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Global Mass Conservation Method for Dual-Continuum Gas

Reservoir Simulation

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Abstract

In this paper, we find that the numerical simulation of gas flow in dual-continuum porous media may generate unphysical or non-robust results using regular finite difference method. The reason is the unphysical mass loss caused by the gas compressibility and the non-diagonal dominance of the discretized equations caused by the non-linear well term. The well term contains the product of density and pressure. For oil flow, density is independent of pressure so that the well term is linear.

For gas flow, density is related to pressure by the gas law so that the well term is non-linear. To avoid these two problems, numerical methods are proposed using the mass balance relation and the local linearization of the non-linear source term to ensure the global mass conservation and the diagonal dominance of discretized equations in the computation. The proposed numerical methods are successfully applied to dual-continuum gas reservoir simulation. Mass conservation is satisfied while the computation becomes robust. Numerical results show that the location of the production well relative to the large-permeability region is very sensitive to the production efficiency. It decreases apparently when the production well is moved from the large-permeability region to the small-permeability region, even though the well is very close to the interface of the two regions. The production well is suggested to be placed inside the large-permeability region regardless of the specific position.

Keywords: Global mass conservation; Numerical method; Dual porosity; Dual permeability; Gas reservoir simulation; Naturally fractured porous media

Nomenclature

C_w well factor $((\text{Pa}\cdot\text{s})^{-1})$

k_M	intrinsic permeability of matrix (m^2)
k_{xxF}	fracture permeability in the x direction (m^2)
k_{yyF}	fracture permeability in the y direction (m^2)
k_{xxM}	matrix permeability in the x direction (m^2)
k_{yyM}	matrix permeability in the y direction (m^2)
L_x	domain length in the x direction (m)
L_y	domain length in the y direction (m)
l_x	fracture spacing in the x direction (m)
l_y	fracture spacing in the y direction (m)
mM	gas mass in matrix (Kg)
mF	gas mass in fracture (Kg)
mT	gas mass transferring from matrix to fracture (Kg)
nx	grid number in the x direction
ny	grid number in the y direction
nt	number of time steps
p_{bh}	bottom hole pressure pressure (Pa)
p_F	fracture pressure (Pa)
p_M	matrix pressure (Pa)
$p_F(0)$	initial condition of fracture pressure (Pa)

$p_M(0)$	initial condition of matrix pressure (Pa)
R	universal gas constant (J/(mol·K))
r_e	equivalent well radius (m)
r_w	well radius (m)
S	source term
S_c	intercept of the linearized source term
S_p	slope of the linearized source term
T	temperature (K)
t	time (s)
u_M	matrix velocity in the x direction (m/s)
u_F	fracture velocity in the x direction (m/s)
v_M	matrix velocity in the y direction (m/s)
v_F	fracture velocity in the y direction (m/s)
W	molecular weight of gas (Kg/mol)
x	horizontal axis in Cartesian coordinate
y	vertical axis in Cartesian coordinate

Greek symbols

α	shape factor of fracture
Δx	grid spacing in the x direction (m)

Δy grid spacing in the y direction (m)

Δt time step (s)

δ_w indicator of well

μ dynamic viscosity (Pa·s)

ϕ_F fracture porosity

ϕ_M matrix porosity

Superscripts

(n) the n th time level, representing the starting moment of a time step
so that variables at this moment are explicitly known

$(n+1)$ the $(n+1)$ th time level, representing the ending moment of a time
step so that variables at this moment need to be solved

$*$ variables obtained by the computation without mass balance

real real value

— mean value

Subscripts

i i th grid point in the x direction

j j th grid point in the y direction

iW the well locates on the iW th grid point in the x direction

jW the well locates on the jW th grid point in the y direction

M matrix

F fracture

1. Introduction

Unconventional oil and natural gas in naturally fractured porous media is an indispensable supplement to conventional petroleum resources (Firoozabadi, 2000; Bourbiaux, 2010). It has been receiving growing attentions in petroleum engineering (Dyke et al., 1995; Allan and Sun, 2003; Delorme et al., 2008; Corbett et al., 2010). Many mathematical models have been proposed to represent flow characteristics in the naturally fractured reservoirs. Among these models, the dual-continuum (i.e. dual-porosity dual-permeability) model is most commonly used (Barenblatt et al., 1960; Warren and Root, 1963; Kazemi, 1969; Pruess and Narasimhan, 1985; Wu and Pruess, 2000; Wu, 2002; Wu et al., 2002; Wu et al., 2004a, 2004b; Wu et al., 2011). Corresponding analytical studies (de Swaan O., 1976; Kazemi and Merrill, 1979; Wu et al., 1999; Wu et al., 2007; Huang et al., 2011; Nie et al., 2012) have been fulfilled to obtain analytical solutions in simple situations such as slight compressibility, homogenization, infinite radial flow, etc. However, flow behaviors in naturally fractured reservoirs is so

complicated that numerical simulations in Darcy scale (Wu and Qin, 2009; Presho et al., 2011) should be used to observe the complex transport phenomena in dual-continuum porous media.

All the above research progresses are for oil flow. Oil density is independent of pressure so that the governing equations are basically linear. Numerical stable solutions can be easily obtained by solving the linear algebraic equation system. For gas flow, gas compressibility makes the density change with pressure, leading to non-linear governing equations. If the numerical method for linear equations of oil flow is used straightforward to solve the non-linear equations of gas flow, numerical stability and mass conservation may be broken so that unphysical or inaccurate results may occur. Thus, it is meaningful to study numerical stable and accurate method for gas reservoir simulation in dual-continuum porous media. To the best knowledge of the authors, numerical methods for dual-continuum gas flow have not been reported in details. Also, study on production characteristics of gas flow in dual-continuum porous media is few, especially on spatial-distributed permeability. Therefore, we firstly study the numerical methods suitable

for dual-continuum gas reservoir simulation and then numerically study the effect of the production well location on gas production.

