
As the tensile stress increases due to increase of upward displacement on top
edge, the load-displacement curve approaches a critical load which is expected
to be equal to tensile strength of rock, 1.0 N/mm^2 , as depicted in Figure 13.
Once, the displacement gets beyond the value corresponding to the maximum
load, the load-displacement plot enters a declining section in which stiffness de-
creases. The decrease in stiffness is due to the ongoing bond breakage at the
contact zone as the vertical displacement increases. The first part of the plot is
in perfect agreement with the reference plot; however, the second portion devi-
ates from it. This difference is attributed to the behavior of the cohesive zone
beyond the damage initiation phase, *i.e.*, when the contacts are subjected to
stress. Another factor that causes the deviation of our result from the reference
result is the additional nodes which are used in the formulation of discontinuous
elements defined by Mergheim *et al.* [52]. Notice that in Figure 12 the contact
stress will be maximum (here 1.0 N/mm^2) when the fracture is about to open
and once the fracture opens, the contact stress drops to zero.

Figure 11: Geometry, loading and boundary condition of the problem. Dimensions are given in mm.

Figure 12 shows the distribution of the stress normal to the fracture face.

Figure 12: Maximum stress of contact point at failure

Figure 13: Verification of the CZM model for tensile mode failure.

3.1.2. Verification Model 2- Wellbore breakdown

The second example we use to verify the rock failure mechanism is the wellbore stability problem. From linear elastic analysis, the minimum hoop stress around the wellbore as a function of the drilling fluid pressure p_w , pore pressure p_p , maximum horizontal stress S_{Hmax} , and minimum horizontal stress S_{hmin} can be calculated using Equation (28);

$$\sigma_{\theta\theta,min} = 3S_{hmin} - S_{Hmax} - p_w - p_p \quad (28)$$

when $\sigma_{\theta\theta,min}$ in Equation (28) becomes zero, wellbore pressure P_w will be equal to breakdown pressure. When $\sigma_{\theta\theta,min}$ exceeds the tensile strength of rock, tensile failure occurs. We use the CZM capability of the finite element tool to test the applicability of this model to obtain the value of breakdown pressure. Using a known analytical solution, we show that our model is capable of obtaining the value of breakdown pressure as well as providing the extent of fracture propagation. The extent of fracture opening is a valuable result that can not be obtained using classic finite element models; but it is trivial when CZM is used. Figure 14 (left) shows the geometry of a wellbore and Figure 14 (right) shows the mesh and fragments of rock that we created for our simulation. Contact

elements are placed at the interface of each two fragments, then each fragment is filled with nine elements. A zone of 10 feet around the wellbore is assigned fragments and contact elements.

Figure 14: Geometry (left), and rock elements and fragments (right)

To investigate the variation of minimum hoop stress versus wellbore pressure, we use a wellbore pressure range of 4000 to 7000 psi, zero pore fluid pressure P_p , the in-situ stresses of $S_{Hmax} = 3,000$ psi and $S_{hmin} = 2,500$ psi. Using these values, the analytical breakdown pressure can be calculated as follows;

$$\sigma_{\theta\theta,min} = 3 \times 2,500 - 3,000 - P_w = 0$$

whereby P_w is obtained as 4,500 psi at breakdown initiation. Figure 15 shows the generation of tension cracks and breakdown using CZM model.

Figure 15: Generation of tension cracks and breakdown using CZM model.

440 The variation of the minimum hoop stress around the wellbore versus well-
 bore pressure can be seen in Figure 16. The left vertical axis shows the minimum
 hoop stress values which are compressive when they are positive. The results
 indicate that as wellbore pressure is increased, the minimum hoop stress is re-
 duced and goes from compressive to tensile stress (from positive to negative
 445 values). Assuming zero tensile strength for rock, when minimum hoop stress is
 zero, breakdown initiates at the wellbore pressure of $P_w=4,500$ psi. Our numer-
 ical method is capable of capturing the tensile hoop stress beyond the onset of
 breakdown. This is not possible in classic finite element methods, because the
 domain becomes fractured. Our numerical results exhibit a very good agree-
 450 ment (error of $\pm 5\%$) with the analytical results (the solid line), shown on the
 right vertical axis.

