Asymmetric Modulation for Hardware Impaired Systems - Error Probability Analysis and Receiver Design

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Abstract—Error probability study of hardware impaired (HWI) systems highly depends on the adopted model. Considering the distinct improper Gaussian features of HWI systems, captured by recent models, HWI-aware receivers are designed. An optimal maximum likelihood (ML) receiver serves as a performance benchmark, and a sub-optimal linear minimum mean square error (LMMSE) introduces a reduced-complexity implementation. Whereas, the conventional HWI-unaware minimum Euclidean distance (MED) receiver, based on the proper noise assumption, exhibits substandard performance. Next, the average error probability of the proposed optimal ML-receiver is analyzed, where several tight bounds and approximations are derived for various HWI systems. Motivated by the benefit of improper Gaussian signaling in mitigating HWI, which is proven in recent studies, asymmetric modulation is adopted and optimized for transmission. The numerical results demonstrate a bit error rate (BER) reduction up to 70% of the proposed HWI-aware receivers over HWI-unaware receivers. Moreover, the asymmetric modulation is shown to reduce the BER by 93%. These results signify the importance of incorporating accurate HWI models, designing appropriate receivers and optimizing signal transmission for BER performance compensation.

Index Terms—Asymmetric modulation, error probability analysis, hardware impairments, in-phase and quadrature-phase imbalance, improper Gaussian signaling, optimal receiver and self-interfering signals.

I. INTRODUCTION

The rapidly growing demand for smart communication networks has imposed tremendous design challenges for modern wireless communication systems such as compactness, least complexity, cost-effectiveness and high energy efficiency [1], [2]. Immense research has been carried out to develop new techniques/configurations to tackle these design challenges [3], [4]. However, the performance of the proposed systems can be highly affected by the non-ideal operation of radio frequency (RF) transceivers [5], [6]. Hardware impairments (HWIs) impose a huge challenge on next-generation network planning and deployment especially at high-frequency [1], [5]–[7]. HWIs emerge in various RF stages including imperfections in analog-to-digital/digital-to-analog converters, non-linear high power amplifier/low noise amplifier, mismatched local oscillator and phase shifter, etc. [5], [6]. These hardware imperfections result not only in phase/amplitude errors and raised noise floor but also in an inevitable mixing of the desired and image signals. This distinct behavior motivates

researchers to develop accurate models and propose effective compensation methods to meet the expected performance.

Numerous efforts have been carried out to accurately model various forms of HWIs. As an example, many studies focused on the statistical modeling of additive hardware distortions at the transmitter and the receiver [8]–[14]. Other studies focused on the modeling of in-phase and quadrature phase (I/Q) imbalance where the self interference (SI) signal incorporates the amplitude and rotational imbalance besides the receiver thermal noise [15]–[20]. Moreover, a few studies like [5] and [21] analyzed multiple RF front-end impairments and their individual baseband equivalent error models. In [5], Schenk studied the modeling procedure, impact of non-ideal hardware on the system performance, and digital compensation schemes of various RF imperfections in high data-rate wireless systems. Similarly, Boulogeorgos *et al.* studied the impact of various HWIs on the energy detection spectrum sensing in cognitive radio systems in [21]. The aforementioned contributions are focused on the modeling, performance analysis, and mitigation strategies. However, [22] considered the combined effect of various HWIs including both transmitter and receiver I/Q mismatch as well as accumulative additive distortions at the transmitter as well as the receiver. This research investigates full characterization of the self-interfering (SI) information signals and improper Gaussian noise components inspired by the statistical signal processing studies, which highlighted the asymmetric characteristics of baseband communication signals due to the I/Q imbalance [23].

Asymmetric/improper Gaussian signaling (IGS) is a generalized complex signaling scheme that relaxes the symmetric characteristics of symmetric/proper Gaussian signaling (PGS) scheme allowing a correlation between the signal components and/or unequal power of each component [24]. Hence, IGS scheme offers an additional degree of design, pertinent to its circularly asymmetric characteristics. Interestingly, IGS was proven to improve the system performance of different interference limited configurations such as cognitive radio systems [25], [26] and full-duplex relaying [27], [28]. Recently, we assumed asymmetric additive distortion and studied the benefits of employing asymmetric transmission on the achievable rate and outage probability of SISO and SIMO systems in [29] and [12], respectively. Moreover, the ergodic rate maximization based on a generalized error model incorporating various forms of HWIs in SIMO and MIMO systems is studied in [22].

The error probability performance analysis of systems subjected to HWIs has recently received few attentions [30]–[33]. In [30], Windisch and Fettweis quantified the impact of the receiver I/Q imbalance in terms of closed-form error

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probability. Similarly, in [31] Qi and Aissa studied the bounds on average symbol error probability (SEP) for the receiver I/Q imbalanced MIMO system. In [32], Krishnan *et al.* derived the SEP for systems suffering from Gaussian phase error. In [33], Bouhlef *et al.* analyzed the pairwise error probability (PEP) over correlated Rayleigh and Ricean fading channel and incorporated both imperfect channel state information and additive hardware distortions. To the best of the author's knowledge, there is no substantial research that considers the aggregate effect of HWIs at both the transmitter and the receiver on the error probability performance. Furthermore, it is interesting to look for the optimal receiver especially after the noise deformation to IGS [34].

In this paper, we analyze the error probability performance of communication systems suffering from HWIs represented in I/Q imbalance and additive distortions at both transmitter and receiver. Moreover, motivated by the theoretical limits results in [22], which demonstrated the benefits of employing improper Gaussian signaling to improve the performance of hardware impaired systems, we adopt asymmetric signaling scheme to minimize the error probability performance. In the following, we summarize the main contributions as:

- Studying and accurately quantifying the asymmetric characteristics of the aggregate HWIs from various impairment sources.
- Deriving the optimal maximum likelihood (ML) detector and the suboptimal linear minimum mean square error (LMMSE) receiver based on the improper interference, SI information signals, and asymmetric signaling transmission.
- Analyzing PEP based on the accurate model and optimal ML detector.
- Analyzing the average error probability and deriving bounds and approximations for various HWI system configurations.
- Designing an asymmetric modulation signal to minimize the error probability.

The rest of the paper is organized as: Section II describes the adopted system along with its complete characterization. In section III, we propose an optimal receiver and a sub-optimal linear receiver for the system under mentioned HWIs. Next, we present error probability analysis based on the PEP for generalized M-ary modulation scheme in section IV. Later, average error probability bounds and approximations are computed for three different adopted systems in section V. Section VI illustrates various forms of transmit signal designs to reduce error probability followed by the numerical results in Section VII and the conclusion in Section VIII.

Notations: In this paper, $|a|$ and a^* represent the absolute and complex conjugate of a scalar complex number a . The probability of an event A is defined as $\Pr(A)$. The notations $f_z(z)$ and $f_{z|y}(z|y)$ denote the probability density function (PDF) of the random variable (RV) z and the conditional PDF of z given y . $\mathbb{E}[\cdot]$ is the expected value operator. In addition, ξ_r and ξ_i represents the real and imaginary components of a complex variable ξ . We use $g^{(n)}(\bar{x})$ for the n^{th} derivative of function $g(x)$ w.r.t x evaluated at $\bar{x}$. $F_1(a; b, b; c; x, y)$ is the

Fig. 1: Transmitter I/Q imbalance

Appell hypergeometric function of two variables [35] whereas the Gauss hypergeometric function of one variable is given as ${}_2F_1\left(\left[\tilde{a}, \tilde{b}\right], \tilde{c}, z\right)$ [36, (15.1.1)].

II. SYSTEM DESCRIPTION

A. Preliminaries

To characterize the difference between symmetric/proper signaling and asymmetric/improper signaling, we introduce following definitions for a zero-mean complex RV x :

Definition 1. The variance and the pseudo-variance of x are defined, respectively, as $\sigma_x^2 = \mathbb{E}[|x|^2]$ and $\tilde{\sigma}_x^2 = \mathbb{E}[x^2]$ [24].

Definition 2. A complex random variable is called symmetric/proper if its pseudo-variance is zero, otherwise it is called asymmetric/improper [24].

Definition 3. Circularity coefficient C_x measures the degree of impropriety and is defined as $C_x = |\tilde{\sigma}_x^2|/\sigma_x^2$, where $0 \leq C_x \leq 1$ [37]. $C_x = 0$ indicates proper signal and $C_x = 1$ indicates maximally improper signal.

Definition 4. The complex Gaussian random variable x is fully described as, $x \sim \mathcal{CN}(\mathbb{E}[x], \sigma_x^2, \tilde{\sigma}_x^2)$.

B. Hardware Impaired Signal Model

Consider a wireless communication system employing synchrodyne architecture transceivers in the RF front-end [38]. At the transmitter, the digital baseband modulated signal undergoes pulse shaping and up-conversion to the desired carrier frequency. During this homodyne up-conversion stage, the imperfect local oscillator and phase shifter introduce I/Q imbalance. Transceivers with I/Q imbalance not only contribute to the amplitude and phase errors but also induce SI signal x_{IQI} , due to the limited image-rejection capability [3]. Based on the practically validated I/Q mismatch model presented in [5], [15], [19], [39]–[43] and references therein, the equivalent quadrature imbalanced transmit signal is expressed as $x_{\text{IQI}} = \nu_1 x + \nu_2 x^*$, where x is a band-pass modulated signal taken from M-QAM, M-PSK or M-PAM constellation, ν_1 and ν_2 account for the amplitude (a_t) and phase (θ_t) errors at the transmitter I/Q mixer stage and are modeled as

$$\nu_1 = (1 + a_t e^{j\theta_t})/2; \quad \nu_2 = 1 - \nu_1^* = (1 - a_t e^{-j\theta_t})/2. \quad (1)$$