2. Governing equations and numerical methods

2.1. Physical model and governing equations

For statement convenience, we only consider ideal gas flow in two-dimensional Cartesian coordinate in this paper but it does not affect the discussion of the numerical methods. The physical model is shown in Fig.1. A production well is located at the center of the square domain and placed in the fracture. We only study dual-continuum porous media in Darcy scale, not in discrete fracture scale. Thus, fracture and matrix are represented by porosity and permeability. Matrix permeability is assumed to be uniform distributed in the domain. Fracture permeability is assumed to have two different regions with large and small permeability respectively. The distribution of fracture permeability will be shown in Section 4. All the boundaries are blocked so that no flow boundary conditions are applied. The domain lengths are L_x and L_y .

Fig.1 Physical model

Gas flow in dual-continuum porous media obeys the mass conservation law (Negara, 2015):

$$\phi_M \frac{\partial \rho_M}{\partial t} = -\nabla \cdot (\rho_M \mathbf{u}_M) - T_{MF} + q_M \quad (1)$$

$$\phi_F \frac{\partial \rho_F}{\partial t} = -\nabla \cdot (\rho_F \mathbf{u}_F) + T_{MF} + q_F \quad (2)$$

where ϕ_M and ϕ_F are porosities of matrix and fracture, ρ_M and ρ_F are gas densities in matrix and fracture, q_M and q_F are source terms in matrix and fracture (mass per unit volume per unit time), $\mathbf{u}_M$ and $\mathbf{u}_F$ are Darcy velocities of gas in matrix and fracture, respectively. T_{MF} is the transfer term from matrix to fracture, which has the following form:

$$T_{MF} = \frac{\alpha \rho_M k_M}{\mu} (p_M - p_F) \quad (3)$$

where k_M is the intrinsic permeability of matrix, μ is the dynamic viscosity of gas, α is the shape factor as follows (Kazemi et al., 1976):

$$\alpha = 4 \left(\frac{1}{l_x^2} + \frac{1}{l_y^2} + \frac{1}{l_z^2} \right) \quad (4)$$

where l_x , l_y and l_z are fracture spacings in the three directions respectively. For two-dimensional cases, l_z disappears.

Darcy velocities $\mathbf{u}_M$ and $\mathbf{u}_F$ obey the Darcy's law:

$$\mathbf{u}_M = -\frac{\mathbf{k}_M}{\mu} \nabla p_M \quad (5)$$

$$\mathbf{u}_F = -\frac{\mathbf{k}_F}{\mu} \nabla p_F \quad (6)$$

where $\mathbf{k}_M$ and $\mathbf{k}_F$ are permeability tensors of matrix and fracture, p_M and p_F are gas pressures in matrix and fracture, respectively.

For a production process, gas flows from matrix to fracture represented by the transfer term in Eqs.(1) & (2) and eventually leaves the dual-continuum naturally fractured reservoir through the production well placed in the fracture. The well is treated as a source term in Darcy scale. Therefore, the source terms in Eq.(1) and Eq.(2) have the following form using Peaceman's well model (Peaceman, 1983; Chen, 1989):

$$q_M = 0 \quad (7)$$

$$q_F = -\delta_w C_w \rho_F (p_F - p_{bh}) \quad (8)$$

Where $C_w = \frac{2\pi k_F}{\mu \Delta x \Delta y \ln(r_e / r_w)}$ is a constant factor of the well, k_F is the fracture permeability at the well location, r_w and $r_e (= 0.20788h, h = \Delta x = \Delta y$ is the grid spacing for a square grid containing the production well) are

the well radius and equivalent well radius, δ_w is the well identifier taking 1 for the well location and 0 for other locations. p_{bh} is the bottom hole pressure. For oil flow, density in Eq.(8) is independent of pressure so that the well term (source term) is a linear function of pressure. However, for gas flow, density is related to pressure according to the gas law:

$$\rho_M = p_M \frac{W}{RT} \quad (9)$$

$$\rho_F = p_F \frac{W}{RT} \quad (10)$$

where W is the molecular weight of gas, R is the universal gas constant, T is temperature. Combining Eq.(1)~Eq.(10), final mass conservation equations can be expressed as:

$$\phi_M \frac{\partial p_M}{\partial t} = \nabla \cdot \left(p_M \frac{\mathbf{k}_M}{\mu} \nabla p_M \right) - \frac{\alpha k_M}{\mu} p_M (p_M - p_F) \quad (11)$$

$$\phi_F \frac{\partial p_F}{\partial t} = \nabla \cdot \left(p_F \frac{\mathbf{k}_F}{\mu} \nabla p_F \right) + \frac{\alpha k_M}{\mu} p_M (p_M - p_F) - \delta_w C_w p_F (p_F - p_{bh}) \quad (12)$$

It can be seen from Eq.(12) that the well term (source term) is a non-linear function of fracture pressure, which is harder to solve than the linear function of oil flow. The reason will be explained in the numerical method section. In this paper, we use the two-dimensional physical model in Fig.1, so that the above governing equations have the following forms:

$$\phi_M \frac{\partial p_M}{\partial t} = \frac{\partial}{\partial x} \left(\frac{k_{xxM}}{\mu} p_M \frac{\partial p_M}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{k_{yyM}}{\mu} p_M \frac{\partial p_M}{\partial y} \right) - \alpha \frac{k_M}{\mu} p_M (p_M - p_F) \quad (13)$$

$$\phi_F \frac{\partial p_F}{\partial t} = \frac{\partial}{\partial x} \left(\frac{k_{xxF}}{\mu} p_F \frac{\partial p_F}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{k_{yyF}}{\mu} p_F \frac{\partial p_F}{\partial y} \right) + \alpha \frac{k_M}{\mu} p_M (p_M - p_F) - \delta_w C_w p_F (p_F - p_{bh}) \quad (14)$$

$$u_M = -\frac{k_{xxM}}{\mu} \frac{\partial p_M}{\partial x}, v_M = -\frac{k_{yyM}}{\mu} \frac{\partial p_M}{\partial y} \quad (15)$$

$$u_F = -\frac{k_{xxF}}{\mu} \frac{\partial p_F}{\partial x}, v_F = -\frac{k_{yyF}}{\mu} \frac{\partial p_F}{\partial y} \quad (16)$$

where k_{xxM} and k_{yyM} are two components of matrix permeability tensor

($\mathbf{k}_M = \begin{bmatrix} k_{xxM} & \\ & k_{yyM} \end{bmatrix}$), k_{xxF} and k_{yyF} are two components of fracture

permeability tensor ($\mathbf{k}_F = \begin{bmatrix} k_{xxF} & \\ & k_{yyF} \end{bmatrix}$), u_M and v_M are two components of

Darcy velocity of gas flow in matrix, u_F and v_F are two components of

Darcy velocity of gas flow in fracture, respectively.