Figure 16: Verification of the minimum hoop stress around the wellbore as a function of wellbore pressure (left axis) and the error between numerical and analytical solutions (right axis) using CZM model.

It should be noted that, the finite element simulation using CZM, can reveal the zones in which smaller cracks are developed. These zones are indicated as tension cracks zones in Figure 15. Moreover, the CZM model can predict the extent of the fracture propagation which is not readily available in classic finite element approach. As a result, we use the CZM feature implemented in the finite element package ANSYS for thermal fracturing purposes to obtain the extent and the pattern of thermal fractures.

3.2. CZM Simulation Results

We are interested in studying the effect of in-situ stress on the initiation and propagation pattern of thermal fractures; therefore, we performed fracturing analysis for four different cases of in-situ stresses. The geometry of the model is the same geometry used in Figure 14. The model initial and boundary conditions are shown in Figure 17. The values of maximum and minimum horizontal stress

for the first two cases are shown in Figure 18 (a_1 , a_2) in which we are using an in-situ horizontal stress anisotropy ratio of 3. The other two cases 3 and 4 are shown in Figure 19 (a_3 , a_4) in which an in-situ horizontal stress anisotropy ratio of 1.5 is used. The stresses in each group are one order of magnitude apart, to reflect the effect of low and high reservoir stresses. The simulation results for these four cases reveal the impact of stress anisotropy and reservoir depth on thermal fracture opening and pattern.

Figure 17: Model setup, mechanical/thermal, and initial/boundary conditions.

The wellbore radius is taken as 6 in and the reservoir size is 100ft $\times$ 100ft. A script was written to create 504 rock fragments and 4,700 elements around the wellbore. The scripts with 560, 2300, 3400, 4700, 6800, and 9400 elements were used for a mesh sensitivity analysis to indicate that beyond a certain mesh density, the numerical results are independent from the number of elements. The mesh sensitivity analyses resulted in an optimum number of 4700 elements for our simulations to provide accurate results. The material parameters for the four simulation cases are listed in Table 2. A random function is used to generate random material properties for fracture energy of rock fragments. Fracture energy of shale is in the range of 5–100 J/m^2 and tensile strength of rock is

in the range of 0–100 psi. We generated values of these two parameters in the given possible ranges for all elements of the model.

Parameter	Value
Young's Modulus, E	100 (GPa)
Poisson's ratio, ν	0.25
Rock tensile strength, f_t	0–100 psi
Rock fracture energy, G_f	5–100 J/m^2
Contact gap at de-bonding, δ_0	$2 \times 10^{-6} m$
Coefficient of thermal expansion, α	$1 \times 10^{-5} \frac{1}{^\circ C}$
Isotropic thermal conductivity, k	$2 \frac{W}{m \cdot ^\circ C}$

Table 2: Material parameters for the three cases of reservoir thermal rock fracturing.

Our past studies [7, 8] show that the zone of heat transfer around the wellbore is typically 3-10 ft. We therefore limited our mesh refinement to a 10 feet zone around wellbore. The CZM simulations are 1) nonlinear due to the presence of contact elements at the interface of rock fragments, and 2) coupled due to thermo-mechanical (TM) interactions. Assigning a 10-foot zone for heat transfer reduces the simulation time drastically. The analysis type is based on 2D plane strain assumption. The reservoir rock is subjected to two different loading conditions: thermal and mechanical. Thermal loading is the cooling process which takes place at the wellbore face. Initially the domain is at 150°C temperature and the wellbore temperature is then set to 50°C. Heat transfer is simulated and the solution of temperature distribution field is mapped to the structural mesh. The mechanical load is supplied by the in-situ stress. Figure 18 shows the thermal fracture patterns (c_1 and c_2) for the horizontal in-situ stress ratio of 3, for two cases of (a_1): high in-situ stress, and (a_2): low in-situ stress. Since the actual fracture openings are not visible at reservoir scale, they

are magnified for visibility purposes in Figure 18 (b₁) and (b₂).