The image rejection ratio (IRR) of transmitter I/Q mixer stages is defined as $\Upsilon_t = |\nu_1|^2/|\nu_2|^2$, resides between 20 dB and 40

dB [15], [21]. The baseband equivalent impact of transmitter I/Q imbalance on 16-QAM and 16-PSK modulations is evident in Fig. 1(a) and 1(b), respectively. I/Q imbalance transmits the filled symbols from a skewed and rotated signal constellation in-place of the ideally modulated hollow symbols. Note that the ideal transmitter I/Q mixer stage exhibits $a_t = 1$ and $\theta_t = 0$, yielding $x_{IQI} = x$ and $\Upsilon_t = \infty$. In addition, non-linear transfer functions of various transmitter RF stages, such as digital-to-analog converter, band-pass filter and high power amplifier result in accumulative additive distortion noise $d_t \sim \mathcal{CN}(0, \sigma_t^2, 0)$ ([5, eq. 7.9] and [9, eq. 3]) raising the noise floor

$$x_{tx} = x_{IQI} + d_t = \nu_1 x_m + \nu_2 x_m^* + d_t. \quad (2)$$

The additive Gaussian model (2) for the aggregate residual RF distortions - after applying existing compensation schemes - is based on various theoretical investigations and measurement results ([8], [9], [11], [13], [14], [21], [44]–[47] and references therein), which can also be motivated analytically by the central limit theorem. The transmitted signal undergoes a slowly varying flat Rayleigh fading channel $h \sim \mathcal{CN}(0, \lambda, 0)$. Moreover, the receiver further induces an additive distortion $d_r \sim \mathcal{CN}(0, \sigma_r^2, 0)$, resulting from the non-linear transfer function of low noise amplifier, band-pass filters, image rejection low pass filter, analog-to-digital converter and thermal noise at the receiver. The received signal is expressed in terms of the average transmit power p as $y_{rx} = \sqrt{p} h x_{tx} + d_r$, where it experiences again I/Q imbalance at the down-conversion stage yielding $y = \mu_1 y_{rx} + \mu_2 y_{rx}^*$ [5, eq. 5.28] and [19, eq. 3]. The parameters $\mu_1 = (1 + a_r e^{j\theta_r})/2$ and $\mu_2 = 1 - \mu_1^* = (1 - a_r e^{-j\theta_r})/2$ captures the receiver's amplitude (a_r) and rotational errors (θ_r). Thus, the receiver's IRR in the down-conversion process is $\Upsilon_r = |\mu_1|^2/|\mu_2|^2$, where the perfect receiver I/Q balance occurs when $\mu_1 = 1$ and $\mu_2 = 0$, which follows from $a_r = 1$ and $\theta_r = 0$. Consequently, the received signal y under the aggregate HWIs model is given as

$$y = \sqrt{p} \tilde{h}_1 x + \sqrt{p} \tilde{h}_2 x^* + z, \quad (3)$$

where $\tilde{h}_1 = \mu_1 \nu_1 h + \mu_2 \nu_2^* h^*$ and $\tilde{h}_2 = \mu_1 \nu_2 h + \mu_2 \nu_1^* h^*$ are modified channel gain parameters accommodating fading and I/Q imbalance characteristics. Moreover, the aggregated noise is given by

$$z = \mu_1 (\sqrt{p} h d_t + d_r) + \mu_2 (\sqrt{p} h^* d_t^* + d_r^*). \quad (4)$$

Theorem 1. *Transceiver I/Q imbalance transforms the symmetric transmitted signal to asymmetric received signal and the proper Gaussian interference to improper Gaussian interference [22].*

Proof. This follows from the non-zero pseudo-variance of the received signal $\tilde{\sigma}_y^2 = 2\tilde{h}_1 \tilde{h}_2 + \tilde{\sigma}_z^2$ and the accumulative interference term under HWI $\tilde{\sigma}_z^2 = 2\mu_1 \mu_2 (p|h|^2 \sigma_t^2 + \sigma_r^2)$. $\square$

The symmetric distribution of the proper additive distortions noise in Fig. 2(a) transforms to the improper aggregated noise in Fig. 2(b) pertaining to the SI additive distortions induced by the I/Q imbalance at receiver.

Fig. 2: Impropriety Characterization

C. Signal-to-Noise Ratio

The useful signal component in (3) comprises of both the actual signal x and the SI signal x^* . Thus, treating the SI term as mere interference will result in a loss of useful information. For a given channel and I/Q parameters, the instantaneous SNR averaged over additive distortions and M signal constellation with $E\{|x_m|^2\} = \sigma_x^2$ and $E\{x_m^2\} = \tilde{\sigma}_x^2$, $m = 1, 2, \dots, M$, is given as

$$\Lambda = \frac{p \left(|\tilde{h}_1|^2 \sigma_x^2 + |\tilde{h}_2|^2 \sigma_x^2 + \tilde{h}_1 \tilde{h}_2^* \tilde{\sigma}_x^2 + \tilde{h}_2 \tilde{h}_1^* \tilde{\sigma}_x^{2*} \right)}{(p|h|^2 \sigma_t^2 + \sigma_r^2) (|\mu_1|^2 + |\mu_2|^2)}. \quad (5)$$

We relax the transmit distortion obtaining reduced complex system and express p in terms of the average SNR $\bar{\Lambda} = E_h[\Lambda]$ as

$$p = \frac{\sigma_r^2}{\lambda} \left[\frac{\bar{\Lambda}}{|\nu_1|^2 \sigma_x^2 + |\nu_2|^2 \sigma_x^2 + 2\Re\{\nu_1 \nu_2^* \tilde{\sigma}_x^2\}} \right]. \quad (6)$$

The transceiver I/Q imbalance and the improper additive noise change the system characteristics and render the traditional minimum Euclidean distance (MED) receiver as a sub-optimal detection solution. MED can neither support the SI structure induced by the I/Q imbalance nor the asymmetric characteristics of the additive noise.

III. RECEIVER DESIGN

In this section, we propose an optimal ML receiver for the adopted HWI system model. We further propose a least complex LMMSE receiver and use the conventional minimum Euclidean distance receiver for performance comparison.

A. Optimal Maximum Likelihood Receiver

Given the equiprobable symbols and improper interference, the conventional MED receiver for Gaussian interference performs sub-optimally as it fails to accommodate the dependent and non-identical real and imaginary components of the improper noise. Therefore, we propose an optimal ML receiver for the presented hardware impaired system model. The aggregated received signal in (3) is equivalent to

$$y = \sqrt{p} \chi^m + z; \quad m = 1, 2, \dots, M \quad (7)$$

$$f_{y_r y_i | \chi^m}(y_r, y_i | \chi^m) = \frac{1}{2\pi\sigma_1\sigma_Q\sqrt{1-\rho^2}} \exp \frac{-1}{2(1-\rho^2)} \left(\frac{(y_r - \sqrt{p}\chi_r^m)^2}{\sigma_1^2} + \frac{(y_i - \sqrt{p}\chi_i^m)^2}{\sigma_Q^2} - \frac{2\rho(y_r - \sqrt{p}\chi_r^m)(y_i - \sqrt{p}\chi_i^m)}{\sigma_1\sigma_Q} \right). \quad (8)$$

where $\chi^m = \tilde{h}_1 x_m + \tilde{h}_2 x_m^*$ depends on the transmitted symbol m and z is the accumulated noise component (4). For a given channel, I/Q imbalance estimates [48] and transmitted signal x_m , the real component y_r and imaginary component y_i of the received signal y are jointly Gaussian with PDF as given in (8), where the useful in-phase signal component χ_r^m and quadrature component χ_i^m are given, respectively, as

$$\begin{aligned} \chi_r^m &= (\tilde{h}_{1r} + \tilde{h}_{2r}) x_r^m + (\tilde{h}_{2i} - \tilde{h}_{1i}) x_i^m, \\ \chi_i^m &= (\tilde{h}_{1i} + \tilde{h}_{2i}) x_r^m + (\tilde{h}_{1r} - \tilde{h}_{2r}) x_i^m, \end{aligned} \quad (9)$$

where $\tilde{h}_{kr}$ and $\tilde{h}_{ki}$ are the real and imaginary components of $\tilde{h}_k$ respectively. In addition, x_r^m and x_i^m are the real and imaginary components of the transmitted symbol x_m , respectively. Moreover, the in-phase z_r and quadrature component z_i of z are zero-mean Gaussian RV with variances σ_1^2 and σ_Q^2 , respectively. $\sigma_Q^2 = a_r^2 \sigma_1^2$ and σ_1^2 is written as

$$\sigma_1^2 = \frac{\sigma_r^2}{2} + \frac{p\sigma_t^2}{2}|h|^2. \quad (10)$$

Interestingly, the noise variance depends not only on the I/Q imbalance parameter and the distortion variance but also on the transmitted power and channel gain. Therefore, an increase in the transmitted power marks a proportional increase in the noise variance owing to the presence of transmit distortions. Using (4), the correlation coefficient ρ between z_r and z_i simplifies to

$$\rho = \frac{\mathbb{E}(z_r z_i)}{\sigma_1 \sigma_Q} = \sin \theta_r. \quad (11)$$

Note that ρ merely depends on the amount of rotational error induced by the receiver I/Q imbalance. Considering the improper Gaussian interference, the optimal ML detection is based on maximizing the following conditional probability

$$\hat{x}_m = \arg \max_{m=1,2,\dots,M} f_{y_r y_i | \chi^m}(y_r, y_i | \chi^m) \quad (12)$$

Using (8), the ML receiver in (12) reduces to the minimization of the argument of $f_{y_r y_i | \chi^m}(y_r, y_i | \chi^m)$

$$\hat{x}_m = \arg \min_{m=1,2,\dots,M} \left\{ \frac{(y_r - \sqrt{p}\chi_r^m)^2}{\sigma_1^2} + \frac{(y_i - \sqrt{p}\chi_i^m)^2}{\sigma_Q^2} - \frac{2\rho(y_r - \sqrt{p}\chi_r^m)(y_i - \sqrt{p}\chi_i^m)}{\sigma_1\sigma_Q} \right\}. \quad (13)$$

Using (10) and (11), the optimal ML receiver is simplified as

$$\hat{x}_m = \arg \min_{m=1,2,\dots,M} \left\{ \frac{(y_r - \sqrt{p}\chi_r^m)^2}{\sigma_1^2} + \frac{1}{a_R^2} (y_i - \sqrt{p}\chi_i^m)^2 - \frac{2 \sin \theta_R}{a_R} (y_r - \sqrt{p}\chi_r^m)(y_i - \sqrt{p}\chi_i^m) \right\}. \quad (14)$$

B. Minimum Euclidean Distance Receiver

Considering the traditional MED receiver with the assumption of circularly symmetric complex Gaussian interference

$$\hat{x}_m = \arg \min_{m=1,2,\dots,M} |y - \sqrt{p}\chi^m|^2, \quad (15)$$

where $\chi^m = hx_m$ under ideal hardware assumption. The disagreement between (14) and (15) is owing to the presence of amplitude translation a_r and rotational error θ_r caused by the receiver I/Q imbalance. The amplitude error is responsible for the non-identical real and imaginary components and the rotational error is accountable for the mutual correlation.