2.2. Numerical methods

We use the cell-centered finite difference method to discretize the above governing equations. Staggered grid, i.e. pressure placed on the center of grid cells and velocity placed on the edges of the cells, is used to avoid unphysical pressure field. Temporal advancement is the semi-implicit scheme so that the time step can be much larger than the explicit scheme to achieve efficient numerical simulation. Using these numerical methods, Eq.(13) and Eq.(14) can be discretized to be the following form:

$$\begin{aligned}
 cp_{Mi,j} p_{Mi,j}^{(n+1)} &= cex_{Mi,j} p_{Mi+1,j}^{(n+1)} + cwx_{Mi,j} p_{Mi-1,j}^{(n+1)} \\
 &\quad + cny_{Mi,j} p_{Mi,j+1}^{(n+1)} + csy_{Mi,j} p_{Mi,j-1}^{(n+1)} + b_{Mi,j}
 \end{aligned} \tag{17}$$

$$\begin{aligned}
 &\left(cp_{Fi,j} + \delta_w C_w \Delta t \left(p_{Fi,j}^{(n)} - p_{bh} \right) \right) p_{Fi,j}^{(n+1)} - \Delta t \frac{\alpha k_{Mi,j}}{\mu} p_{Mi,j}^{(n)} p_{Mi,j}^{(n+1)} \\
 &= cex_{Fi,j} p_{Fi+1,j}^{(n+1)} + cwx_{Fi,j} p_{Fi-1,j}^{(n+1)} + cny_{Fi,j} p_{Fi,j+1}^{(n+1)} + csy_{Fi,j} p_{Fi,j-1}^{(n+1)} + b_{Fi,j}
 \end{aligned} \tag{18}$$

where $cp_{Mi,j} = \phi_M + cwx_{Mi,j} + cex_{Mi,j} + csy_{Mi,j} + cny_{Mi,j} + \Delta t \frac{\alpha k_{Mi,j}}{\mu} p_{Mi,j}^{(n)} \left(1 - \frac{p_{Fi,j}^{(n)}}{p_{Mi,j}^{(n)}} \right)$,

$$cp_{Fi,j} = \phi_F + cwx_{Fi,j} + cex_{Fi,j} + csy_{Fi,j} + cny_{Fi,j} + \Delta t \frac{\alpha k_{Mi,j}}{\mu} p_{Mi,j}^{(n)},$$

$$b_{Mi,j} = \phi_M p_{Mi,j}^{(n)} + swx_{Mi,j} + sex_{Mi,j} + ssy_{Mi,j} + sny_{Mi,j},$$

$$b_{Fi,j} = \phi_F p_{Fi,j}^{(n)} + swx_{Fi,j} + sex_{Fi,j} + ssy_{Fi,j} + sny_{Fi,j},$$

$$cwx_{Mi,j} = \begin{cases} \frac{\Delta t}{(\Delta x)^2} \left(\frac{k_{xxM}}{\mu} p_M^{(n)} \right)_{i-\frac{1}{2},j} & i = 2 \sim nx \\ 0 & i = 1 \end{cases}, \quad cex_{Mi,j} = \begin{cases} \frac{\Delta t}{(\Delta x)^2} \left(\frac{k_{xxM}}{\mu} p_M^{(n)} \right)_{i+\frac{1}{2},j} & i = 1 \sim nx-1 \\ 0 & i = nx \end{cases},$$

$$csy_{Mi,j} = \begin{cases} \frac{\Delta t}{(\Delta y)^2} \left(\frac{k_{yyM}}{\mu} p_M^{(n)} \right)_{i,j-\frac{1}{2}} & j = 2 \sim ny \\ 0 & j = 1 \end{cases}, \quad cny_{Mi,j} = \begin{cases} \frac{\Delta t}{(\Delta y)^2} \left(\frac{k_{yyM}}{\mu} p_M^{(n)} \right)_{i,j+\frac{1}{2}} & j = 1 \sim ny-1 \\ 0 & j = ny \end{cases},$$

$$cwx_{Fi,j} = \begin{cases} \frac{\Delta t}{(\Delta x)^2} \left(\frac{k_{xxF}}{\mu} p_F^{(n)} \right)_{i-\frac{1}{2},j} & i = 2 \sim nx \\ 0 & i = 1 \end{cases}, \quad cex_{Fi,j} = \begin{cases} \frac{\Delta t}{(\Delta x)^2} \left(\frac{k_{xxF}}{\mu} p_F^{(n)} \right)_{i+\frac{1}{2},j} & i = 1 \sim nx-1 \\ 0 & i = nx \end{cases},$$

$$csy_{Fi,j} = \begin{cases} \frac{\Delta t}{(\Delta y)^2} \left(\frac{k_{yyF}}{\mu} p_F^{(n)} \right)_{i,j-\frac{1}{2}} & j = 2 \sim ny \\ 0 & j = 1 \end{cases}, \quad cny_{Fi,j} = \begin{cases} \frac{\Delta t}{(\Delta y)^2} \left(\frac{k_{yyF}}{\mu} p_F^{(n)} \right)_{i,j+\frac{1}{2}} & j = 1 \sim ny-1 \\ 0 & j = ny \end{cases},$$