Figure 18: Thermal fracture patterns (c₁ and c₂) for the horizontal in-situ stress ratio of 3, for two cases of (a₁): high in-situ stress and (a₂): low in-situ stress. (b₁ and b₂) are the magnified displacement fields.

It can be seen from Figure 18 that the fracture propagation direction is along the maximum in-situ stress. Moreover, the pattern of thermal fractures in case 2 shows more branching; in other words, the reservoir rock is more efficiently stimulated when in-situ stress is lower. Similar simulations are performed for stress anisotropy ratio of 1.5 to investigate the impact of stress ratio in the thermal fracture opening and pattern, as shown in Figure 19. The first observation is that fracture propagation direction is along the maximum in-situ stress as

expected. Also, the stimulation zone is larger for the lower in-situ stress case 4 than that of case 3.

Figure 19: Thermal fracture patterns (c₃ and c₄) for the horizontal in-situ stress ratio of 1.5, for two cases of (a₃): high in-situ stress and (a₄): low in-situ stress. (b₃ and b₄) are the magnified displacement fields.

510 The maximum fracture opening values are crucial in estimation of rock permeability enhancement. Figure 20 shows the fracture width values and compares all four cases. The fracture width at a stress ratio of 3 ranges from 0.36 mm for the high in-situ stress case (1) to 0.64 mm for the low in-situ stress case (2). At a stress ratio of 1.5, fracture openings are smaller and within the range of

0.25 mm (high stress case 3) to 0.38 mm (low stress case 4). This demonstrates the effect of stress anisotropy in fracture opening values. The higher the stress anisotropy, the bigger the fracture width.

It should be noted that the thermal fractures for stress ratio of 1.5 and 3 are totally different. This is due to the fact that thermal stresses compete against the combination of $3S_{hmin} - S_{Hmax}$ (as discussed in Equation (28)) between the maximum and minimum horizontal stresses. As the in-situ stress ratio increases from 1.5 to 3, the term $3S_{hmin} - S_{Hmax}$ gets smaller and creation of thermal fracture is easier; hence, wider fractures are observed and fractures extend further from the cooling source.

Figure 20: Comparison of thermal fracture opening for all four cases of stress ratio of 1.5 and 3 for high and low values of in-situ stress.

The element size and domain discretization are of crucial significance to the

validity of a finite element model. In particular, the numerical results must be independent from the number of elements beyond a certain mesh density. Figure 21 shows the convergence of results at a given point at $\theta = 0$ on the wellbore wall for case 3 (a representative case among the four cases presented above). Two types of elements, low order and high order elements, were used in our simulations. Figure 21-a shows the convergence results for low order elements and Figure 21-b shows the convergence results for high order elements. In both cases, the stress values from the Finite Element simulation converged to the analytical value of stress when mesh density increases. Beyond 4000 elements, the numerical results get to a plateau, indicating that the results are mesh independent. We used 4700 elements in our simulations based on these mesh sensitivity analysis results. In addition, the error plots are added to the mesh sensitivity plots. In all cases, they show a declining pattern of error as mesh density increases. At above 4000 elements, the error is always less than 2%, which indicates good agreement between the numerical results and analytical values.

(a)

(b)

Figure 21: Mesh sensitivity analysis for case 3 at $\theta = 0$. (a) Low order elements, (b) High order elements.

Similar results were obtained for $\theta = 90$ and are presented in Figure 22.

(a)

(b)

Figure 22: Mesh sensitivity analysis for case 3 at $\theta = 90$. (a) Low order elements, (b) High order elements.

Creation of small gaps in tight formations has a significant impact on the conductivity of such formations to gas flow. The values of gaps we obtained from CZM simulations indicate that thermal fracturing has the potential to meaningfully improve the recovery in tight formations. Recent observation of samples taken from Eagle Ford stimulated rock volumes following hydraulic stimulation shows the existence of a great number of open un-propped fractures [53]. This indicates the significance of thermal fractures as they can stay open and con-

tribute to permeability enhancement even if they may be un-propped. It should be noted that thermal fractures propagate only a short distance from the source of cooling, but as soon as initial fractures open, fluid penetrates into these opening and further propagation is expected to continue. Further work is necessary to study the pressure driven fracture propagation.