C. Linear Minimum Mean Square Error Receiver

Linear receivers are popular for the ease of implementation and least complex receiver design. Therefore, the LMMSE receiver is derived for the adopted HWI system model. Consider the linear estimator $\hat{x} = c^* y$, we aim to design the estimator in order to minimize the mean square error $\mathbb{E}\{\|\hat{x} - x\|^2\} = \mathbb{E}\{\|c^* y - x\|^2\}$. Given the convex nature of norm minimization problem, second order sufficient condition renders the stationary point as the global optimal solution. Thus, the optimal c is given as $\bar{c} = R_{yy}^{-1} R_{yx}$, where R_{yy} is the covariance of the received signal y given in (3) and R_{yx} is the cross-covariance of the received signal y and the transmitted signal x . Considering the asymmetric transmission, the $\bar{c}$ is derived as:

$$\bar{c} = \frac{\sqrt{p}h_1\sigma_x^2 + \sqrt{p}h_2\tilde{\sigma}_x^{2*}}{p(|h_1|^2 + |h_2|^2)\sigma_x^2 + 2p\Re\{h_1h_2^*\tilde{\sigma}_x^2\} + \sigma_z^2}, \quad (16)$$

where the aggregate noise variance $\sigma_z^2 = (p|h|^2\sigma_t^2 + \sigma_r^2)(|\mu_1|^2 + |\mu_2|^2)$ from (4). Note that the obtained $\hat{x}$ is LMMSE estimate of the transmitted symbol x_m . Thus, the ultimate detection relies on finding the closest possible symbol in a reference constellation using slicers.

The optimal ML receiver serves as the performance benchmark for the conventional MED receiver, which is considered optimal under the ideal hardware and AWGN assumption. Moreover, the performance analysis of LMMSE receiver in Section VIII justifies its suitability for reduced complexity practical implementation.

IV. ERROR PROBABILITY ANALYSIS

Symbol error probability analysis has been carried out based on the proposed optimal ML receiver presented in section III. This work focuses on the derivation of pairwise error probabilities in higher order modulation schemes for the simplified and tractable analysis of the adopted system model under HWI. We further analyze the asymptotic behavior of the derived error probability expression with increasing SNR.

A. Symbol Error Probability

Symbol error probability (P_s) is defined as the probability of detecting χ^n given χ^m was transmitted with prior probability $\Pr(\chi^m)$ for all $m \neq n$. Thus, P_s is expressed as

$$P_s \leq \sum_{m=1}^M \sum_{\substack{n=1 \\ n \neq m}}^M \Pr(\chi^m \rightarrow \chi^n | \chi^m) \Pr(\chi^m) \quad (17)$$

where the term $\Pr(\chi^m \rightarrow \chi^n | \chi^m)$ is the well-known PEP expression. Under the assumption of equal likely transmitted symbols of M^{-1} , (17) reduces to

$$P_s \leq \frac{1}{M} \sum_{m=1}^M \sum_{\substack{n=1 \\ n \neq m}}^M \Pr(\chi^m \rightarrow \chi^n | \chi^m). \quad (18)$$

At high SNR, most error events occur with neighboring symbols, where the probability of making error with neighboring symbol is higher than that with a farther point which is not located at minimum distance ($d_{\min}$) from the transmitted symbol. Thus, we can upper bound P_s by the PEP between two closest possibilities as $P_s \leq (M-1) \Pr(\chi^m \rightarrow \chi^n | \chi^m)$, where x_n is assumed to be the neighbor located at $d_{\min}$ from x_m . This upper bound is the loose union bound as it assumes all neighbors to be located at $d_{\min}$, which is only valid for orthogonal modulation schemes and invalid for other modulation schemes. Therefore, we employ the nearest neighbor union bound (NNUB) for M-ary modulation.

$$P_s \leq \psi \Pr(\chi^m \rightarrow \chi^n | \chi^m) \quad (19)$$

where ψ represents the average number of neighbors located at $d_{\min}$ distance to each other. For example, for M-PSK $\psi = 2$ and for M-QAM square constellation, it is found to be

$$\psi = \frac{1}{M} \left(4x_2 + 4 \left(\sqrt{M} - 2 \right) x_3 + \left(\sqrt{M} - 2 \right)^2 x_4 \right). \quad (20)$$

Based on the optimal ML receiver in (13), PEP is defined by the conditional probability expressed in (21). Given that x_m was transmitted, it is justified to replace all occurrences of y_r with $\sqrt{p}\chi_r^m + z_r$ and y_i with $\sqrt{p}\chi_i^m + z_i$. Thus, we obtain the following simplified PEP expression

$$\Pr(\chi^m \rightarrow \chi^n | \chi^m) = \Pr \left\{ \eta > \frac{p\zeta_r^2}{\sigma_1^2} + \frac{p\zeta_i^2}{\sigma_Q^2} - \frac{2p\rho\zeta_r\zeta_i}{\sigma_1\sigma_Q} \right\}, \quad (22)$$

where η is obtained by the superposition of z_r and z_i

$$\eta = \left(\frac{2\rho\zeta_i}{\sigma_1\sigma_Q} - \frac{2\zeta_r}{\sigma_1^2} \right) \sqrt{p}z_r + \left(\frac{2\rho\zeta_r}{\sigma_1\sigma_Q} - \frac{2\zeta_i}{\sigma_Q^2} \right) \sqrt{p}z_i, \quad (23)$$

One can show that η is Gaussian RV with zero mean and a variance σ_η^2 that is expressed as

$$\sigma_\eta^2 = 4(1-\rho^2) \left[\frac{p\zeta_r^2}{\sigma_1^2} + \frac{p\zeta_i^2}{\sigma_Q^2} - \frac{2p\rho\zeta_r\zeta_i}{\sigma_1\sigma_Q} \right]. \quad (24)$$

Moreover, $\zeta_r = \chi_r^m - \chi_r^n$ and $\zeta_i = \chi_i^m - \chi_i^n$ are the distances between real and imaginary components of the transmitted and received useful signal component in an error event, respectively. Consequently, the probability in (22), is

the cumulative density function (CDF) of the Gaussian RV η and is given by

$$\Pr(\chi^m \rightarrow \chi^n | \chi^m) = Q \left(\sqrt{\frac{1}{4(1-\rho^2)} \left(\frac{p\zeta_r^2}{\sigma_1^2} + \frac{p\zeta_i^2}{\sigma_Q^2} - \frac{2p\rho\zeta_r\zeta_i}{\sigma_1\sigma_Q} \right)} \right). \quad (25)$$

The instantaneous PEP expression depends on the distance between the transmitted and received erroneous signal component under Rayleigh fading, I/Q imbalance and transceiver additive distortions. It further relies on the average transmit power and statistical characteristics of the non-identical and dependent I/Q phase improper interference components.

B. Asymptotic Analysis

Throughout the following discussion, we investigate the system performance of HWIs systems under high SNR assumption. The dependence of the statistical characteristics of the improper interference components (10) on the average SNR motivates us to analyze the asymptotic behavior of the PEP expression with respect to the parameter p . Substituting (10) and (11) in (25) yields

$$\text{PEP} = Q \left(\sqrt{\frac{a_r^2 p \zeta_r^2 + p \zeta_i^2 - 2a_r p \sin \theta_r \zeta_r \zeta_i}{2a_r^2 \cos^2 \theta_r (\sigma_r^2 + p \sigma_t^2 |h|^2)}} \right). \quad (26)$$

The asymptotic behavior of the instantaneous PEP depicts an irreducible error floor, which is found from the following limit,

$$\lim_{p \rightarrow \infty} \text{PEP} = Q \left(\sqrt{\frac{a_r^2 \zeta_r^2 + \zeta_i^2 - 2a_r \sin \theta_r \zeta_r \zeta_i}{2a_r^2 \sigma_t^2 \cos^2 \theta_r |h|^2}} \right). \quad (27)$$

It is important to note that out of all the aforementioned impairments, transmitter distortions are mainly responsible for the error floor. Intuitively, increasing SNR also increases the transmitter distortions, thus rendering it ineffective to reduce the error probabilities. From (26), it is evident that assuming $\sigma_t^2 = 0$ will result in $\lim_{p \rightarrow \infty} \text{PEP} = 0$.

V. AVERAGE PROBABILITY OF ERROR

In this section, we analyze the average SEP for three different scenarios; system under transmitter and receiver HWIs, system with only receiver HWIs and system with only transmitter HWIs.

A. System with Transmitter and Receiver Hardware Impairments

Consider the generalized system model under transmitter and receiver I/Q imbalance as well as the transmitter and receiver additive distortions. The PEP of the underlying system is given by (26). The average SEP ($\bar{P}_s$) of the generalized model is investigated assuming small θ_r , where it has been shown experimentally that $\theta_r \leq 5^\circ$ is a valid assumption for the I/Q imbalance rotational errors in [5]. As a result, we can assume $\sin \theta_r \approx 0$, which reduces the PEP expression to

$$\text{PEP} = Q \left(\sqrt{\frac{\alpha h_r^2 + \beta h_i^2}{1 + \gamma (h_r^2 + h_i^2)}} \right), \quad (28)$$

$$\Pr(\chi^m \rightarrow \chi^n | \chi^m) = \Pr \left\{ \frac{(y_r - \sqrt{p} \chi_r^m)^2}{\sigma_r^2} + \frac{(y_i - \sqrt{p} \chi_i^m)^2}{\sigma_Q^2} - \frac{2\rho(y_r - \sqrt{p} \chi_r^m)(y_i - \sqrt{p} \chi_i^m)}{\sigma_1 \sigma_Q} > \frac{(y_r - \sqrt{p} \chi_r^n)^2}{\sigma_r^2} + \frac{(y_i - \sqrt{p} \chi_i^n)^2}{\sigma_Q^2} - \frac{2\rho(y_r - \sqrt{p} \chi_r^n)(y_i - \sqrt{p} \chi_i^n)}{\sigma_1 \sigma_Q} \right\}. \quad (21)$$

where $\alpha = p(a_r^2 \varsigma_1^2 + \varsigma_2^2)/(2\sigma_r^2 a_r^2 \cos^2 \theta_r)$ and $\beta = p(a_r^2 \varsigma_3^2 + \varsigma_4^2)/(2\sigma_r^2 a_r^2 \cos^2 \theta_r)$. In addition, γ is the scaled ratio of transmitter distortion variance to receiver distortion variance, i.e., $\gamma = p\sigma_t^2/\sigma_r^2$. Moreover, $\varsigma_s, s \in \{1, 2, 3, 4\}$ are obtained using I/Q imbalance parameters and the respective modulation scheme characteristics as $\varsigma_1 = \xi_r$, $\varsigma_2 = a_t a_r \cos \theta_t \cos \theta_r \xi_i$, $\varsigma_3 = a_t \cos \theta_t \xi_i$ and $\varsigma_4 = a_r \cos \theta_r \xi_r$. They particularly depend on the symbol separation between two closest possibilities with $\xi_r = (x_r^m - x_r^n)$ being the separation between the real components and $\xi_i = (x_i^m - x_i^n)$ being the separation between the corresponding imaginary components. Let us denote the argument of the Q-function in (28) as

$$\vartheta = \frac{\alpha h_r^2 + \beta h_i^2}{1 + \gamma(h_r^2 + h_i^2)}. \quad (29)$$