$$swx_{Mi,j} = \begin{cases} 0 & i = 2 \sim nx \\ \frac{\Delta t}{\Delta x} (u_M p_M^{(n)})_{i-\frac{1}{2},j} & i = 1 \end{cases}, \quad sex_{Mi,j} = \begin{cases} 0 & i = 1 \sim nx-1 \\ \frac{\Delta t}{\Delta x} (u_M p_M^{(n)})_{i+\frac{1}{2},j} & i = nx \end{cases},$$

$$ssy_{Mi,j} = \begin{cases} 0 & j = 2 \sim ny \\ \frac{\Delta t}{\Delta y} (v_M p_M^{(n)})_{i,j-\frac{1}{2}} & j = 1 \end{cases}, \quad sny_{Mi,j} = \begin{cases} 0 & j = 1 \sim ny-1 \\ \frac{\Delta t}{\Delta y} (v_M p_M^{(n)})_{i,j+\frac{1}{2}} & j = ny \end{cases},$$

$$swx_{Fi,j} = \begin{cases} 0 & i = 2 \sim nx \\ \frac{\Delta t}{\Delta x} (u_F p_F^{(n)})_{i-\frac{1}{2},j} & i = 1 \end{cases}, \quad sex_{Fi,j} = \begin{cases} 0 & i = 1 \sim nx-1 \\ \frac{\Delta t}{\Delta x} (u_F p_F^{(n)})_{i+\frac{1}{2},j} & i = nx \end{cases},$$

$$ssy_{Fi,j} = \begin{cases} 0 & j = 2 \sim ny \\ \frac{\Delta t}{\Delta y} (v_F p_F^{(n)})_{i,j-\frac{1}{2}} & j = 1 \end{cases}, \quad sny_{Fi,j} = \begin{cases} 0 & j = 1 \sim ny-1 \\ \frac{\Delta t}{\Delta y} (v_F p_F^{(n)})_{i,j+\frac{1}{2}} & j = ny \end{cases}.$$

The superscripts (n) and $(n+1)$ represent the starting and ending moments of a time step respectively. Thus, the variables with the superscript (n) are known while the ones with the superscript $(n+1)$ are unknown. nx and ny are grid numbers in the x and y directions. Matrix pressure ($p_{Mi,j}^{(n+1)}$) and fracture pressure ($p_{Fi,j}^{(n+1)}$) can be obtained by directly solving the equation system of Eq.(17) and Eq.(18).

3. Discussions on numerical methods

The above equations are solved using the physical model with square uniform mesh in Fig.1. Computational parameters are listed in Table 1. Numerical results show that the fracture pressure at the well is always negative for the first 10000 time steps (Fig.2). However, pressures must be always larger than the bottom hole pressure ($p_{bh} = 202650 Pa$ in Table 1) in this case so that gas can be produced through the well. This can also be seen from Eq.(14). Thus, the negative pressure in Fig.2 is unphysical.

Table 1 Computational parameters

Parameter	Value	Unit
ϕ_M	0.5	/
ϕ_F	0.001	/
$p_M(0)$	10132500	Pa
$p_F(0)$	10132500	Pa
p_{bh}	202650	Pa
k_{xxF}	100	md

k_{yyF}	100	md
k_{xxM}	1	md
k_{yyM}	1	md
k_M	1	md
W	16×10^{-3}	kg/mol
R	8.3147295	J/(mol·K)
T	298	K
μ	11.067×10^{-6}	Pa·s
nx	101	/
ny	101	/
L_x	100	m
L_y	100	m
l_x	0.2	m
l_y	0.2	m
r_w	0.2	m
Δt	864	s

*1md= $9.8692327 \times 10^{-16} \text{ m}^2$.

Fig.2 Fracture pressure at the well

To reveal the reason of the unphysical pressure through the numerical computation, we analyze the mass of gas in the whole domain.

It is the summation of the masses in all the grid cells:

$$m = \sum_{j=1}^{ny} \sum_{i=1}^{nx} m_{i,j}^c = \sum_{j=1}^{ny} \sum_{i=1}^{nx} (\rho_{i,j} V_{i,j}) \quad (19)$$

Where m is the mass in the whole computational domain either for gas in matrix or for gas in fracture, $m_{i,j}^c$ is the gas mass in the (i,j) th grid cell, $\rho_{i,j}$ is the gas density in the (i,j) th grid cell, $V_{i,j}$ is the volume of the (i,j) th grid cell which is a constant for the uniform square mesh ($V_{i,j} = \Delta x \Delta y$). Substituting the gas law (Eq.(9) and Eq.(10)) to the above equation, we can obtain:

$$\begin{aligned} m &= \sum_{j=1}^{ny} \sum_{i=1}^{nx} \left(p_{i,j} \frac{W}{RT} \Delta x \Delta y \right) = \frac{W}{RT} \Delta x \Delta y \sum_{j=1}^{ny} \sum_{i=1}^{nx} p_{i,j} \\ &= \frac{W}{RT} \frac{L_x}{nx} \frac{L_y}{ny} \sum_{j=1}^{ny} \sum_{i=1}^{nx} p_{i,j} = \frac{W}{RT} L_x L_y \frac{\sum_{j=1}^{ny} \sum_{i=1}^{nx} p_{i,j}}{nx \cdot ny} = \frac{W}{RT} L_x L_y \bar{p} \end{aligned} \quad (20)$$

From Eq.(20), gas masses in matrix and fracture can be expressed as:

$$mM = \frac{W}{RT} L_x L_y \bar{p}_M \quad (21)$$

$$mF = \frac{W}{RT} L_x L_y \bar{p}_F \quad (22)$$

From the well term (Eq.(8)) and the gas law (Eq.(10)), gas mass produced through the well (placed in one grid cell with side lengths Δx and Δy) in one time step can be expressed as:

$$mQ = -q_F \Delta x \Delta y \Delta t = \delta_w C_w \rho_F (p_F - p_{bh}) \Delta x \Delta y \Delta t = \Delta x \Delta y \Delta t \frac{W}{RT} \delta_w C_w p_F (p_F - p_{bh}) \quad (23)$$

Theoretically, the gas mass for the whole computational domain should obey the mass conservation law:

$$mM + mF - mQ = m_0 \quad (24)$$

Where m_0 is the gas mass in the dual-continuum system in the initial condition. We check the total mass $mM + mF - mQ$ in every time step and compare it with m_0 . It can be clearly seen in Fig.3 that total mass of gas is decreasing with time, indicating that the mass is lost only due to the numerical computation so that the result is unphysical. This means the so-called mass conservation equation of the dual-continuum system (Eq.(13) and Eq.(14)) cannot guarantee the mass conservation law to be satisfied numerically. A numerical method ensuring mass conservation over the domain should be developed.