3.2.1. Effect of Cooling on Fracture Initiation Pressure

The analytical form of the wellbore cooling effect on fracture initiation pressure (FIP) was shown in Equation (17). To study the sensitivity of FIP to temperature reduction, the model of Figure 14 was used in which reservoir rock temperature is kept at 150°C and wellbore temperature is maintained at several temperature levels of 150°C, 125°C, 100°C, 75°C, and 50°C. The analytical minimum hoop stress $\sigma_{\theta\theta}$ value is obtained from Equation (17) and the numerical values of FIP using CZM are obtained and plotted in Figure 23. The same values of pore pressure, maximum and minimum horizontal in-situ stresses used in validation model were used here *i.e.*, zero pore fluid pressure $P_p = 0$, $S_{Hmax} = 3,000$ psi and $S_{Hmin} = 2,500$ psi.

Figure 23: Reduction of the Fracture Initiation Pressure versus reservoir/wellbore temperature difference at steady state condition.

Both analytical and numerical results show that the FIP decreases (Figure 23, left axis), as the temperature difference between the wellbore and reser-

voir increases. The error between numerical and analytical solutions are less
 570 than 0.5% for all five cases and shown on Figure 23 (right axis).

3.2.2. Effect of Cooling on Fracture Width

Figure 24 shows the variation of maximum fracture width versus temperature difference between reservoir and well bore for one of the four cases of in-situ stress presented earlier (*i.e.*, $S_{Hmax} = 3,000$ psi and $S_{hmin} = 1,000$ psi.)

Figure 24: Fracture width increase versus reservoir/wellbore temperature difference at steady state condition.

575 The results indicate a nonlinear response of the system, *i.e.*, the rate of change of the maximum fracture width versus temperature reduction is nonlinear. Maximum fracture width increases at a low rate for a temperature difference below 50°C; from 50 to 70°C, fracture opening increases at a higher rate; beyond 70°C, the rate of increase lowers and the plot levels off and reaches
 580 a plateau. This nonlinear trend can be tentatively attributed to the combined effect of the reservoir rock stiffness, the pattern of existing in-situ fractures, thermal conditions and in-situ stresses. It should be noted that the CZM simulation is a nonlinear simulation; in other words, once part of the reservoir rock breaks in shear or tension, the stiffness of rock in the vicinity of the fractured
 585 zone reduces, leading to an unpredictable nonlinear solution. The behavior of

rock after failure can be studied using the fracture width distribution and deformation contour plots. Figure 25 shows the evolution of fracture width versus cooling. Blue line segments in the left column of figures indicate the maximum opening between rock segments. The temperature difference keeps increasing
590 from 0°C to 100°C and thermal fractures develop. Maximum fracture width of 0.37mm is obtained when the temperature reduction is 100°C. The trend of fracture propagation direction is in the direction of maximum compressive in-situ stress. At smaller temperature difference values, the continuous fracture openings are limited in terms of magnitude and extent of propagation. The
595 plots of Figure 25 clearly demonstrate the discontinuous nature of rock failure, which features significantly in CZM simulation.

Figure 25: Contour plots of deformation (R) and fracture width distribution (L) versus reservoir/wellbore temperature difference at steady state condition for the case of $S_{Hmax} = 3,000$ psi and $S_{hmin} = 1,000$ psi.

To investigate the effect of stress ratio further, new simulations for stress ra-

tio of 1.5 are performed to compare the pattern and width of thermal fractures with those of Figure 25. Simulation of fracture opening for the stress ratio of 1.5 exhibits the same effect in terms of creating thermal fractures for various thermal reductions shown in Figure 24; however, the pattern and magnitude of fracture openings will be different (Figure 26).

Figure 26: Contour plots of deformation (R) and fracture width distribution (L) versus reservoir/wellbore temperature difference at steady state condition for the case of $S_{H\max} = 4,500$ psi and $S_{H\min} = 3,000$ psi.