Under the assumption of Rayleigh fading channels, the RV ϑ for a given signal constellation and I/Q imbalance parameters has a CDF of $F_\vartheta(\vartheta)$ that is derived in Appendix A as

$$F_\vartheta(\vartheta) = \frac{1}{k_1(\vartheta)} \sum_{m=0}^{\infty} \frac{2^{-2m} k_3(\vartheta)^{2m}}{[m!]^2 k_2(\vartheta)^{2m+1}} [2m! - \Gamma(2m+1, \vartheta k_2(\vartheta))]. \quad (30)$$

Using (19), $\bar{P}_s$ is obtained by averaging the PEP expression in (28) with respect to ϑ as

$$\bar{P}_s \leq \psi \mathbb{E}_\vartheta \left[Q(\sqrt{\vartheta}) \right] = \psi \int_0^\delta Q(\sqrt{\vartheta}) f_\vartheta(\vartheta) d\vartheta. \quad (31)$$

1) *Average PEP Bound:* Using the Chernoff bound $Q(x) \leq 0.5 \exp(-x^2/2)$, yields

$$\bar{P}_s \leq \frac{\psi}{2} \int_0^\delta e^{-\frac{\vartheta}{2}} f_\vartheta(\vartheta) d\vartheta = \frac{\psi}{2} e^{-\frac{\delta}{2}} + \frac{\psi}{4} \int_0^\delta e^{-\frac{\vartheta}{2}} F_\vartheta(\vartheta) d\vartheta. \quad (32)$$

The equality in (32) is obtained after applying integration by parts, which enables us to represent the presented bound in terms of $F_\vartheta(\vartheta)$. Average PEP bound can be accurately evaluated using numerical integration techniques such as Gaussian quadrature. Existence of the definite integral with finite limits motivates us to employ the popular n^{th} -order Legendre orthogonal polynomial $P_N(t)$ [36, (25.4.29-30)]. We obtain the required form by following parametrization,

$$\bar{P}_s \leq \frac{\psi}{2} e^{-\frac{\delta}{2}} + \frac{\delta \psi}{8} \int_{-1}^{+1} s(0.5\delta(t+1)) dt. \quad (33)$$

where $s(\vartheta) = e^{-\frac{\vartheta}{2}} F_\vartheta(\vartheta)$. Gaussian quadrature enables us to numerically approximate the integral in (33) with weights ω_n and $s(\vartheta)$ evaluation at the corresponding instances.

$$\bar{P}_s \leq \frac{\psi}{2} e^{-\frac{\delta}{2}} + \frac{\delta \psi}{8} \left[\sum_{n=1}^N \omega_n(t_n) s(0.5\delta(t_n+1)) + R_N \right], \quad (34)$$

where t_n is the n^{th} zero of the N^{th} -order Legendre polynomial $P_N(t)$ and the corresponding weights are obtained using [36, (25.4.29)].

$$\omega_n(t_n) = \frac{2}{(1-t_n^2) [P_N^{(1)}(t_n)]^2}. \quad (35)$$

The approximation error is given by residual term R_N [36, (25.4.30)], which decreases significantly with the increasing order of the Legendre polynomial and expressed by

$$R_N = \frac{\delta^{2N+1} (N!)^4}{(2N+1) [(2N)!]^3} f^{(2N)}(\phi) \quad ; -1 < \phi < 1. \quad (36)$$

This numerical integration technique accurately evaluates the intractable integral in (32) as (34).

2) *Average PEP Approximation:* The Craig's formula representation of the Q-function [49, (4.2)] can be employed to approximate the average probability of error as

$$\bar{P}_s \leq \psi \int_0^\delta \left(\frac{1}{\pi} \int_0^{\pi/2} e^{-\frac{\vartheta}{2\sin^2 \varphi}} d\varphi \right) f_\vartheta(\vartheta) d\vartheta. \quad (37)$$

Based on uniform convergence, swapping the integrals and employing integration by parts yields the $\bar{P}_s$ as a function of $F_\vartheta(\vartheta)$ as

$$\bar{P}_s \leq \frac{\psi}{\pi} \int_0^{\pi/2} e^{-\frac{\delta}{2\sin^2 \varphi}} d\varphi + \frac{\psi}{2\pi} \int_0^{\pi/2} \int_0^\delta \frac{e^{-\vartheta/(2\sin^2 \varphi)}}{2\sin^2 \varphi} F_\vartheta(\vartheta) d\vartheta d\varphi. \quad (38)$$

The first integral in (38) reduces to the complementary error function and the second term involving double integration leads to an intractable analysis. Therefore, we employ Gaussian Quadrature to numerically integrate this definite integration as

$$\bar{P}_s \leq \frac{\psi}{2} \text{erfc} \left(\sqrt{\frac{\delta}{2}} \right) + \frac{\psi}{2\pi} \int_0^{\pi/2} \int_0^\delta g(\vartheta, \varphi) d\vartheta d\varphi. \quad (39)$$

Choosing the following parametrization for numerical integration enables us to employ n^{th} -order Legendre orthogonal polynomial in the square interval $[-1, 1]$ as

$$\bar{P}_s \leq \frac{\psi}{2} \text{erfc} \left(\sqrt{\frac{\delta}{2}} \right) + \frac{\delta \psi}{8} \int_{-1}^1 \int_{-1}^1 g \left(\frac{\delta}{2} (y+1), \frac{\pi}{4} (x+1) \right) dy dx, \quad (40)$$

where $y = \frac{2}{\delta} \vartheta - 1$ and $x = \frac{4}{\pi} \varphi - 1$. Eventually the numerical approximation of double integrals is obtained using K^{th} -order

and L^{th} – order Legendre polynomial approximations for integration over dy and dx respectively as

$$\bar{P}_s \leq \frac{\psi}{2} \operatorname{erfc} \left(\sqrt{\frac{\delta}{2}} \right) + \frac{\delta\psi}{8} \sum_{l=1}^L \sum_{k=1}^K \omega_l \omega_k g \left(\frac{\delta}{2} (y_k + 1), \frac{\pi}{4} (x_l + 1) \right), \quad (41)$$

where the weights $\omega_k(y_k)$ and $\omega_l(x_l)$ are obtained using (35). Moreover, y_k and x_l are the k^{th} and l^{th} zeros of the K^{th} –order Legendre polynomial $P_K(y)$ and L^{th} –order Legendre polynomial $P_L(x)$ respectively. Higher order K and L yield better approximation with minimal residual errors at the cost of computational overhead.

B. Zero-Distortion Transmitter

In this subsection, we propose a closed form bound and approximation for the average PEP expression of the adopted system model under transmitter and receiver I/Q imbalance along with the thermal noise at receiver but negligible transmitter distortion. This simplifies the PEP expression in (28) to $\text{PEP} = Q(\sqrt{\varrho})$ where $\varrho = \alpha h_r^2 + \beta h_i^2$. Following similar steps as in Appendix A, the PDF of $f_\varrho(\varrho)$ for the Rayleigh fading channel is given as

$$f_\varrho(\varrho) = \frac{1}{\lambda\sqrt{\alpha\beta}} e^{-\frac{\varrho}{2\lambda}(\frac{1}{\alpha} + \frac{1}{\beta})} \mathcal{I}_0 \left\{ \frac{\varrho}{2\lambda} \left(\frac{1}{\alpha} - \frac{1}{\beta} \right) \right\}; \varrho \geq 0. \quad (42)$$

Thus the average PEP can be evaluated by solving $\bar{P}_s \leq \psi \mathbb{E}_\varrho [Q(\sqrt{\varrho})]$.

1) *Average PEP Bound:* Employing the Chernoff bound on the Q-function, we obtain

$$\bar{P}_s \leq \frac{\psi}{2} \int_0^\infty e^{-\frac{\varrho}{2}} f_\varrho(\varrho) d\varrho. \quad (43)$$

The uniform convergence and the interchangeable integral and summation enable us to evaluate the closed form expression for the $\bar{P}_s$ bound as shown in Appendix B obtaining

$$\bar{P}_s \leq \frac{\sqrt{\alpha\beta}\psi}{|\beta - \alpha|} \sum_{m=0}^\infty \frac{(2m)!}{[m!]^2 2^{2m}} \left[\frac{|\beta - \alpha|}{(\alpha + \beta + \alpha\beta\lambda)} \right]^{2m+1}. \quad (44)$$

This yields a closed form upperbound on the average SEP using (18) or (19).