Fig.3 Total mass of gas

By revisiting Eq.(20), we can find the gas mass is proportional to the mean pressure. The unphysical mass loss should come from the wrong $\bar{p}$. Therefore, we propose a new method to correct gas pressures in matrix and fracture in every time step as flows:

$$p_{Mi,j}^{(n+1)} = p_{Mi,j}^{*(n+1)} - \overline{p_M^{*(n+1)}} + \overline{p_M^{(n+1)}} \quad (25)$$

$$p_{Fi,j}^{(n+1)} = p_{Fi,j}^{*(n+1)} - \overline{p_F^{*(n+1)}} + \overline{p_F^{(n+1)}} \quad (26)$$

where $p_{Mi,j}^{(n+1)}$ and $p_{Fi,j}^{(n+1)}$ are the matrix pressure and fracture pressure to be solved in each time step, $p_{Mi,j}^{*(n+1)}$ and $p_{Fi,j}^{*(n+1)}$ are matrix pressure and fracture pressure obtained from the regular numerical methods in Section 2.2 which cannot guarantee global mass conservation, $\overline{p_M^{*(n+1)}}$ and $\overline{p_F^{*(n+1)}}$ are mean pressures in matrix and fracture calculated from $p_{Mi,j}^{*(n+1)}$ and $p_{Fi,j}^{*(n+1)}$, $\overline{p_M^{(n+1)}}$ and $\overline{p_F^{(n+1)}}$ are correct mean pressures in matrix and fracture which are unknown because they are actually mean values of $p_{Mi,j}^{(n+1)}$ and $p_{Fi,j}^{(n+1)}$ which need to be solved in every time step. Thus, the calculation of $p_{Mi,j}^{(n+1)}$ and $p_{Fi,j}^{(n+1)}$ turns to the calculation of $\overline{p_M^{(n+1)}}$ and $\overline{p_F^{(n+1)}}$.

$\overline{p_M^{(n+1)}}$ and $\overline{p_F^{(n+1)}}$ in each time step should satisfy the mass conservation law mentioned above. It can be fulfilled by using the above mass balance relation in matrix and fracture in each time step:

$$mM^{(n)} - mT^{(n+1)} = mM^{(n+1)} \quad (27)$$

$$mF^{(n)} + mT^{(n+1)} - mQ^{(n+1)} = mF^{(n+1)} \quad (28)$$

where $mM^{(n)}$ and $mF^{(n)}$ are the masses of gas in matrix and fracture at the beginning moment of each time step, $mM^{(n+1)}$ and $mF^{(n+1)}$ are the masses of gas remained in matrix and fracture at the ending moment of each time

step, $mQ^{(n+1)}$ is the mass leaving the fracture via the production well in each time step induced by the well term in Eq.(14). These terms can be easily obtained via Eqs.(21)~(23) as follows:

$$mM^{(n)} = \frac{W}{RT} L_x L_y \overline{p_M^{(n)}} \quad (29)$$

$$mF^{(n)} = \frac{W}{RT} L_x L_y \overline{p_F^{(n)}} \quad (30)$$

$$mM^{(n+1)} = \frac{W}{RT} L_x L_y \overline{p_M^{(n+1)}} \quad (31)$$

$$mF^{(n+1)} = \frac{W}{RT} L_x L_y \overline{p_F^{(n+1)}} \quad (32)$$

$$mQ^{(n+1)} = \Delta x \Delta y \Delta t \frac{W}{RT} \delta_w C_w p_F^{(n+1)} (p_F^{(n+1)} - p_{bh}) \quad (33)$$

$mT^{(n+1)}$ in Eq.(27) and Eq.(28) is the gas mass transferring from the matrix to the fracture in each time step. It can be derived from the transfer term (Eq.(3)) as follows:

$$\begin{aligned} mT^{(n+1)} &= \sum_{j=1}^{ny} \sum_{i=1}^{nx} T_{MFi,j}^{(n+1)} \Delta x \Delta y \Delta t \\ &= \Delta x \Delta y \Delta t \sum_{j=1}^{ny} \sum_{i=1}^{nx} \frac{\alpha \rho_{Mi,j}^{(n+1)} k_{Mi,j}}{\mu} (p_{Mi,j}^{(n+1)} - p_{Fi,j}^{(n+1)}) \\ &= \frac{\alpha}{\mu} \Delta x \Delta y \Delta t \frac{W}{RT} \sum_{j=1}^{ny} \sum_{i=1}^{nx} \left[k_{Mi,j} p_{Mi,j}^{(n+1)} (p_{Mi,j}^{(n+1)} - p_{Fi,j}^{(n+1)}) \right] \end{aligned} \quad (34)$$

Eq.(29) and Eq.(30) can be used to directly calculate the values of $mM^{(n)}$ and $mF^{(n)}$ explicitly because the variables at the starting moment of each time step is always known. Eq.(31)~Eq.(34) should be substituted to Eq.(27) and Eq.(28) so that the mass balance equations in matrix and fracture become:

$$mM^{(n)} - \frac{\alpha}{\mu} \Delta x \Delta y \Delta t \frac{W}{RT} \sum_{j=1}^{ny} \sum_{i=1}^{nx} \left[k_{Mi,j} p_{Mi,j}^{(n+1)} \left(p_{Mi,j}^{(n+1)} - p_{Fi,j}^{(n+1)} \right) \right] = \frac{W}{RT} L_x L_y \overline{p_M^{(n+1)}} \quad (35)$$

$$\begin{aligned} mF^{(n)} + \frac{\alpha}{\mu} \Delta x \Delta y \Delta t \frac{W}{RT} \sum_{j=1}^{ny} \sum_{i=1}^{nx} \left[k_{Mi,j} p_{Mi,j}^{(n+1)} \left(p_{Mi,j}^{(n+1)} - p_{Fi,j}^{(n+1)} \right) \right] \\ - \Delta x \Delta y \Delta t \frac{W}{RT} \delta_w C_w p_F^{(n+1)} \left(p_F^{(n+1)} - p_{bh} \right) = \frac{W}{RT} L_x L_y \overline{p_F^{(n+1)}} \end{aligned} \quad (36)$$