3.3. Comparison of XFEM and CZM

605 A comparison is added in Table 3 which compares the normalized CPU time for the validation cases of two simulations performed using CZM and XFEM ([54] and [55]). Both validation cases were compared to KGD solution.

	XFEM	CZM
Real time (s)	5.71	22.3
Core time (hr)	20	48
Element count	186,562	66,000
Number of utilized cores	96	48

Table 3: Comparison of CZM vs. XFEM validation cases reported in [54] and [55].

To compare the two cases, the computational effort is defined as shown in Equation (29);

$$\text{Computational effort} = \frac{\text{Core time} \times \text{Number of utilized cores}}{\text{Real time} \times \text{Element count}} \quad (29)$$

610 The computational effort calculated for XFEM and CZM are 0.00180 and 0.00157 which results in XFEM simulation time to be 1.149 times more expensive than CZM for such a validation case. This ratio may change significantly as the number of fractures increases. Here in the validation cases, only one fracture is simulated.

3.4. Application to Cemented Micro-fracture Reactivation

615 We believe that the CZM framework as described in this article will be able to simulate the cemented micro-fracture reactivation scenario. More complex phenomena such as coupled thermo-mechanical fracture reactivation can be dealt with, using CZM simulation. Complexities associated with nonlinearity of rock stress-strain equation and the impact of fracture and formation fluid pressures

can also be handled using coupled elements with displacement and fluid pressure degrees of freedom. Note that in the paper we have focused exclusively on the aspect of thermal rock stimulation by thermal fracturing and are not yet addressing other aspects such as cemented micro-fracture reactivation. This could be discussed in future work.

4. Conclusions

The work presented here deals with thermal fracturing to amplify hydrocarbon recovery from tight rock formations such as shales. More specifically, cooling/freezing of such tight rocks can make hydraulic fracturing more efficient by lowering fracture initiation and propagation pressures, and by opening thermal cracks. The latter is of particular interest here. The efficacy of thermal fracturing is a function of 1) the induced temperature reduction in the reservoir rock that is being stimulated, 2) the rock mechanical properties and 3) prevailing in-situ stress conditions. The thermal fracturing process is simulated using the CZM finite element method. It is shown that embedded multiple-site CZM is particularly well-suited for thermal fracturing modeling. The merits of the CZM approach include:

- CZM computation time is fast compared to large-scale molecular dynamics simulations (such as using a Large-scale Atomic/Molecular Massively Parallel Simulator (LAMMPS)).
- Tight reservoir rock such as shale is inhomogeneous and rock and stress properties are typically anisotropic. Such features, important to a realistic simulation of the thermal fracturing process in actual tight rock formations, are relatively easily implemented in CZM analysis. For instance, in the simulations presented here, inhomogeneity of fracture distribution is represented by assuming a random distribution of fracture energy (which varies in the range of 50-100 J/m²).

- In general, its ease of handling material non-linearity and rock constitutive law make CZM a preferred method for rock thermal fracture analysis.
- CZM allows for simulation of both tensile and shear failure modes, using two independent traction-separation properties associated with these two modes. Using this feature, we can investigate whether the mode of failure is tensile-dominant or shear-dominant.
- CZM simulation can furthermore reveal zones in which smaller cracks are developed. These zones are indicated predominantly by the presence of tension cracks.
- In addition, a CZM simulation can predict the extent of fracture propagation, which is more difficult to determine in a classic finite element approach. CZM can thereby readily be used to obtain the extent and the pattern of induced thermal fractures.
- Stress anisotropy and magnitude affect the thermal fracture pattern and opening values. Our simulations show that, wider thermal fractures are created at higher stress anisotropy and lower in-situ stress values.

ACKNOWLEDGMENT

Authors would like to express their appreciation to the Petroleum and Geosystems Engineering Department of the University of Texas at Austin for computational resources and financial support of this research.