2) *Exact Average PEP using Craig Representation:* The Craig representation of Q-function yields exact average PEP expression and a close approximation for $\bar{P}_s$ using NNUB as

$$\bar{P}_s \leq \frac{\psi}{\pi} \int_0^{\pi/2} \int_0^\infty e^{-\frac{\varrho}{2\sin^2\varphi}} f_\varrho(\varrho) d\varrho d\varphi. \quad (45)$$

Using similar steps to the presented in Appendix B, we solve the integral over ϱ obtaining

$$\bar{P}_s \leq \frac{2\sqrt{\alpha\beta}\psi}{\pi|\beta - \alpha|} \sum_{m=0}^\infty \frac{(2m)!}{(m!)^2 2^{2m}} \int_0^{\pi/2} \left(\frac{|\beta - \alpha| \sin^2\varphi}{\alpha\beta\lambda + (\alpha + \beta) \sin^2\varphi} \right)^{(2m+1)} d\varphi. \quad (46)$$

Furthermore, the Wolfram Mathematica integrator solves the complex integral in (46) as

$$\bar{P}_s \leq \frac{2\sqrt{\alpha\beta}\psi}{\pi|\beta - \alpha|} \sum_{m=0}^\infty \frac{(2m)!}{(m!)^2 2^{2m} (3+4m)} \left(\frac{|\beta - \alpha|}{\alpha\beta\lambda} \right)^{(2m+1)} \dots F_1 \left(2m + \frac{3}{2}; \frac{1}{2}, 2m + 1; 2m + \frac{5}{2}; 1, \frac{-(\alpha + \beta)}{\alpha\beta\lambda} \right). \quad (47)$$

Equivalently, $\bar{P}_s$ can be represented with Gauss hypergeometric function for easy implementation as given in (48).

3) *Exact Average PEP using Moment Generating Function (MGF):* The lack of correlation between the real and imaginary channel coefficients in $\varrho = \alpha h_r^2 + \beta h_i^2$ enables us to compute approximately $\bar{P}_s$ using MGF approach. The average PEP is given in terms of MGF of ϱ as

$$\mathbb{E}_\varrho [Q(\sqrt{\varrho})] = \frac{1}{\pi} \int_0^{\pi/2} M_\varrho \left[\frac{-1}{2\sin^2\varphi} \right] d\varphi. \quad (49)$$

where $M_\varrho(s)$ is expressed as, see Appendix C for derivation,

$$M_\varrho(s) = \frac{1}{\sqrt{1 - \alpha s} \sqrt{1 - \beta s}}. \quad (50)$$

Therefore, the bound on $\bar{P}_s$ is given as

$$\bar{P}_s \leq \frac{\psi}{\pi} \int_0^{\pi/2} \frac{1}{\sqrt{1 + \frac{\alpha}{2\sin^2\varphi}} \sqrt{1 + \frac{\beta}{2\sin^2\varphi}}} d\varphi. \quad (51)$$

The Chernoff, Craig and MGF bounds provide tight bound and approximation on the average PEP, respectively, which can then be employed to evaluate NNUB on average SEP as well as average BER for the adopted HWI system.

C. System with Negligible Transmitter I/Q Imbalance

This section deals with the receiver I/Q imbalance under the assumption of minimal transmitter I/Q imbalance. An example of such scenario is the cellular downlink case when the BTS employs sophisticated signal processing techniques rendering minimal I/Q imbalance. However, the mobile station employs homodyne RF front-end architecture to assist compactness and energy efficiency. In such scenarios, the system only suffers from the receiver I/Q imbalance.

In the presence of the thermal noise, we further consider two scenarios with and without transmitter distortion to evaluate average PEP. In the first case, system suffers from receiver I/Q imbalance as well as non-trivial transmitter and receiver distortions. In this case the average PEP has the same bound as given in eqs. (32) to (34) and a close approximation as given in eqs. (40) and (41) with $F_\vartheta(\vartheta)$ and the corresponding parameters are given in Equations (74) to (75). However, the parameters α and β are now defined as $\alpha = p(\xi_r^2 + \cos^2\theta_r \xi_i^2)/(2\sigma_r^2 \cos^2\theta_r)$, and $\beta = p(\xi_i^2 + \cos^2\theta_r \xi_r^2)/(2\sigma_r^2 \cos^2\theta_r)$. In the second scenario, when the transmitter exhibits negligible transmit distortion, the receiver I/Q imbalanced system follows the same closed form bound as given in (44) and exact average PEP in (48), with the corresponding parameters α and β .

$$\bar{P}_s \leq \frac{2\psi}{\pi\lambda\sqrt{\alpha\beta}} \sum_{m=0}^{\infty} \frac{(2m)!}{(m!)^2(3+4m)} \left(\frac{|\beta-\alpha|}{2\alpha\beta\lambda}\right)^{2m} \Gamma\left(\frac{1}{2}\right) \frac{\Gamma(2m+\frac{5}{2})}{\Gamma(2m+2)} {}_2F_1\left([2m+\frac{3}{2}, 2m+1], 2m+2, \frac{-(\alpha+\beta)}{\alpha\beta\lambda}\right) \quad (48)$$

D. System with Negligible Receiver I/Q Imbalance

After neglecting I/Q imbalance at the receiver, the PEP expression in (28) reduces to

$$\text{PEP} = Q\left(\sqrt{\frac{\alpha|h|^2}{1+\gamma|h|^2}}\right) = Q\left(\sqrt{\Omega}\right), \quad (52)$$

where $\alpha = \frac{p}{2\sigma_r^2}(\tau_1^2 + \tau_2^2)$, $\tau_1 = \xi_r$ and $\tau_2 = (a_t \sin \theta_t) \xi_r + (a_t \cos \theta_t) \xi_i$. To evaluate the average PEP, we first need to investigate the PDF of Ω that is derived in Appendix D as

$$f_{\Omega}(\Omega) = \frac{\alpha\lambda}{(\alpha - \gamma\Omega)^2} e^{\frac{-\lambda\Omega}{\alpha - \gamma\Omega}}; \quad 0 \leq \Omega \leq \alpha/\gamma. \quad (53)$$

1) *Average PEP Bound:* The average PEP is upper bounded using the Chernoff bound as

$$\bar{P}_s \leq \frac{\alpha\lambda\psi}{2} \int_0^{\alpha/\gamma} \frac{e^{\frac{\gamma\Omega^2 - (\alpha+2\lambda)\Omega}{2(\alpha-\gamma\Omega)}}}{(\alpha - \gamma\Omega)^2} d\Omega. \quad (54)$$

Again, we employ Gaussian quadrature numerical integration to evaluate the presented upper bound using Legendre polynomial, which simplifies the average PEP as

$$\bar{P}_s \leq \frac{\lambda\psi}{\gamma} \int_{-1}^{+1} g(t) dt = \frac{\lambda\psi}{\gamma} \sum_{k=1}^n \omega_k g(t_k) + R_n, \quad (55)$$

where the nodes t_k are the zeros of n^{th} -order Legendre polynomial and the weights ω_k and the residual term R_n are defined in (35) and (36) respectively. In addition, the function $g(t)$ is derived from the integrand in (54) by choosing the parametrization $t = -1 + 2\Omega\gamma/\alpha$ as

$$g(t) = \frac{1}{(t-1)^2} \exp\left(\frac{-\alpha(t+1)}{4\gamma} + \frac{\lambda(t+1)}{\gamma(t-1)}\right). \quad (56)$$

2) *Average PEP Approximation:* Using the Craig representation of the Q-function, the average error probability of the transmitter only I/Q imbalance reduces to

$$\bar{P}_s \leq \frac{\psi}{\pi} \int_0^{\alpha/\gamma} \int_0^{\pi/2} e^{-\frac{\Omega}{2\sin^2\phi}} \frac{\alpha\lambda}{(\alpha - \gamma\Omega)^2} e^{\frac{-\lambda\Omega}{\alpha - \gamma\Omega}} d\phi d\Omega. \quad (57)$$

which cannot be evaluated in a tractable way. However, by incorporating similar procedure to approximate the $\bar{P}_s$ bound in (57) using numerical integration Gaussian Quadrature technique by choosing the following parameterization yields

$$\bar{P}_s \leq \frac{\alpha^2\lambda\psi}{8\gamma} \int_{-1}^1 \int_{-1}^1 h\left(\frac{\alpha}{2\gamma}(u+1), \frac{\pi}{4}(v+1)\right) du dv, \quad (58)$$

where $u = \frac{2\gamma}{\alpha}\Omega - 1$ and $v = \frac{4}{\pi}\phi - 1$. The numerical approximation of double integrals in the square region is

obtained using K^{th} -order and L^{th} -order Legendre polynomial approximations for integration over du and dv , respectively.

$$\bar{P}_s \leq \frac{\alpha^2\lambda\psi}{8\gamma} \sum_{l=1}^L \sum_{k=1}^K \omega_l \omega_k h\left(\frac{\alpha}{2\gamma}(u_k+1), \frac{\pi}{4}(v_l+1)\right), \quad (59)$$

where the weights $\omega_k(u_k)$ and $\omega_l(v_l)$ are obtained using (35). Moreover, u_k and v_l are the k^{th} and l^{th} zeros of the K^{th} -order and L^{th} -order Legendre polynomials, respectively.

The average BER can be derived as $\bar{P}_b \leq \bar{P}_s/\log_2 M$ assuming gray coding in the high SNR regime. The average PEP bounds and approximations are given in (34) and (41) for the system under dual I/Q imbalance with transmit distortion respectively. Similarly, the average PEP bounds and approximations for the dual I/Q imbalance without transmit distortion are given in (44) and (48) respectively. The same results hold for the system with negligible transmitter I/Q imbalance with and without transmit distortion given modified parameters α and β . Similarly, the average PEP bounds and approximations for the system with negligible receiver I/Q imbalance are proposed in (55) and (59), respectively.