Then, Eq.(25) and Eq.(26) are substituted to the above two equations

to obtain the following expressions:

$$\begin{aligned} & - \left(\frac{\alpha}{\mu} \Delta x \Delta y \Delta t \sum_{j=1}^{ny} \sum_{i=1}^{nx} k_{Mi,j} \right) \left(\overline{p_M^{(n+1)}} \right)^2 + \left(\frac{\alpha}{\mu} \Delta x \Delta y \Delta t \sum_{j=1}^{ny} \sum_{i=1}^{nx} k_{Mi,j} \right) \overline{p_M^{(n+1)}} \overline{p_F^{(n+1)}} \\ & - \left[L_x L_y + \frac{\alpha}{\mu} \Delta x \Delta y \Delta t \sum_{j=1}^{ny} \sum_{i=1}^{nx} k_{Mi,j} \left(2p_{Mi,j}^{*(n+1)} - 2\overline{p_M^{*(n+1)}} - p_{Fi,j}^{*(n+1)} + \overline{p_F^{*(n+1)}} \right) \right] \overline{p_M^{(n+1)}} \\ & - \frac{\alpha}{\mu} \Delta x \Delta y \Delta t \sum_{j=1}^{ny} \sum_{i=1}^{nx} k_{Mi,j} \left(p_{Mi,j}^{*(n+1)} - \overline{p_M^{*(n+1)}} \right) \left(p_{Mi,j}^{*(n+1)} - \overline{p_M^{*(n+1)}} - p_{Fi,j}^{*(n+1)} + \overline{p_F^{*(n+1)}} \right) \\ & + \left[\frac{\alpha}{\mu} \Delta x \Delta y \Delta t \sum_{j=1}^{ny} \sum_{i=1}^{nx} k_{Mi,j} \left(p_{Mi,j}^{*(n+1)} - \overline{p_M^{*(n+1)}} \right) \right] \overline{p_F^{(n+1)}} + mM^{(n)} \frac{RT}{W} = 0 \end{aligned} \quad (37)$$

$$\begin{aligned} & \left(\frac{\alpha}{\mu} \Delta x \Delta y \Delta t \sum_{j=1}^{ny} \sum_{i=1}^{nx} k_{Mi,j} \right) \left(\overline{p_M^{(n+1)}} \right)^2 - \delta_w C_w \Delta x \Delta y \Delta t \left(\overline{p_F^{(n+1)}} \right)^2 - \left(\frac{\alpha}{\mu} \Delta x \Delta y \Delta t \sum_{j=1}^{ny} \sum_{i=1}^{nx} k_{Mi,j} \right) \overline{p_M^{(n+1)}} \overline{p_F^{(n+1)}} \\ & + \left[\frac{\alpha}{\mu} \Delta x \Delta y \Delta t \sum_{j=1}^{ny} \sum_{i=1}^{nx} k_{Mi,j} \left(2p_{Mi,j}^{*(n+1)} - 2\overline{p_M^{*(n+1)}} - p_{Fi,j}^{*(n+1)} + \overline{p_F^{*(n+1)}} \right) \right] \overline{p_M^{(n+1)}} \\ & - \left[L_x L_y + \frac{\alpha}{\mu} \Delta x \Delta y \Delta t \sum_{j=1}^{ny} \sum_{i=1}^{nx} k_{Mi,j} \left(p_{Mi,j}^{*(n+1)} - \overline{p_M^{*(n+1)}} \right) + \delta_w C_w \Delta x \Delta y \Delta t \left(2p_{Fi,j}^{*(n+1)} - 2\overline{p_F^{*(n+1)}} - p_{bh} \right) \right] \overline{p_F^{(n+1)}} \\ & + \alpha \Delta x \Delta y \Delta t \sum_{j=1}^{ny} \sum_{i=1}^{nx} \left(\frac{k_M}{\mu} \right)_{i,j} \left(p_{Mi,j}^{*(n+1)} - \overline{p_M^{*(n+1)}} \right) \left(p_{Mi,j}^{*(n+1)} - \overline{p_M^{*(n+1)}} - p_{Fi,j}^{*(n+1)} + \overline{p_F^{*(n+1)}} \right) \\ & - \delta_w C_w \Delta x \Delta y \Delta t \left(p_{Fi,j}^{*(n+1)} - \overline{p_F^{*(n+1)}} \right) \left(p_{Fi,j}^{*(n+1)} - \overline{p_F^{*(n+1)}} - p_{bh} \right) + mF^{(n)} \frac{RT}{W} = 0 \end{aligned} \quad (38)$$

The above two equations are final expressions of mass balance equations and can be directly solved to obtain $\overline{p_M^{(n+1)}}$ and $\overline{p_F^{(n+1)}}$. Then they are substituted back to Eq.(25) and Eq.(26) to obtain $p_{Mi,j}^{(n+1)}$ and $p_{Fi,j}^{(n+1)}$ so that temporal advancement for one time step is completed. Fracture pressure at the well and total mass of gas using the proposed global mass

conservation method are shown in Fig.4 and Fig.5. Fracture pressure at the well becomes always larger than the bottom hole pressure. Computational total mass of gas is always equivalent to the real total mass of gas, indicating that the mass conservation law is obeyed in computation by using the proposed method. However, another problem arises. The fracture pressure at the well in Fig.4 appears increase between $1300 \Delta t$ and $1434 \Delta t$. This phenomenon is unphysical because gas is continuously produced from the well so that the fracture pressure at the well should be decreasing monotonically.