NOMENCLATURE

T_{ult}	= Maximum tensile stress
m	= Behavior of cohesive law
$\vec{t}$	= Traction vector
$\vec{\delta}$	= Separation vector
F_t	= Tensile strength
G_c	= Fracture energy
δ_0	= Displacement at failure
δ_f	= Displacement at rupture
L	= Fracture length
H	= Fracture height
K	= Traction stiffness matrix
D	= Damage parameter
E	= Young's modulus
ν	= Poisson's ratio
S_{hmin}	= Minimum horizontal in-situ stress
S_{HMax}	= Maximum horizontal in-situ stress
S_v	= Vertical in-situ stress
γ	= Fracture surface energy
T	= Normal contact (tensile) force
Δ_n	= Contact gap or normal separation
Δ_n^c	= Contact gap at the completion of de-bonding
T_{max}	= Maximum normal contact (tensile) force
σ_{max}	= Maximum normal contact (tensile) stress
δ_n	= Contact gap at the maximum normal contact (tensile) stress
d_n	= Damage or de-bonding parameter, 0 at A and 1 at point C
k	= Normal contact stiffness
$k_{Reduced}$	= Reduced normal contact stiffness
$\bar{\Delta}$	= Dimensionless displacement, $\frac{\Delta_n}{\delta_n}$

References

- 670 [1] C. L. Cipolla, E. Lolon, M. J. Mayerhofer, N. R. Warpinski, et al., Fracture design considerations in horizontal wells drilled in unconventional gas reservoirs, in: SPE Hydraulic Fracturing Technology Conference, Society of Petroleum Engineers, 2009.
- [2] P. Rijken, M. L. Cooke, Role of shale thickness on vertical connectivity of fractures: application of crack-bridging theory to the austin chalk, texas, *Tectonophysics* 337 (1) (2001) 117–133.
- 675 [3] Z. Haifeng, C. Hang, L. Guohua, L. Yawei, S. Jun, R. Peng, New insight into mechanisms of fracture network generation in shale gas reservoir, *Journal of Petroleum Science and Engineering* 110 (2013) 193–198.
- 680 [4] R. Beckwith, et al., Hydraulic fracturing: the fuss, the facts, the future, *Journal of Petroleum Technology* 62 (12) (2010) 34–40.
- [5] J. I. Adachi, E. Detournay, Plane strain propagation of a hydraulic fracture in a permeable rock, *Engineering Fracture Mechanics* 75 (16) (2008) 4666–4694.
- 685 [6] J. F. Gale, R. M. Reed, J. Holder, Natural fractures in the barnett shale and their importance for hydraulic fracture treatments, *AAPG bulletin* 91 (4) (2007) 603–622.
- [7] S. Enayatpour, T. Patzek, Thermal shock in reservoir rock enhances the hydraulic fracturing of gas shales, *Unconventional Resources Technology Conference*.
- 690 [8] S. Enayatpour, E. van Oort, T. Patzek, Freezefrac improves the productivity of gas shales, *SPE ATCE Conference* (SPE 166482).
- [9] S. Enayatpour, R. Khaledialidusti, T. Patzek, Assessment of thermal fracturing in tight hydrocarbon formation using dem, the 48th US Rock Mechanics / Geomechanics Symposium held in Minneapolis, MN, USA, 1-4
- 695 June 2014.

- [10] C. E. Inglis, Stresses in a plate due to the presence of cracks and sharp corners, Spring Meetings of the Fifty-fourth Session of the Institution of Naval Architects (1913) 219–241.
- 700 [11] G. Irwin, Analysis of stresses and strains near the end of a crack traversing a plate, *Journal of Applied Mechanics* 24 (1957) 361–364.
- [12] A. Griffith, The phenomena of rupture and flow in solids, *Philosophical Transactions of the Royal Society of London* 221 (1921) 163–198.
- 705 [13] H. M. Westergaard, Bearing pressure and cracks, *Journal of Applied Mechanics* 61 (1939) 49–53.
- [14] G. N. Wells, L. J. Sluys, A new method for modeling cohesive cracks using finite elements, *International Journal of Applied Mechanics* 50 (2001) 2667–2682.
- 710 [15] J. R. Rice, Path-independent integral and the approximate analysis of strain concentration by notches and cracks, *Journal of Applied Mechanics, Transaction ASME* 35 (2) (1968) 379–386.
- [16] H. A. Elliott, An analysis of the conditions for rupture due to griffth cracks, *Proc. Phys. Soc.* 59 (2) (1947) 208 – 223.
- 715 [17] G. I. Barenblatt, The formation of equilibrium cracks during brittle fracture: General ideas and hypotheses, axially symmetric cracks, *Appl. Math. Mech.* 23 (3) (1959) 622 – 636.
- [18] G. I. Barenblatt, The mathematical theory of equilibrium cracks in brittle fracture, *Adv. Appl. Mech* 7 (1962) 55 – 129.
- 720 [19] D. S. Dugdale, Yielding of steel sheets containing slits, *J. Mech. Phys. Solids* 8 (2) (1960) 100 – 104.
- [20] E. Smith, A generalization of elliot’s model of a crack tip, *Int. J. Fract* 11 (2) (1975) 295 – 299.