VI. TRANSMIT SIGNALING DESIGN

Transmit signaling can be optimized to reduce the probability of error by effectively mitigating HWI effects. Practically, the signal constellation can be modified within power constraints to achieve a lower probability of error. The rotation and translation in-variance of error probability is a well-known fact. However, this is only true in the presence of proper additive noise. We propose an asymmetric modulation scheme where effective rotation and optimal scaling of a symmetric signal constellation is performed to mitigate the HWIs impact in distorting signals and deforming the aggregate noise to improper Gaussian one. The PEP expression in (25) can be presented in a matrix form as follows

$$\Pr(\mathbf{x}^m \rightarrow \mathbf{x}^n | \mathbf{x}^m) = Q\left(\sqrt{\frac{p}{4(1-\rho^2)} \mathbf{x}_{mn}^T \tilde{\mathbf{H}} \mathbf{Y} \tilde{\mathbf{H}}^T \mathbf{x}_{mn}}\right), \quad (60)$$

where $\mathbf{x}^m = [\Re\{x_m\} \ \Im\{x_m\}]^T$, $\tilde{\mathbf{H}}$ and $\mathbf{Y}$ are expressed as

$$\tilde{\mathbf{H}} = \begin{bmatrix} \tilde{h}_{1r} + \tilde{h}_{2r} & \tilde{h}_{2i} - \tilde{h}_{1i} \\ \tilde{h}_{1i} + \tilde{h}_{2i} & \tilde{h}_{1r} - \tilde{h}_{2r} \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} 1/\sigma_1^2 & -\rho/\sigma_1\sigma_Q \\ -\rho/\sigma_1\sigma_Q & 1/\sigma_Q^2 \end{bmatrix} \quad (61)$$

and $\mathbf{x}_{mn}$ represent the distance vector between any two information symbols x_m and x_n and is written as

$$\mathbf{x}_{mn} = \begin{bmatrix} \Re\{x_m\} - \Re\{x_n\} \\ \Im\{x_m\} - \Im\{x_n\} \end{bmatrix}. \quad (62)$$

In order to design the rotation and scaling, consider the information bearing signal $s = s_r + is_i$ and the vector representation of m^{th} symbol as $\mathbf{s}^m = [\Re\{s_m\} \ \Im\{s_m\}]^T$. Thus, the suggested rotation and translation transforms the

$$\begin{aligned}
\mathbf{P3}: \quad & \min_{\eta, \tau} \quad \tau \\
\text{s. t.} \quad & \frac{1}{M} \sum_{m=1}^M \sum_{\substack{n=1 \\ n \neq m}}^M Q \left(\sqrt{\frac{p}{4(1-\rho^2)}} \mathbf{s}_{mn}^T \mathbf{R}^T \mathbf{A}^T \tilde{\mathbf{H}} \mathbf{Y} \tilde{\mathbf{H}}^T \mathbf{A} \mathbf{R} \mathbf{s}_{mn} \right) \leq \tau \quad \forall \mathbf{s}^m \neq \mathbf{s}^n \\
& 0 \leq \eta \leq 1
\end{aligned}$$

TABLE I: Rotation angles for maximal symbol separation in one-dimension

Modulation Scheme	θ	Modulation Scheme	θ
4-QAM	26.56 ⁰	16-QAM	14.0375 ⁰
8-QAM	26.56 ⁰	64-QAM	7.0 ⁰

symmetric modulated symbols $\mathbf{s}^m$ to asymmetric modulated symbols $\mathbf{x}^m$ as

$$\mathbf{x}^m = \mathbf{A} \mathbf{R} \mathbf{s}^m \quad (63)$$

where, the rotation matrix $\mathbf{R}$ with rotation angle θ is:

$$\mathbf{R} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \quad (64)$$

Moreover, the translation matrix $\mathbf{A}$ is given in (65). It is important to highlight that $\eta \in [0, 1]$. It can be easily derived using Definition 3; where $C_x = |\tilde{\sigma}_x^2|/\sigma_x^2$, and $C_x \in [0, 1]$ with $|\tilde{\sigma}_x^2| = \eta \sigma_x^2$ given translation $\mathbf{x} = \mathbf{A} \mathbf{s}$,

$$\mathbf{A} = \begin{bmatrix} \sqrt{1+\eta} & 0 \\ 0 & \sqrt{1-\eta} \end{bmatrix} \quad (65)$$

We aim to design the rotation angle θ and scaling η to dampen error probability by effectively mitigating various HWI effects. Table I contains the intuitive θ for various modulation schemes to attain maximum separation between any two adjacent symbols in one-dimension and can be introduced as $\mathbf{x} = \mathbf{R} \mathbf{s}$. Next, we propose several design schemes for the translation matrix $\mathbf{A}$.

A. Special Cases:

The existing symmetric signaling schemes induce $\theta = 0$ and $\eta = 0$ which render identity rotational and scaling matrices. The symmetric signaling scheme fails to attain lower error probability in the presence of improper impairments. Therefore, a sub-optimal approach for the asymmetric transmission i.e., maximal asymmetric transmission with $\eta = 1$ can be employed. This η renders the following scaling matrix

$$\mathbf{A} = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix} \quad (66)$$

This scheme reduces computational complexity to solve any optimization problem and outperforms the existing symmetric signaling scheme in some scenarios as discussed in Section VII. Next, we propose two optimization schemes for the design of optimal translation matrix.

B. Maximum Pairwise Error Probability Minimization

In this subsection, we design the scaling matrix $\mathbf{A}$ for asymmetric signal transmission to minimize the maximum PEP. The PEP in (60) after the requisite transformation in (63) can be presented as

$$\Pr(\mathbf{s}^m \rightarrow \mathbf{s}^n | \mathbf{s}^m) = Q \left(\sqrt{\frac{p}{4(1-\rho^2)}} \mathbf{s}_{mn}^T \mathbf{R}^T \mathbf{A}^T \tilde{\mathbf{H}} \mathbf{Y} \tilde{\mathbf{H}}^T \mathbf{A} \mathbf{R} \mathbf{s}_{mn} \right) \quad (67)$$

The following optimization problem, **P1**, minimize the maximum PEP based on Definition 3 for all possible error events between any two different symbols in a given signal constellation:

$$\begin{aligned}
\mathbf{P1}: \quad & \min_{\eta, \kappa} \quad \kappa \\
\text{s. t.} \quad & \Pr(\mathbf{s}^m \rightarrow \mathbf{s}^n | \mathbf{s}^m) \leq \kappa, \quad \forall \mathbf{s}^m \neq \mathbf{s}^n, \forall m, n \\
& 0 \leq \eta \leq 1.
\end{aligned}$$

The minimization problem **P1** can be equivalently reformulated to the following maximization problem by disregarding the Q-function in (67) for the ease of implementation,

$$\begin{aligned}
\mathbf{P2}: \quad & \max_{\eta, \tau} \quad \tau \\
\text{s. t.} \quad & \mathbf{s}_{mn}^T \mathbf{R}^T \mathbf{A}^T \tilde{\mathbf{H}} \mathbf{Y} \tilde{\mathbf{H}}^T \mathbf{A} \mathbf{R} \mathbf{s}_{mn} \geq \tau, \quad \forall \mathbf{s}^m \neq \mathbf{s}^n, \forall m, n \\
& 0 \leq \eta \leq 1.
\end{aligned}$$

P2 is prove to be a concave problem as shown in Appendix E, thus it can be optimally solved using solvers like CVX.

C. Maximum Symbol Error Rate Minimization

The asymmetric modulation can also be designed based on another optimization criteria, where the maximum SEP has to be minimized. To this end, we employ the SEP bound in (18) for the information bearing symbols $\mathbf{s}^m$, $m \in \{1, 2, \dots, M\}$ as

$$P_s \leq \frac{1}{M} \sum_{m=1}^M \sum_{\substack{n=1 \\ n \neq m}}^M \Pr(\mathbf{s}^m \rightarrow \mathbf{s}^n | \mathbf{s}^m). \quad (68)$$

The minimization problem can be formulated as in **P3** along with the box constraint on the design parameter η based on Definition 3. The problem **P3** is proven to be convex in Appendix F and thus it can be optimally solved using any available solver which supports Q-function such as fmincon in MATLAB. The optimal η renders an optimal scaling matrix $\mathbf{A}$ using (65). Therefore, symmetric signaling based on the traditional modulation schemes is transformed to asymmetric transmission using (63) to effectively combat HWI effects.

Fig. 3: HWI-Aware Receivers

The proposed asymmetric transmission schemes are compared in the following section while keeping the existing symmetric transmission scheme as a benchmark and an upper-bound on the average error probability of HWIs.

VII. NUMERICAL RESULTS

In this section, we numerically investigate the BER or symbol error rate (SER) performance of the proposed HWIs system model where I/Q imbalance results in self-interference. Furthermore, we analyze the performance of the two proposed receivers relative to the conventional MED receiver in the presence of improper additive noise. Moreover, we investigate the tightness of the designed bounds and approximations on the average probability of error. Eventually, we compare various suggested asymmetric transmission schemes to effectively combat HWIs in order to attain lower error probability. The numerical investigations are carried out for the HWI systems under dual I/Q imbalance with or without transmitter distortion as well as the individual I/Q imbalances at the transmitter or receiver. Moreover, the results for average error probability are validated only for gray coded QAM modulation scheme for brevity. However, same results can be extended to other modulation schemes.

A. Optimal Receiver

In this subsection, we analyze the performance gain obtained by employing the proposed HWI-aware optimal ML and sub-optimal LMMSE receivers in-place of the conventional HWI-unaware MED receiver for various system configurations. As for the system parameters, we assume normalized 16-QAM modulation with HWI parameters $a_t = a_r = 0.835$, $\theta_t = \theta_r = 5^\circ$, $\sigma_r^2 = 1$, $\sigma_t^2 = 0.05$, $\lambda = 1$, the average SNR in 0-36dB range and $\psi = 3$, unless otherwise stated.

Firstly, we analyze the average BER of the HWIs system with dual I/Q imbalance as well as additive distortions at both transmitter and receiver as shown in Fig. 3(a). We investigate average BER versus average SNR for two different symmetric amplitude I/Q imbalance estimates $a_t = a_r = 0.835$ & 1.195 . Evidently, the impaired system depicts a high error probability especially for the lower value of I/Q amplitude scaling. The increasing SNR helps to improve the average BER performance but eventually undergoes saturation. It is important to

highlight that this error floor is due to the presence transmit distortions as discussed in (27). The HWI-aware ML and LMMSE receivers clearly outperform the HWI-unaware MED receiver, which was the optimal choice for negligible HWIs. The performance gain is particularly significant for higher amplitude errors and higher order SNR. Although the optimal ML receiver demonstrates superior performance relative to the LMMSE receiver for any given SNR and I/Q amplitude error but the LMMSE receiver is a fairly good candidate for least complex practical receiver design.

Secondly, the performance gain of the optimal receiver is observed for a receiver impaired system under receiver I/Q imbalance and additive distortions at the receiver only. Fig. 3(b) represents the average BER versus average SNR for three different receiver distortion levels $\sigma_r^2 = 0.2, 0.6$ & 1 . Evidently, the drastic effect of receiver HWIs significantly decreases with increasing SNR. Moreover, the HWI-aware optimal ML receiver outperforms suboptimal HWI-aware LMMSE receiver and HWI-unaware MED receiver at all distortion levels, especially at higher SNR. Similarly, the average BER performance of the proposed receivers of the transmitter impaired system is observed for three different transmit distortion levels $\sigma_t^2 = 0.001, 0.04$ & 0.1 in Fig. 3(c). The average BER depicts an error floor for significant transmit distortion levels but decreases with increasing SNR for negligible σ_t^2 .