After checking the coefficients of Eq.(17) and Eq.(18), we find that the coefficient matrix of Eq.(18) cannot always keep diagonally dominant (Table 2). This is the reason why the fracture pressure at the well appears unphysical increase and the computation is collapsed after $1434 \Delta t$.

Fig.4 Fracture pressure at the well considering mass conservation

Fig.5 Total mass of gas considering mass conservation

Table 2 Examination of diagonal dominance

Time	$cp_{Fi,j} + \delta_w C_w \Delta t (p_{Fi,j}^{(n)} - p_{bh})$	$cex_{Fi,j} + cwx_{Fi,j} + cny_{Fi,j} + csy_{Fi,j}$
1435 Δt	20.6414	21.9179
	$cp_{Fi,j} + \delta_w C_w \Delta t (p_{Fi,j}^{(n)} - p_{bh}) < cex_{Fi,j} + cwx_{Fi,j} + cny_{Fi,j} + csy_{Fi,j}$ diagonal dominance is violated	

To avoid the unphysical pressure and the interruption of computation, diagonal dominance of coefficient matrix of Eq.(18) should be ensured (it is automatically satisfied in Eq.(17)). It is clear in Table 2 that only the well term $\delta_w C_w \Delta t (p_{Fi,j}^{(n)} - p_{bh})$ may cause the diagonal coefficient decreasing. Thus, the non-linear well term in Eq.(14) ($S = -\delta_w C_w p_F (p_F - p_{bh})$) should be transformed to be linear source term in the form of $S = S_c + S_p p_F$ with $S_p \leq 0$ (Tao, 2002). S_c and S_p can be obtained by the following Taylor expansion:

$$\begin{aligned}
 S &= S^{(n)} + \left. \frac{dS}{dp_F} \right|^{(n)} (p_F^{(n+1)} - p_F^{(n)}) \\
 &= -\delta_w C_w p_F^{(n)} (p_F^{(n)} - p_{bh}) - \delta_w C_w (2p_F^{(n)} - p_{bh}) (p_F^{(n+1)} - p_F^{(n)}) \quad (39) \\
 &= \delta_w C_w (p_F^{(n)})^2 - \delta_w C_w (2p_F^{(n)} - p_{bh}) p_F^{(n+1)}
 \end{aligned}$$

Eq.(39) is used in the discretization of Eq.(14) instead of $S = -\delta_w C_w p_F^{(n+1)} (p_F^{(n)} - p_{bh})$ so that Eq.(18) can be modified to be the following form:

$$\begin{aligned}
 &\left(cp_{Fi,j} + \delta_w C_w \Delta t (2p_{Fi,j}^{(n)} - p_{bh}) \right) p_{Fi,j}^{(n+1)} - \Delta t \frac{\alpha k_{Mi,j}}{\mu} p_{Mi,j}^{(n)} p_{Mi,j}^{(n+1)} \\
 &= cex_{Fi,j} p_{Fi+1,j}^{(n+1)} + cwx_{Fi,j} p_{Fi-1,j}^{(n+1)} + cny_{Fi,j} p_{Fi,j+1}^{(n+1)} + csy_{Fi,j} p_{Fi,j-1}^{(n+1)} + b_{Fi,j} + \delta_w C_w (p_F^{(n)})^2 \quad (40)
 \end{aligned}$$

After the local linearization of the well term, numerical results show that the fracture pressure at the well is decreasing monotonically and always higher than the bottom hole pressure (Fig.6). The total mass of gas is also constant (Fig.7). Therefore, current numerical methods are completely correct for gas flow in dual-continuum porous media.

Fig.6 Fracture pressure at the well considering mass conservation & linearization of source term

Fig.7 Total mass of gas considering mass conservation & linearization of source term

From the above discussions, the proposed numerical methods can be summarized as follows:

(a) Governing equations (Eq.(13) and Eq.(14)) should be discretized into the form of Eq.(17) and Eq.(40) respectively, using the local linearization for the source term to ensure robust computation.

(b) Mass balance equations of matrix and fracture (Eq.(37) and Eq.(38)) should be established respectively. They are solved to correct the mean pressures of matrix and fracture from $\overline{p_M^{*(n+1)}}$ and $\overline{p_F^{*(n+1)}}$ to $\overline{p_M^{(n+1)}}$ and $\overline{p_F^{(n+1)}}$ in every time step. Then, matrix pressure ($p_M^{(n+1)}$) and fracture pressure ($p_F^{(n+1)}$) can be calculated via Eq.(25) and Eq.(26) correctly.

4. Prediction of gas production using proposed numerical methods

The proposed global mass conservation method in Section 3 is applied to study the gas production process in dual-continuum porous

media. Four cases in Fig.8 are designed using the same parameters in Table 1 except fracture permeability, which is defined as follows: the blue area represents the region occupied by small fractures with permeability of 100md in Darcy scale; the red area (20m×20m) represents the region occupied by large fractures with permeability of 1000md in Darcy scale. The black circle still represents the production well in the center of the domain as shown in Fig.1. In Case 1, the production well locates in the center of the large-permeability region. In Case 2, the production well is adjacent to the left interface between the large- and small- permeability regions but in the large-permeability region. In Case 3, the production well is still adjacent to the left interface but just out of the large-permeability region. In Case 4, the production well is far from large-permeability region. Computations for the four cases are all stopped when 90% of the gas has been produced. Thus, analyses on numerical results of the four cases can provide valuable suggestions for selecting drilling location of the gas well.