- [21] A. Hillerborg, M. Modeer, P. Petersson, Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements, *Cem Concr Res* (6) (1976) 773 – 782.
- [22] T. J. Boone, P. A. Wawrzynek, A. R. Ingraffea, Simulation of the fracture process in rock with application to hydrofracturing, *Int. J. Rock Mech. Min. Sci* 22 (3) (1986) 255 – 265.
- [23] M. Elices, G. V. Guinea, J. Gomez, J. Planas, The cohesive zone model: Advantages, limitations and challenges, *Eng. Fract. Mech* 69 (2) (2002) 137 – 163.
- [24] Z. P. Bažant, E. Becq-Giraudon, Statistical prediction of fracture parameters of concrete and implications for choice of testing standard, *Cem. Concr. Res* 32 (4) (2002) 529 – 556.
- [25] J. Roesler, G. H. Paulino, K. Park, C. Gaedicke, Concrete fracture prediction using bilinear softening, *Cem. Concr. Compos* 29 (4) (2007) 300 – 312.
- [26] S. Rangaraj, K. Kokini, A study of thermal fracture in functionally graded thermal barrier coatings using a cohesive zone model, *ASME J. Eng. Mater. Technol* 126 (1) (2004) 103–115.
- [27] B. Eftekhari, A lattice model for gas production from hydrofractured shale, Ph.D. thesis, The University of Texas at Austin (2016).
- [28] M. Marder, C.-H. Chen, T. Patzek, Simple models of the hydrofracture process, *Phys. Rev. E* 92 (2015) 062408.
- [29] L. Brely, F. Bosia, N. M. Pugno, A hierarchical lattice spring model to simulate the mechanics of 2-d materials-based composites, *Frontiers in Materials* 2 (2015) 51.
- [30] R. M. Reed, R. Loucks, D. M. Jarvie, S. C. Ruppel, Nanopores in the mississippian barnett shale: Distribution, morphology, and possible gene-

- 750 sis (abs.), Geological Society of America Abstracts with Programs 39 (6)
(2007) 358.
- [31] D. Dugdale, Yielding of steel sheets containing slits, *Journal of the Mechanics and Physics of Solids* 8 (2) (1960) 100–104.
- [32] G. I. Barenblatt, The mathematical theory of equilibrium cracks in brittle
755 fracture, *Advances in applied mechanics* 7 (1) (1962) 55–129.
- [33] A. Needleman, A continuum model for void nucleation by inclusion debonding, *Journal of applied mechanics* 54 (3) (1987) 525–531.
- [34] J. F. Gale, J. Holder, et al., Natural fractures in the barnett shale: Constraints on spatial organization and tensile strength with implications for
760 hydraulic fracture treatment in shale-gas reservoirs, in: *The 42nd US Rock Mechanics Symposium (USRMS)*, American Rock Mechanics Association, 2008.
- [35] B. Carrier, S. Granet, Numerical modeling of hydraulic fracture problem in permeable medium using cohesive zone model, *Engineering fracture mechanics* 79 (2012) 312–328.
765
- [36] M. F. Kanninen, C. H. Popelar, *Advanced Fracture Mechanics*, Oxford University Press, New York, 1985.
- [37] Z. P. Bazant, L. Cedolin, *Stability of Structures: Elastic, Inelastic, Fracture, and Damage Theories*, Oxford University Press, New York, 1991.
- [38] T. L. Anderson, *Fracture Mechanics: Fundamentals and Applications*, CRC
770 Press, Boca Raton, FL, 1995.
- [39] S. Suresh, *Fatigue of Materials*, Cambridge University Press, New York., 1998.
- [40] K. B. Broberg, *Cracks and Fracture*, Academic Press, Cambridge, 1999.