Conclusively, the HWI-aware receivers outperform HWI-unaware receiver in all presented scenarios especially at higher amplitude errors, lower transmitter distortions, and higher SNR. This signifies the importance of incorporating HWIs in the system model and receiver design. Moreover, the optimal ML receiver sets the performance benchmark for sub-optimal LMMSE receiver. Although the superior performance of ML receiver is unprecedented, but the LMMSE receiver is suitable for practical implementation with subsidiary performance. Additionally, the transmitter distortions are mainly responsible for the error floor at higher SNR levels. Increasing SNR amplifies transmit distortion as well and thus rendering it ineffective to reduce error probability. An important observation is the quantification of the deteriorating effects of various HWIs. Fig. 3(a) demonstrates that the dual I/Q impairments and distortions are more drastic than the individual impaired systems in Fig. 3(b) and 3(c). Moreover, the transmitter impairments are far more degrading than receiver impairments.

Fig. 4: Bounds and Approximations on $\bar{P}_b$

B. Bounds and Approximations

This subsection deals with the evaluation of Chernoff bounds and numerical integration approximations on the average probability of error for two system configurations. They include dual I/Q imbalanced system with negligible transmit distortions and transmitter impaired system for brevity. We assume the following system parameters, unless otherwise stated, $a_t = a_r = 0.835$, $\theta_t = \theta_r = 5^\circ$, $\sigma_r^2 = 1$, $\sigma_t^2 = 0$, $\lambda = 1$, $\psi = 3$ for NNUB, normalized 16-QAM modulation scheme.

Firstly, the average BER is analyzed for IRR ranging from 3.5-27dB ($a_t = a_r = 0.2 - 1.0$) in a dual I/Q imbalanced system with receiver distortion but negligible transmitter distortion. The bounds and approximations are observed for two different SNR conditions (20dB and 30dB) in Fig. 4(a). The derived closed form Chernoff bound and the MGF bound are in great agreement with the average BER trend and become tighter with increasing IRR. The MGF bound is remarkably tight in the region of interest when $\text{IRR} \geq 20\text{dB}$. Similarly, Fig. 4(b) investigates the similar system with ($a_t = a_r = 0.835$) for three different receiver distortion levels $\sigma_r^2 = 0.1, 0.6 \& 1.0$. Evidently, lower receiver distortion guarantees lower probability of error. In addition, we observe fairly tight bounds for all distortion and SNR levels. Moreover, the NNUB bound on average BER using closed form $\bar{P}_s$ approximation (48) closely follows the simulated average BER for $\text{IRR} = 20\text{dB}$. Interestingly, the closed form Chernoff bound and Craig's approximation given by (44) and (48), respectively can be accurately computed by summing few initial terms exhibiting negligible percentage errors, as given in Table II(a).

Lastly, the derived bounds and approximations are investigated for a transmitter impaired system under transmitter I/Q imbalance and transmitter additive distortions with thermal noise at the receiver and negligible receiver I/Q imbalance in Fig. 4(c). The results are analyzed for three different transmit distortion levels $\sigma_t^2 = 0.001, 0.01 \& 0.1$. The Chernoff bound (54) is examined using the proposed Gaussian quadrature numerical integration employing 30 - order Legendre polynomial. Analysis depicts that employing lower order $P_N(t)$ exhibits promising results for lower SNR ranges. However, for higher SNR ranges, we need to incorporate a higher order Legendre polynomial to achieve a close bound. Fig. 4(c) depicts that employing 30 - order $P_N(t)$ follows the same trend as the simulated average probability of error. In

TABLE II: Percentage Error of Gaussian Quadrature Legendre Polynomial Approximation

(a) Dual I/Q: Zero-Distortion Transmitter		
Sum Terms	Chernoff Bound Equation (44)	Craig's Approx Equation (48)
1	2.84898e-16	3.41016e-17
2	6.54392e-24	3.28161e-25
3	1.57825e-31	3.44822e-33
4	3.86638e-39	4.97314e-40

(b) Transmitter Impaired System		
Order (N, L, K)	Chernoff Bound Equation (54)	Craig's Approx Equation (57)
1	4.66701e-01	9.51256 e-01
5	1.33278e-03	2.16892e-02
10	8.65744e-05	2.28766e-03
20	1.95911e-06	3.32592e-04

order to achieve a close approximation, 30 - order $P_K(u)$ and 32 - order $P_L(v)$ is employed to numerically integrate the $\bar{P}_s$ approximation intractable double integral in (57). The approximation shows promising results especially for medium to high SNR and higher impairment levels. In order to evaluate the accuracy of the proposed Gaussian quadrature numerical integration method for intractable Chernoff bound and Craig's approximation on average SEP, we derived the percentage error for varying ordered Legendre Polynomials as shown in Table II (b). We assumed the system with negligible receiver I/Q imbalance as adopted in Fig. 4c with 25dB SNR, 0.01 transmitter distortion variance and varying Legendre polynomial orders. It can be safely concluded that 10-20 ordered Legendre Polynomials can accurately approximate the intractable integrals with negligible percentage error.

C. Asymmetric Transmission

The drastic effects of HWIs can be partially mitigated by employing optimal detection scheme. However, we can attain better system performance using asymmetric transmission scheme to effectively dampen the improper interference. This subsection studies the effectiveness of various proposed transmit optimization schemes relative to the traditional symmetric

Fig. 5: Asymmetric Transmission Schemes

transmission scheme with or without optimal receiver for various impairments. We assume medium impairment levels of $\theta_t = \theta_r = 5^\circ$, $a_{I/Q} = 0.5$, $\sigma_r^2 = 1$, $\sigma_t^2 = 0.01$, $\lambda = 1$ and normalized 4-QAM modulation scheme, unless otherwise specified, in a dual impaired transmission system.

Firstly, we analyze the average SER for the range of symmetric amplitude errors $a_{I/Q}$ ($a_t = a_r$) = 0 – 0.9 for two given SNR levels; 10 dB and 20 dB in Fig. 5(a). We compare traditional symmetric scheme with or without optimal receiver with the three proposed asymmetric transmission schemes to quantify the error performance gain. Evidently, symmetric transmission with suboptimal receiver performs far worst than the symmetric transmission with the optimal receiver, especially for higher SNR and lower amplitude scaling (higher impairment level). Moreover, the error performance of the adopted system under symmetric transmission improves with increasing $a_{I/Q}$. On contrary, the asymmetric transmission schemes depict a significantly lower error probability even for lower $a_{I/Q}$. The suboptimal asymmetric transmission i.e. maximal asymmetric performs equally good as any other optimized asymmetric transmission for higher impairment levels but renders higher error probability than symmetric transmission for $a_{I/Q} \geq 0.6$. However, the optimal asymmetric transmission schemes MinmaxPEP and MinmaxSEP equally outperform all other transmission schemes. Thus based on this analysis, one can conclude that the application of mere optimal receiver with symmetric transmission does not reduce the error probability to a greater extent but a better performance can be achieved by employing asymmetric transmission schemes along with the optimal receiver. Moreover, Maximal asymmetric transmission is a fairly good choice for highly impaired systems as it performs equally good as any other optimized asymmetric scheme without inducing additional computational overhead.

Furthermore, we investigated the average SER for a viable range of transmitter distortions $\sigma_t^2 = 0.01 - 0.1$ for three different SNR values 10 dB, 20 dB and 30 dB in Fig. 5(b). We observe a similar pattern between symmetric signaling with the suboptimal receiver and symmetric signaling with the optimal receiver for all distortion levels and SNR values. However, a different trend is observed for the maximal asymmetric trans-

mission scheme. It clearly outperforms symmetric signaling scheme at all distortion levels and renders suboptimal performance as compared to the optimized asymmetric transmission schemes (MinmaxPEP and MinmaxSEP). Thus, it depicts a trade-off between the error probability performance gain and computational expenses of the optimization problems.

VIII. CONCLUSION

In this paper, we demonstrated the significance of incorporating HWIs in accurate system modeling and analysis. We mainly focused on the detailed modeling of I/Q imbalance and several distortion noises at the transceiver. We also proposed an optimal and a sub-optimal linear receiver which incorporates improper interference characteristics. Both transmitter and receiver I/Q imbalance render SI information bearing signal whereas only receiver I/Q imbalance is responsible for transforming AWGN to improper Gaussian noise. The transmitter distortion is subject to channel fading while the receiver added impairment is not. Analytical results are validated using simulations where the derived Chernoff bounds and numerical integration approximations are in close agreement with the simulated average BER trend and are significantly tight for lower impairments levels. Further performance improvement can be achieved using the proposed asymmetric modulation schemes, which outperform the existing symmetric signaling with or without the optimal receiver. Moreover, the maximal asymmetric scheme can be a fairly good candidate to achieve better performance without rendering any optimization expenses for highly impaired systems.

A possible extension to this paper would be to consider the impact of aggregated HWIs on the massive MIMO systems, which is a key concept to attain higher area throughput in future wireless networks. Interestingly, the studies have shown that the huge degrees of freedom offered by the massive densification provide robustness to only some of the impairments. For example, [50] proved that the concentrated antennas deployment offers immunity to the hardware distortions but not the phase drifts through closed-form achievable rate performance analysis. Similarly, [51] demonstrated that the effects of impairments and noise at the massive-antenna

fusion center vanish while the sensor impairment dominates the achievable distributed detection performance, in the limit of an infinite number of antennas and infinite sensors reporting power budget.