Fig.8 Four cases of gas production for different distributions of fracture permeability: blue region-100md; red region-1000md

First of all, fracture pressure at the wells and mass conservations are checked. Total masses of gas are all constant in the whole transient process for the four cases, indicating that the mass conservation is satisfied very well (Fig.9). As shown in Fig.10, fracture pressures at the wells for the four cases are all decreasing monotonically. The pressures between Case 1 and Case 2 have no apparent differences. The same

situation also occurs for Case 3 and Case 4. This indicates that fracture pressure at the well is not sensitive to the locations of the production well when it is in the large permeability region (Case 1 and Case 2) or in the small permeability region (Case 3 and Case 4). However, fracture pressure at the well have a large gap between Case 2 and Case 3, reflecting that the pressure is sensitive to the relative position of the production well to the permeability interface even though the distances from the well to the interface are the same in Case 2 and Case 3. The fracture pressure at the well in Case 3 is much larger than the one in Case 2, indicating that much more gas remains in the porous media for Case 3 (verified in Fig.11) so that the production of the same amount of gas need much more time (verified in Fig.12). From Fig.11 and Fig.12, we can also find that the production time increases with increasing distance of the well to the large-permeability region, i.e. production of 90% gas needing 42.6 days for Case 1, 44.3 days for Case 2, 55.2 days for Case 3 and 68.1 days for Case 4. The reason is the slower pressure decrease for the longer distance of the well to the large permeability region, as shown in Fig.10. It should be noted that the production time from Case 2 to Case 3 increases about 25% but the wells in these two cases are both adjacent to

the permeability interface. The time increase from Case 1 to Case 2 (about 4%) is much smaller than 25%. Thus, the production efficiency will be largely reduced even if the production well is just moved from the inside to the outside of the large permeability region.

Fig.9 Mass conservation check

Fig.10 Fracture pressure at the well

of the four cases

of the four cases

Fig.11 Gas remained in porous media

Fig.12 Cumulative gas production

The details of gas behaviors in the dual-continuum porous media are helpful for understanding the transport mechanism of gas production. Thus, pressure fields and velocity fields in matrix and fracture at the end

of production are shown in Fig.13~Fig.16. The units for the pressures and for the velocities are Pa and m/s respectively. The contours in Fig.15 and Fig.16 are the distributions of the velocity values ($\sqrt{u_M^2 + v_M^2}$ for matrix and $\sqrt{u_F^2 + v_F^2}$ for fracture).

Pressures in matrix (Fig.13) are one order of magnitude higher than pressures in fracture (Fig.14). On the other hand, velocities in matrix (Fig.15) are 4~5 orders of magnitude lower than velocities in fracture (Fig.16). This reflects the slow response of gas in matrix with small permeability and the fast response of gas in fracture with larger permeability. When the production well is near the large permeability region, large velocity in fracture is always in the large permeability region (Case 1~Case 3 in Fig.16(a)~(c)). Velocity in Case 2 is apparently larger than velocity in Case 3, confirming that the gas production can be greatly promoted even if the production well is just moved from the outside to the inside of the permeability interface. When the production well is far from the large permeability region, the pressure and velocity distributions are very similar to those of Case 1 but the velocity is about half of the velocity in Case 1 (Case 4 in Fig.16(d)). In this condition, the role of the large permeability region on gas production is very weak.

Fig.13 Matrix pressure field at the end of the production

Fig.14 Fracture pressure field at the end of the production

Fig.15 Matrix velocity field at the end of the production

Fig.16 Fracture velocity field at the end of the production

5. Conclusions

Numerical simulation of gas flow in dual-continuum porous media is different from oil reservoir simulation: i) direct calculation of the mass conservation equations (Eq.(13) and Eq.(14)) may not guarantee mass conservation, leading to unphysical mass loss and incorrect pressures in matrix and fracture; ii) source term induced by the production well is non-linear. Direct calculation of the non-linear well term, even if the semi-implicit scheme is used, leads to unphysical results and interruption of computation because diagonal dominance of the discretized equations cannot be ensured. To avoid these two problems, we proposed a global mass conservation method as follows:

(a) Mass balance equations for matrix and fracture are established to calculate the correct mean pressures so that the mass conservation law is completely satisfied to ensure correct pressure. The temporal advancement splits three fractional steps: 1) obtain uncorrected pressures that not satisfying mass conservation; 2) solve the mass balance equations to obtain mean pressures in matrix and fracture; 3) correct matrix and fracture pressures using the mean pressures.

(b) Local linearization should be made for the non-linear source term to ensure diagonal dominance of the coefficient matrix of the discretized equation system, so that the computation is always robust.

The global mass conservation method is successfully used to the computation of gas flow in the dual-continuum porous media with spatially distributed permeability of fracture. From the analyses of the numerical results, we find that the production time is very sensitive to the position of the production well being adjacent to the interface between large-permeability region and small-permeability region. The production time becomes much longer even if the production well is just moved from the inside to the outside of the large permeability region, but it has no big difference for different locations of the well when it is inside or outside the large permeability region. Thus, high efficient gas production is mainly determined by the well inside the large-permeability region.

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Table 1 Computational parameters

Parameter	Value	Unit
ϕ_M	0.5	/
ϕ_F	0.001	/
$p_M(0)$	10132500	Pa
$p_F(0)$	10132500	Pa
p_{bh}	202650	Pa
k_{xxF}	100	md
k_{yyF}	100	md
k_{xxM}	1	md
k_{yyM}	1	md
k_M	1	md
W	16×10^{-3}	kg/mol
R	8.3147295	J/(mol·K)
T	298	K
μ	11.067×10^{-6}	Pa·s
nx	101	/
ny	101	/
L_x	100	m
L_y	100	m
l_x	0.2	m
l_y	0.2	m
r_w	0.2	m
Δt	864	s

*1md= $9.8692327 \times 10^{-16}$ m².

Table 2 Examination of diagonal dominance

Time	$cp_{Fi,j} + \delta_w C_w \Delta t (p_{Fi,j}^{(n)} - p_{bh})$	$cex_{Fi,j} + cwx_{Fi,j} + cny_{Fi,j} + csy_{Fi,j}$
1435 Δt	20.6414	21.9179
	$cp_{Fi,j} + \delta_w C_w \Delta t (p_{Fi,j}^{(n)} - p_{bh}) < cex_{Fi,j} + cwx_{Fi,j} + cny_{Fi,j} + csy_{Fi,j}$ diagonal dominance is violated

Mass conservation method is proposed for dual-continuum gas reservoir simulation.

Source term linearization should be used to avoid unphysical results.

Effect of well location on gas production is studied based on proposed method.