- 775 [41] K. Park, G. H. Paulino, Cohesive zone models: A critical review of traction-separation relationships across fracture surfaces, *Applied Mechanics Reviews* 64 (6) (2011) 061002.
- [42] V. Tvergaard, Effect of fibre debonding in a whisker-reinforced metal, *Mater. Sci. Eng A125* (2) (1990) 203 – 213.
- 780 [43] M. Ortiz, A. Pandolfi, Finite-deformation irreversible cohesive elements for three dimensional crack-propagation analysis, *Int. J. Numer. Methods Eng* 44 (9) (1999) 1267 – 1282.
- [44] J. W. Foulk, D. H. Allen, K. L. E. Allen, Formulation of a three-dimensional cohesive zone model for application to a finite element algorithm, *Comput. Methods Appl. Mech. Eng* 183 (1) (2000) 51 – 66.
- 785 [45] S. Enayatpour, T. Patzek, Finite element solution of nonlinear transient rock damage with application in geomechanics of oil and gas reservoirs, *Proceedings of the 2012 COMSOL Conference*, Boston.
- [46] G. Stephens, B. Voight, Hydraulic fracturing theory for conditions of thermal stress 19 (1982) 279–284.
- 790 [47] R. Ritchie, A. Sakakura, Asymptotic expansions of solutions of the heat conduction equation in internally bounded cylindrical geometry, *Journal of Applied Physics* 27 (12) (1956) 1453–1459.
- [48] P. A. Charlez, *Rock mechanics*, Vol. 1, Editions Technip, 1991.
- 795 [49] P. A. Charlez, *Rock mechanics: petroleum applications*, Vol. 2, Editions Technip, 1997.
- [50] M. E. Gonzalez, J. B. Bloys, J. E. Lofton, G. P. Pepin, J. H. Schmidt, C. J. Naquin, S. T. Ellis, P. E. Laursen, et al., Increasing effective fracture gradients by managing wellbore temperatures, in: *IADC/SPE Drilling Conference*, Society of Petroleum Engineers, 2004.
- 800

- [51] T. Diehl, On using a penalty-based cohesive-zone finite element approach, part i: Elastic solution benchmarks, *International Journal of Adhesion and Adhesives* 28 (4) (2008) 237–255.
- [52] J. Mergheim, E. Kuhl, P. Steinmann, A finite element method for the computational modeling of cohesive cracks, *International Journal for Numerical Methods in Engineering* 63 (2) (2005) 276 – 289.
- [53] K. T. Raterman, H. E. Farrell, O. S. Mora, A. L. Janssen, G. A. Gomez, S. Buseti, M. Warren, Sampling a stimulated rock volume: An eagle ford example, *Unconventional Resources Technology Conference*.
- [54] M. Haddad, K. Sepehrnoori, XFEM-based CZM for the simulation of 3D multiple-stage hydraulic fracturing in quasi-brittle shale formations, *American Rock Mechanics Association, 49th U.S. Rock Mechanics/Geomechanics Symposium*, 28 June–1 July, San Francisco, California.
- [55] M. Haddad, J. Du, S. Vidal-Gilbert, Integration of dynamic microseismic data with a true 3D modeling of hydraulic-fracture propagation in the vaca muerta shale, *Society of Petroleum Engineers*.

Highlights

- Shale hydrocarbon reservoir rock is a prime candidate for thermal fracturing
- CZM is particularly well-suited for shale thermal fracturing analysis
- CZM handles material non-linearity, constitutive law, tensile and shear modes
- CZM computational effort is less compared to XFEM and large MD simulations
- Stress anisotropy and magnitude affect thermal fracture opening and pattern