APPENDIX A DERIVATION OF $F_\vartheta(\vartheta)$

Consider the Rayleigh fading channel where real and imaginary components of the channel coefficient are distributed as $h_j \sim \mathcal{CN}(0, \lambda/2, 0); j \in \{r, i\}$. This implies h_j^2 to be distributed as unnormalized chi-square variable with first degree of freedom i.e. $f_{h_j^2}(x) \sim \frac{1}{\sqrt{\pi\lambda x}} e^{-\frac{x}{\lambda}}; x \geq 0$. Using (29), the CDF of ϑ is defined as $F_\vartheta(\vartheta) = \Pr\left\{\frac{\alpha h_r^2 + \beta h_i^2}{1 + \gamma(h_r^2 + h_i^2)} \leq \vartheta\right\}$ which can also be represented as the probability of the event $F_\vartheta(\vartheta) = \Pr\{(\alpha - \gamma\vartheta)h_r^2 + (\beta - \gamma\vartheta)h_i^2 \leq \vartheta\} = \Pr\{S \leq \vartheta\}$ (69)

PDF of $(\alpha - \gamma\vartheta)h_r^2$ is evaluated using careful transformations.

$$(\alpha - \gamma\vartheta)h_r^2 \sim \frac{1}{\sqrt{\pi\lambda(\alpha - \gamma\vartheta)x}} e^{-\frac{x}{(\alpha - \gamma\vartheta)\lambda}}; \quad x \geq 0 \quad (70)$$

Similarly, the distribution of $(\beta - \gamma\vartheta)h_i^2$ is identical to (70) and can be obtained by replacing α with β in (70) pertaining to iid channel coefficients. Next, we convolve the derived PDFs under the assumption of uncorrelated real and imaginary channel coefficients. Then employing Mathematica integrator to compute the convolution, we obtain

$$f_S(s) = \frac{1}{k_1(\vartheta)} e^{-sk_2(\vartheta)} \mathcal{I}_0\{sk_3(\vartheta)\}; s \geq 0 \quad (71)$$

where, $k_1(\vartheta) = \frac{\lambda\sqrt{(\alpha - \gamma\vartheta)(\beta - \gamma\vartheta)}}{2[k_1(\vartheta)]^2}$, $k_2(\vartheta) = \frac{\lambda(\alpha + \beta - 2\gamma\vartheta)}{2[k_1(\vartheta)]^2}$, $k_3(\vartheta) = \frac{\lambda(\beta - \alpha)}{2[k_1(\vartheta)]^2}$ and $\mathcal{I}_0\{x\}$ is the zero-order modified Bessel function, which has the following power series representation [52]

$$\mathcal{I}_0\{x\} = \sum_{m=0}^{\infty} \frac{x^{2m}}{[m!]^2 2^{2m}}. \quad (72)$$

From (69), the distribution function $F_\vartheta(\vartheta)$ is given as

$$F_\vartheta(\vartheta) = \frac{1}{k_1(\vartheta)} \int_0^\vartheta e^{-sk_2(\vartheta)} \mathcal{I}_0\{sk_3(\vartheta)\} ds, \quad (73)$$

Eventually, substituting $t = sk_3(\vartheta)$ and the power series representation of $\mathcal{I}_0\{t\}$ yields

$$F_\vartheta(\vartheta) = \frac{1}{k_1(\vartheta)k_3(\vartheta)} \int_0^{\vartheta k_3(\vartheta)} e^{-t \frac{k_2(\vartheta)}{k_3(\vartheta)}} \sum_{m=0}^{\infty} \frac{t^{2m}}{[m!]^2 2^{2m}} dt \quad (74)$$

where $\delta = \max(\alpha, \beta)/\gamma$ and $0 \leq \vartheta \leq \delta$. Based on the uniform convergence [53], swapping the order of integral and summation yields the closed form distribution function $F_\vartheta(\vartheta)$ as

$$F_\vartheta(\vartheta) = \frac{1}{k_1(\vartheta)} \sum_{m=0}^{\infty} \frac{2^{-2m} k_3(\vartheta)^{2m}}{[m!]^2 k_2(\vartheta)^{2m+1}} [\Gamma(2m+1, \vartheta k_2(\vartheta))]. \quad (75)$$

where $\Gamma(a, x)$ is the upper incomplete gamma function [36, (6.5.12)].

APPENDIX B CHERNOFF BOUND ON AVERAGE PEP

Substituting $f_\varrho(\varrho)$ from (42) to (43) yields

$$\bar{P}_s \leq \frac{\psi}{2\lambda\sqrt{\alpha\beta}} \int_0^\infty e^{-\frac{\varrho}{2\lambda}(\lambda + \frac{1}{\alpha} + \frac{1}{\beta})} \mathcal{I}_0\left\{\frac{\varrho}{2\lambda}\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)\right\} d\varrho \quad (76)$$

Substituting $t = \varrho(\beta - \alpha)/(2\lambda\alpha\beta)$ yields

$$\bar{P}_s \leq \frac{\sqrt{\alpha\beta}\psi}{|\beta - \alpha|} \int_0^\infty e^{-\frac{t(\alpha + \beta + \alpha\beta\lambda)}{|\beta - \alpha|}} \mathcal{I}_0\{t\} dt \quad (77)$$

Using the power series representation of $\mathcal{I}_0\{t\}$ from (72) in (77) and swapping integration and summation based on the uniform convergence yields

$$\bar{P}_s \leq \frac{\sqrt{\alpha\beta}\psi}{|\beta - \alpha|} \sum_{m=0}^{\infty} \frac{1}{(m!)^2 2^{2m}} \int_0^\infty e^{-\frac{t(\alpha + \beta + \alpha\beta\lambda)}{|\beta - \alpha|}} t^{2m} dt \quad (78)$$

Finally, we utilize the well-known integral substitution as given in (79) to comprehend (78).

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}} \quad (79)$$

Thus, (78) reduces to (80) and we can obtain the desired closed form expression as given in (44) after some trivial simplifications.

$$\bar{P}_s \leq \frac{\sqrt{\alpha\beta}\psi}{|\beta - \alpha|} \sum_{m=0}^{\infty} \frac{(2m)!}{(m!)^2 2^{2m}} \left(\frac{\alpha\beta(\alpha + \beta + \alpha\beta\lambda)}{|\beta - \alpha|} \right)^{-(2m+1)} \quad (80)$$

APPENDIX C MOMENT GENERATING FUNCTION OF ϱ

Let $\varrho = \Theta_r + \Theta_i$ where $\Theta_p = k_p \hat{h}_p^2$ with $p \in \{r, i\}$, $k_r = \frac{\alpha}{2}$ and $k_i = \frac{\beta}{2}$. Thus, $\hat{h}_p^2$ is a chi-square RV $\chi^2(1)$ and the MGF of Θ_p is given as $M_{\Theta_p}(s) = \frac{1}{\sqrt{1 - k_p s}}$. Furthermore, the independent Θ_r and Θ_i dictate $M_\varrho(s) = M_{\Theta_r}(s) M_{\Theta_i}(s)$ as given in (50).

APPENDIX D DERIVATION OF $f_\Omega(\Omega)$

Using (52), the CDF of Ω is $F_\Omega(\Omega) = \Pr\left\{\frac{\alpha|h|^2}{1 + \gamma|h|^2} \leq \Omega\right\} = \Pr\{(\alpha - \gamma\Omega)|h|^2 \leq \Omega\}$. Rayleigh fading channel assumption yields $f_{|h|^2}(x) = \lambda e^{-\lambda x}$. This implies $(\alpha - \gamma\Omega)|h|^2 \sim \text{Exp}[(\alpha - \gamma\Omega)/\lambda]$. Thus, $F_\Omega(\Omega) = 1 - e^{-\frac{\lambda\Omega}{\alpha - \gamma\Omega}}$, that can be differentiated giving $f_\Omega(\Omega)$ as given in (53).

APPENDIX E OPTIMALITY OF PROBLEM P2

Considering $\mathbf{D} \in \mathbb{C}^{2 \times 2}$ and $\mathbf{D} = \tilde{\mathbf{H}}\mathbf{Y}\tilde{\mathbf{H}}^T$. Clearly $\mathbf{Y}$ is a symmetric positive definite matrix owing to the positive trace $\text{Tr}(\mathbf{Y}) = 1/\sigma_1^2 + 1/\sigma_Q^2$ and the positive determinant

$|\mathbf{Y}| = (1 - \rho^2) / \sigma_I^2 \sigma_Q^2$. This implies that $\mathbf{D}$ is also a symmetric positive definite matrix. Assuming $\mathbf{q}_{mn} = \mathbf{R}\mathbf{s}_{mn}$, the constraint becomes

$$\mathbf{s}_{mn}^T \mathbf{R}^T \mathbf{A}^T \tilde{\mathbf{H}} \mathbf{Y} \tilde{\mathbf{H}}^T \mathbf{A} \mathbf{R} \mathbf{s}_{mn} = \mathbf{q}_{mn}^T \mathbf{A}^T \mathbf{D} \mathbf{A} \mathbf{q}_{mn} = (1 + \eta) \times (\mathbf{q}_{mn}^1)^2 \mathbf{D}_{11} + (1 - \eta) (\mathbf{q}_{mn}^2)^2 \mathbf{D}_{22} + 2\sqrt{1 - \eta^2} \mathbf{q}_{mn}^1 \mathbf{q}_{mn}^2 \mathbf{D}_{12} \quad (81)$$

where $\mathbf{q}_{mn}^1$ and $\mathbf{q}_{mn}^2$ are the first and second element of vector $\mathbf{q}_{mn}$ respectively. They represent the distance between the real and imaginary components of symbols m and n in a rotated signal constellation respectively. Conclusively, (81) demonstrates the concave nature of the **P2** in η .

APPENDIX F OPTIMALITY OF PROBLEM **P3**

Considering the constraint in **P3**,

$$f(\eta) = Q(\sqrt{h(\eta)}) = Q\left(\sqrt{\frac{\mathbf{p}\mathbf{s}_{mn}^T \mathbf{R}^T \mathbf{A}^T \tilde{\mathbf{H}} \mathbf{Y} \tilde{\mathbf{H}}^T \mathbf{A} \mathbf{R} \mathbf{s}_{mn}}{4(1 - \rho^2)}}\right) \quad (82)$$

where $h(\eta)$ is proven to be concave in η in Appendix E. Furthermore, $\sqrt{h(\eta)}$ is also concave owing to the positive nature of $h(\eta)$. The composite function in (82) is proven to be convex as

$$f''(\eta) = Q''(\sqrt{h(\eta)}) \left((\sqrt{h(\eta)})' \right)^2 + Q'(\sqrt{h(\eta)}) (\sqrt{h(\eta)})'' \quad (83)$$

where, $Q''(\sqrt{h(\eta)}) \geq 0$ and $Q'(\sqrt{h(\eta)}) \leq 0$ owing to convex decreasing nature of $Q(x)$ in $\mathbb{R}_+$. Moreover, $(\sqrt{h(\eta)})'' \leq 0$ yields $f''(\eta) \geq 0$ proving convex constraints and **P3** optimality.

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